

Soil Dynamics
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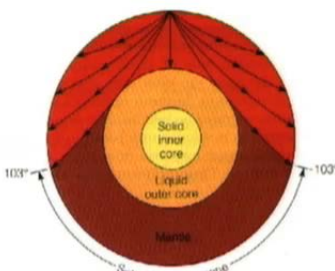
Module - 4
Dynamic Soil Properties
Lecture - 18
Stresses in Soil Element, Field Tests,
Seismic Reflection Test

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SOIL DYNAMICS

Secondary Wave

- S waves (secondary)
- Typical crustal velocity: 3 km/s (~6,750 mph)
- Behavior: Cause shearing and stretching of the earth material through which they pass. Generally cause the most severe shaking; very damaging to structures.
- Travel through solids only
- shear waves - move material perpendicular to wave movement
- Arrival: Second on a seismogram.

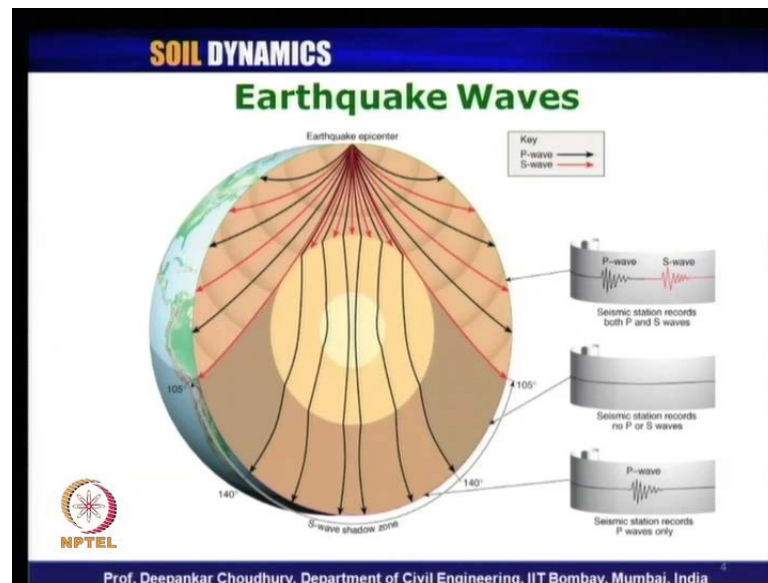


S-wave velocity drops to zero at the core-mantle boundary or *Gutenberg Discontinuity*

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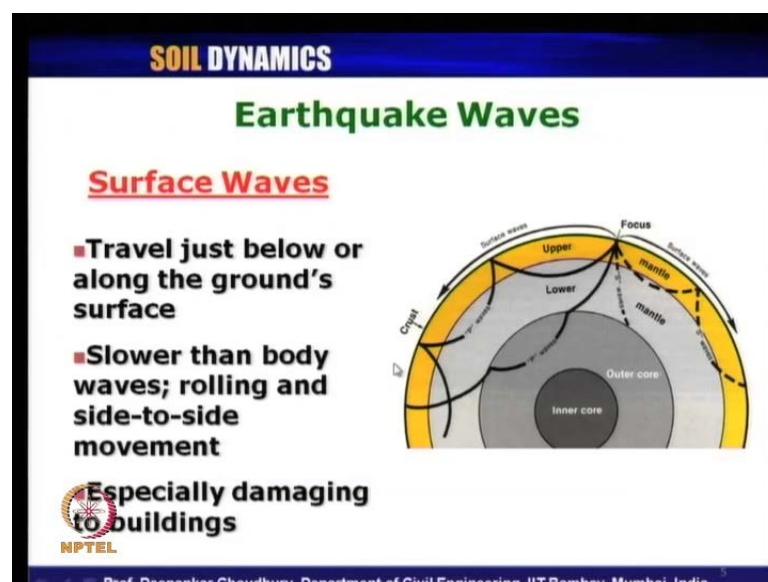
Let us start our lecture today on soil dynamics. In module 3 of wave propagation, in the previous lecture we have seen what are the characteristics of secondary wave and what is called Gutenberg discontinuity and what is the shadow zone for secondary wave inside the earth.

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And then if any earthquake occurs at earthquake epicenter and waves are travelling through all around through the inner of the earth. Then we have seen at different stations P-wave and S-wave can arrive. In some station none of the waves are arriving; that is the complete shadow zone for both P- and S-wave and in some of the zone only P-wave arrive not the S-wave due to this portion of the core which is liquid in nature.

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Then, we have seen surface waves which are travelling only within the earth crust and on the surface media.

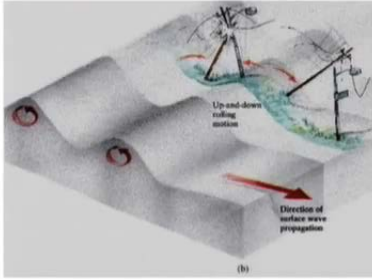
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SOIL DYNAMICS

Earthquake Waves

Rayleigh Wave

- Typical velocity: ~ 0.9 that of the S wave
- Behavior: Causes vertical together with back-and-forth horizontal motion. Motion is similar to that of being in a boat in the ocean when a swell moves past.
- Arrival: They usually arrive last on a seismogram.



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Then among the surface wave we have seen the characteristics of Rayleigh wave; the typical velocity of Rayleigh wave is about 0.9 times of shear wave velocity and the behavior also we have seen.

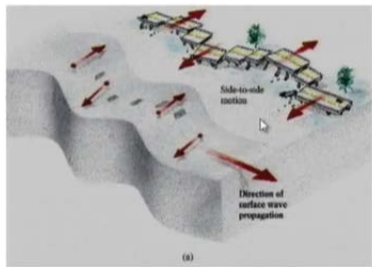
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SOIL DYNAMICS

Earthquake Waves

Love Wave

- Typical velocity: Depends on earth structure, but less than velocity of S waves.
- Behavior: Causes shearing motion (horizontal) similar to S waves.
- Arrival: They usually arrive after the S wave and before the Rayleigh wave.

