Soil Dynamics
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Module - 3
Wave Propagation
Lecture - 15
3 Dimensional Wave Propagation, Waves in Semi-infinite Media, Rayleigh Wave

Let us start with today's lecture of soil dynamics. We are continuing with our module three, that is on wave propagation. A quick recap what we had studied in the previous lecture.

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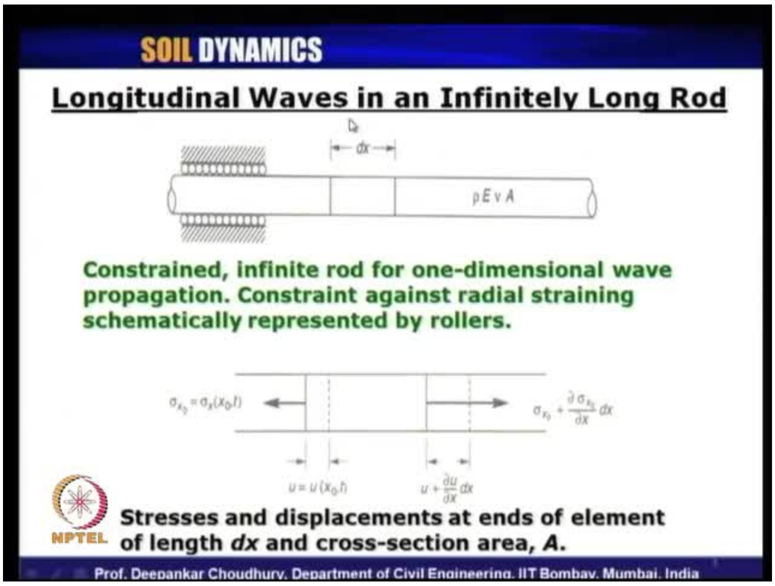

We have considered the longitudinal wave equation in an infinitely long rod, which is constrained in this lateral direction with material property like this.

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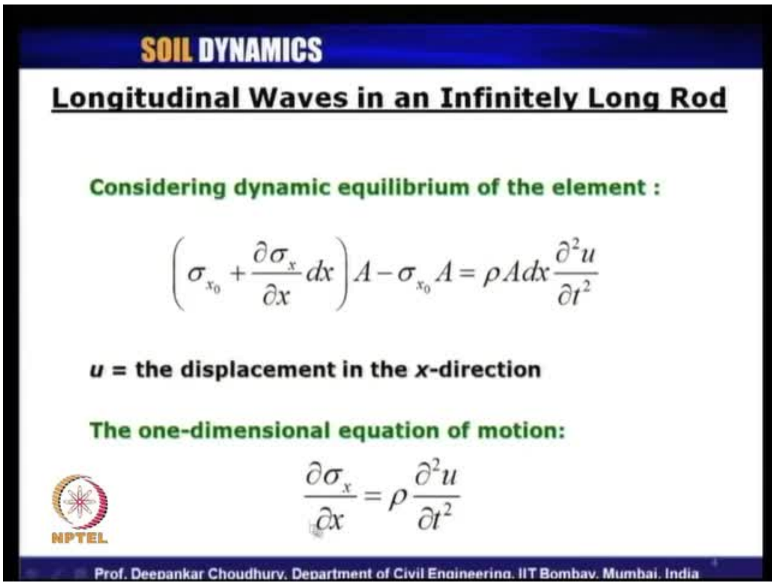

And we have seen how the one-dimensional equation of motion can be derived in terms of stress and displacement of the particle.

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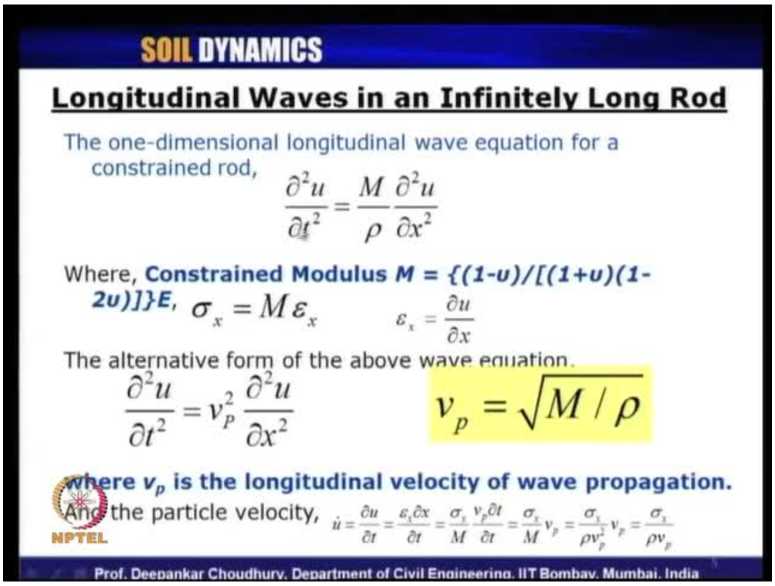

And finally, we have simplified that in this form where v p is called the primary wave or p wave velocity, which is nothing but equals to root over constrained modulus by rho; rho is the density of the material.

