

**Soil Dynamics**  
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**Module - 4**  
**Dynamic Soil Properties**  
**Lecture - 20**  
**Centrifuge Tests Stress Strain**  
**Behavior of Cyclically Loaded Soils**

Let us start today's lecture of soil dynamics.

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The slide features a blue header with the text "SOIL DYNAMICS" in yellow. Below the header, the title "Field Tests" is written in large blue font, followed by the subtitle "For Determination of Dynamic Soil Properties" in a smaller black font. The content is organized into two main sections: (A) Direct Tests, listed in black text, and (B) In-direct Tests, listed in red text. A small NPTEL logo is visible at the bottom left of the slide content. The footer of the slide contains the text "Prof. Deepankar Choudhury, Department of Civil Engineering, IIT Bombay, Mumbai, India" in white on a dark blue background.

**SOIL DYNAMICS**

**Field Tests**

**For Determination of Dynamic Soil Properties**

**(A) Direct Tests:**

- Seismic Reflection Test**
- Seismic Refraction Test**
- Spectral Analysis of Surface Wave (SASW) Test**
- Seismic Cross-hole Test**
- Seismic Down-hole/Up-hole Tests etc.**

**(B) In-direct Tests:**

- Standard Penetration Test (SPT), Cone Penetration Test (CPT), Becker Penetration Test (BPT) etc.**

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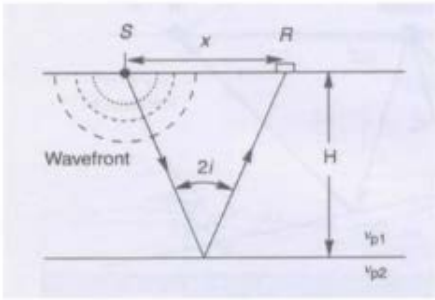
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In the previous lecture, let us quickly do a recap of what we had learnt in the previous lecture, like for determination of the soil properties similar to the static case; for dynamic case also, it can be classified in two major categories, field test and laboratory test. Within field test, we have sub classified mainly in two broad categories; one is called direct test, another is in-direct test. Within direct test we have seen seismic reflection test, seismic refraction test, SASW or MASW test, seismic cross-hole test, seismic down-hole, up-hole test, etc. And in-direct test are which are common for static test field test like SPT, CPT, BPT pressure meter test, dilatometer test; all these are tests for static property determination for the soil which can be also used for determination of dynamic soil properties using some empirical relations.

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**SOIL DYNAMICS**

### Seismic Reflection Test



Path for incident ray and reflected ray of p-wave from horizontal layer boundary

Time for wave energy to come from S to R:

$$t_d = \frac{\text{distance of travel}}{\text{wave velocity}} = \frac{x}{v_{p1}}$$


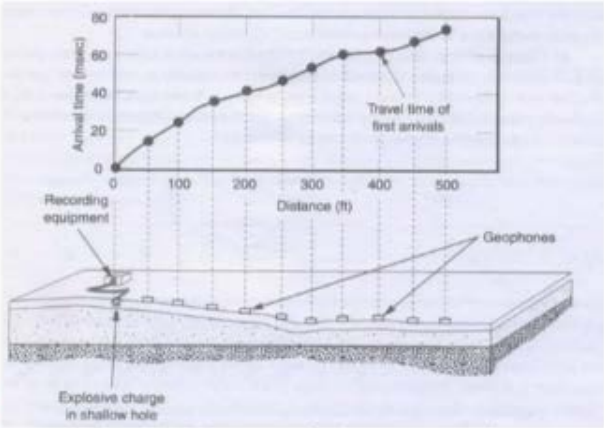
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Then seismic reflection test, we have seen what are the basis of the seismic reflection test.

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**SOIL DYNAMICS**

### Seismic Refraction Test



Arrival time (msec)

Distance (ft)


Travel time of first arrivals

Recording equipment

Geophones

Explosive charge in shallow hole

Schematic Diagram of the test procedure



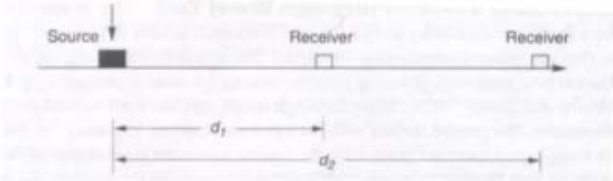
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Also for seismic refraction test.

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**SOIL DYNAMICS**

### Spectral Analysis of Surface Waves (SASW) Test



Typical configuration of source and receivers in SASW test. Receiver spacing is changed such that  $d_1 + d_2$  remains constant.

Travel time between receivers calculated for each frequency as:

$\phi(f)$  is the phase difference function). 
$$\Delta t(f) = \frac{\phi(f)}{2\pi f}$$

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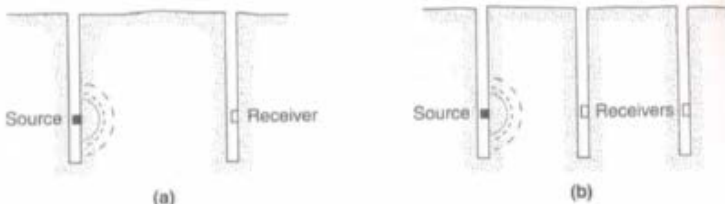
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Then SASW test.

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**SOIL DYNAMICS**

### Seismic Cross-hole Tests



Schematic diagram of test

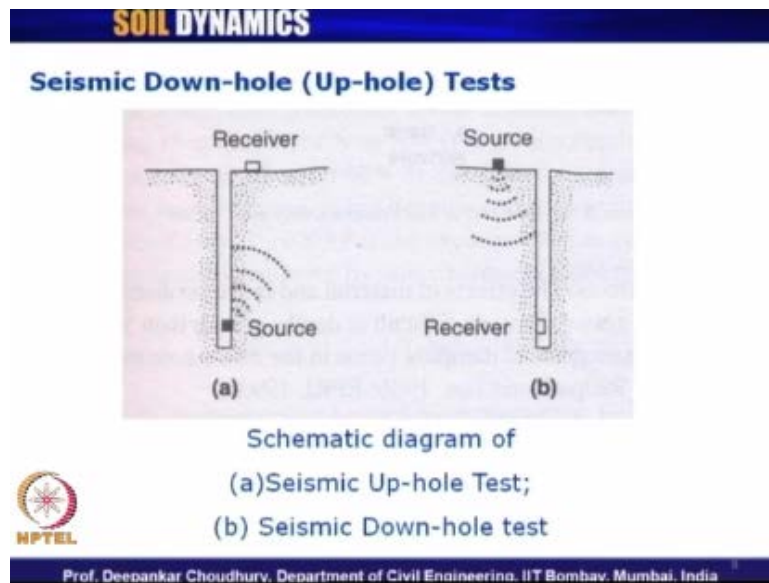
(a) Direct Measurement using two-hole configuration;  
(b) interval measurement using three-hole configuration

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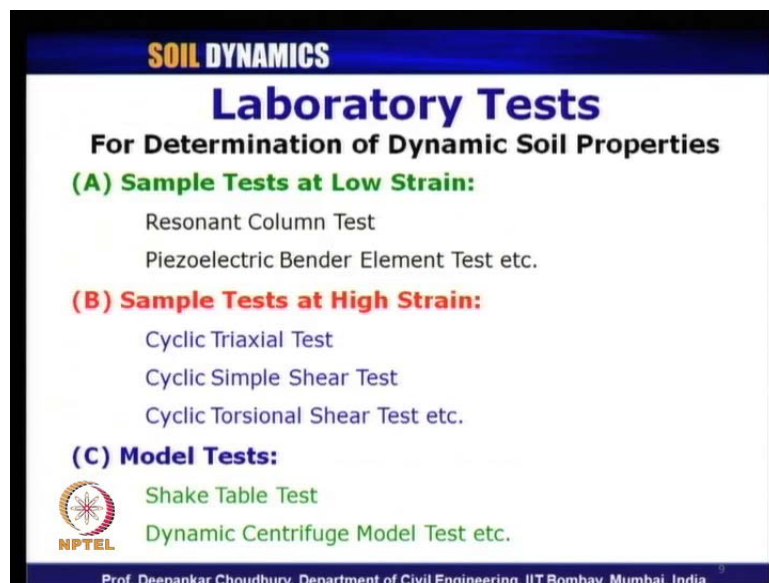
Then seismic cross-hole test.