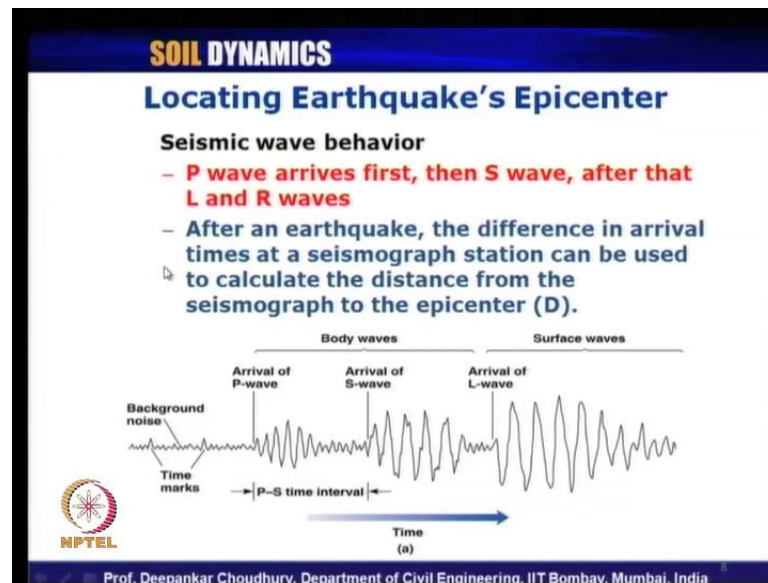


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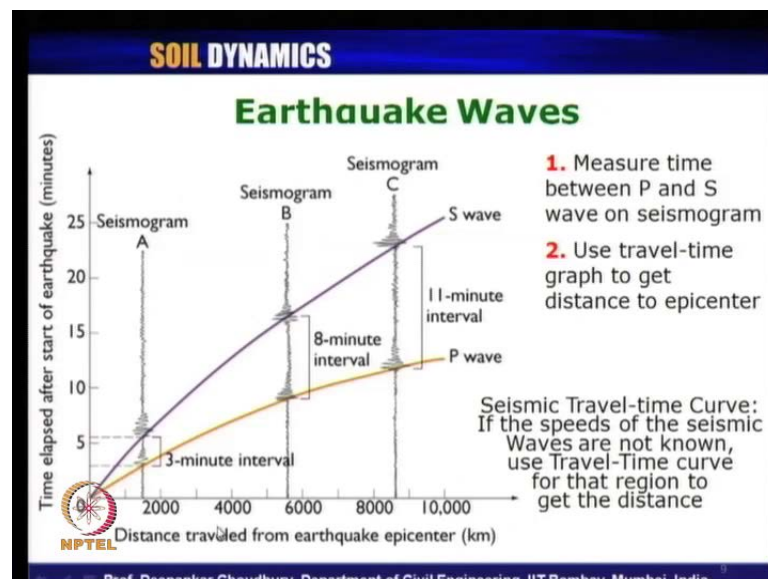
Then another type of surface wave is love wave. The wave velocity for love wave is again lesser than secondary wave or shear wave, but it is higher than the Rayleigh wave.

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Then how to locate earthquakes epicenter, we have seen different methods because in the seismogram where all the waves are arriving, we have seen that first P-wave will arrive, then is followed by S-wave. There will be sufficient time lag between the arrival time of this P-wave and S-wave and then all the surface waves will arrive; that is first the love wave and followed by Rayleigh wave.

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We have seen how the distance can be computed from the epicenter for different seismogram stations and using this travel time curve method, if the velocity of this P-

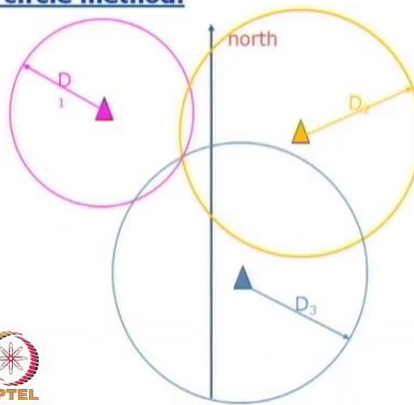
wave and S-wave are not known in the earth crust, we can find out the intuitive distances for the epicenter from those stations.

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SOIL DYNAMICS

Earthquake Waves

3-circle method:



- 1) Read S-P time from 3 seismograms.
- 2) Compute distance for each event/recording station pair (D_1 , D_2 , D_3) using S-P time formula.
- 3) Draw each circle of radius D_i on map.
- 4) Overlapping point is the event location.

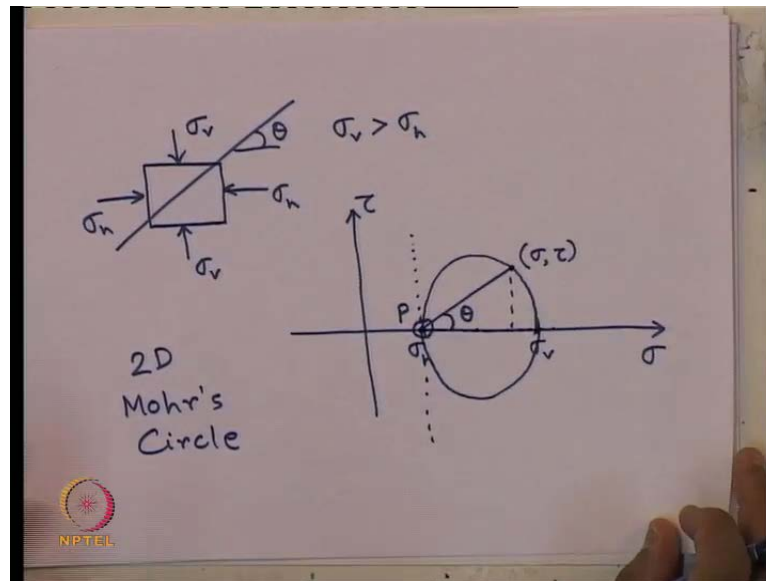
Assumption: Source is relatively shallow; epicenter is relatively close to hypocenter.

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Then using those concepts, we can plot it by using the 3-circle method for different seismological stations and where those circles are meeting, that is the point of earthquake epicenter. So with that, we have come to the end of the module 3 and now let us start with our next module; going to our module four which is dynamic soil properties. Now to start the dynamic soil properties, let me give some background information or let us brush up some memories about how to represent the stresses in a soil element through Mohr's circle which is known to all of us from our undergraduate studies.