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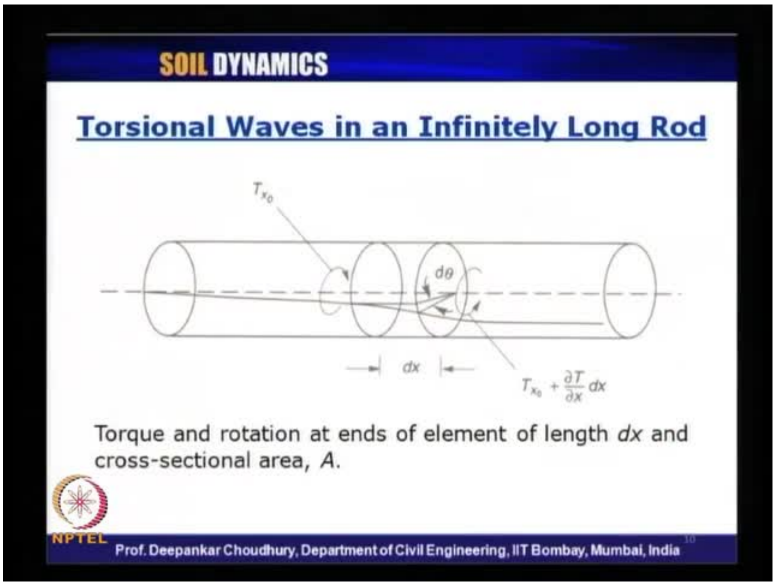

Now, let us take the next case. Let us look at the slide here for torsional wave in an infinitely long rod. Last time we have taken the longitudinal wave, now we are taking the torsional wave. Suppose, a torsional wave is travelling through an infinitely long rod, then what we are doing? We are doing the same thing, we are taking a small segment within the rod of length d x, and the torque at the two ends of that element. Let us say at this end the torque is in this direction T x naught on the other end of the element, due to the movement of the particle in this direction is T x naught plus del T del x into d x, when the torsional rotation of the element is given by del theta.

So, torque and rotation at the ends of the element of length d x with A cross sectional area a is shown. What we can do here? The same thing the difference between the torque at the two ends of this small element the total torque can be equated or total moment can be equated with respect to the inertia force, due to the dynamic movement of the particle within this element.

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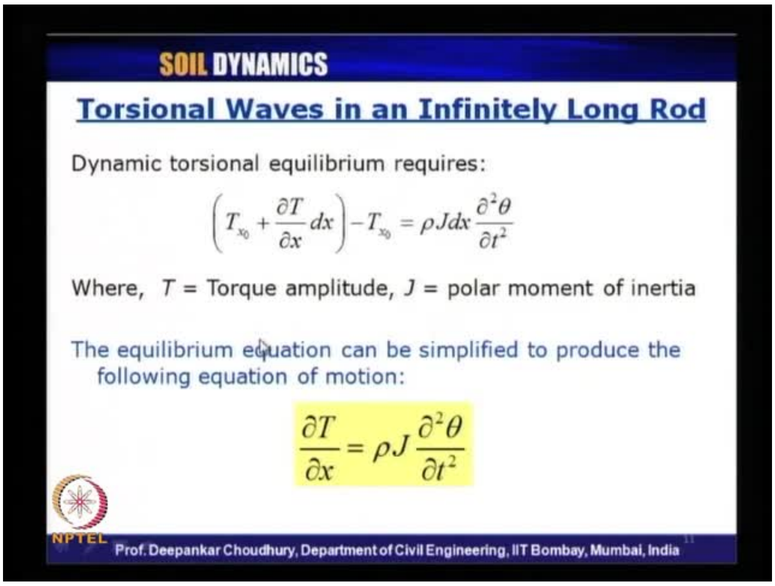

So, if we do that what we can write the dynamic torsional equilibrium will give us, this T x naught plus del T del x into d x minus T x naught. So, this is the resultant torque remains within the unit length of d x that should be equated with respect to, the inertia force generated due to movement of the particle which is nothing but density of the material, times polar moment of inertia, d x, length, times the rotational acceleration which is del 2 theta by del T square.

So, this T is torque amplitude, J is polar moment of inertia now, again we can simplify it. If you do that, the equilibrium equation can be represented as del T del x equals to rho J del square theta by del T square. So, this is the torque equation with respect to the rotational component. Similarly, what we got for the longitudinal case, the stress equation with respect to the displacement function.

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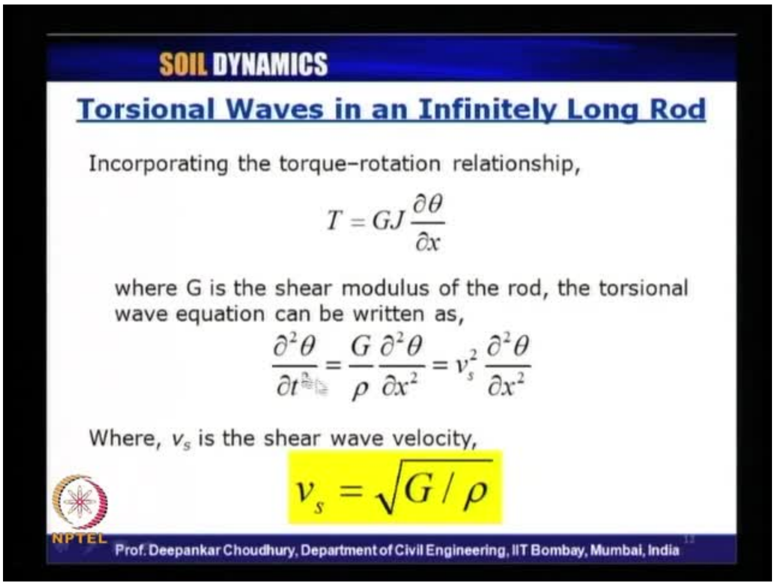

Now, let us put it in terms of the acceleration del 2 theta del T square with respect to del 2 theta by del x square, as we have done for the previous case of longitudinal wave. So, now we have torque relationship with respect to del theta del x in this form T equals to given by G J del theta del x, where G is the shear modulus of the rod. So, the torsional wave equation can be re-written in this form that is del 2 theta del T square equals to G by rho del 2 theta del x square, where this G by rho is represented as v s square.

This v s is called the shear wave velocity, which is represented as root over G by rho, so the primary wave velocity or the longitudinal wave velocity is root over m by rho, constant modulus by rho and the shear wave velocity or the torsional wave velocity is given as the shear modulus by the density of the material under root of that. So, if we want to generalize this two cases, that is for any wave equation whether it is longitudinal wave or a torsional wave, what is the basic form of the equation of motion?