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Then seismic up-hole and down-hole test.

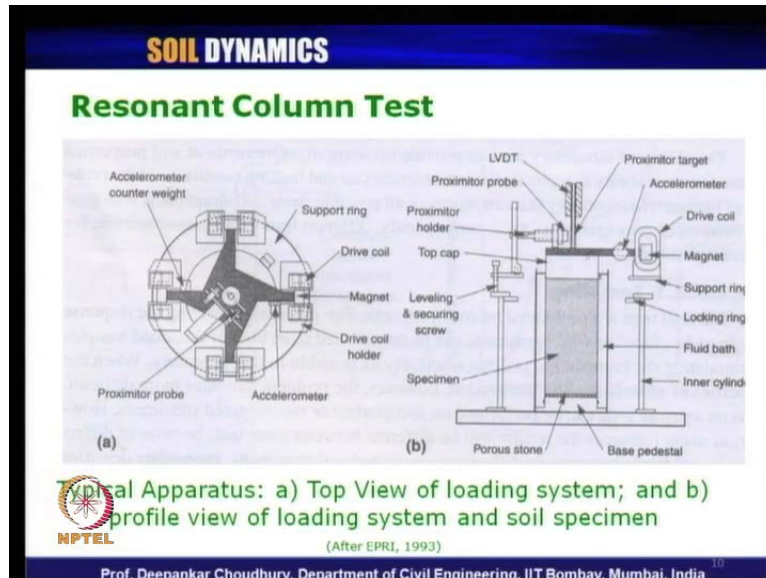
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Then, coming to the next major category of test; that is laboratory test determination for dynamic soil properties, we have seen the sub classification of the testing procedures like when we do the sample testing at low strain level, the another one is sample test at high strain level, and the third category is model test. So, within the category (A) that is sample test at low strain level, we have same different types of tests like resonant column test, piezoelectric bender element test, etc. And for sample test at high strain, we have seen cyclic triaxial test,

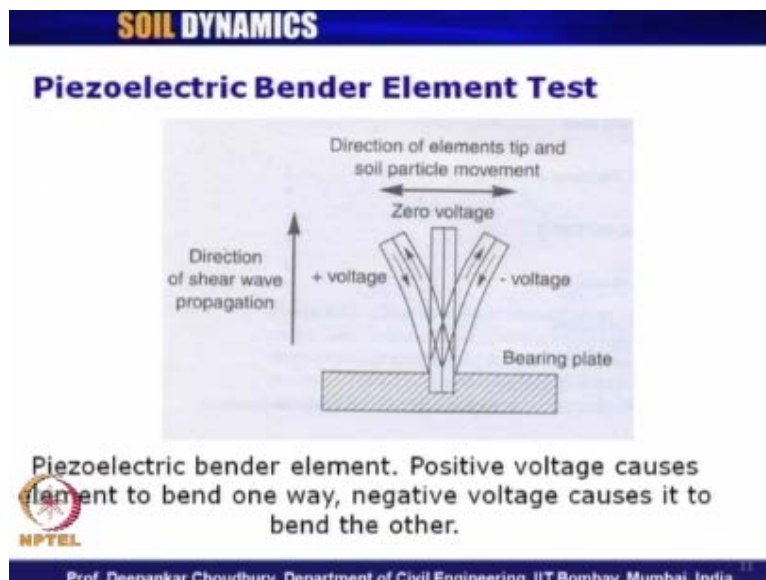
cyclic simple shear test, cyclic torsional shear test, etc and model test we have seen shake table test, dynamic centrifuge model test, etc.

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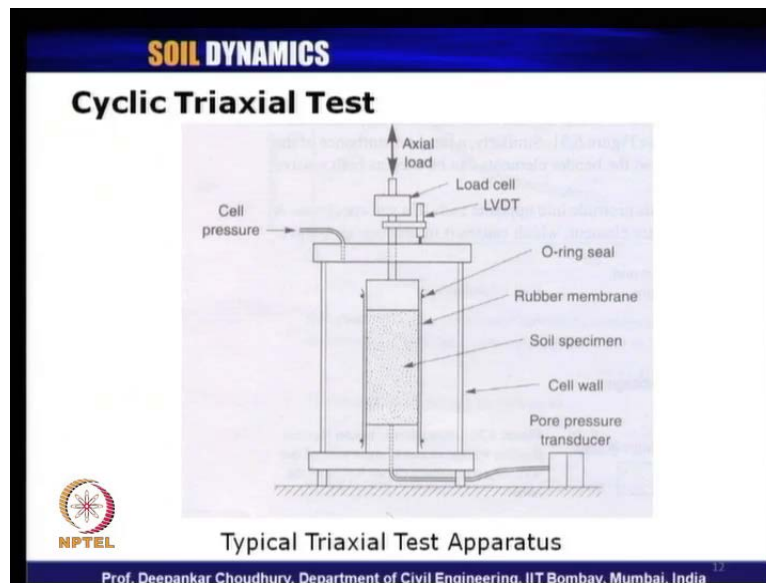
So, the schematic diagram for the resonant column test we have seen and the basic concept of resonant column test we have understood.

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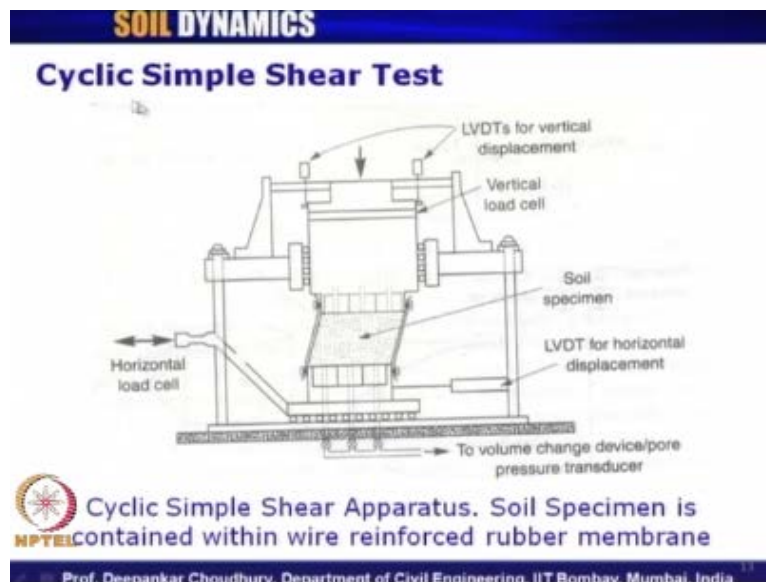
Then piezoelectric bender element test also we have seen.

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Again for the cyclic triaxial test, we have seen what is the basic methodology or basic difference between the conventional triaxial test with the cyclic triaxial test; only we use this deviatoric load as cyclic in nature. And also in this test we have seen, we can do not only the isotropic C D and CU test, but also an isotropic test.

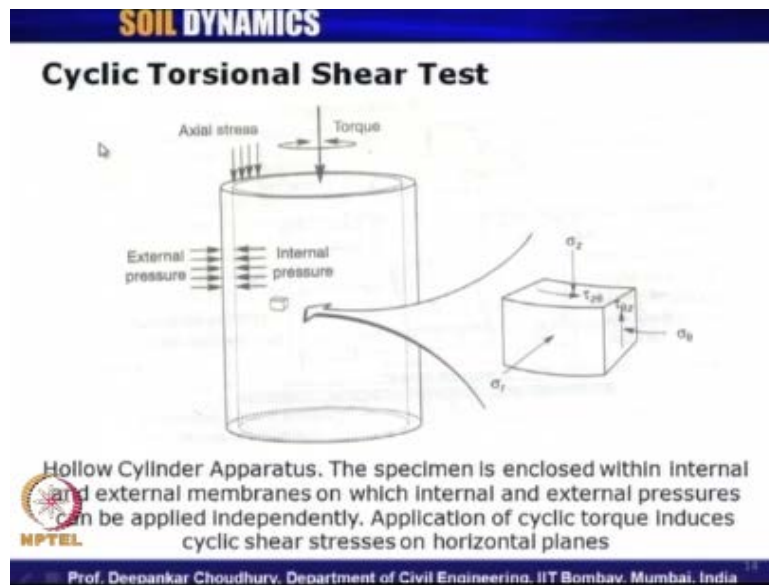
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Then, we have seen what is the basic concept of cyclic simple shear test in direct shear apparatus when we apply the shearing load in the horizontal direction instead of static load; we apply in the case of cyclic simple shear test, the dynamic load or cyclic load.