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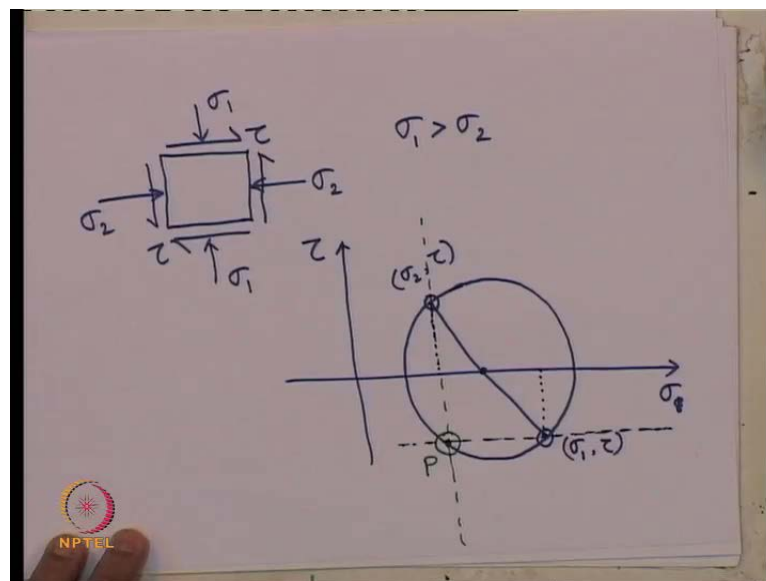
So, we know if we take a small soil element in two-dimension, let us say this is sigma vertical, this is sigma horizontal, and we are considering principle stress conditions. So, they are principle stresses; obviously, there will not be any shear stress. And let us say sigma v is greater than sigma h. Now how to draw this in terms of Mohr's circle that we know; this is x axis normal stress, y axis shear stress, and these are compressive stresses we are considering as positive. So, sigma v is greater than sigma h and these are principal stresses, this is the value of sigma v, this is the value of sigma h. Then using these two extreme points the middle point is center, we can draw a Mohr's circle easily like that. So, this is the Mohr's circle.

Now what is the pole of this circle, how to obtain the pole of a Mohr's circle. Pole is nothing but, let us look at this phase where it acts the horizontal sigma h is acting on a plane which is vertical. So, sigma h is acting on a vertical plane. Let us draw a vertical plane for sigma h and sigma v is acting on a horizontal plane. So, sigma v is acting on the horizontal plane where these two planes meet each other; the junction point is nothing but the pole of Mohr's circle. So, this is P. The junction of these two planes where they meet together that is nothing but pole of the Mohr's circle.

Why pole is so important? Because suppose if we want to know the stress condition at any inclined plane; what are the stresses. So obviously that will not be a principle stress; and they are normal stress and shear stress both will be present at an angle say theta with

respect to this direction. What does it mean? Pole is that point from which we should draw an inclined line theta where it intersects the Mohr's circle, that value of sigma and tau will give us the normal and shear stress at that inclined plane theta. So, that is the use of pole; why pole is so important? This is the reason; that graphically in a very easy way using Mohr's circle in two-dimension, it is a 2D Mohr's circle. We can obtain the stresses on a soil element very easily at any inclination.

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Suppose we have another soil element which is not of principle stress conditions. So, we have shear stresses also let us say. Let us say this is sigma 1, sigma 2, and tau is the shear stress, sigma 1, sigma 2, tau, and let us say sigma 1 is greater than sigma 2. If we want to draw this in a form of Mohr's circle, this is sigma, this is tau. So, sigma 1 is greater than sigma 2 and the convention we can consider anticlockwise moment is positive. So, this one on the phase where sigma 1 is acting, this tau is creating a clockwise moment about the center of the soil element; that means, that tau is negative, right. So, the point will be sigma 1 up to here, then here. So, this is the point with value sigma 1 tau as per our convention as I said and sigma 2 and this tau is creating anticlockwise moment. So, this will be positive, say, this is our sigma 2.

So, this point will be sigma 2 tau. Now to draw the Mohr's circle we know just simply we join them by a straight line; this will be the center of the circle and this one is the radius. So, this is how we draw the Mohr's circle for any stress condition like this. Now

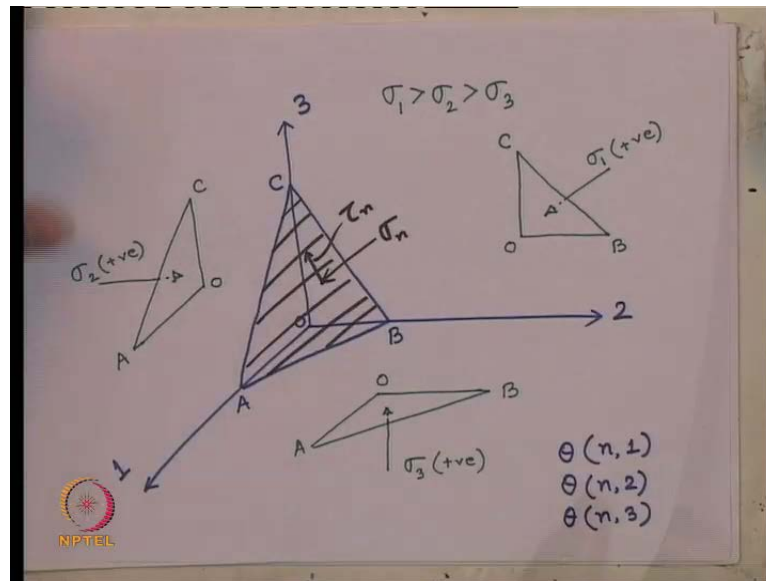
how to obtain the pole for this Mohr's circle? In the same way as I have explained, all I remember, the σ_1 is acting on this horizontal plane. So, σ_1 is acting at horizontal plane. So, draw a horizontal plane like this. Let me use another color and the σ_2 is acting on a vertical plane. So, σ_2 is acting on a vertical plane. So, where σ_2 is there draw a vertical plane and where these two planes are meeting each other on the circumference of the circle; obviously, it will meet on the circumference of the circle. That point is nothing but the pole of the Mohr's circle.

So, this is the pole of the Mohr's circle. Again why it is important? As I have said for any inclined plane to obtain the normal and shear stress, we can draw in the Mohr's circle that inclined plane with respect to the pole; that is how we can get the normal and shear stress for any soil element. So, this is the representation of two-dimensional stresses, but having two-dimensional stresses in our static problem or assumptions in the static problem by considering two-dimensional stresses for soil is pretty well valid, because when we take any soil element, we consider the vertical stress is one major principle stress and the horizontal throughout the cross section remains uniform we assume in the static problems. So, because of that we can very well take a two-dimensional problem and represent it in the form of Mohr's circle.

But in case of dynamics, it is not true always. In most of the cases it is not true, that the stresses in the all around cross sectional area of the soil element should be same because it may happen many a times because of your propagation of the wave; basically the S-wave, your deduction of particle movement perpendicular to the deduction of S-wave can be in a particular direction, say, in the East-West direction instead of North-South direction. In that case, the stresses in that East-West direction will be different than the stresses in the North-South direction for the soil element.

So for dynamic problems, it is not correct to consider a two-dimensional stress problem or two-dimensional stress element and draw it in form of Mohr's circle like this and to get stresses for soil element at any inclined plane. So, what to do? To address this issues of three orthogonal different stresses for a soil element in actual problem of dynamics, that is where σ_1 , σ_2 , σ_3 are not same; they are altogether different. How to address that problem let us see.