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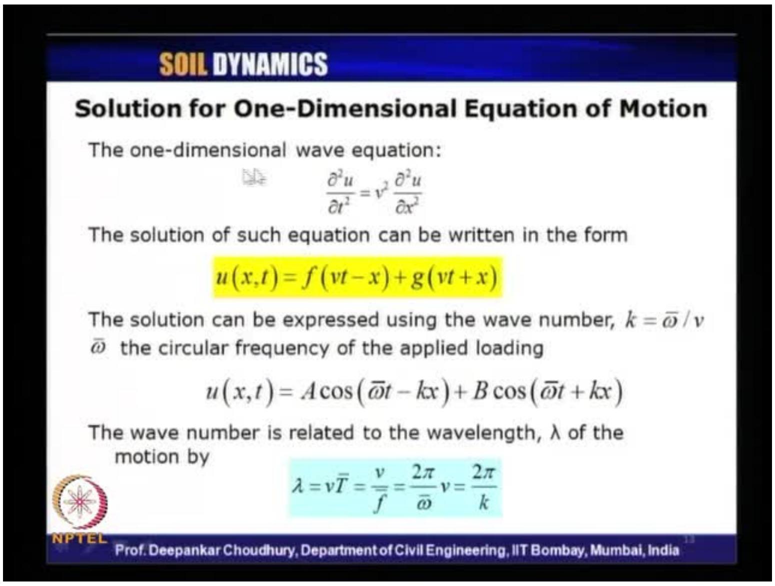

This is the basic form of equation of motion for one-dimensional wave equation, that is del 2 u by del t square equals to v square del 2 u by del x square, right? So, if it is longitudinal wave then it is v p, if is it torsional wave then it is v s, otherwise the form of the equation is exactly same. So, for longitudinal wave it is displacement, for rotational wave it is rotation theta that is the only difference, otherwise form of the equation is this one. And the solution of that type of equation can be represented by in function form like this, u of x of t can be given as a one function, v t minus x that is before little phase of x and plus x that is after phase of x, the solution can be expressed with respect to another terminology is given here, which is called as wave number.

Wave number is generally denoted by kappa, so this is the symbol similar to our small k it is kappa actually. Kappa is nothing but omega bar by v, where omega bar is nothing but the circular frequency of the applied loading. What we have considered earlier actually the omega, omega is nothing but the circular natural frequency. So, the wave number is related to the natural frequency in this form, wave number you can compute by dividing the natural frequency by velocity.

So, this solution can be represented in this format in terms of wave numbers and this A and B constants again can be obtained using the initial conditions given to us. And the wave numbers is related to another parameter which is known as wave length, it is generally denoted by the symbol lambda of the motion. So, if I want to simplify this for you, what I can say that, let me draw it here. So, it will simplify the understanding of our two system, when the displacement function u of t we are plotting with respect to the variation in time.

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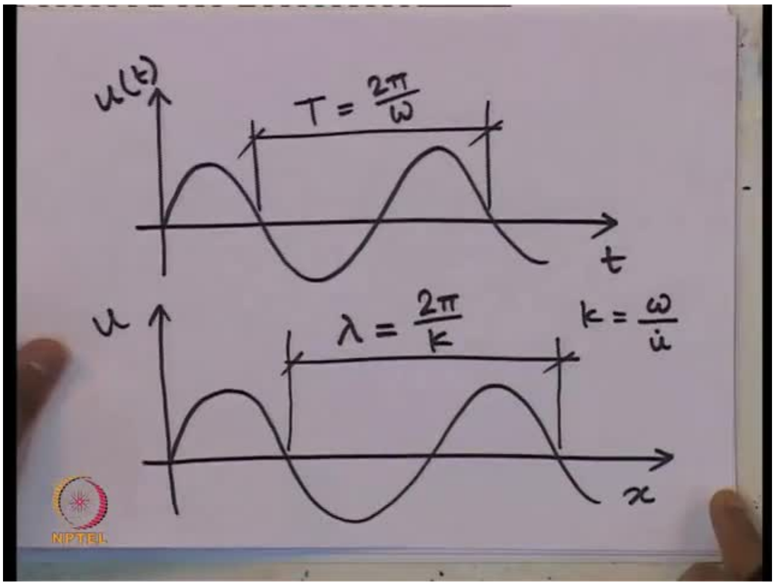

Let us say this was like this some variation, in that case what was our time period? If it is a un damped free vibration, this was our time period natural time period, right? T was expressed as 2 pi by omega, omega was natural frequency of the system. The same response now we are representing it as, the same u we are representing it with respect to the distance x, that is displacement u we are representing it with respect to its distance x. The same thing, in this case when we are representing it with respect to distance, to complete one cycle the distance covered is known as wave length, which you must have read in your physics also earlier.

This is nothing but lambda equals 2 pi by that kappa. So, what does it mean from this simple figure? It will make you understand this is a simple way that the natural frequency circular natural frequency of any vibrating system is equivalent to, the wave number of the system and the natural period of the system is equivalent to the wave length of the system. And how they are related, that also I have mentioned, this kappa is nothing but this omega by the velocity, so u dot. So, that way both the terms are related to each other.

So, if we look at this slide you can see here, lambda is nothing but v times the T, but I have written just now v by f 2 pi by omega times v it equal to 2 pi by kappa. So, the same relation between T and omega holds good for lambda and kappa that is wave length and wave number.

