Soil Dynamics
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Module - 2
Vibration Theory
Lecture - 2
Degrees of Freedom, SDOF System, Types of Vibrations

Let us start today's lecture on soil dynamics. This is our second lecture on this NPTEL phase two video course on soil dynamics. Before starting today's topic, let me recap what we have learnt from the first lecture. In first lecture we have covered module one, which is containing the introduction of the course. We have seen, what is the need for studying this course, and also the objective of studying this course on soil dynamics.

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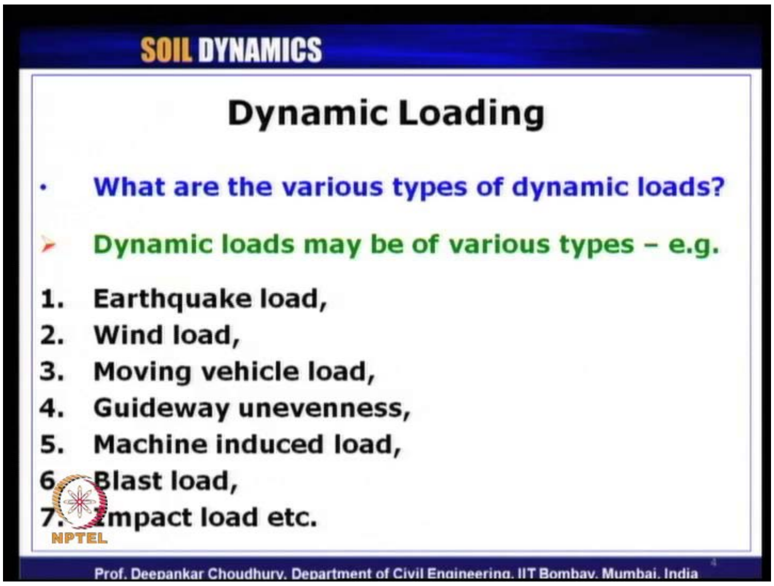

Then, we have seen, what is called dynamic loading; any load which is varying with respect to time are need not to be dynamic, but the dynamic load must vary with time. And we have seen the examples of various types of dynamic loads, like earthquake load, wind load, moving vehicle load, guide way unevenness, machinery induced load, blast load, impact load, etcetera. But that does not mean we have seen that all time dependent loads are dynamic, there can be live load also which are not dynamic in nature.

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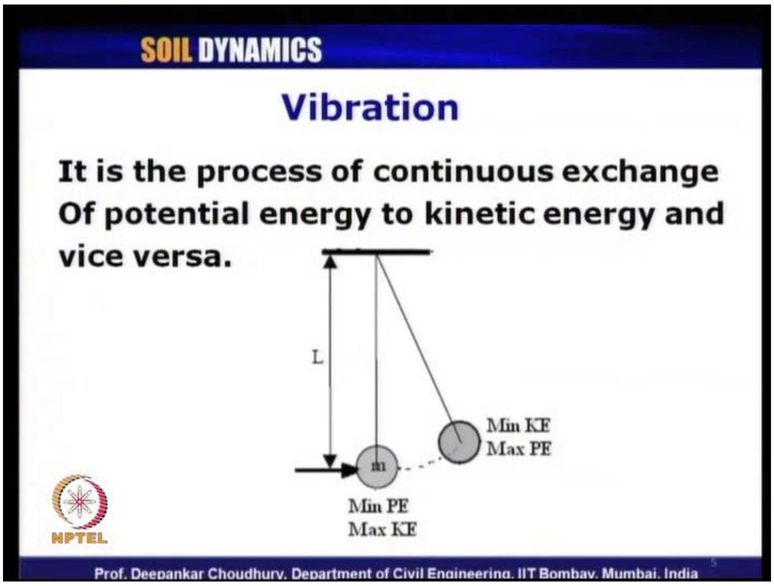

So, to comply with the loads which are dynamic in nature and also varying with respect to time, the criteria is, it has to vibrate the system. And the vibration is the process of continuous exchange of potential energy to kinetic energy and vice versa. And we have seen the example of this simple pendulum, where the exchange of potential energy to kinetic energy occurs. And another component of energy is also involved in the process, we have seen, that is the dissipation due to the loss of energy in form of sound or heat, etcetera.

So, with that we completed our previous lecture on module one, which is the introduction of the topic on soil dynamics. Let us continue our discussion with our today's lecture on lecture 2 of soil dynamics, which contains module 2 on vibration theory. So, to go with the details of the module 2 on vibration theory, let us define some of the parameters which will be very much required for us, to understand the concept of vibration in this module.

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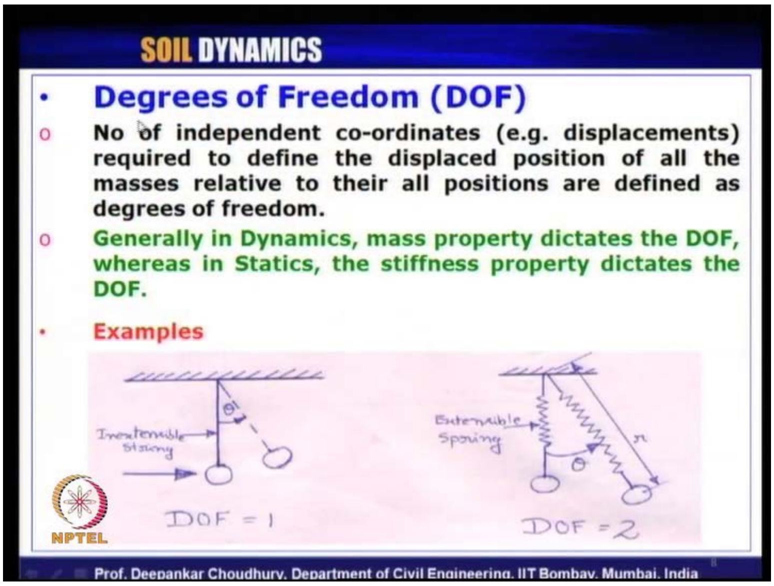

The first important parameter which we want to define now, is called degrees of freedom. And generally, it is denoted by this abbreviation DOF; DOF means degrees of freedom, it is nothing but it is defined as number of independent coordinates, for example, it can be displacement, required to define the displaced position of all the masses relative to their original positions or all positions are defined as degrees of freedom. So, in this definition, the major highlights, or the word which we should emphasize here, or which should underline, or taken note specially, it is nothing but number of independent coordinates. So, one coordinate should not be dependent on the other one, it is the number of independent co-ordinate which defines the all the position of the system, and that defines the degrees of freedom of the system.

Generally, in dynamics, mass property dictates the degrees of freedom; whereas, in the problem related to static loading, the stiffness property dictates the degrees of freedom. So, in the dynamic problem, as we are concerned about here, the soil dynamics course, let us see few examples when we are considering the vibration of different systems, how to obtain the degrees of freedom and why? Let us look at this example of simple pendulum which is hanging from a thin string or rod which is inextensible string, and the mass of the bob is concentrated here. If we apply a load here, a dynamic load or a tap, it will start vibrating like this. So, it is a new displaced position.