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So, what we are now considering let me draw this first. Three axis system first I am drawing; three orthogonal axis system, say, 1, 2, 3. Three principle stress direction let us consider. Now one element in three-dimension now I am taking; this is one element in three-dimension. Let us say this origin is o, this point is A, this is B, this one is C. As we have taken these are the principle stress direction, what does it mean? I am considering on this plane; let me draw it individually so that it is clear to us OBC on this plane. Let us say we have normal stress sigma 1 and this is the direction of positive we are considering, compression positive.

So, sigma 1 is what; the principle stress in direction one acting on this plane. Now on this plane OCA, here we are considering sigma 2 as the principle stress in the direction of two and on this plane OAB; let us say this is sigma 3, this is the positive direction. So, these are the three principle stresses acting on the soil element and let us say sigma 1 is greater than sigma 2 is greater than sigma 3. So, what we call them? This is major principle stress sigma 1; sigma 3 the lowest one is minor principle stress; and sigma 2 is called intermediate principle stress. Now if I want to find out the normal and shear stresses on any inclined plane with respect to these three orthogonal axes, how we can determine that.

So, let us say on this plane that is the plane ABC, can you see this ABC; on this plane any inclined plane, I want to find out how much is normal stress, how much is shear

stress. Shear stress, say, τ_n and normal stress, say, σ_n where n is the n th plane I am considering this inclined plane. So, on this inclined plane how much normal and shear stresses are acting we want to find out and inclination of this plane on which we want to find out the normal. And shear stress, this arbitrary plane n th plane is such that its angle with respect to axis one is, say, given by θ with respect to n th plane and axis one is given by like this. The another angle it makes with axis two is given by, say, θ_2 with respect to axis two and θ_3 with respect to axis three. These are the three angles made by this plane n th plane with respect to axis one, axis two, and axis three. So, what we can write.

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The image shows a whiteboard with handwritten mathematical equations. At the top, three equations define the areas of triangles formed by projecting a triangle ABC onto the coordinate axes: $\Delta OBC = \Delta ABC \cos(n,1)$, $\Delta OAC = \Delta ABC \cos(n,2)$, and $\Delta OAB = \Delta ABC \cos(n,3)$. Below these, an equation for the normal stress σ_n is derived: $\sigma_n \cdot \Delta ABC = \sigma_1 \Delta OBC \cos(n,1) + \sigma_2 \Delta OAC \cos(n,2) + \sigma_3 \Delta OAB \cos(n,3)$. Finally, the normal stress σ_n is expressed as $\sigma_n = \sigma_1 \cos^2(n,1) + \sigma_2 \cos^2(n,2) + \sigma_3 \cos^2(n,3)$, which is enclosed in a hand-drawn box. The NIPTEL logo is visible in the bottom left corner of the whiteboard.

$$\begin{aligned} \Delta OBC &= \Delta ABC \cos(n,1) \\ \Delta OAC &= \Delta ABC \cos(n,2) \\ \Delta OAB &= \Delta ABC \cos(n,3) \end{aligned}$$

$$\sigma_n \cdot \Delta ABC = \sigma_1 \Delta OBC \cos(n,1) + \sigma_2 \Delta OAC \cos(n,2) + \sigma_3 \Delta OAB \cos(n,3)$$

$$\therefore \sigma_n = \sigma_1 \cos^2(n,1) + \sigma_2 \cos^2(n,2) + \sigma_3 \cos^2(n,3)$$

From this we can write down the Triangle OBC, let us skip the figure here, so it will be easy for us to follow. This triangle OBC, this area of the triangle is nothing but how much we can express it as this inclined plane area ABC times cosine of n_1 , because n_1 is the angle between this n th plane with respect to axis one. So, the projection of this plane on this two, three axis will give us the projection of this area on this plane will give us the area OBC, right. Similarly, the other areas like area OAC this area, this area should be area ABC times cosine of that angle n_2 , because this is projection on one and three axis; one and three axis it will be the projection cosine of angle this one makes with respect to axis two, projection of that on this will be the area.

And triangle OAB this area should be triangle ABC times cosine of angle between that n th plane and axis three. So, n th plane that direction is nothing but perpendicular to this one. So, that axis n th axis, n with respect to one, n with respect to two, n with respect to three; these are the three angles. Now the normal force acting on this inclined plane ABC is σ_n . So from which what we can write, the total normal force will be the normal stress times the area of that inclined plane which will be nothing but equals to σ_1 times OBC cosine n_1 plus σ_2 times OAC cosine n_2 plus σ_3 times OAB cosine n_3 . Total normal force I am obtaining; total normal force on n th axis or n th plane that should be equal to this one that is component of the normal force what we are having on plane OBC.

So, on plane OBC we have total normal force is σ_1 times this area, then its projection on the n th plane. So, that is why cosine of n_1 ; similarly for the other two, σ_2 OAC times projection of it on this and σ_3 OAB times projection on this. So, now if we put OBC, OAC, and OAB expressions from here from this relation in this expression, then ABC from both the sides will get cancelled. So, final expression from this relation we will get σ_n equals to σ_1 cosine square n_1 plus σ_2 cosine square n_2 plus σ_3 cosine square n_3 . So, this is the expression by using which mathematically we can obtain the normal stress on any n th plane from the given three principle stresses; σ_1 , σ_2 , σ_3 , and knowing the angle of that normal plane with respect to those principle stress directions. Now, mathematically we have obtained this.

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Resultant stress on ΔABC .

$$\sigma_R = \sqrt{\sigma_n^2 + \tau_n^2}$$
$$F_R = \sigma_R \cdot \Delta ABC$$
$$F_{\Delta OBC} = \sigma_1 \cdot \Delta OBC = \sigma_1 \Delta ABC \cos(n,1)$$
$$F_{\Delta OAC} = \sigma_2 \cdot \Delta OAC = \sigma_2 \Delta ABC \cos(n,2)$$
$$F_{\Delta OAB} = \sigma_3 \cdot \Delta OAB = \sigma_3 \Delta ABC \cos(n,3)$$
$$F_R^2 = F_{\Delta OBC}^2 + F_{\Delta OAC}^2 + F_{\Delta OAB}^2$$

Let us see how the shear stress we can obtain now. So, resultant stress on that inclined plane ABC; let us say that is sigma r. The resultant stress is nothing but normal stress square plus shear stress square and the resultant force F R should be this resultant stress times the area of that plane Now, what is the force acting on this plane OBC, that is principle planes; that is sigma one times area OBC which is nothing but sigma 1 ABC cosine n 1. Similarly the other three also we can write like force acting on OAC is sigma 2 times OAC which is sigma 2 ABC cosine of n 2 and area OAB should be sigma 3 times OAB sigma 3 ABC cosine of n 3. Now, this resultant force F R square is how much; summation of squares of the three orthogonal forces, right. So, it should be equals to F OBC square plus F OAC square plus F OAB square. So, on simplification what we will get.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $F_R^2 = [\sigma_1 \Delta ABC \cos(n,1)]^2 + [\sigma_2 \Delta ABC \cos(n,2)]^2 + [\sigma_3 \Delta ABC \cos(n,3)]^2$. Below it, a boxed equation states $\therefore \sigma_R^2 = \sigma_1^2 \cos^2(n,1) + \sigma_2^2 \cos^2(n,2) + \sigma_3^2 \cos^2(n,3)$. A second boxed equation below that states $\cos^2(n,1) + \cos^2(n,2) + \cos^2(n,3) = 1$. In the bottom left corner of the whiteboard, there is a logo for NIPTEIL.