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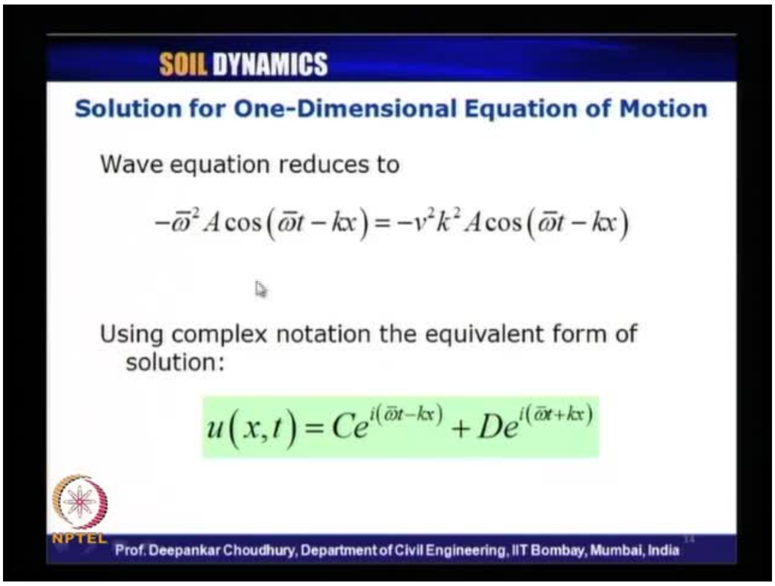

So, wave equation if we put in this form, it takes a shape of this and using complex notation the equivalent solution can be written in the form of a wave number like this.

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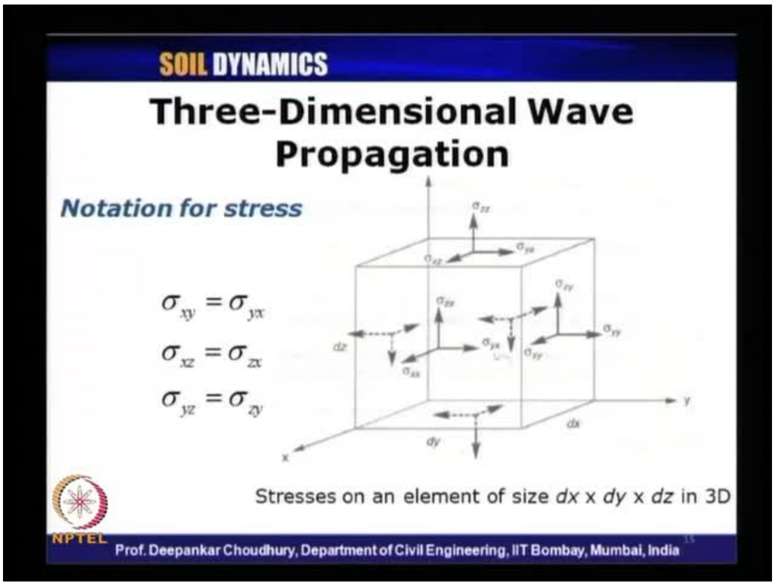

Now, let us start with the three-dimensional wave propagation. We have seen the one dimensional wave propagation, how the equation of motion we are getting? Now, we are trying to consider the wave propagation in all the three dimension, that is generalized case we are considering. So, if you draw this, this is a small element we are taking of size d x in the x direction, d y in the y direction and d z is the length of that element in the z direction. And what are the stresses acting?

On one side of this x, let us say sigma x x here, and these are the shear stresses. On the y, at one phase sigma y y and the these are shear stresses and this is the normal stress, and here on the z sigma z z normal stress and the shear stresses these. Let me draw it in a better way for you, so that it will be more clear.

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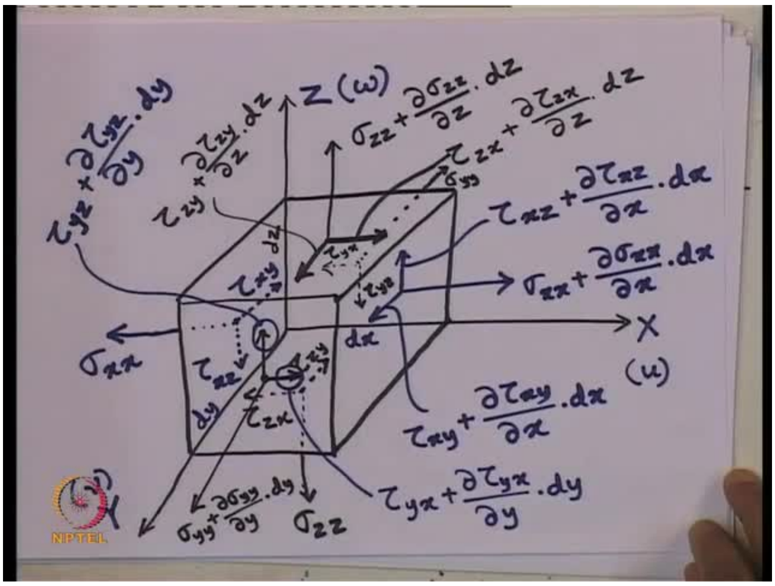

So, this is 3 d element, which we are considering. So, this is our x axis, this is our y axis, this is the z axis we are considering. And stresses as I said on the left side, when waves are travelling through this element, what happens correspondingly? The particle velocity in X direction is let us say u, in Y direction it is let us say v, and in Z direction it is say w. So, what at u, v, w? The corresponding displacement of the particle, because of the movement of the wave. Now, on this phase on the left hand side of the element, the normal stress, let us say sigma x x and shear stresses in this direction, and shear stress in this direction this will be tau x z, this will be tau x y.

One convention for writing the stresses, you must be knowing, when we write any stress about 3 d this two axis system, the first one shows on which plane the stress is acting and the second one shows in which direction the stress is acting, clear? So, for normal stress sigma x x it is acting on x plane in the direction of X, because it is a normal stress so that is why sigma x x, for shear stress this is acting on X plane but in Z direction, so that is why tau x z, and this shear stress is acting on X plane once again so tau x but in Y direction, so that is why tau x y.