So, for this system, entire system, if we know this angle theta at any point of time then we can define the position or the displacement or the coordinate of the entire system at any time. So, that is why this theta is the that independent coordinate as in the definition we have seen, this theta is the independent coordinate which is required to define its displaced position at any point of time. So, for this system, the degrees of freedom is 1, and this is the minimum number of or independent number of coordinate required to define its position at any point of time.

Let us take another example of that same pendulum problem; the only change is instead of having an inextensible string if we consider an extensible string, so, something like this. So, as if a spring, and to the spring a mass or pendulum bob is attached. So, with respect to time if we apply a load to it, and it starts vibrating like this. So, at any point of time how many coordinates we require to define its position, minimum number of coordinate, one is this angle theta we should know, and another one is this distance by which the spring has been extended or compressed whatever it may be. So, that dimension is also required.

So, in this case, only theta will not be sufficient enough to define the position of the system at any point of time, but the minimum number of independent co-ordinates we require, this distance as well as this theta. So, that is why the degrees of freedom for this problem becomes 2.

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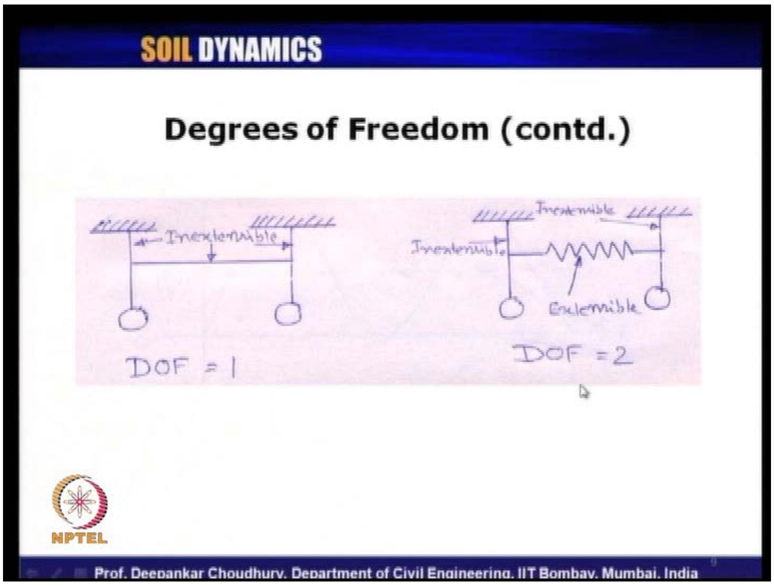

Let us take some more example on this, like if we take 2 simple pendulum. And let us look at the figure here; 2 simple pendulum connected by a rigid rod or rigid string, and all these 3 rods are inextensible in nature; in that case, what is the degrees of freedom of the system? That is 1. Why? Because if we apply a dynamic load to the system at any point of time, its new or displaced position we should know the angle theta, as all these are inextensible whatever theta angle this pendulum will make, the same angle it should maintain for this pendulum as well, being this connecting rod a rigid one. So, this problem is nothing but a single degree of freedom problem, because number of minimum coordinate required to define its position is 1.

Now, if we change the same problem, if we change the same problem to this problem that is, 2 simple pendulum connected by an extensible spring or by a linear spring, in that case, what will happen? Suppose these rods are still inextensible, in that case we require how many coordinates minimum to define the position of the system? One is, suppose if we apply a dynamic load to the system here, we need this angle to define the position of this pendulum bob at any point of time; also we need to know this theta, because this theta 1 and this theta 2 will not necessarily be same, because they in between there is an extensible string because of which either this spring will expand or compress; so, that means, this theta 1 and theta 2 will be different, or these are the two independent coordinate we require to define the system at any point of time, and that is why the degrees of freedom for this problem is 2. Let us look at few more examples of, how to obtain the degrees of freedom, of different system?

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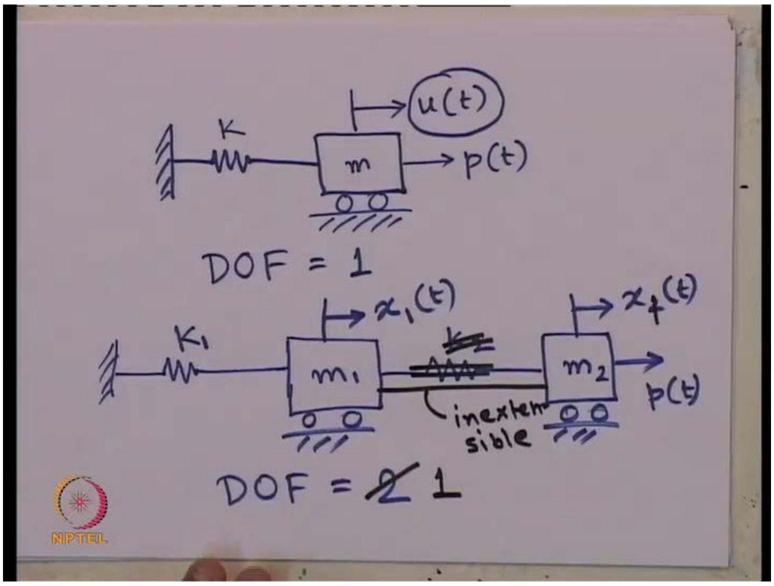

If we take a system like this; it is a rigid support, and then one spring connected to a mass like this. So, if any dynamic load is applied to the system, how many independent coordinate is required to define its position at any point of time that is this u of t, because of application of say p of t. So, in this case, the degrees of freedom will be 1, because only 1 parameter or co-ordinate is required to know its position at any point of time.

Now, if we take another example; suppose, if we have one spring then another mass, then another spring then another mass, say this is k 1, this is m 1, this is another spring k 2, this is mass m 2. And to this system if we apply some dynamic load, so, how many degrees of freedom this problem is having? That is, DOF is 2. Where from we got this value 2? Because, at any point of time this displacement, say x 1 of t; and this displacement of x 2 of t, these two are independent parameters or independent co-ordinate which we require to find at any point of time, because they are connected through this spring, that is why, x 1 and x 2 will not be same.

But, instead of this, suppose if we connect them through inextensible string instead of K 2, then the DOF will become 1. Because, in that case essentially, this x 1 and x 2 will become same if this rod is inextensible, then the problem boils down to degrees of freedom 1.