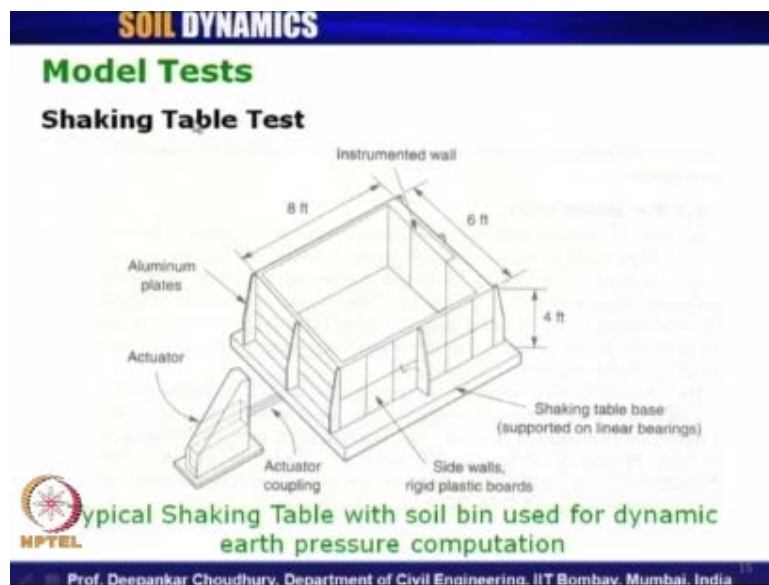


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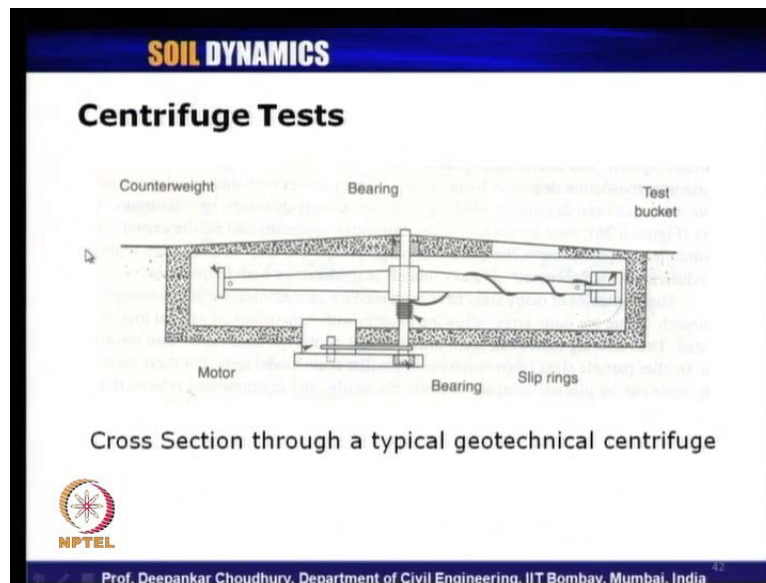
Also we have seen the cyclic torsional shear test.

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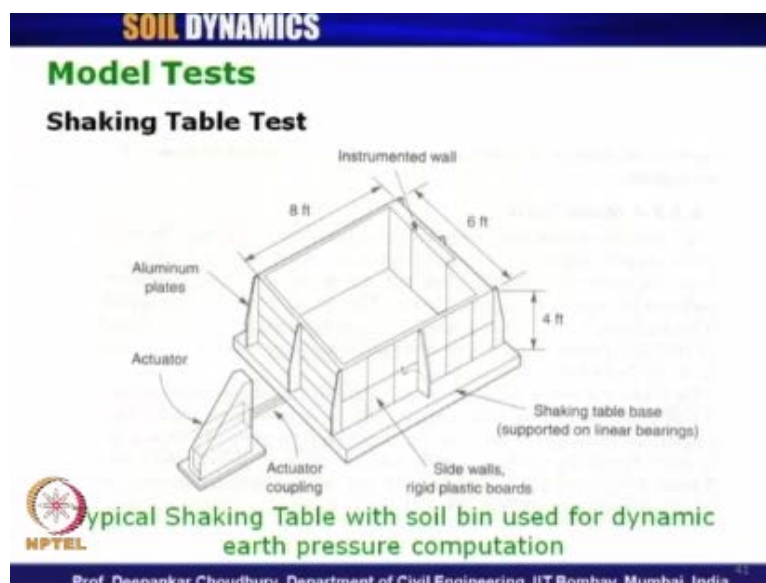
Within model test, we have seen what are the basic fundamentals about the shaking table test and what are its direction of shaking; depending on that we can have one-directional, two-directional or bidirectional or multidirectional shaking table facility testing facility.

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Now, another model test is centrifuge test for obtaining the dynamics soil properties. So, in centrifuge test this is the cross section of a typical geotechnical centrifuge; we have bearing, slip rings, we have a bucket test bucket, here counterweights are provided about this point. This keeps on rotating and the centrifugal force is getting generated and that is how the principle is used to scale up the model dimension which we use in our geotechnical engineering to a prototype or actual field dimensions.

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So, you remember or those of you were aware of the centrifuge testing facility, they know the drawbacks of one gravity one G test model test for our geotechnical engineering problems. Why? Because in structural engineering, it is pretty common to do one gravity or one G model test; that is you scale down your 10-storied actual building to a small model 10-storied building and do the test. You will get fairly accurate values of stresses displacement and so on; from the model test you apply the scaling loss, etc. Whereas if you apply to do the same thing for geotechnical problems, that is where you are handling with soil samples or a problems where suppose you have a soil slope, you have earthen dam, what happens? The concept of that scaling down from the actual prototype or field problem to a small model dimension is not giving you always the correct value of the stresses and displacement getting generated in model to prototype.

Why? Because in this case, the scaling loss are not that simple, not that linear, scaling loss. In this case many a times we have the concept of pressure bulb generated; in case of suppose we want to find out the bearing capacity or settlement of footings. So, if we do the test of a actual foundation, say, a raft foundation of 10 meter by 15 meter of raft size; that if you scale down in the model say 1 meter by 1.5 meter to maintain the ratio and do the test with the soil media also you have changed. Suppose in actual case we have just below the raft about 20 meter depth one particular soil, then below that another 10 meter depth of another particular soil, and if you scale it down with respect to prototyped model what you will do? You will put the 2 meter depth of that particular soil below the footing, then again another 1 meter of next layer of soil.

But when you are doing the test, doing the bearing capacity failure test or finding out the settlement of the footing under some particular load, your pressure bulbs getting generated in this small one gravity one g test in laboratory for the model test, the pressure bulb it may go through the other layers also in model tests; whereas, it may did not be necessary for your actual prototype case where you are getting sufficient 20 meter depth. So, these are the major cases. Also suppose if you want to test a pile. Now scaling down big actual pile, say, of 20 meter length; in prototype you are scaling it down in the laboratory say to 1 meter depth and doing the test in a soil profile. Again the same theory of scaling loss is not accurately used for geotechnical problems because for piles also, the zone of influence will come into picture which will not be commensurate with your model and prototype. So, these are the basic

disadvantages of using one gravity model test in geotechnical problems compared to other areas like structural engineering or other areas.