What happens on this side of the element, that is when it is travelled by a distance d x in X direction, the normal stress is sigma x x plus del sigma x x del x into over the distance d x, whatever the increment in stresses occurred, on this phase of the element. What about the shear stresses? To keep equilibrium the direction of shear stresses should be now, this one will be vertically up, this should be towards this direction, fine? Now, what is this one? This should be tau x acting in Z direction tau x z because here also tau x z in the opposite direction tau x z plus del tau x z by del x into the change over the length d x, and this will be tau x y plus del tau x y by del x into d x.

So this is also trying to balance on the other direction, shear stress tau x y. So, these are the changes of stresses on the two phases of the element in the x direction. Let us see what happens in the other phases? Let me draw on this phase, here the normal force will be sigma z z plus del sigma z z del z over a distance of d z, so this distance is d z. In this direction one shear stress, in this direction another shear stress. This shear stress should be, how much? Tau, now what is it? It is acting on Z plane, in the direction of x plus del tau z x del z d z and this one will be, tau how much? This is acting on Z plane in Y direction, so z plane y direction plus del tau z y del z into d z.

This is on upper phase, on the lower plane should be sigma z z in this direction one and in this direction another one, because they have to be on the opposite of this phase. So, this one should be tau on Z plane X direction, this one should be tau on Z plane Y direction, still we have two more phases that is, on the Y plane, that is this plane and the other side of the plane. So, on this plane let me draw it, in this direction, this should be sigma y y plus del sigma y y del y into d y, that is the normal stress. Now, shear stresses should be in this direction and in this direction, how we are taking this directions?

You know, we have to take in such a way this and this in the opposite directions this and this in the opposite direction, that balance you know the equilibrium. And this values should be, let me write it in blue, I am writing for this one. This is tau on Y plane in X direction plus tau del tau y x del y into over into over a length of d y, so d y is this distance actually. And this force, now let me draw, will be tau on Y plane in Z direction plus del tau y z del y into d y. Whereas, on the other plane it should be, on the other plane this should be sigma y y, there should be in this direction, it should be downward direction. These are tau y z and this one is tau y x. On the other phase of y plane, this plane normal one sigma y y tau y z tau y x.

Now, with all this forces, what we can do now? We can write down the equilibrium equation in each of these direction x, y and z. If we write the equilibrium equation in X direction, what are the forces involved? Let us look at it and try to put a tick on that now, let us see, the equilibrium of the forces in X direction. So, in X direction what are the forces are involved, let me mark them first.

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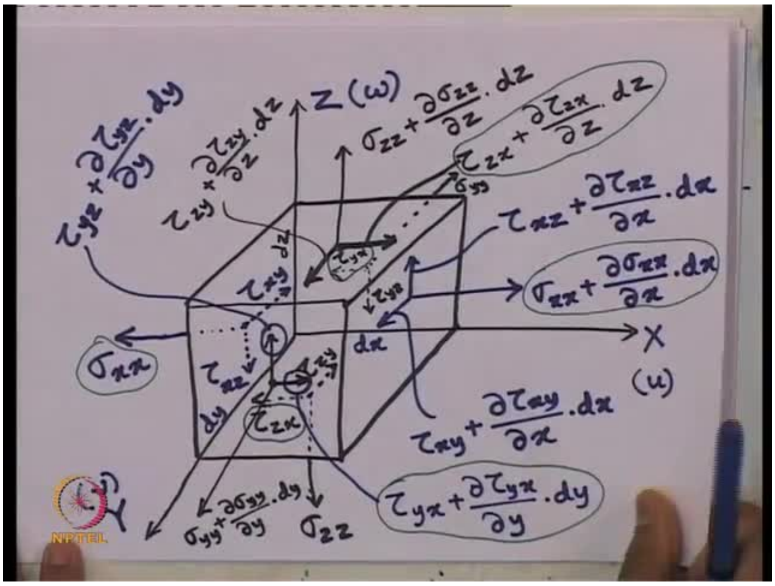

This normal force is in X direction, this normal force is in X direction, what else? This shear force on Z plane is acting in X direction. Similarly, on the other side this shear force on Z plane is acting in X direction. Any other force acting in x direction, let us look here carefully what about Y plane, yes. On Y plane this force is acting in X direction similarly, this force is acting or this stress is acting in X direction, clear? If it is clear to us, let us write down the equilibrium equation.

First let us take this two normal forces in x direction or normal stresses in x direction, total normal force in x direction should be multiplied with respect to the area cross sectional area. And what is the cross sectional area for this normal force on x plane? It is nothing but this is d y this is d z, so d y times d z is the cross sectional area. So, let me write down this equilibrium equation in x direction.

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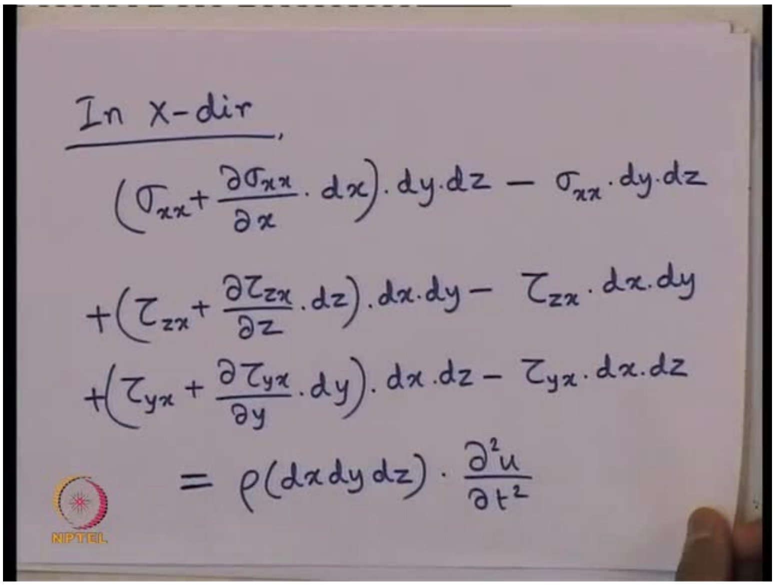

So, in X direction by considering equilibrium we are writing sigma x x plus del sigma x x del x into d x times d y d z, is the force acting on the right hand side, right hand direction minus sigma x x times d y d z, it is acting on the left ward direction. So, net is on the right ward direction, this much due to the normal force or normal stress acting on X plane in X direction. Plus next, whatever is in act what are the stresses acting in the x direction, let us see.