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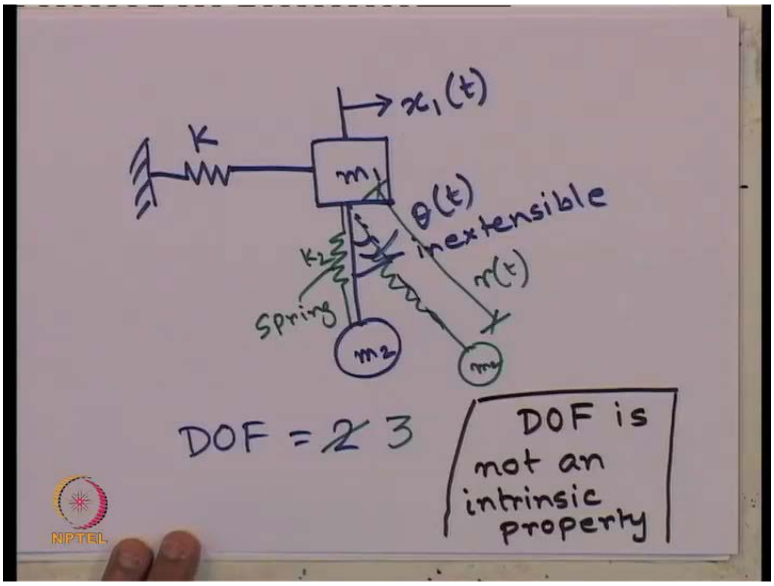

Let us take another example. If we have a system with combination of this linear spring connected to a mass, then connected to a bob or pendulum kind of arrangement, then how many degrees of freedom this problem is having? This is mass, say m 1; this is mass, say m 2; this is spring k, and this rod lets us say inextensible. In this case, the degree of freedom for the problem is 2, why? Because, if we apply a dynamic load to the system, in that case the minimum number of co-ordinate we require to define its position at any point of time is this x 1 t, and this theta of t. So, these two are the 2 degrees of freedom for the problem, under any dynamic loading.

Now, the same problem if we change a little bit; suppose instead of this inextensible string, if it is connected by a spring, another spring, say k 2. How many degrees of freedom the problem we are dealing with? In that case, it will become then 3. One degrees of freedom is this one, another is this theta, and the other one will be this length, say r of t. So, these 3 will the degrees of freedom for the new problem, what we have shown in the green color. So, that way, we have to find out how many number of independent coordinates are involved in any system when we are handling with the dynamic loading acting on the system, to define its position at any point of time. So, that defines as the degrees of freedom.

And it is extremely important to find out the correct degrees of freedom for any problem, before we start modelling any system in practice. Because, then onwards, depending on its degrees of freedom, the solution technique, and all other things will depend on. So, that is why the importance of obtaining correct degrees of freedom is very much for any dynamic problem.

But, from all these examples, what we can mention? So, let me write it down. DOF, degrees of freedom is not an intrinsic property; it is not an intrinsic property of the system, why? Because we have seen, for the same system, depending on the conditions of different connections or boundaries or things like that, the degrees of freedom, the degrees of freedom of the problem keep changing for the same system. So, that is why degrees of freedom is not at all an intrinsic property of the system, but it depends on the boundary conditions, and other loading conditions, etcetera.

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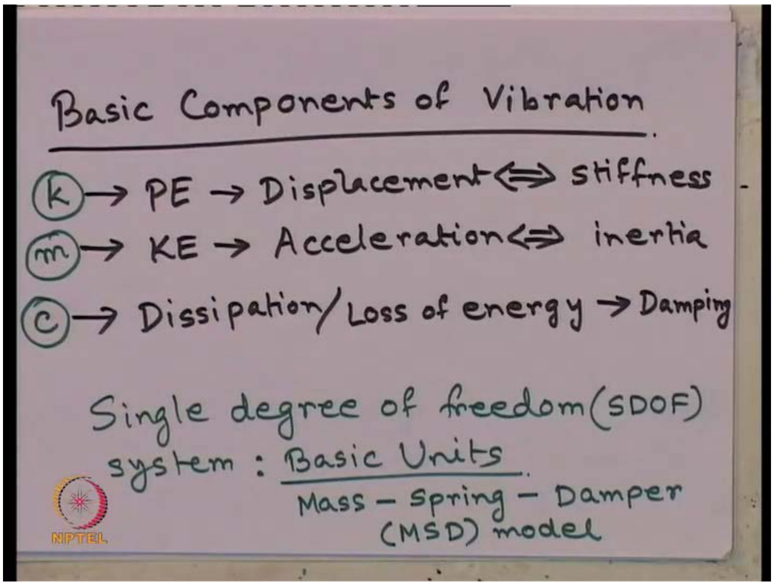

Now, let us look, what we have studied in the previous lecture. Let us move back, the basic components of vibration. In the previous lecture we have seen, there are three basic components of vibration: one is potential energy in which the displacement and stiffness are related to each other, then kinetic energy in which acceleration and inertia is related to each other, and then dissipation or loss of energy where damping is involved.

So, if we take any simple vibrating system, then what are the basic units for a simple vibrating system? A single degree of freedom system, single degree of freedom, in short, generally we write it as SDOF, so, single degree of freedom system, and if the condition of simple vibrating system, the basic units, what are the basic units? We have three basic units which will allow us to consider all the basic components of the vibration that is potential energy, kinetic energy and dissipation or loss of energy. So, potential energy it can be represented by a spring, the kinetic energy can be represented by a mass, and the dissipation or loss of energy can be represented by a damper. So, the basic units of a single degree of freedom system is mass, spring, and damper, which is in short it is called MSD model.

So, it is clear, why in a simple vibrating system we need these 3 essential components, because these 3 actually defines all the basic components of vibration. The dissipation, actually it is optional, it may be present, may not be present, whether there is any loss of energy or not, but these 2 components are essential, that potential energy and kinetic energy is essential in this simple vibrating system.

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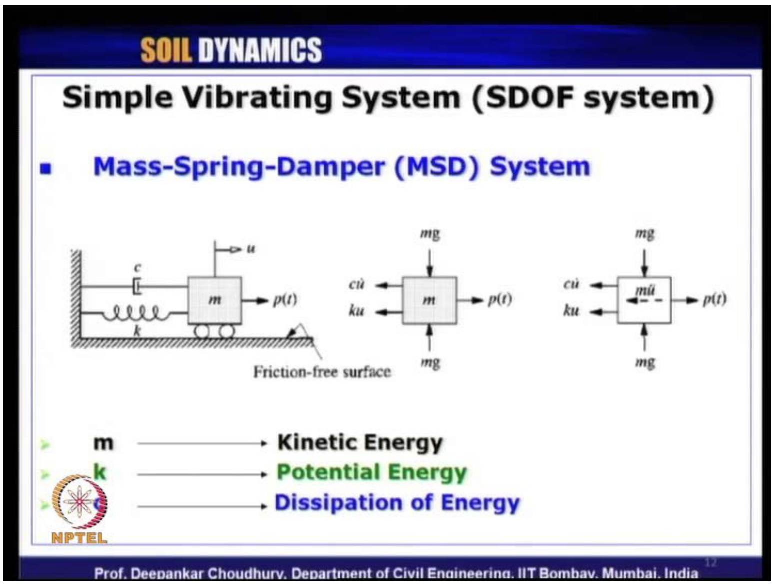

So, if we now look at the pic slide here, it says, for a simple vibrating system with single degree of freedom system, mass spring damper system is the basic unit to consider a single degree of freedom system. So, let us take a single degree of freedom system, as I said, with a mass m with a spring, with spring constant k, and a damper with damping coefficient c, which represents these three basic units of any vibration, simple vibration. And the degrees of freedom, the single degree of freedom, let us say, it is defined as u. So, u at any point of time, u of t we should know to know the position or coordinate of the system at any time, under this dynamic load, externally applied dynamic load on the system, let us say p of t, with. And let us assume, this is frictionless surface, and these are rollers.

So, next, what we should do? We should draw the free body diagram of this mass subjected to different types of forces. Now, what are the forces are acting on the system? If we try to draw the free body diagram, this will be the free body diagram of the mass. On this, we have the weight of the mass m g acting vertically downwards; And of course, from these 2 rollers we will get the static equilibrium condition, to satisfy that we will get the roller reactions, 2 roller reactions equal to that weight. So, that will balance the static loading condition, in this direction.