This F_R square is equals to sigma 1 delta ABC cosine of n 1 whole square plus sigma 2 ABC cosine of n 2 whole square plus sigma 3 ABC cosine of n 3 whole square. Therefore, sigma r square, this ABC square we can cancel from both the sides; it boils down to sigma 1 square cosine square n 1 plus sigma 2 square cosine square n 2 plus sigma 3 square cosine square n 3. This is another relation we can obtain. So, two relations we got; we need three relations to draw Mohr's circle because we have to obtain the three center of the circles. Now how to represent this 3D stresses in the form of 2D Mohr's circle; that is we are representing now three-dimensional stresses in two-dimensional form. How to do that let us see.

So, the third relation which must follow is the cosine rule of angles. What is that? Cosine square n 1 plus cosine square n 2 plus cosine square n 3 must equals to 1, right. This is the cosine rule of any inclined plane with respect to the three orthogonal axes system. In some of the books they denote it by l m and n; l square plus m square plus n square equals to 1. So, using these three relations; what are those three relations? I have highlighted it here. Let me put it once again so that now we know. Yes, this is one relation, this is second relation, this is the third relation.

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$$\cos^2(n,1) = \frac{\tau_n^2 + (\sigma_n - \sigma_2)(\sigma_n - \sigma_3)}{(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)} \geq 0$$
$$\cos^2(n,2) = \frac{\tau_n^2 + (\sigma_n - \sigma_3)(\sigma_n - \sigma_1)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)} \geq 0$$
$$\cos^2(n,3) = \frac{\tau_n^2 + (\sigma_n - \sigma_1)(\sigma_n - \sigma_2)}{(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)} \geq 0$$

Using them now we can simplify this three and write that cosine square n 1 tau n square plus sigma n minus sigma 2 sigma n minus sigma 3 by sigma 1 minus sigma 2 sigma 1 minus sigma 3. On simplification, one can get this expression using those 3 relations. Similarly cosine square n 2 should be tau n square plus sigma n minus sigma 3 sigma n minus sigma 1 by sigma 2 minus sigma 3 sigma 2 minus sigma 1 and cosine square n 3 can be solved as tau n square plus sigma n minus sigma 1 sigma n minus sigma 2 by sigma 3 minus sigma 1 times sigma 3 minus sigma 2. Now how does it help us to obtain the radius and center of Mohr's circle? What is the nature of these three values? They must be greater than or equals to 0; they cannot be negative, am I right? These are cosine of an angle square.

So, it never can be negative. This also must be equal to greater than equal to 0; this also must be equal to greater than equal to 0. Now what we can look here that sigma 1 is greater than sigma 2 is greater than sigma 3. So, this parameter is always this is positive, this is positive, positive; sigma 2 is greater than sigma 3. This is positive, but sigma 2 is less than sigma 1. So, this is negative. So, this product is negative. So, to obtain this as positive this also must be negative, whereas, this one, sigma 3 minus sigma 1 will be negative; sigma 3 minus sigma 2 will also be negative, but product will be positive. So, no problem; we will get a positive. What does it mean?

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$$\cos^2(\theta, 2) = \frac{\tau_n + (\sigma_n - \sigma_3) \cos(2\theta)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)} \geq 0$$

$$-\sin^2(\theta, 3) = \frac{\tau_n + (\sigma_n - \sigma_1)(\sigma_n - \sigma_2)}{(\sigma_2 - \sigma_3)(\sigma_1 - \sigma_2)} \geq 0$$

$$\tau_n^2 + \left(\sigma_n - \frac{\sigma_2 + \sigma_3}{2}\right)^2 \geq \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2$$

$$\tau_n^2 + \left(\sigma_n - \frac{\sigma_1 + \sigma_3}{2}\right)^2 \leq \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2$$

$$\tau_n^2 + \left(\sigma_n - \frac{\sigma_1 + \sigma_2}{2}\right)^2 \geq \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

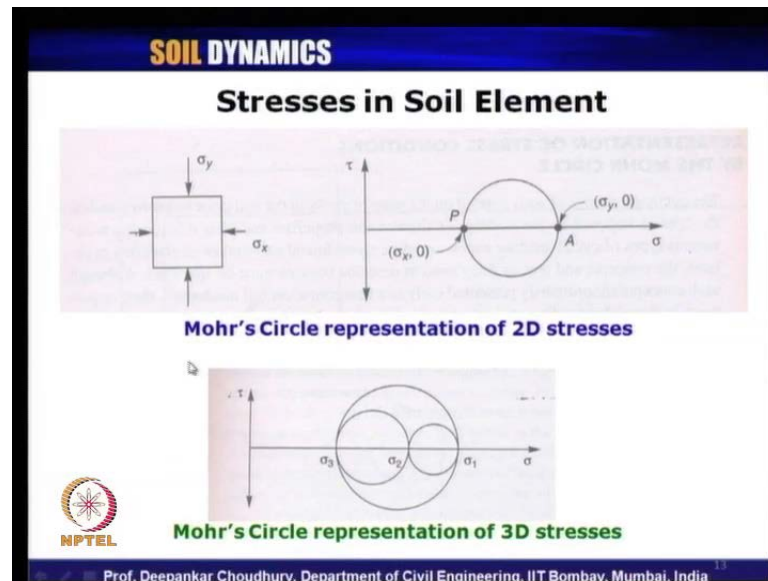
If we simplify this expression further, it can be written in this form tau n square plus sigma n minus sigma 2 plus sigma 3 by 2 whole square should be greater than or equals to sigma 2 minus sigma 3 by 2 whole square. The second relationship gives us tau n square plus sigma n minus sigma 1 plus sigma 3 by 2 whole square should be less than equals to sigma 1 minus sigma 3 by 2 whole square. And the third relationship will gives us tau n square plus sigma n minus sigma 1 plus sigma 2 by 2 whole square should be greater than equals to sigma 1 minus sigma 2 by 2 whole square.