We have this tau z x plus this one and tau z x in this direction and this direction. So, net resultant is in this direction, and on which plane it is acting? It is acting on this Z plane, so what is the cross sectional area for this? d x d y, so the force due to this shear stress will be? Tau z x plus del tau z x del z into d z times area is d x d y minus tau z x into d x d y plus, let us see what else we have in X direction. We have this force acting on the Y plane and this force acting on the Y plane, now net resultant of that should be on the right ward direction. And on which plane it is acting on Y plane?

What is the dimension on this Y plane d x d z. So, if we multiply this shear stress with respect to the area, what we will get? The force tau y x plus del tau y x del y into d y times the area is d x d z, minus tau y x into d x d z, these should be equal to the inertia of force which is nothing but mass time acceleration. Now, in X direction how much is the displacement? That is u we are consider in X direction the displacement is u, so the corresponding acceleration is del 2 u by del t square. So, the equating with the inertia we will get rho times what is the volume of the element d x d y d z is the volume, times density will give us the mass into del 2 u by del t square mass times acceleration will give us the inertia force, that will balance the all this forces arising because of the travel of the wave, within the element, okay?

Now, if we simplify this one, what we will get? This term gets vanished, del x del y del z is coming everywhere, that we can divide both the sides because they are non-zero. So, finally, on simplification we can write it like This, that del sigma x x del x plus del tau z x del z plus del tau y x del y, equals to rho del 2 u del t square.

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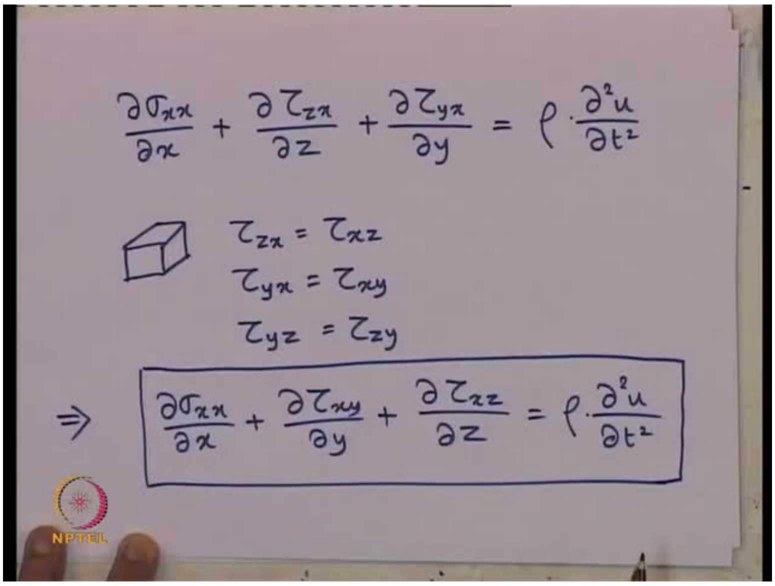

Now, if we consider a very small, infinitively small element, what we know? For the equilibrium of infinitively small element, the moment created by the shear forces should be equal about it centre, what does it mean? Because of the shear the couple getting generated from the two phases, what we can write? For one small element this tau z x is nothing but tau x z to maintain equilibrium similar tau y x is equals to tau x y, and tau y z equals to tau z y. These are the conditions for a small element to maintain equilibrium.

So, these symbols are interchangeable with its plane and direction, in which it is applying. So, what we can write then del sigma x x del x plus del tau x y del y plus del tau x z del z equals to rho del 2 u by del t square, this is the governing equation of motion in x direction, okay? This is the basic governing equation of motion in three dimensional wave propagation, for X direction only. So, similarly in other two direction, Y direction and Z direction we can write the equation in this form, you can note down.

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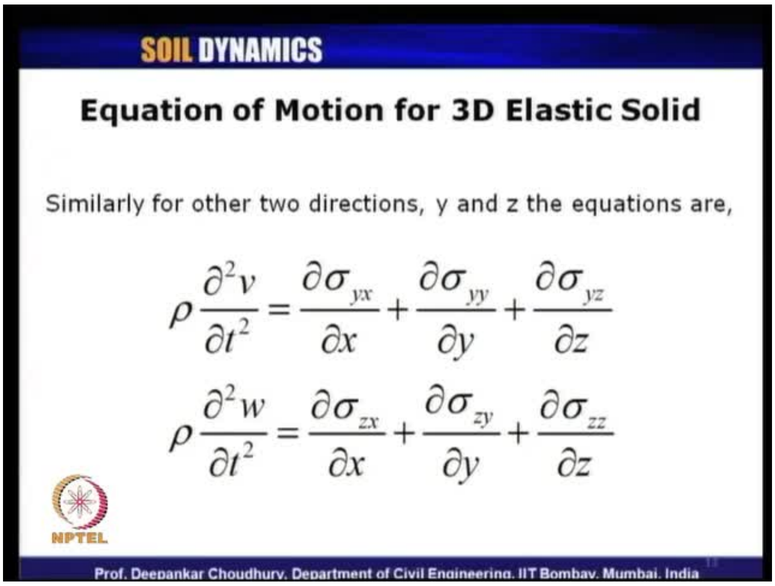

So, we can simply compare in case of X direction this was u in case of Y direction the corresponding displacement is v. So, acceleration is del 2 v by del t square times rho. We are getting, we have used the symbol tau for shear stress, you can write here it tau also. Del tau y x del x plus del sigma y y del y, this is normal force component, delta sigma y y by del y this is normal force component, plus del tau y z by del z this is another shear stress component acting in the Y direction. This is equation of motion for Y direction similarly, in Z direction the equation of motion, is in Z direction. We have considered the displacement is noted by w. So, corresponding acceleration will be del 2 w by del t square.