What are the other loads we have in this? We have p of t, that is externally applied dynamic load; then, for the free body diagram, we have some damper load c u dot, u dot is nothing but the velocity of this body or the system; if u is the displacement, the first differential of that u with respect to time that is d u of d t is nothing but our u dot. So, the damping coefficient times the velocity, will give us the damper force; and k u is the spring force; And within the mass, as it is prompted to act under the dynamic load in this direction, there will be a self resistance developed within the mass itself, which is called the inertia force. The inertia force will be nothing but mass times acceleration. So, acceleration is u double dot, u double dot is nothing but the second differential of this displacement. So, it is d 2 u by d t square that will act in this direction.

So, as I have already mentioned, these 3 are the basic components of any vibrating system: mass to denote the kinetic energy component, stiffness to denote the potential energy component, and the damper to denote the dissipation of energy component.

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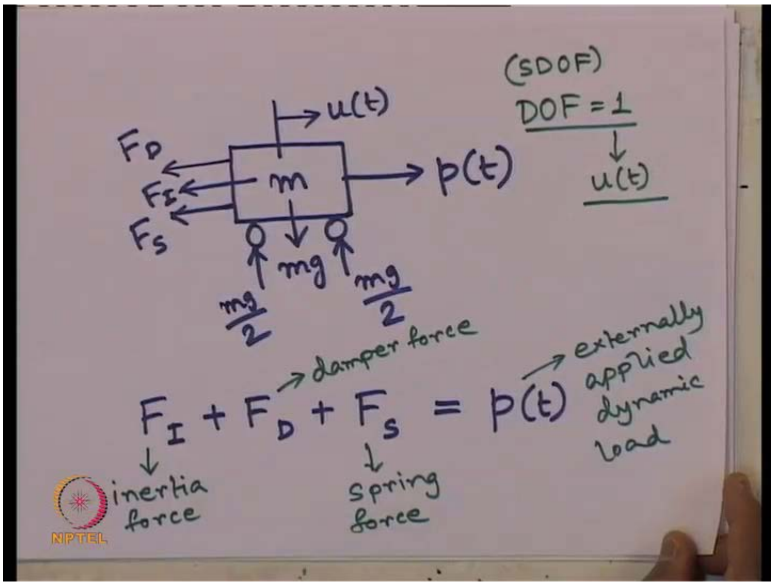

Now, what we were discussing just now, let me redraw it here further, the free body diagram. This is the mass on which the dynamic load externally applied, p of t was acting; And we have the weight acting vertically downward, which is getting balanced from the static reactions from these rollers. So, this will be m g by 2, this will be m g by 2. And as at any point of time, it is prompted to move, at any instant of time, we have taken its direction of motion in this way. So, obviously, the forces of resistances will try to pull it back to its original position. So, that is why the damper force, spring force, and the inertia force, these are the 3 forces of resistance are getting developed in the system.

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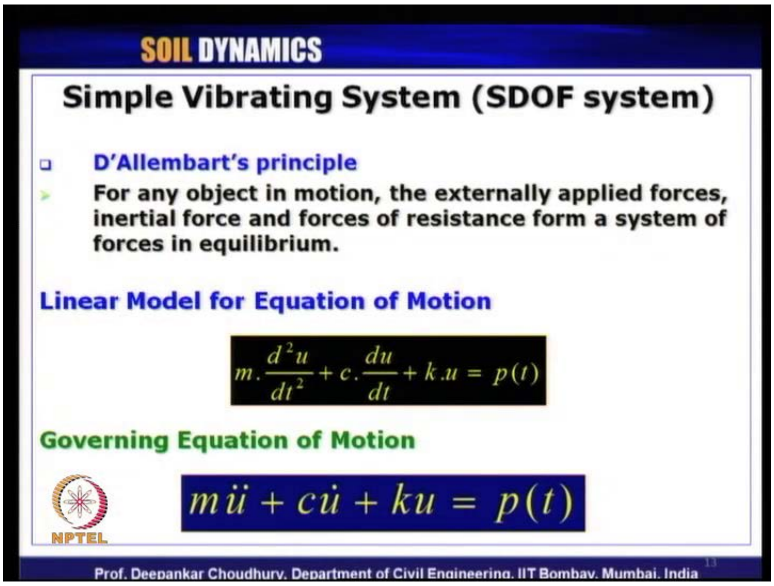

Then, what we can see, let us look at the slide. This principle, D Allembart's principle is very much useful to satisfy the equilibrium of a dynamic system. What it says? That, for any object in motion, the externally applied forces, inertial force and forces of resistance form a system of forces in equilibrium.

So, in simple words, if we look at here, what it means? That, F I plus F D plus F S equals to p of t, using D'Allembart's principle. Because it says, externally applied dynamic load for any body which is in motion, that should balance the inertia force and other forces of resistance, that is damper force, spring force, etcetera. So, in this case, we have these 3 units. These are the 3 forces of resistance that must balance the externally applied force, to maintain the equilibrium of the system, that is the condition from we are getting from D'Allembart's principle.

Now, let us look at behavior of each of these 3 forces. So, here, what are these 3 forces? Let us write it once again. This F I is called inertia force, F D is called damper force, and F S is called spring force, and p of t is externally applied dynamic load, and in this case it is a problem of degree of freedom 1. So, that is why single degree of freedom of system, we have already defined; And that degree of freedom is nothing but u of t, that is we should know that displacement at any point of time, which is the coordinate minimum required to define its position at time.

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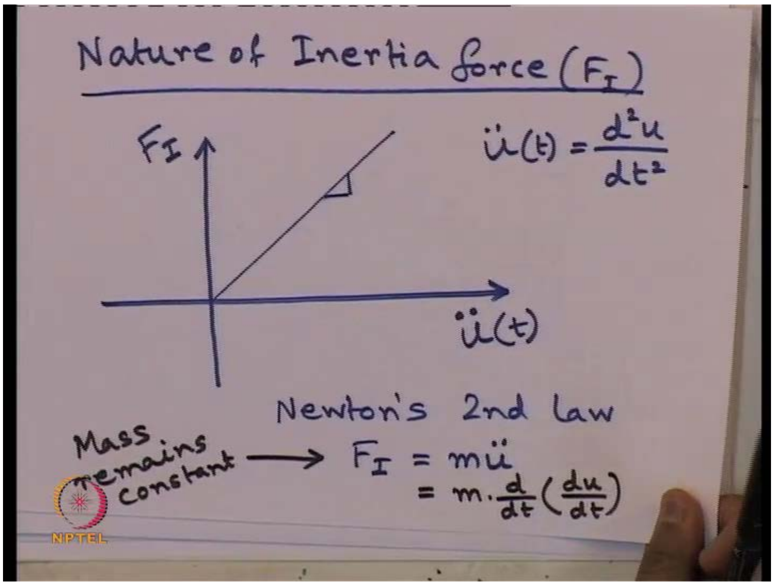

Now, let us look at the nature of each of these forces. So, first, let us take the nature of inertia force, F of I. So, how the inertia force varies with respect to acceleration? It was already in the previous lecture we have mentioned, it is a, inertia is related to the accelerator component. So, F of I, how it varies with respect to u double dot t? In this case, as I have already mentioned, u double dot is nothing but, it is d 2 u by d t square, second differential of the displacement, with respect to time.