What does it mean, where from they have come? Let us put it back here again. From this relation as I said, this is always positive; it will give us this one. This if you simplify if you take it here, it will be positive; you will get this greater than equal to sign for this one. Whereas for second one to satisfy this relation this is being negative, the relationship will come like this if you simplify you will get. And the third relation one, this is positive, so if you take it here, you will finally obtain this relationship greater than equals to like this. So from these three relationships, now we have to see how we can draw this 3D stresses in two-dimensional Mohr's diagram form; so, that we will continue in our next lecture.

End of part A

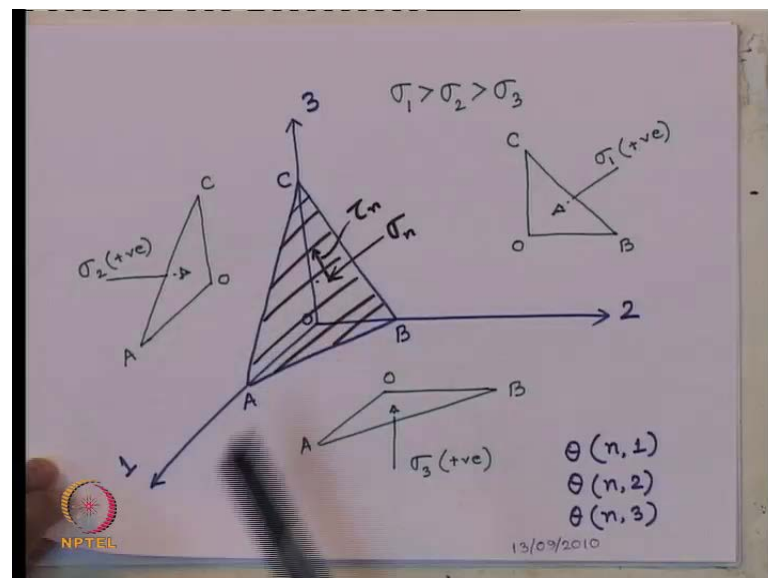
Lecture - 18(B)

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Now coming to the three-dimensional stresses in soil element, we have seen this in the previous lecture.

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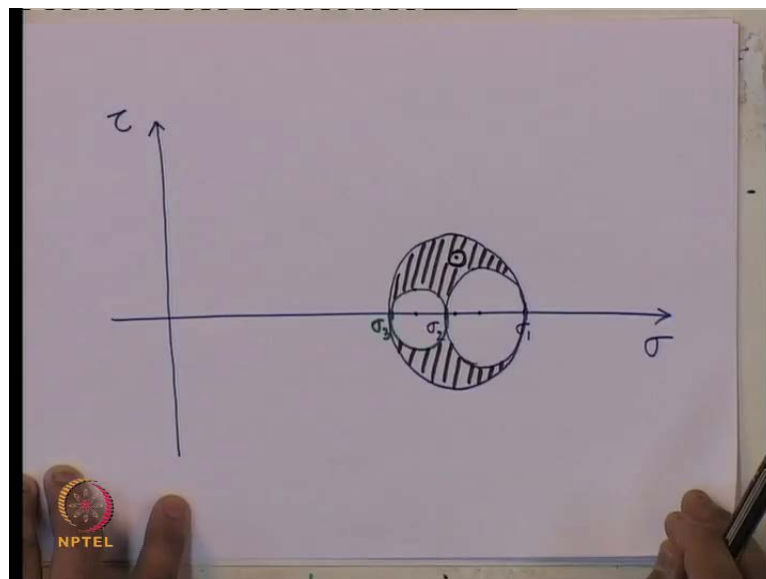


That the three-dimensional soil element is we have represented like this with three directional principle stresses σ_1 , σ_2 , and σ_3 ; major principle stress, intermediate principle stress, and minor principle stress, and the corresponding direction cosines with respect to any plane n th plane with respect to those principle stress

directions we have seen. And from which we had derived the expressions for angle of this cosines square of that n th plane with respect to principle stress direction one, two, and three to satisfy this conditions.

From which finally we got the expressions like this where this τ_n is nothing but the shear stress on that n th plane on which we want to find out what is the normal and shear stress; σ_n is the normal stress on the n th plane and other things we have already defined. So, from these expressions what we can say if we want to represent it in graphical form in the form of a circle, if we look at these equations very carefully, these are nothing but equations of a circle; this is also equation of a circle, equation of a circle, equation of a circle with which these are the center of the circle and these are the radius of the circle, right. If we recap our memories of equations for general equation for circle, we can easily remember these things.

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So, if we want to draw these circles in a 2D plane how they should look like; let us draw it. So, this is our normal stress direction, shear stress direction. Now first equation, let us look at it; let us take one by one. This is σ_1 and σ_2 . So, these are the two principle stresses and what we have considered, σ_1 is greater than σ_2 is greater than σ_3 ; that is the condition we have taken. So, let us plot those principle stresses, say, this is our σ_1 , this is σ_2 . So with these two principle stresses, we can draw

Mohr's circle with the center of the circle at $\frac{\sigma_1 + \sigma_2}{2}$. So, this is one Mohr's circle which is satisfying this equation.

Let us look at here. This is the center of the circle $\frac{\sigma_1 + \sigma_2}{2}$ and this is the radius of the circle $\frac{\sigma_1 - \sigma_2}{2}$; that is what we have drawn. Then let us plot another principle stress which is, let us say σ_3 . So, this point is σ_3 . Our order we have seen; σ_1 is greater than σ_2 , σ_2 is greater than σ_3 . Now we these two points σ_2 and σ_3 , we can draw another circle another Mohr's circle. So, for this circle what is the center; that is $\frac{\sigma_2 + \sigma_3}{2}$ and the radius is $\frac{\sigma_2 - \sigma_3}{2}$ which is nothing but coming from this first equation.

Now we can draw another Mohr's circle by considering this two principle stresses σ_1 and σ_3 and the center of which will be $\frac{\sigma_1 + \sigma_3}{2}$ and the radius will be $\frac{\sigma_1 - \sigma_3}{2}$. So, the third circle should be something like this. Now the state of stress of any point which we are trying to find out on n th plane. So, that point if we want to put on this Mohr's circle graphically, where it should come; the point must lie in this zone. So, it must be somewhere here. So let me shade the area first, then I will explain why. The state of stress on any plane any n th plane must lie on this shaded area, not anywhere else.