So, that is why using this shaking table test for a dynamic soil properties or the behavior of dynamic, say, earth pressure or dynamic bearing capacities things like that are not always correct or not always giving us the true result. So, we have to be very very careful to understand what are the limitations of the shaking table test for geotechnical engineering problems. Though people worldwide they use it, but it is not giving us accurate results and when we will get more accurate; obviously, as bigger the size as we can use. So, if we can use actual prototype sizes for our testing in shaking table test; then obviously, yes, it will give you the correct results.

So, to avoid that kind of scaling effect for the geotechnical problem, the best solution is to apply the geotechnical centrifuge facility where the stress conditions are first maintained, then all other dimensions are getting obtained, so that your small model can be scaled up to your actual prototype by using at high gravity level. So, it is something like you are doing the testing of your material in some other planet where you have, say, high value of  $g$  level and that centrifugal force, etc will create those kind of facility.

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And as you all know in India the national centrifuge facility is only available at IIT Bombay. So, this is the picture of IIT Bombay's geotechnical centrifuge, the beam type centrifuge. Its

name is called as “Sudarshan” when the bucket moves up during the spinning it goes up and then it rotates about its axis. So, this is during the movement process.

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Technical Specifications of Geotechnical Centrifuge at IIT Bombay	
Radius from the axis of rotation to basket base	: 4.5 m
Acceleration range at 4.25 m radius	: 10 g to 200 g
Payload at 100g	: 2500 kg
Payload at 200g	: 625 kg
Soil model size	: 0.9 m x 1.0 m x 0.65 m or 0.7 x 0.7 x 0.8m
Basket base dimensions (clear)	: 1000 mm x 1200 mm
Electrical sliprings	: 112
Hydraulic rotary joints	: 6
Run-up time	: 6 minutes to reach 200 g

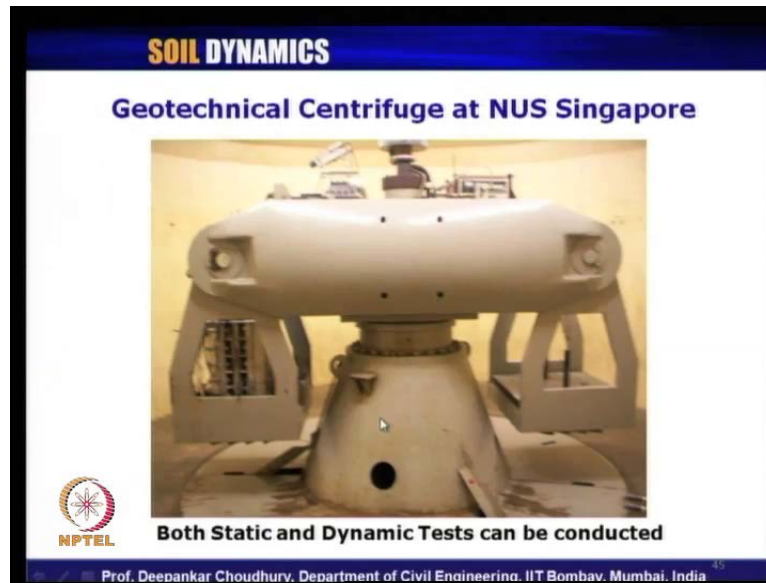
**Only Static Tests can be conducted till date**

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And what are the technical specifications of geotechnical centrifuge at IIT Bombay. Its radius from the axis of rotation of basket base is 4.5 meter and acceleration range at 4.25 meter radius is we can apply from 10 g to 200 g. Payload at 100 g is about 2500 kg; payload at 200 g is 625 kg. Soil model size can be used depending on your size of the container box of course, but there is a basket size. So, based on the basket clear dimensions, you can create your container size and then within that you can put your model sizes. And run-up time is about 6 minutes to reach 200 g.

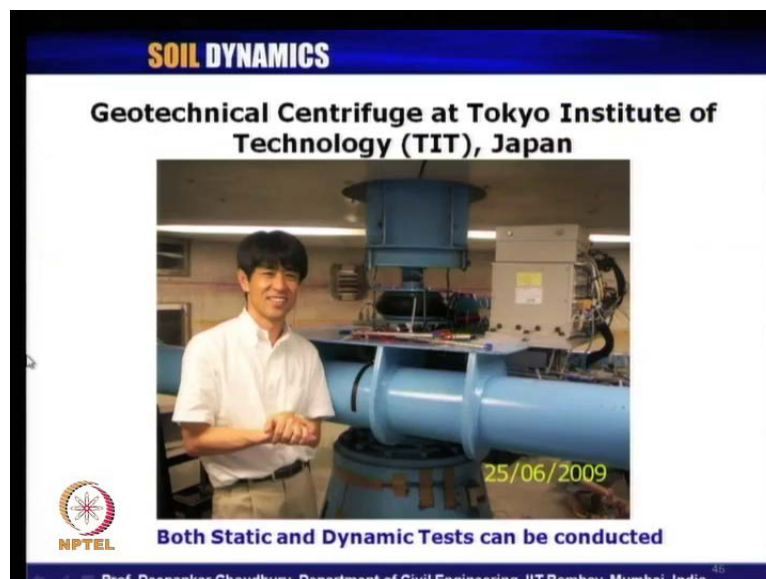
But this is one important note that till date, only the static test can be conducted in this IIT Bombay geotechnical centrifuge facility; no dynamic test can be conducted. Why? Because for doing the dynamic test, what is the basic principle let me come back again now. The basic principle of dynamic geotechnical centrifuge test is nothing but a shaking table test conducted in the centrifuge. So, this shake table has to be placed in the basket which is spinning during the centrifugal force. So, when we are keeping in this basket, the soil model like this; this container cannot be placed directly on this basket, it has to be placed on another shaking table; that shaking has to be applied during the flight or during the rotation of the sample. So, still it is not available with us.

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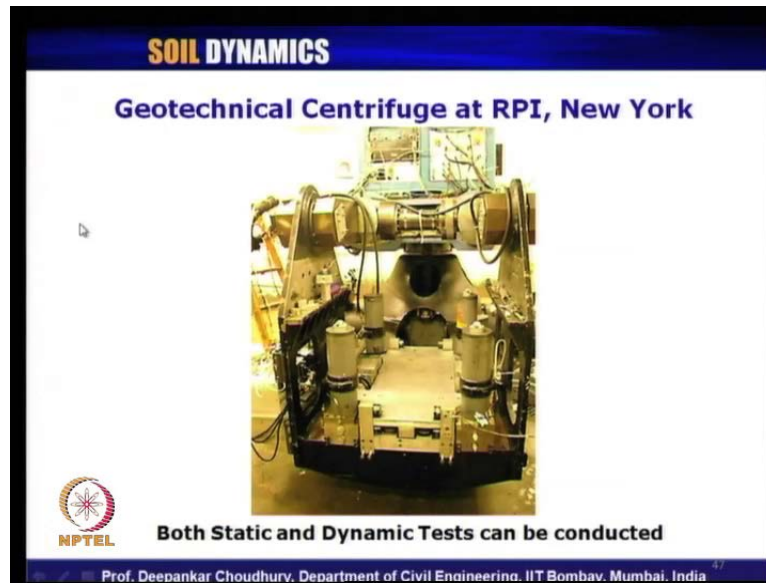
Some of the world class centrifuge testing facilities where dynamic test can be done; only a very very few and limited organizations can do the dynamic centrifuge test, some of them I am listing here for your knowledge. At National University of Singapore, this is their drum type centrifuge, this is counterweight part, and this is the bucket where the soil sample with container wheels placed. So, here they can do the dynamic test and as well as the static test.

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Another facility at Tokyo Institute of Technology at Tokyo, Japan; they also can do the dynamic geotechnical centrifuge test.

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Then another geotechnical centrifuge test which can conduct the dynamic test is at RPI, Rensselaer Polytechnic Institute which is in New York state of USA. And remember country like USA also cannot have many dynamic centrifuge testing facilities because of its several complications; I will come to that complication soon. So, in US they have major two equipments which can do this dynamic centrifuge test; those are in eastern US. This institute that is Rensselaer Polytechnic Institute, New York and in western USA, it is University of California at Davis UC Davis and these two are their national facility like ours.