Now, we can assume a variation like this, which is coming from basically if we use Newton's second law of motion; what it says, that F of I is equals to m u double dot, that is inertia force is nothing but mass times the acceleration; mass into acceleration will give us the inertia force. And in this case, the slope of this graph is nothing but the mass; and this variation is true only when, if we re-write it in different way, actually it is coming from m d of d t, then d u by d t. So, if mass remains constant, then only this is constant, and this behavior is linear. So, this model, or this equation is correct for mass remains constant, m remains constant then only we can use this relation.

Suppose if any dynamic system, the mass does not remain constant, in that case it is not the relation of F I with respect to the acceleration, but it will be something different. it can be curvilinear, some variation, some non-linear variation of F of I with respect to u double dot, in that case need not be this simple relation. For example, when we take some aerodynamic problem, where with respect to time there is loss of mass. So, mass does not remain constant in many of the aerodynamic problem. In those cases, we cannot use these relations of F I equals to m u double dot. It is valid only for the system where there is no change in the mass. So, mass remains constant, then only this equation is holding good. So, this is the linear model what we have considered for inertia force.

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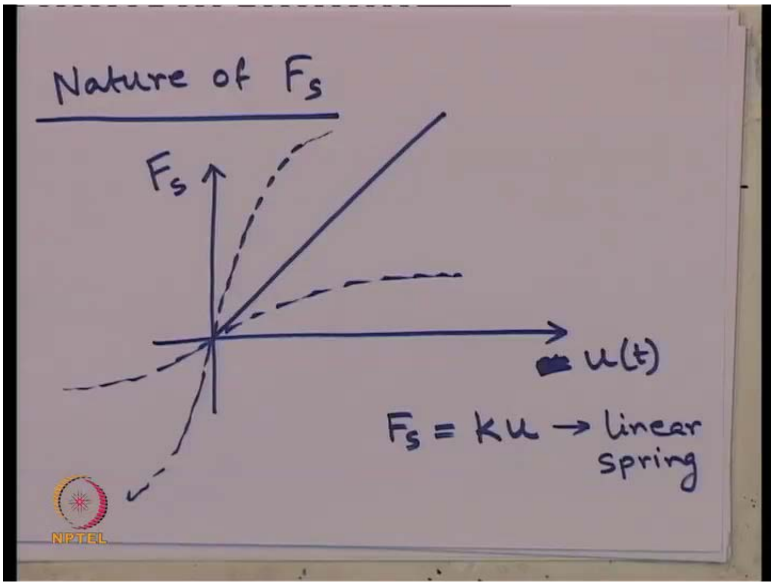

Similarly, let us see, what are the nature of other forces? So, nature of F S that is spring force. In previous lecture we have studied that the spring force is varying or related to the displacement x, or we have taken actually u, so, let us denote at u of t. So, F S varies with respect to u, a displacement. It may vary like this that is linear variation. In that case, what we write? F S equals to k times u. So, stiffness constant, times the displacement will give us the stiffness force. This is valid only for linear spring that is where the force displacement relation is linear like this, then only the slope of the graph is nothing but the spring constant, we know this.

But, there are several examples, where it is need not be like this; it can behave something like this; it may behave something like this; so, these are called, this is like strain softening effect mostly for cable we consider, this is the hardening effect mostly we consider for beams, rigid beams. So, there the relation of force displacement need not necessarily be a linear one, but it can be a curvilinear in nature. In that case, this relation of F S equals to k times u, is not holding good. So, we have to be careful when we are using this expression for forces. The spring force equals to k of u is valid only for the linear spring, if we are considering.

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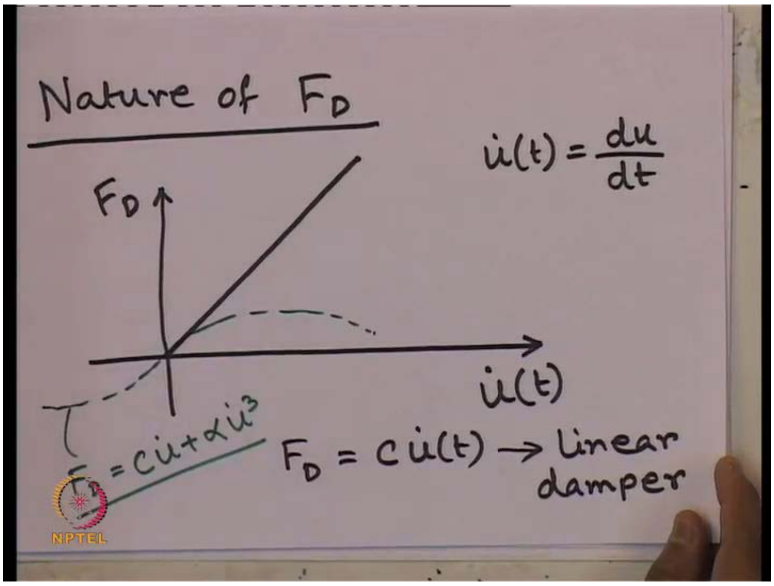

Now, coming to the nature of the damper force; nature of the damper force, F of D. The damper force is related to the velocity component u dot of t, where u dot t is nothing but du dot of dt. So, how it varies? It can vary like this that is a linear starting from this 00 point; in that case we write F of D equals to c times u dot t, which is valid for linear damper once again, if it varies this linear relation of the damper force with respect to the velocity; then slope of this line is nothing but the damping constant or damping co-efficient.

But, there may be several other different variation of F of D with respect to u dot, say some non-linear variation something like this, which is pretty common in some design, for example, chimney subjected to wind load etcetera. In that case suppose F of D can have some non-linear relationship say, c u dot plus alpha times u dot cube, something like this, some non-linear relationship; then we cannot use this linear damper equation so easily for our model. So, we have to be careful, what type of variation of these each forces of F S, F D and F I are considered for a particular model or particular system when we are analyzing a dynamic problem.

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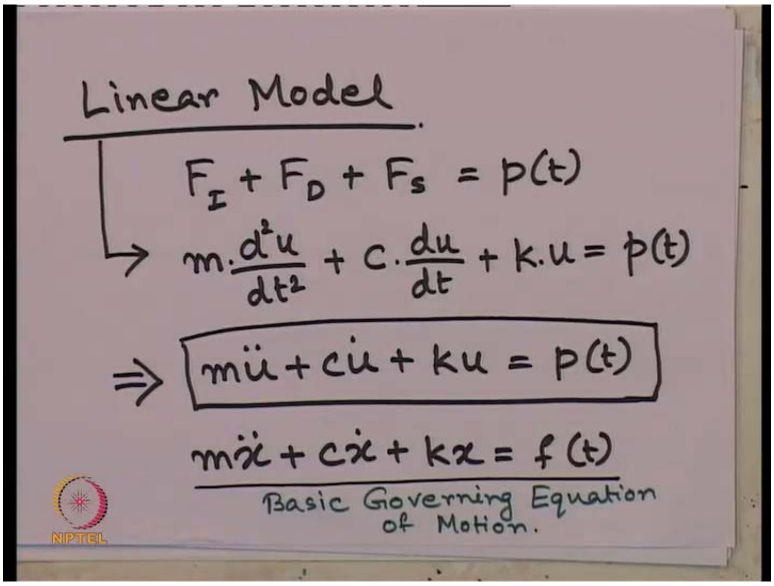

So, for the simplest case, what we can mention? That by considering the linear model, what we got, the equation F I plus F D plus F S is equals to p of t that was the equation from the D'Allembarts principle. If we use the linear model, F I is nothing but m times d 2 u by d t square that is the inertia force; what is the damper force? c times d u d t; what is the spring force? k times u that equals to p of t.