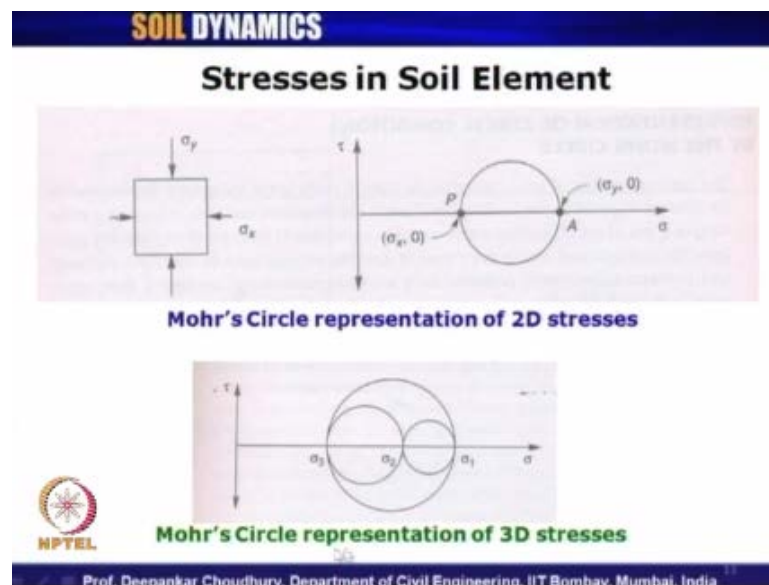
Why? If we look back to our equations of this circle, so what does it mean? The state of stress on that n th plane from this equation between σ_2 and σ_3 , that is the Mohr's circle between σ_2 and σ_3 , whatever the circle we have drawn, the state of stress that point must be outside or maximum on the periphery of this circle or circumference of the circle; that is what this inequality signs denotes. Equal sign shows that point can lie on the circumference of the circle; the greater than sign denotes the point can lie outside this smaller circle between σ_2 and σ_3 .

What about the second equation? Second equation gives us that the point must lie within the circle of σ_1 and σ_3 with the maximum boundary condition of equality sign where the point can lie on this circumference of the circle. So when the equality sign holds, at that time the point will be somewhere on this circumference of the circle; otherwise, always it must be within the circle of between σ_1 and σ_3 . What are the criteria we are getting from the third equation? That the point on that n th plane again

they must be within the circle; sorry. The point on n th plane must be outside the circle between σ_1 and σ_2 .

The equality sign again says that it can lie maximum on this circumference of the circle; otherwise, the greater than sign denotes it should be outside or at this point. So from this three equations, what the graphical representation of the three-dimensional stresses in a soil element; this is the graphical representation in two-dimension; means three-dimensional stresses we are representing in two-dimension using this concept of Mohr's circle where any state of stress must lie in this shaded zone not in any other areas. And obviously, if you find out the values of σ and τ for that point, you will get your normal stress and shear stress for any n th plane. So, this is the way graphically we find out the state of stress on any n th plane for a case of three-dimensional soil element.

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So, let us look at the slide here; that is what it is given Mohr's circle representation of three-dimensional stresses.

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SOIL DYNAMICS

Field Tests

For Determination of Dynamic Soil Properties

(A) Direct Tests:

- Seismic Reflection Test
- Seismic Refraction Test
- Spectral Analysis of Surface Wave (SASW) Test
- Seismic Cross-hole Test
- Seismic Down-hole/Up-hole Tests etc.

(B) In-direct Tests:

- Standard Penetration Test (SPT), Cone Penetration Test (CPT), Becker Penetration Test (BPT) etc.

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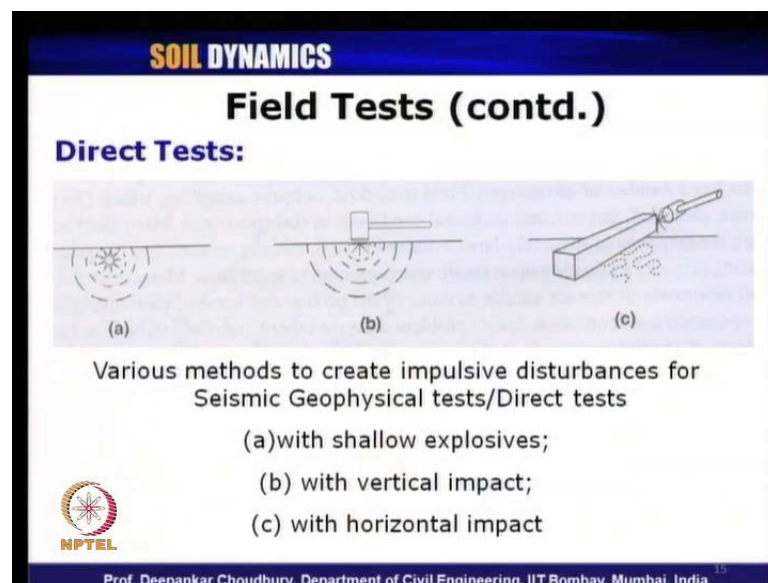
Now when we are talking about the determination of dynamic soil properties; as we know similar to our static determination, determination of static soil properties, we can broadly classify the tests in two categories. One is field test or intuitive test; another one can be laboratory test. So, among field test for the determination of dynamic soil properties, we can again sub classify them into two categories; one is called direct test, another is called indirect test. What are the different direct tests? One is seismic reflection test; another one is seismic refraction test; another one is Spectral Analysis of Surface Wave Test which is in short called SASW.

If you look at here, Spectral this S, Analysis A, Surface S, Wave W. So, SASW test and a modified version of this SASW test are also available nowadays that is called MASW. What is MASW? The same spectral analysis of surface wave in that case that is multichannel. So, that is why MASW. But the methodology of the testing and principle everything remains same; only difference is instead of a single channel we use multichannel. So, that is why MASW also you can note down with this. Then another direct test is called seismic cross-hole test and another one is called seismic down-hole or up-hole test. So, these are the different types of direct field test for determination of dynamic soil properties.

And what are the common indirect tests? Like SPT that is Standard Penetration Test, Cone Penetration Test or CPT, Becker Penetration Test BPT, etc, Vane Shear Test, etc,

etc. As you know these are the common test field tests for static condition also. So, to determine the static soil properties, these are the test. So, that is why we have mentioned here or classify it under the category of indirect test, because this static field test also helps us to correlate with some of the dynamic soil properties. So as we progress in the lecture, we will see how the correlation holds good for determination of dynamic soil properties from these static tests.

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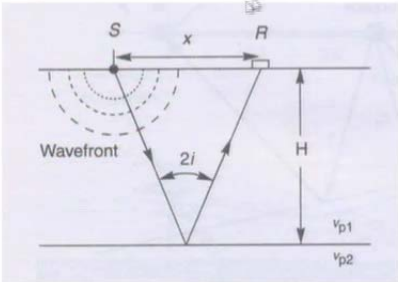
So now, for different types of direct test for this field testing, what is the basic methodology that we have to create a vibration in the soil; that is the basic criteria to do a direct test for the dynamic soil properties. So, to create an impulse or vibration within the soil, there are various types of methods; these are shown in the picture here. The first one is by using shallow explosives; if we use some explosives at a very shallow depth below ground, then the waves can get generated and then we record them using different direct test method for the determination of dynamic soil properties.