So, whoever from other US university they want to do dynamic centrifuge test, they have to go either to this western extreme part or eastern extreme part to conduct the experiment on dynamic problems. All other centrifuge which is available in US those are static; they can conduct only the static test. Another testing facility which I have no data here am not showing. This place is I have visited and at some of these places I have worked. So, I can provide you the details. Other places like, for example University of Cambridge at London, they also have the facility of centrifuge which can do the dynamic test as well and there are few more others.

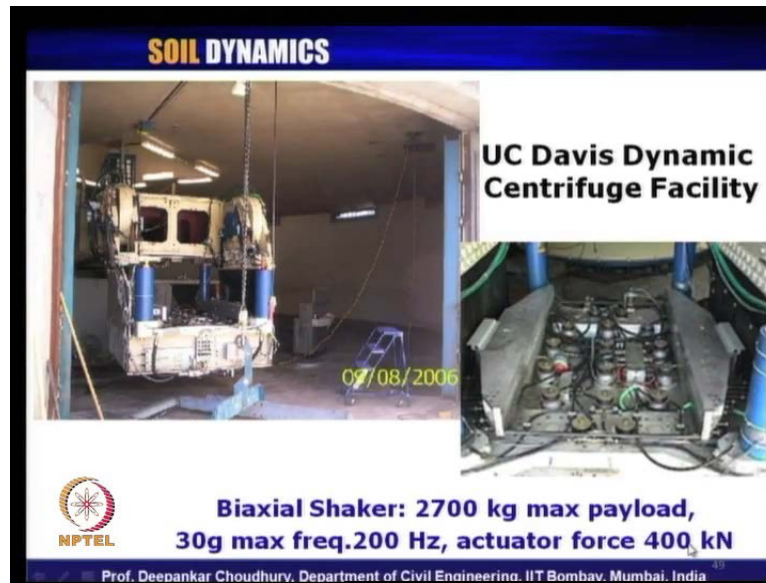
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Now, at present world's biggest centrifuge or largest centrifuge is located at University of California, Davis, USA. This is the world's biggest size of centrifuge. You can see its major specification is 9.1 meter radius. So, diameter is about 18 meter; 18 meter means about, say, conventional 6-storied building in the horizontal direction. It is just the double the size of our centrifuge. So, that is the world's biggest centrifuge. Also they can do the dynamic testing. So, in this bucket when they put the sample here, when they are doing the dynamic test, in this plate they provide the shaking table arrangements. Other specification is their capacity is 240g-ton capacity; ours is 200g-ton capacity, maximum payload is 4500 kg and the bucket area is 4 meter square.



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So, these are interior details when I used this facility during my visit to Berkley; these were the testing facilities of Biaxial Shaker they have, in this bucket. So, in this exaggerated picture is shown here. This can provide all the three directional of movement; that is both the horizontals using these accelerometers and also the verticals and on the top of it you place your container with the soil and the biaxial shaker capacity is 2700 kg maximum payload and 30 g is the maximum frequency with 200 hertz at 30 g. They can operate the machine with the actuator force of 400 kilo Newton.

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And how the testing container; this test of dynamic soil test is not using the same continual like static test. In static, we use a continuous container; a single container with the continuous boundary in all the sides. But in dynamic test it should not be; it should be a layered type of container which is called flexible-shear-beam-model container which are used for dynamic test. It is across the world wherever the dynamic testing can be done, all these individual layers they are hinged between these two layers, so that when axial shaking is applied, one layer can move with respect to the other layer similar to the behavior of your soil media; that is when waves are propagating within this you have your soil model and when it is getting shaken at this bottom, different layers also at different mode shapes they vibrate. So, that is why your model container cannot be shear for rigid continuously, but it has to be sliced like this using the concept of flexible-shear-beam-model and the soil material is kept here. So, that way the dynamic centrifuge testing is done.

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Type of Event	Quantity	Model Dimension Prototype Dimension
All events	Stress	1
	Strain	1
	Length	$1/N$
	Mass	$1/N^3$
	Density	1
	Force	$1/N^2$
Dynamic events	Gravity	$N$
	Time	$1/N$
	Frequency	$N$
	Acceleration	$N$
Diffusion events	Strain rate	$N$
	Time	$1/N^2$
	Strain rate	$N^2$

Source: After Kutler and James (1989).  
 \*Values are based on the assumption that the same soils and fluid are used in the model and the prototype and that the soil properties are not rate dependent.

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Now, coming to the typical scaling factors which are used for centrifuge modelling; these are also known to many of you those who are aware of centrifuge testing facility, but that is not the scope of this test. So, I am just highlighting only the component of only dynamic event; that is when we are doing the dynamic test, what happens to the scaling loss? As we know, the centrifuge the major aspect we are following is we are keeping stress strain conditions same in both model and prototype; that is the basic principle we have to follow and from that we arrive at other scaling parameters.

So, stresses model to prototype dimension ratio is one; strain also one. But dynamic event, the time is 1 by N. What is N? N is the number at which you are rotating your centrifuge, that is this is the g level. Suppose your centrifuge is rotating as 30 g, then N is equal to 30; then if it is rotating at 35 g, then N is 35, like that. So, model dimension if the time is one, then for your prototype dimension dynamic time becomes N times of model dimension. So for example, if you do a dynamic centrifuge test in UC Davis dynamic centrifuge testing facility, what happens? As I said, generally the dynamic testing they are doing in the range of 30 to 35 g level. So, if we do a testing on a model at 30 g level. So, your earthquake whatever input value or whatever excitation you are giving, suppose it is given for say for a long time of 30 seconds in the prototype, that is, it is actually acting on your prototype or actual structure for 30 second long earthquake.

So, when you are modelling it in the centrifuge and running it at 30 g, how much time you should take to give that input to that model that is divided by thirty; that is only one second. So, within one second you have to give the input excitation to your model. So, this is the most complex part of the application. Now, you can understand probably why the dynamic centrifuge testing facility is not so common worldwide, though it is extremely important. Because people cannot afford it first of all due to its heavy cost and the second part is its operational complicacy, instrumentational complicacy is another major criteria where here earthquake response even sometimes it may not occur for 30 seconds; it can occur for 10 seconds only, 15 seconds only the major earthquakes shock. How are you going to put that in your model? It will be only for one-third second; within one-third second, you have to put your earthquake input in your model and you have to get the corresponding response.

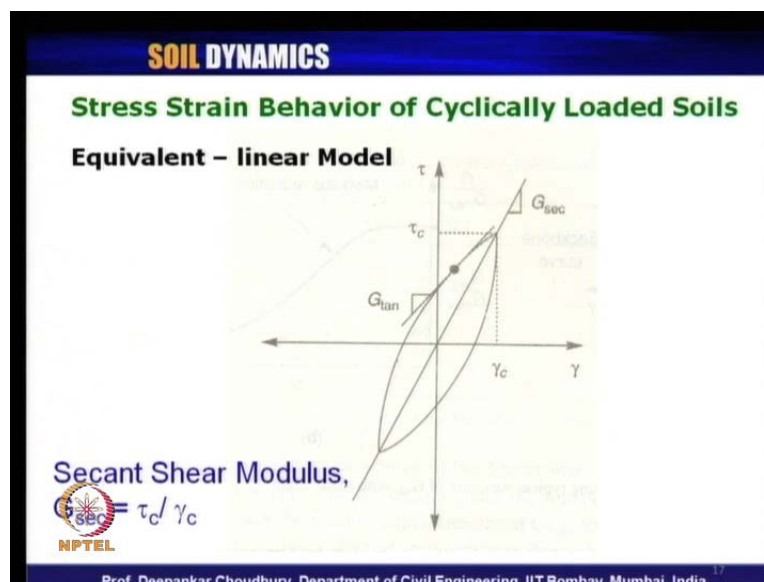
Also the frequency if you look at the slide, frequency is model to prototype is N; that means, if the frequency of your earthquake is say 1 hertz, it should be 30 hertz frequency you have to apply; such a very high frequency and such a very small time you are getting. So, what kind of dangerous situation you are handling you see because, remember applying a very very high frequency is dangerous both to your equipment as well as your sample and everything in the surrounding vicinity. So, that is why all this centrifuge facilities you know it is closed in a very well designed container kind of environment, so that it is very well protected. Acceleration scale effect gets N times in the model. So, that again is another major problem.