So, this equation we have written based on the nature of all these inertia force, damper force and spring force by considering them to follow the linear model that is linear system where mass is not changing, then this equation is valid; the linear damper then this relation is valid; a linear spring then this equation is valid; which in other form we write the governing equation of motion m u double dot plus c u dot plus k u equals to p of t. The same thing in some book can be written as m x double dot plus c x dot plus k x equals to f of t, the same thing actually in written using different notations that is instead of u for the displacement, x is considered as displacement, in that case the same relationship can be written in this format. So, this is called the basic governing equation of motion. So, basic governing equation of motion for a single degree of freedom mass spring dash spot model vibrating system is expressed as m u double dot plus c u dot plus k u equals to p of t.

So, now, let us look at the slide here, what I have discussed till now, the same thing is written here. The linear model by considering that is considering for mass, damper and stiffness, the linear module, the equation of motion takes the shape like this. So, governing equation of motion is like this.

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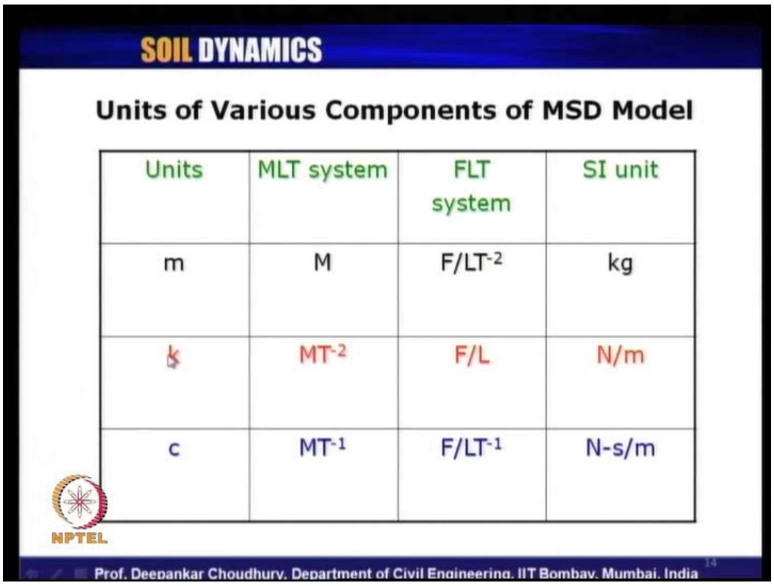

Units	MLT system	FLT system	SI unit
m	M	F/LT^{-2}	kg
k	MT^{-2}	F/L	N/m
c	MT^{-1}	F/LT^{-1}	N-s/m

Now, let us discuss about various units of these components for our basic mass spring dash spot model. If we take the unit for mass, stiffness, and damper, in different system that is MLT system that is mass length time system, FLT system force length time system, and in SI unit; then unit of mass in MLT system is of course, the mass m itself, for stiffness the unit is mass by time square that is mass T to the power minus 2, for the damper the unit is mass by time that is mass T to the power minus 1.

In FLT system unit for mass is force by acceleration, so, L to the power minus 2 is the unit for mass, for spring constant the unit in FLT system is force by length, and for the damper the unit in FLT system is force by velocity, so, length T to the power minus 1. So, the commonly used SI units which is now worldwide used commonly, are for mass generally we use k g unit, for the spring stiffness we use the unit Newton per meter, and for the damper we generally use the SI unit Newton second per meter. So, these are the standard units which are used for these basic 3 components of the MSD model.

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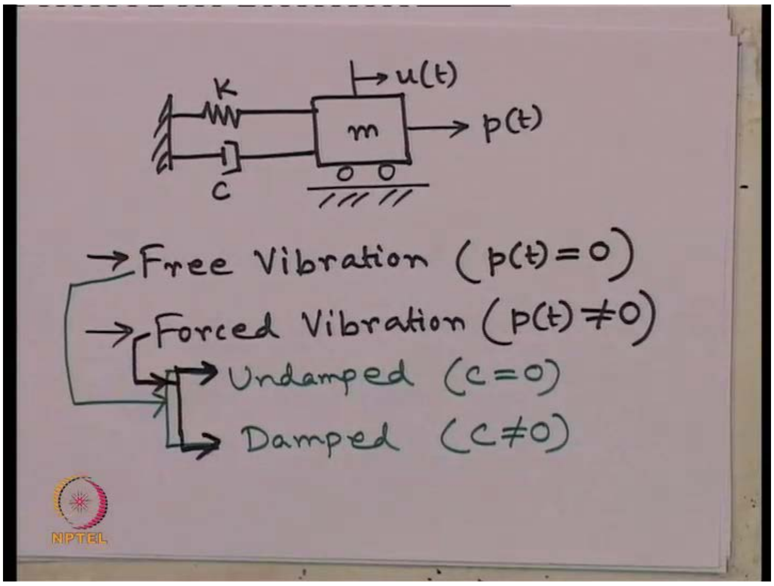

Now, let us see what are the different types of vibration we can have, by considering linear model? So, for different types of vibration what we may consider in a single degree of freedom system we are taking. Let me draw that single degree of freedom system once again; mass, spring, damper, we have externally applied load dynamic load, and this is our degree of freedom.

So, for this single degree of freedom system, the type of vibration can be, major classification is one is called free vibration; what is free vibration? If there is no externally applied load on the system. So, this p of t if it is not present, then the system is called the free vibration. But, if that externally applied dynamic load is present in the system, in that case we call it as forced vibration, in that case p of t is not equal to 0. Now, within each of them we can have again two separate subcategories, those are like within free vibration we can have two categories: one is undamped, undamped means when this damper is not present, so there is no damping effect c is equal to 0; and another is damped that is c is present, not equals to 0. Similarly, for forced vibration also we can have the same two conditions that is this undamped and damped.