So, times rho will be equated with respect to del tau z x the shear stress in z direction by del x, plus del tau z y by del y plus the normal stress in the Z direction result in normal stress in the Z direction del sigma z z by del z. So, this is the easy way to remember this equation, the easiest way to remember this equation that, the normal component of the corresponding stress along that axis and the corresponding shear stresses along that axis to be considered, which has to be equated with respect to the acceleration, or the in other words the inertia component in that particular direction. Now, before simplify further this equations, let us go through quickly our known expressions for various modulus which we have studied in our solid mechanics courses, definitely in our undergraduate curriculum.

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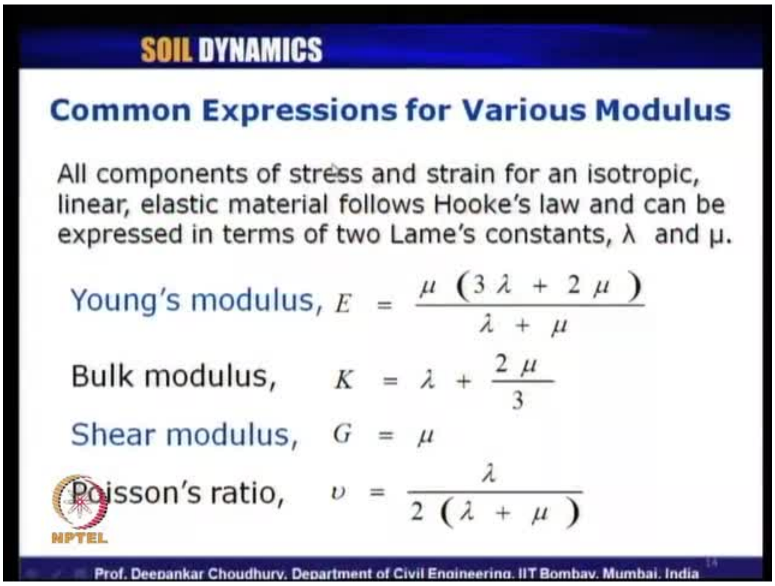

So, all components of stress and strain for an isotropic linear elastic material follows Hooke's law and that can be expressed in terms of two Lame's constants. Lame's constants are denoted by these two symbols, lambda and mu. Lambda and mu are the two Lame's constant. So, in terms of Lame's constant the corresponding expressions for various modulus which we use are Young's modulus E, which is given by this expression new times 3 lambda plus 2 mu by lambda plus mu. The expression for bulk modulus K is given by lambda plus 2 mu by 3.

The expression for shear modulus G is nothing but that mu, and the Poisson's ratio expression in terms of Lame's constant mu is given by lambda 2 times lambda plus mu.

So, these are the standard expressions which we can obtain in any solid mechanics book, which we have studied in our undergraduate. Now, using this standard notation of various modulus, what we can do?

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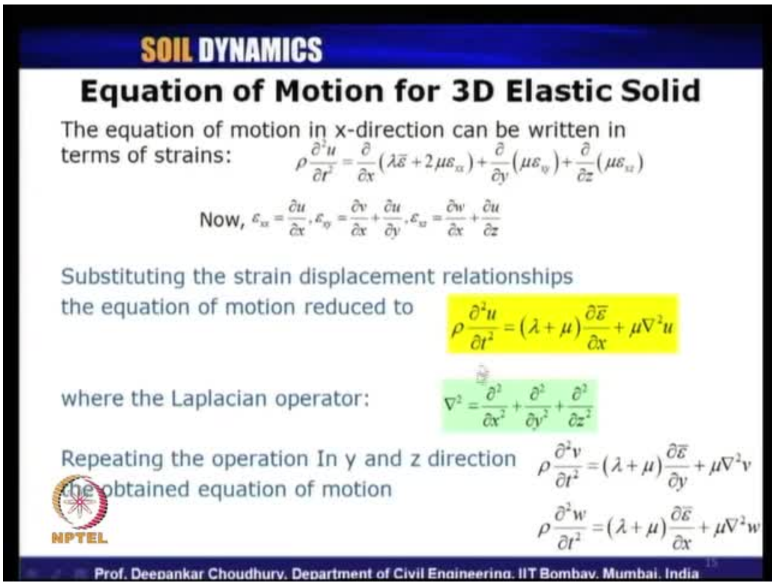

In terms of lame's constant the equation of motion, what we have derived just now in three dimension in the X direction that we can rewrite in terms of the strains, which can be written in this form. So, this is coming from the normal stress component, these two are coming from the shear stress component and this was as usual on the left hand side remains same. That equation of motion what we have derived in X direction, if we rewrite them in terms of strains, that is from stresses we have converted them in terms of strains using the Lame's constant because we are assuming the Hooke's law is applicable for the material.

So, that is why we can write those stress expressions in these form, now the normal strain and shear strain they can be expressed in these forms, that we know. The normal strain in X direction is nothing but del u by del x, where as the shear strain epsilon x y can be written in terms of del v del x plus del u by del y, that is the partial differential of the corresponding displacement with respect to the other axis system. If you note it properly you can easily remember, when we are taking the shear strain in X direction we are differentiating the displacement in Y direction with respect to x, and when we are considering the shear strain with respect to Y direction we are differentiating it with respect to y, but displacement in the direction of x. That sum will give us the corresponding shears strain, right?