Suppose earthquake acceleration of 0.2 g or 0.3 g peak acceleration is occurring at your actual site of prototype during the earthquake. So, when you are rotating it at 30 g, how much

acceleration you should provide for 0.2 g. So, 6 g; so obviously, it is trying to lift up. So, that times of acceleration you have to provide within the soil sample, but it will not lift up because it is not at 1 g level; it is actually at 30 g level spinning. Strain rate, the rate at which you are applying your cyclic strain; that also should be n times from your prototype to model. So, these are the scaling formulas which are proposed by Bruce Kutter and James in long back in 1989. Because Bruce Kutter is the main professor who is the incharge of the UC Davis dynamic centrifuge facility and these are the other dimensions or scaling factors. So, it automatically gives us some idea, though it is extremely important to do the dynamic testing model testing but how complex it can be actually.

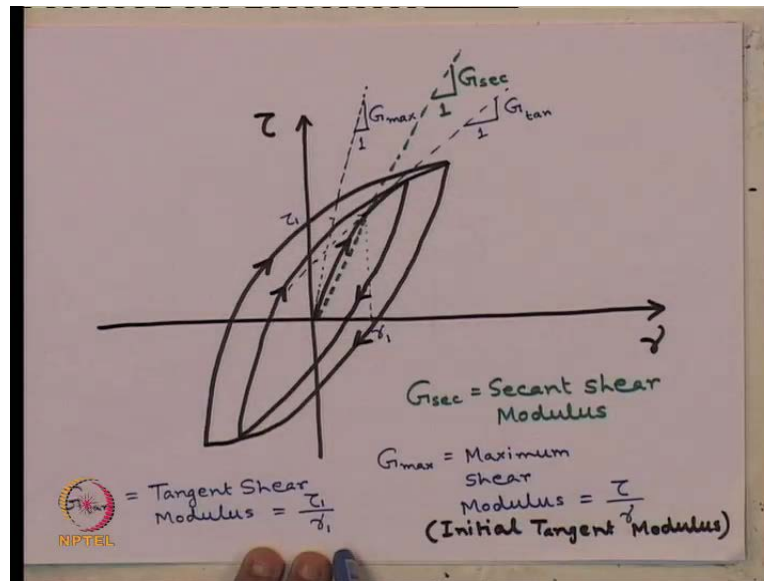
End of Part (A)

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Now coming to today's lecture, let us first discuss about when we conduct in laboratory, suppose the second category of test; that is the sample test with high strain level. So, suppose we are conducting in laboratory the cyclic triaxial test or cyclic simple shear test, what finally we can get the result or what we can draw, let us draw here.

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So, the output which we are getting from the test is shear stress versus shear strain. So, tau is shear stress and gamma is shear strain. So, what is the behavior of shear stress development for different cyclic shear strain, we want to measure through the laboratory test of cyclic triaxial test or cyclic simple shear test like that. So, how the variation for any particular sample can go. So, suppose the path followed is like this, when we keep on applying the load cyclic load on the sample. So, cyclic strain is also increasing and shear stress is also increasing. Now after reaching certain point, let us say we withdraw the load. So, what will happen? it will come back, but as the soil material is not fully elastic as we know; so obviously, during the unloading process the path it will follow is not exactly this path but some other path. So, it will follow this path and if we do the stress reversal that is in other cycle or in other direction we are applying load. So, it will go in this other direction.

Then again we start applying load to the sample; it will go like this and suppose it reaches at this point. After that we again do the unloading; then again we carry out the reloading. So, this is the typical variation of any loading unloading; that is any cyclic test if we conduct in the laboratory on any soil sample, this is the typical behavior of shear stress versus shear strain. And from the slope of this curve what we will get? We will get the modulus which is nothing but the shear modulus. So, slope of the curve at any point we will get different modulus. If we draw a tangent at this point, then the slope of this line that is which is tangent at the initial position of the curve, that is called  $G_{max}$  that is maximum shear modulus;  $G$

max is called maximum shear modulus which is nothing but slope of the curve, that is  $\tau$  by  $\gamma$  at a very low strain level.

Now if we take any other point. Suppose this point if we draw a tangent at this point, we will then get another value of  $G$ . What is that  $G$  called? It is called tangent shear modulus  $G_{tan}$ . So,  $G_{tan}$  is nothing but tangent shear modulus. This is expressed again by  $\tau$  by  $\gamma$  at any particular point; suppose at this point we have, say,  $\gamma_1$  and  $\tau_1$ . So, it will be  $\tau_1$  by  $\gamma_1$  at that particular point and another slope of the  $\tau$ - $\gamma$  relationship we can get. Suppose at this same point, if we join this point with the initial point zero-zero and draw the slope of it that is called  $G_{sec}$ . So,  $G_{sec}$  is nothing but secant shear modulus.

Now between these three values of shear modulus, what clearly we can understand from this figure, which value of the shear modulus for the same soil sample will be maximum and which one will be minimum? The maximum value of course will be this value of shear modulus at very low strain level that is at initial. So,  $G_{max}$  is nothing but we can rewrite it as initial tangent modulus. Initial tangent modulus is nothing but  $G_{max}$  or maximum shear modulus.

So, the maximum value of the shear modulus we will get, if we take the value of this  $G_{max}$  at very low strain level very small strain level. And if we keep on going at the higher value of the strains, what we can see; this slope of this line that is tangent at any point on this curve will keep on becoming flatter and flatter because soil obviously will go from elastic range to plastic range. So, that is why when we compute the  $G_{tan}$ , remember that for a soil material tangent shear modulus is not a constant value; it is fully dependent on at which shear strain level you are measuring it. So, this value of shear strain must be specified to mention that at this level we are mentioning about the shear modulus of the soil.

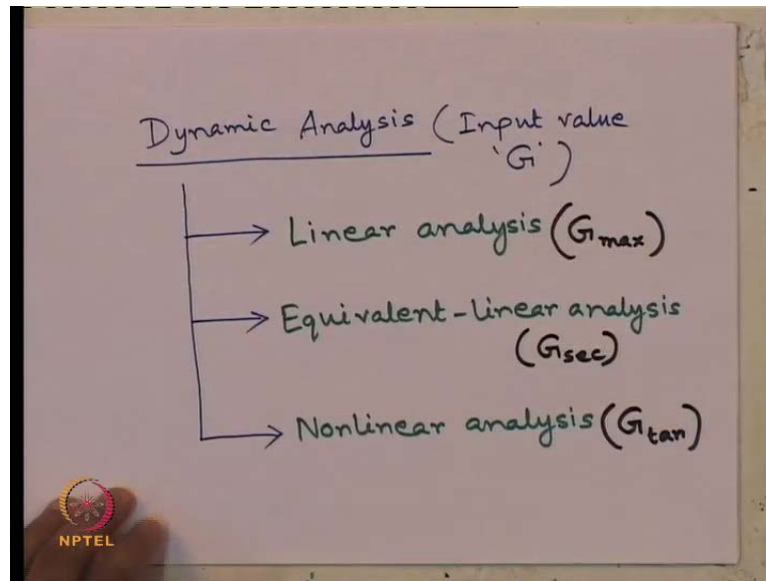
So,  $G_{tangent}$  is very much dependent on the cyclic shear strain applied to the sample; but  $G_{max}$  is not dependent on the shear strain, it will remain constant for a particular type of soil material, but  $G_{tan}$  will keep on changing. Similarly another slope this green colored line what we have drawn which is called secant shear modulus, this is nothing but we are joining the initial point that is zero-zero to a particular point at a particular strain level where we want to obtain the shear modulus.

So by joining them linearly, we are getting the slope of the curve as the secant shear modulus. So, note it here that secant shear modulus for a soil sample is again not a constant value, but



it keeps changing depending on what is your final shear strain level. Suppose if we join this point and this point, obviously the slope of the secant modulus will change. However if we want to do a proper analysis the dynamic analysis which value of the  $G$  we should consider do you think looking at this three different types of shear modulus that is maximum shear modulus, tangent shear modulus, and secant shear modulus, which one we should consider.