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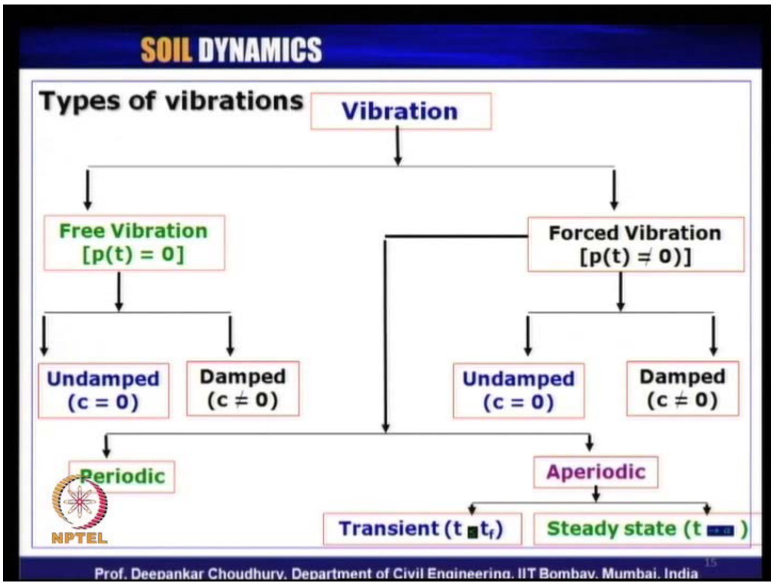

So, if we look at the slide here for different types of vibration, what we are discussed just now. Vibration can be classified in two major categories: one is called free vibration, when there is no applied dynamic load that is called free vibration; And the other one is called forced vibration when the externally applied dynamic load is present in the system that is this is not equals to 0. Within free vibration there can be again 2 sub categories: one is called undamped free vibration, when c is equals to 0 that is damper is not present in that case it is called undamped free vibration, so, c will be equal to 0 as well as p of t equals to 0 that is undamped free vibration; And damped free vibration is that where the damper is present that is c is not equals to 0, but there is no externally applied dynamic loads. So, p of t is equal to 0. So, that is called damped free vibration.

Similarly, the forced vibration also can be sub classified into two categories: undamped forced vibration that is when c is equal to 0, but there is a externally applied dynamic load to the system. So, p of t is non-zero; another category is damped forced vibration, where c is also present, p of t is also present that is called damped forced vibration. Another way to classify the forced vibration can be in these 2 categories: one is called periodic forced vibration, another is called aperiodic forced vibration.

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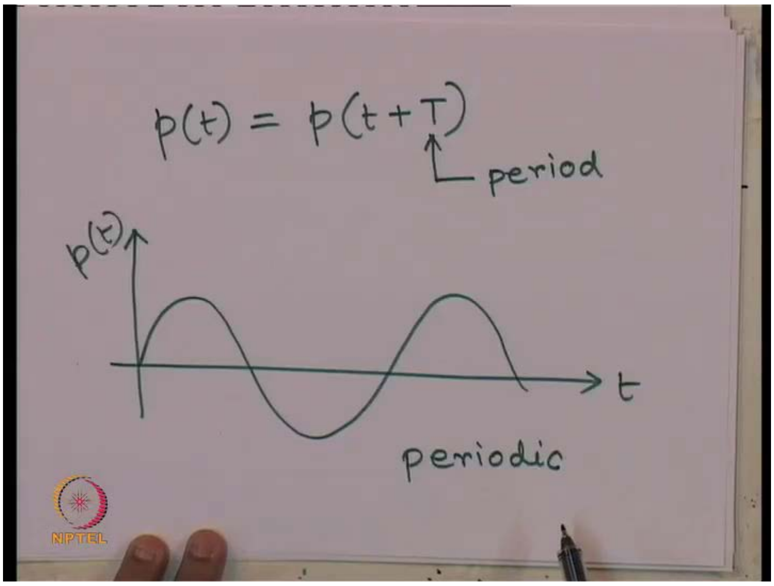

What is periodic forced vibration? Periodic forced vibration is such that when we have a system where this p of t is such that it repeats its nature after certain period, in the previous lecture we have discussed this capital T is generally denoted as natural period, so, this if we mention as period, then this is the type of force on the system when the dynamic load is present on the system. For example, it can be sign load or cosign load something like this, where the nature of this p of t is repeating after certain period or certain time. In that case, it is called the periodic forced vibration. And suppose this relation is not holding good that is if this variation of the dynamic externally applied dynamic load is not repeating after a certain interval of time or after a certain period, in that case it is called aperiodic.

So, let us look at the slide once again. So, that is why the another 2 ways of classification of forced vibration is one is called periodic forced vibration, another can be aperiodic forced vibration. Again, within aperiodic forced vibration we can have 2 more sub classification: one is called transient type forced vibration; another is called steady state type aperiodic forced vibration.

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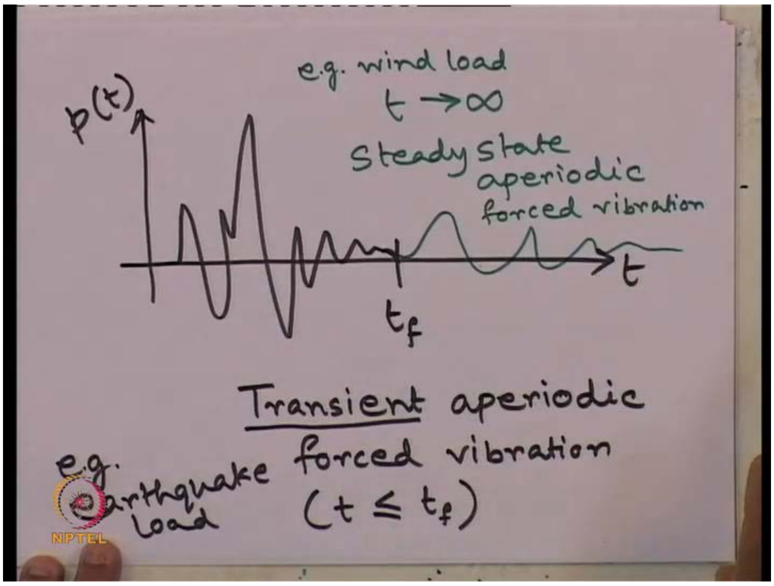

What is transient type aperiodic forced vibration? When this externally applied dynamic load that is p of t with variation with respect to t, it is aperiodic in nature. Say, suppose it is starting somewhere here, and its random actually, it stops here at certain time say, t f, after that p of t is not present in the system. In that case, we call it as transient aperiodic forced vibration. Because, why transient? This time t should be less than or equals to this finite time, this time should be a finite value of time then we call this type of aperiodic forced vibration as a transient type aperiodic forced vibration. What is the example of this transient type aperiodic forced vibration? For example, earthquake load; that is a good example of transient type a periodic forced vibration, because earthquake occurs for a finite duration of time, earthquake load.

Whereas, suppose this dynamic load, externally applied dynamic load if it keeps continuing for time t tends to infinity, in that case we call it as steady state aperiodic forced vibration. So, steady state aperiodic forced vibration, the example for that can be wind load. Because, generally, we consider wind load keep on acting on the system that is the dynamic load, where the load varies with respect to time for an infinite time. So, that is why, in the slide if we look here, aperiodic forced vibration also has been sub classified as transient when for time for which the dynamic load is acting is a finite amount of time; whereas, it is called steady state aperiodic forced vibration when that applied time duration tends to infinity.

Now, let us take one by one each of these different types of vibration, their basic solution from the governing equation of motion for single degree of freedom system, and how to get the response of the system that is the displacement profile at any point of time.