The second one is by using vertical impact on the soil directly like this and another type can be by using horizontal impact like this; if you put a log, so a wooden log is kept, then on that you create or hammer it, then by providing that you are creating a vibration in your soil which gets propagated. So, these are sources of vibration; these are called sources of vibration for different seismic geophysical test or the direct test. So now, let us come to the major criteria or characteristic of various direct tests.

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SOIL DYNAMICS

Seismic Reflection Test



Path for incident ray and reflected ray of p-wave from horizontal layer boundary

Time for wave energy to come from S to R:

$$t_d = \frac{\text{distance of travel}}{\text{wave velocity}} = \frac{x}{v_{p1}}$$

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The first one is seismic reflection test. As the name suggests, here we use the principle of reflection of wave. We have seen wave propagation; the three component reflection refraction, and the incident wave, three components of waves. So, here the concept of reflection of waves is used. This S is nothing but source; that is where you are creating the disturbance or vibration in the soil by using any of the above three methods just now what we have described and the point R is called receiver. So, these are receiving the disturbance created at this point followed as a waves through the soil media and which is finally recorded at the receiver R. Suppose if we have layered media layered soil media like this and these are horizontal layers; let us assume first and the top layer is having a primary wave velocity or p-wave velocity, say, v_{p1} and the second layer is having primary wave velocity or p-wave velocity of v_{p2} .

Then obviously, the waves which are travelling some of them will get reflected back in the first media and some of them will get transmitted in the second media. So, and i is the angle of incidence with respect to vertical. So, this angle between them is $2i$. So, it shows path for incident ray and reflected ray of p-wave from horizontal layer boundary and how much time is required for the wave energy to come from this point is $2R$. If they are horizontal distance on the ground is given by x , then the time t_d is given by distance of travel by the wave velocity in that media. Because primary wave as we know, this is the wave which will travel fast; so obviously, fast one which we are recording in our time receiver that t_d is nothing but x by v_{p1} .

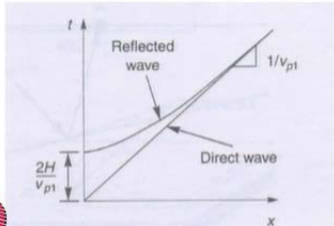
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SOIL DYNAMICS

Angle of incidence of impulsive energy Traveling downwards:

$$i = \tan^{-1} \frac{x}{2H}$$

Time taken by the reflected wave:

$$t_r = \frac{\text{distance of travel}}{\text{wave velocity}} = \frac{2\sqrt{H^2 + (x/2)^2}}{v_{p1}} = \frac{\sqrt{4H^2 + x^2}}{v_{p1}}$$


Hence thickness of upper layer,

$$H = \frac{1}{2} \sqrt{t_r^2 v_{p1}^2 - x^2}$$

Travel time variation for direct and reflected waves

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And the angle of incidence of the impulsive energy travelling downwards, that i can be expressed as $\tan^{-1} x / 2H$ which is very simple from this triangular distribution; from the geometry we can write it very easily. And the time taken by the reflected wave by putting all this relationship t_r , that can be, the previous time was for the direct wave and this is for the reflected wave. So, the direct wave actually goes directly from here to here. So, t_d is this one, whereas reflected wave is the next one which will arrive at your receiver. So, that time t_r will be something different from t_d ; that will not be same. So, you can easily compute from the geometry of the figure what was shown. This is the expression for distance of travel by wave velocity that is reflected wave time taken.

If you plot the time versus the distance relationship, obviously the direct wave will be at complete linear distribution with respect to time versus x . Whereas the reflected wave, it will depend on your thickness of the media and of course the wave velocity within that media. So, it will take a shape like this with an intercept on the t axis like this. So, the thickness of the upper layer can be computed if you simplify this expression and put it in terms of H , you will get the expression in terms of H like this in terms of time taken by the reflected wave and primary wave velocity in the first media and the x distance between the source and the receiver.

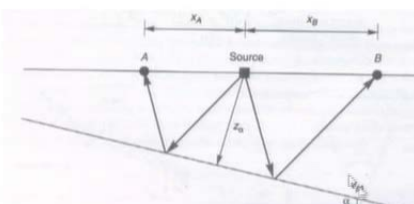
Now, suppose if we do not have the horizontal layer; so nice horizontal layers but we have inclined boundaries or irregular boundaries, then how we can measure the thickness

of a particular layer and of course the v_{p1} either of these two. In the previous case in the time travel variation for direct and reflected wave, we can measure the v_{p1} from the slope of the curve; that is direct wave and reflected wave when the slope of the two curves are matching that is the slope which will give us the value of v_{p1} and then using that, this values is known to you, this value is known to you, you can compute the thickness of the upper layer.

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SOIL DYNAMICS

For Inclined boundary layers:



Layout of Source and receiver for seismic reflection test with inclined layer boundary

Where $v_{p1} = x_A/t_{dA} = x_B/t_{dB}$ and t_{dA}, t_{dB} = direct wave travel times at A & B respectively.

$$\sin \alpha = \frac{v_{p1}^2(t_{RA} + t_{RB})(t_{RB} - t_{RA})}{4z_{\alpha}(x_B - x_A)} = \frac{x_A + x_B}{4z_{\alpha}}$$

If receiver A is placed at source ($x_A = 0$) then,

$$z_{\alpha} = \frac{t_{RA} v_{p1}}{2}$$

$$\sin \alpha = \frac{v_{p1}^2(t_{RA} + t_{RB})(t_{RB} - t_{RA}) - x_B^2}{2t_{RA} v_{p1} x_B}$$

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But if the layers are not horizontal, but inclined like this, what will happen? So, this is the layout of source and receiver for that seismic reflection test with inclined layer boundary. In this case, suppose this is the source point and A and B are the two receivers at the both the sides of the source at a distance of x_A and x_B on the ground surface horizontal distances. And α is the inclination of this layer with respect to horizontal and v_{p1} is the primary wave velocity of this top layer. Then this v_{p1} if we talk about the direct wave that will be equal to this distance by time taken for the direct wave to arrive at station A and the time taken to arrive at station B. They must be equal to v_{p1} ; they must equate each other.

And if receiver A is placed at the source when x_A is 0 and then the equation $\sin \alpha$, this expression for α can be expressed like this which on simplification will give us this expression. In this case also, we can plot the travel time curve; that is time versus distance for two cases and at one place they will meet each other; that is you plot the

time for receiver A at left hand side of the y axis and for receiver B on right hand side of the y axis, then start drawing the t versus x plot. At the junction of them, the slope of the curves will give you the value of v_p from which you can find out the alpha inclination of the layer. So, with this we have come to the end of today's lecture. We will continue our lecture in the next class.