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Like we know this dynamic analysis can be classified in three major categories. So, whenever we are doing any dynamic analysis we have to give some input value of the soil properties as  $G$  shear modulus which is the most important dynamic soil property the shear modulus. Now depending on the input value of  $G$ , we can sub classify the dynamic analysis into three major categories, what are those three? One is called linear analysis, another one is called equivalent-linear analysis, and the third one is called nonlinear analysis.

Now, how this three major classification of any dynamic analysis has been made; that is nothing but because of your input value of  $G$  which you are using for your analysis. When we call the analysis is linear analysis; when we are using the value of  $G$  as  $G_{max}$  because for a particular soil this is the only modulus which is not changing and  $G_{max}$  is that value of the shear modulus in which the soil remains in the elastic zone completely elastic zone because this modulus is at very very low strain level.

So, that is the reason why we call this analysis as linear analysis; that is linear behavior of the soil material is considered for the dynamic analysis, that is why the name linear analysis

which is obviously the simplest type of dynamic analysis because your  $G$  value remains constant and the  $G_{max}$  value we can use from several test as we have seen SASW or several field test; by measuring the shear wave velocity you can get the  $G_{max}$  value of any soil.

But in reality, the dynamic problems not necessarily will remain in the linear zone of the behavior of the soil material; it is not necessarily be within the elastic zone. If it crosses its elastic boundary, then the logic behind assumption of  $G_{max}$  is not correct. Then something we are missing or something we are doing wrong in our analysis even then if we take the value of  $G_{max}$  as our input value. So, that is why the correct analysis is fully nonlinear analysis. So, in nonlinear analysis this third category, there we are considering the fully nonlinear behavior because if we look at the variation of this tau versus gamma plot for the soil material typically, its behavior is completely nonlinear. This curve is completely nonlinear except this initial portion which is only linear which gives us the  $G_{max}$  value.

So apart from that, entire behavior of the soil at any shear strain level will be completely nonlinear. So, it is always better to use the nonlinear analysis for an actual dynamic analysis. So, if we want to use nonlinear analysis which value of  $G$  we should use? Because in that case, we cannot use  $G_{max}$  only because  $G_{max}$  remains constant. But as we have seen the curve of tau versus gamma, the slope is keep on changing as we change the shear strain or as we go from one cycle to another cycle of loading. So, which value of  $G$  we should use in this case; that should be tangent shear modulus because tangent shear modulus is that modulus which is giving you the actual slope of the tau versus gamma plot at any point of shear strain or any state of loading.

So if we look back here, at this point this slope will give the shear modulus of the soil; at this point if we draw the tangent at this point it will give this modulus of the soil material. So that way, even at this point also in the other side of the cycle if we draw the tangent at this point we will get another  $G_{tan}$  value. So, depending on your shear strain at which you are doing the analysis, corresponding  $G_{tan}$  value has to be used. So, complete nonlinear analysis means when the sample goes from this state to at any level of shear strain, at each of them you should consider the change in the value of the  $G$  due to the curvature of this behavior, which is taken care by if you take the input value as tangent shear modulus.

Now, what is in between? As we have seen, linear analysis is the simplest analysis because  $G_{max}$  value remains constant and we have also understood fairly that nonlinear analysis is

most complex analysis because at each shear strain level you need to change the value of the  $G$  during the analysis process.  $G$  no longer remains constant; it keeps on changing from point to point.

So, to compromise between these two extreme that is the simplest analysis of linear model and complex analysis of nonlinear model, something in between has been formulated which is known as equivalent-linear analysis. So, what do we understand by equivalent linear? That is instead of capturing the entire nonlinear behavior of the curvature of tau versus gamma; if we take at high shear strain level, if we join that point to the initial point of zero-zero, that is nothing but a straight line we are assuming. So, that is why the name equivalent-linear. So, we are basically considering the linear slope of the curve, but equivalent means we are also taking care of the possibility of higher strain.

So, let us look back at this picture again. So, at this point as I said if we want do the analysis at gamma 1, we have joined this point to this basic origin point and we have joined them by a simple straight line. So, we are considering equivalent-linear behavior of the soil as if that tau-gamma is following this path though it is not so. So, the slope of the curve is  $G$  secant. Remember if we take in our analysis  $G$  max, obviously we are overestimating the value of shear modulus or strength of the soil under the dynamic load which might be dangerous in many cases; that is you are assuming that your soil can take a high strength, but soil is having a high strength under a dynamic loading condition, but it may not be necessarily true in the linear analysis. So, that is why linear analysis is the dangerous one to consider.

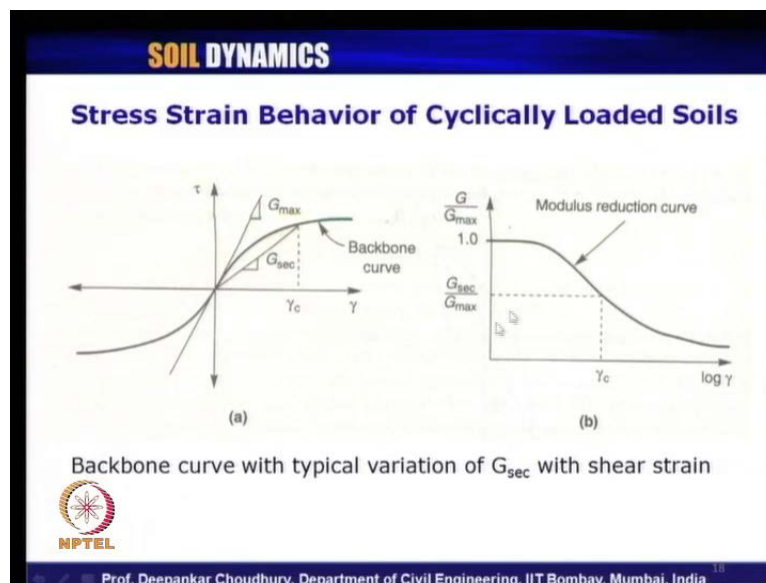
On the other side if we take  $G$  tan, there we are taking lowest value of the  $G$  which is taking actually the accurate picture of the behavior of the soil sample. If we can do the nonlinear analysis, there is nothing better in it, but as I have mentioned the complexity arise. So, you have to see just for some simple problem if you take too much of time in doing or proposing the design aspect, nobody will accept your design due to its huge complexity.

So, that is why the in between thing has been identified which is known as equivalent-linear analysis where we consider this  $G$  secant. And here if you notice the value of  $G$  here is not as high as  $G$  max or not as low as  $G$  tan; it is in between and also it can be very close to  $G$  tan if your final range of shear strain up to which your sample can go through is pretty fairly on a lower side. So, then it almost replicate the actual position of the behavior of the soil sample. So, that is why the equivalent linear model is very very commonly used where we are not

compromising with linear analysis; also we are not entering into the complexity of nonlinear analysis. But truly speaking for important problems we must do the nonlinear analysis.

So, for equivalent-linear analysis we use secant shear modulus. So, these are the basic three types of dynamic analysis based on the different input value of the  $G$ . So, now let us look at the slide here what it says; that for a cyclically loaded soil, this is the common typical stress strain behavior as we have discussed just now tau versus gamma plot and tangent shear modulus and secant shear modulus have been defined here.

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Now another terminology is used for this curve of tau versus gamma for any soil sample when it is cyclically loaded; that curvature that curve of tau versus gamma is called backbone curve. So, what is backbone curve refers to in a dynamic soil properties problem related to? It is nothing but the output of shear stress versus shear strain plot of any cyclically loaded soil; the curvature that curve is called backbone curve.

And another common way to represent the results of the reduction in shear modulus because just now we have seen in this picture, if I put it here again, let us look at here. The shear modulus keeps on decreasing with increase in the strain level. So,  $G_{max}$  value keeps on decreasing and at each shear strain level it is keep on decreasing and some lower value than  $G_{max}$  we are getting. So, another way to represent the ratio or reduction of the  $G$  value with respect to the shear strain is called by using modulus reduction curve.