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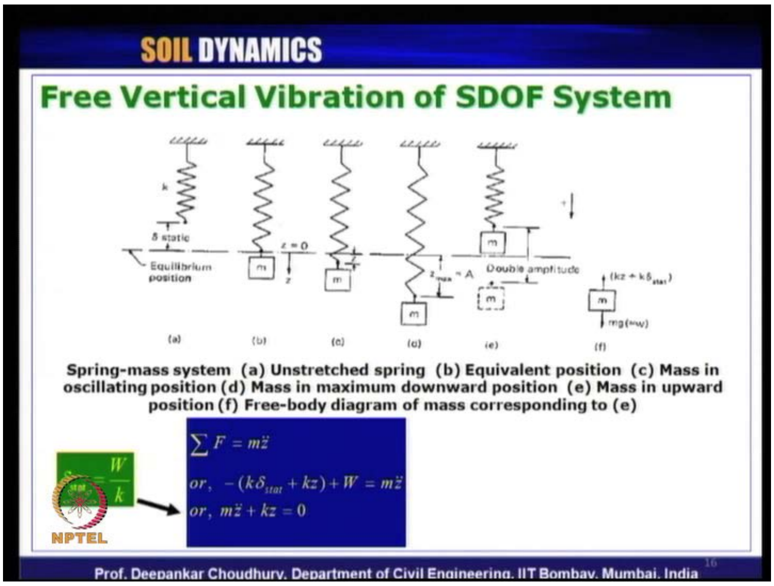

Before starting that, let me take up this another sub topic that is when we are considering free vertical vibration of single degree of freedom system. Just now, what we have studied that, for a single degree of freedom system which is subjected to an externally applied dynamic load in the horizontal direction, we have taken the degrees of freedom one in the horizontal direction; instead of that suppose if we have a system which is vibrating again, the single degree of freedom system only, that is one degree of freedom we are considering, and vibrating in the vertical direction, what happens? What is the equation of motion, governing equation of motion for that system? Let us see.

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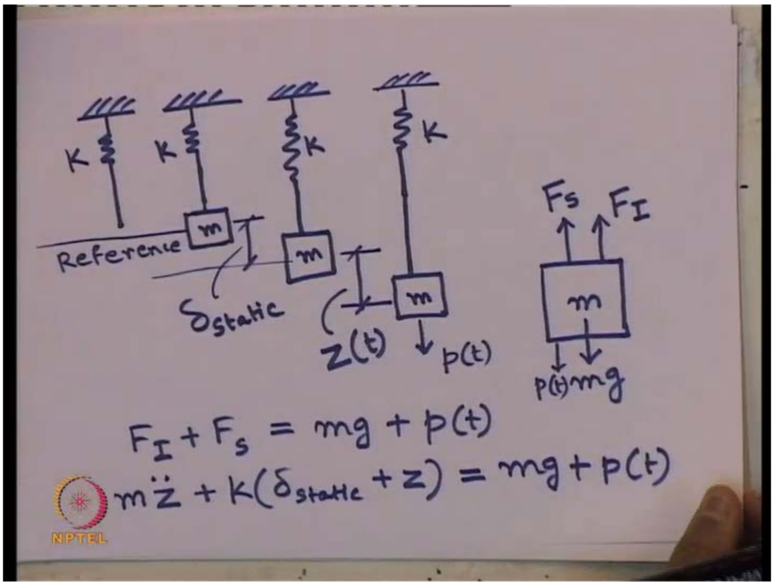

Let us say, initially we have a spring, and it was hanging like this. Then to that system we have attached a mass at one instant of time, due to attaching this mass to this system, due to its own weight, it has come down to this place, by expanding the spring. So, this is the new position after it attends its static equilibriam condition. Now, about this point let us say, here we apply some vibration to the system. So, now, it vibrates with respect to this access and the new position let us say, is some were here.

So, if we take this initial position as a reference line; this is the position after having a displacement, static displacement, delta static; this is due to the own weight of the mass; the spring has come down here. And from this line to this line, let us define this as say, Z of t. So, this component is nothing but the dynamic displacement. So, this is the single degree of freedom we are considering, because at any point of time this Z of t in one instance it will come down, in another instant it will go up. So, about this line, it is vibrating.

Now, if we write the equation of motion for this system, before that what we should do? We should draw the free body diagram of the system. What we can see? There will be a spring force F S, there will be a inertia force F I, there will be m g that is weight of the body, and suppose there is some dynamic load applied to it say, p of t; so, p of t. So, what will be the equation? By using D'Allembarts principle in this direction, we can write F I plus F S equals to m g plus p of t. Now, how much is our F I? F I is nothing but mass times the acceleration. How much is acceleration? This is the dynamic displacement.

So, only this portion of the displacement we can differentiate twice with respect to time. So, Z double dot, by considering linear model we can write F I is m Z double dot plus F S; spring force is how much? Spring force is spring constant considering linear spring times the total distant from its basic reference point, which is nothing but delta static plus this Z of t. So, the static displacement plus this dynamic displacement equals to m g plus p of t. Here, note, we have not considered the damper; if we taken the damper the damper force also would have been added here.

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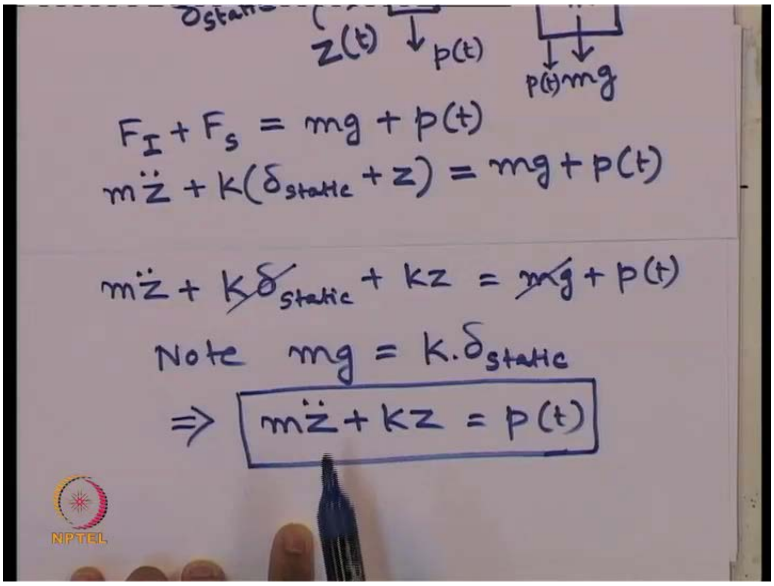

So, what we can continue from here, let me put it here, so, we can follow it up; m Z double dot plus k delta static plus k z equals to m g plus p of t. Now, in this case, note that this m g, weight of the mass is nothing but this k times delta static, because that adding of the mass, due to its own weight it come down; and this is the static displacement; so this component are equal. So, we can cancel, this can cancel, therefore, governing equation of motion we are getting m Z double dot plus k z equals to p of t.

So, what we can see? The equation of motion, the dynamic equation of motion, governing equation of motion remains exactly same as the horizontal vibration case. And in this case we have to remember that this Z denotes only the dynamic component of the displacement; if somebody wants to compute the total displacement, they must add the static component of displacement also, to this dynamic component of displacement. Otherwise, the dynamic equation of motion remains same in any of the cases of vibration, whether it is horizontal vibration, vertical vibration, etcetera. So, we will stop here today's lecture, we will continue our lecture in the next class.