So, now let us look at the slide here what is modulus reduction curve? In y-axis we plot the value of G by G max; G means the shear modulus at any point of shear strain level. So, basically this is G tan. So, G tan by G max versus we are plotting; in log scale, the shear strain. So, log of shear strain we are plotting. So, the variation of the curve will look like for any typical soil sample is something like this; that is initially it will be about one when the shear strain is very low. So, at very low shear strain as we know it will behave linearly. So, that is why the initial tangent shear modulus will be nothing but the maximum shear modulus.

So, their ratio will be one. So, that is why it is starting from this value of one and then keeps on decreasing as we keep on going on the higher values of the shear strain. So, at any point of shear strain, higher value of shear strain, if we want to find out what is the G value from this modulus reduction curve, we should be able to compute the G secant. So, that is why it is also a very popular or common way to represent the laboratory soil test data in terms of modulus reduction curve.

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**SOIL DYNAMICS**

**Maximum Shear Modulus (Empirical)**

Sand:  $G_{\max} = 1000 K_{2,\max} (\sigma'_m)^{0.5}$

Where,  $K_{2,\max}$  is determined from the void ratio or relative density  
and  $\sigma'_m$  is in lb/ft<sup>2</sup>

$e$	$K_{2,\max}$	$D_r$ (%)	$K_{2,\max}$
0.4	70	30	34
0.5	60	40	40
0.6	51	45	43
0.7	44	60	52
0.8	39	75	59
0.9	34	90	70

Source: Adapted from Seed and Idriss (1970).

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Now there are several empirical relations available in the literature to compute the maximum shear modulus. For example, as given by Seed and Idriss in 1970, for sand the G max value can be computed using this empirical relations 1000 times K 2 max times sigma m dash to the power 0.5, where this K 2 max is a function of void ratio or relative density of the sand and sigma m dash is nothing but effective vertical stress which is expressed in terms of pound

per feet square. So, keep a note of this; as you know, whenever you are using any empirical relation we have to be very very particular that what is the unit of the parameters involved.

So parameter involved here, this  $\sigma_m$  dash should be in pound per feet square unit and for different values of void ratio, what are the values of  $K_{2 \max}$  or for different values of relative density of the sand, what are the values of  $K_{2 \max}$  was proposed by Seed and Idriss in 1970. One caution you must note down that this empirical relationship whenever you are using; first thing as I said we have to be very careful about what are the units. If unit are changing, then this relation, this coefficients, etc will change if you change the unit from this FPS to, say, SI unit or other CGS unit. So, remember that one. And another drawback or limitation, I should say limitation of empirical relationship, because this empirical relationships were derived from a number of laboratory test. Now with time, this was proposed long back in 1970; with time, several 100s and 1000s of more laboratory tests were conducted by several other researchers and it is not necessarily that the sand of any particular location will follow this relationship will be truly universal; that is sand of Seed and Idriss did experiments mostly with the California sand, US sand; that may not be true for our Indian sand or some other country sand.

So, that is why later on several other researchers from different countries have proposed different empirical relationship to compute the  $G_{\max}$  value from different countries or different regions. So, whenever you are using this equation or empirical expression, you have to make sure whether really it is valid for your sand condition or soil condition or not. So, this is one caution for using empirical relations; unless it takes care of entire globally available sand or sand material.




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**SOIL DYNAMICS**

**Empirical Relationship between  $G_{max}$  and In Situ Test Parameters**

In Situ Test	Relationship	Soil Type	References	Comments
SPT	$G_{max} = 20,000(N_{60})^{0.53}(\sigma'_v)^{0.5}$	Sand	Ohta and Goto (1976), Seed et al. (1986)	$G_{max}$ and $\sigma'_v$ in lb/ft <sup>2</sup>
	$G_{max} = 325(N_{60})^{0.48}$	Sand	Imai and Tonouchi (1982)	$G_{max}$ in kip/ft <sup>2</sup>
CPT	$G_{max} = 1634(q_c)^{0.230}(\sigma'_v)^{0.333}$	Quartz sand	Rix and Stokoe (1991)	$G_{max}$ , $q_c$ , and $\sigma'_v$ in kPa; Based on field tests in Italy and on calibration chamber tests
	$G_{max} = 406(q_c)^{0.085}e^{-1.130}$	Clay	Mayne and Rix (1993)	$G_{max}$ , $q_c$ , and $\sigma'_v$ in kPa; Based on field tests at worldwide sites

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Some more empirical relations between  $G_{max}$  and other in situ test parameters are given here like from SPT test static SPT test this relationship is given in terms of  $N_{60}$ ,  $N_{60}$  is correct SPT value. For only sand type, it was proposed by Ohta and Goto by 1976 and Seed et al. in 1986. Here also  $G_{max}$  and  $\gamma_m$  dash values are in FPS unit that is pound per feet square unit.

Then another set of researchers Imai and Tonouchi in 1982 they proposed another relationship of  $G_{max}$  for sand only, but this is different sand; this is sand from Japan and that one sand from US. So, you can easily see; if you use for any soil sample or sand sample, this relation and this relation,  $G_{max}$  value obtained by this two empirical relations will differ. It need not be the same value. So, when you are doing or handling any design problem you have to make a proper judgment; that is your soil sample is close to which type of sand. Otherwise, you need to do the testing for your own soil sample before coming to use of any such empirical relationship.

Again the use of CPT test Cone Penetration Test, the CPT test value  $q_c$  can be used to obtain  $G_{max}$  value for Quartz type of sand which is proposed by Rix and Stokoe in 1991 and units for  $G_{max}$ ,  $q_c$ , and the  $\sigma'_v$  dash that is effective vertical stress is in kPa; that is SI unit based on field test in Italy and on calibration chamber test. So, they already specified that it is valid for the Quartz sand available in Italy.

Then another relationship is proposed given here for Clay type of soil Mayne and Rix in 1993; they have proposed based on field test and worldwide site. So, when they have proposed this equation they have considered the soil from almost all regions of the world. So, when you are using this kind of generic equation or generalized equation; obviously, it will be giving you little better results than comparing to using equation which is specific to a particular region or particular soil.

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**SOIL DYNAMICS**

**Empirical Relationship between  $G_{max}$  and In Situ Test Parameters**

In Situ Test	Relationship	Soil Type	References	Comments
DMT	$G_{max} / E_d = 2.72 \pm 0.39$	Sand	Babb et al. (1986)	Based on calibration chamber tests
	$G_{max} / E_d = 2.2 \pm 0.7$	Sand	Bellotti et al. (1986)	Based on field tests
	$G_{max} = \frac{530}{(\sigma'_v / p_{at})^{0.25}} \frac{7p / \gamma_s - 1}{2.7 - \gamma_s / \gamma_w} K^{0.25} (\rho_s \sigma'_v)^{0.5}$	Sand, silt, clay	Hryciw (1990)	$G_{max}$ , $p_{at}$ , $\sigma'_v$ in same units; $\gamma_s$ is dilatometer-based unit weight of soil, based on field tests
PMT	$3.6 \leq \frac{G_{max}}{G_{pm}} \leq 4.8$	Sand	Bellotti et al. (1986)	$G_{pm}$ is corrected unloading-reloading modulus from cyclic PMT
	$G_{max} = \frac{1.68}{\alpha_p} G_{pm}$	Sand	Hryciw et al. (1991)	$G_{pm}$ is secant modulus of unloading-reloading portion of PMG; $\alpha_p$ is factor that depends on unloading-reloading stress conditions; based on theory and field test data

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Similarly, use of DMT what is DMT dilatometer test dilatometer test is a common test field test for static determination of soil property which can be again used to obtain the  $G_{max}$  value from this  $E_d$  value for different sand or sand or silt different researchers have proposed how it can be computed which are listed here Then again by using PMT PMT is pressure meter test pressure meter test is another type of common field test used for static test determination of soil property there also you can find out  $G_{max}$  value from the pressure meter test results for different types of sand as proposed by different researchers as listed here.

In the next lecture we will continue our discussion with dynamic soil property.