Similarly, for epsilon x z, we can get at del w by del x plus del u by del z. Now, if we substitute this strain displacement relationship in this previous equation, how this equation will be further simplified and reduced to? That is the reduce form, that is rho del 2 u del t square left hand side remains same, the right hand side now we are putting the expression for all these shear strain, and normal strain so this is shear normal strain, these are shear strain. We can express them in this term of Lame's constant and simplify it further like this, where this grad square, this symbol we call grad square u, grad square is nothing but the Laplacian operator which is nothing but del square by del x square plus del square del y square plus del square del z square.

So, what this equation is actually? It is rho del square u by del t square equals to lambda plus mu times del epsilon bar by del x, what is epsilon bar? That is the principle strain, the combination of the three strains in the three directions, plus mu times del square u by del x square plus del square u by del y square plus del square u by del z square. That is what is the total equation in X direction, in terms of using this Laplacian operator grad square. So, similarly, what we can write, the equation takes the shape for other two directions of y and z like this. So, we can simply directly write this in Y direction it will be rho del 2 v by del t square equals lambda plus mu times del epsilon bar by del y plus mu times grad square v.

Now, this operator has to be means, this second order differential partial differential has to be executed with respect to the v because we are considering this equation for Y direction, Y axis direction. Similarly, for Z axis direction the equation will take the form of rho del 2 w by del t square equals to lambda plus mu times del epsilon bar by del x plus mu times grad square w. Now, we are taking this Laplacian operator with respect to the component of displacement in Z direction that is w, okay?

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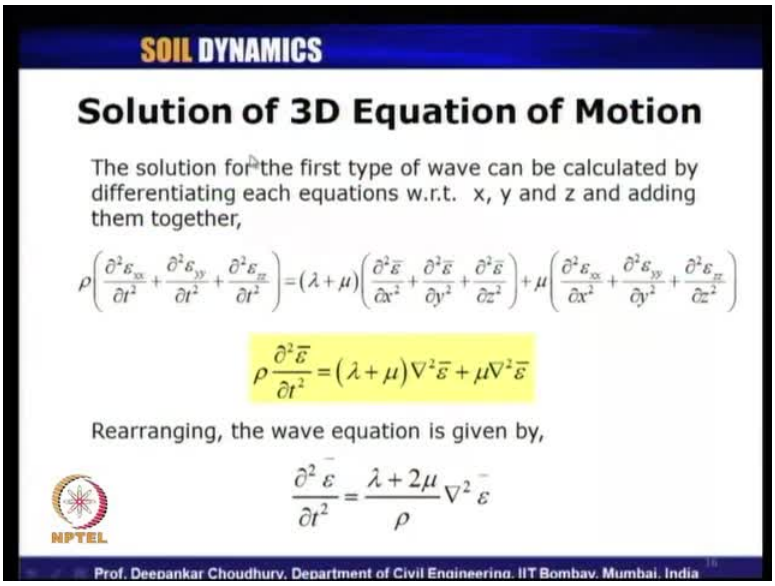

So, now the solution of this 3 d equation of motion for the first type of wave, it can be calculated, how? By differentiating each of the above three equations with respect to x y and z, and then adding them together. So, what we are doing? The previous three equations what we got in the direction of X, in the direction of Y and in direction of Z. Now, we are differentiating with respect to x y z and adding them together to get the first type of wave which finally, will give us on simplification. This is the equation takes the shape in the simplified form, further on rearrangement we can write it like this, del square epsilon bar by del t square equals to lambda plus 2 mu by rho grad square epsilon bar, which will now help us to further simplify the equation in this form.

That the dilation of wave propagates through the body at a velocity v p, which is nothing but primary wave velocity, which is expressed as root over lambda plus 2 mu by row in terms of lame’s constant.

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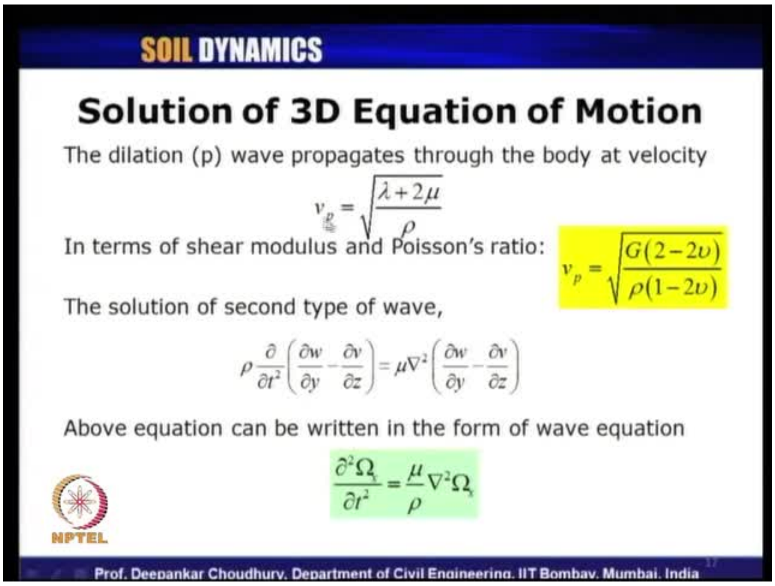

Now, we know the Lame's constant in terms of other parameters, that is shear modulus and Poisson's ratio. If you put those expressions, what I have shown just few minutes back, then in terms of shear modulus and Poisson's ratio. This is the most useful expression, which as a geo technical engineer or in our soil mechanics, we generally use the expression for primary wave velocity v p is given by root over G by rho into 2 minus 2 mu by 1 minus 2 mu. G is the shear modulus of the soil, rho is the density of the soil and mu is the Poisson's ratio of the soil.

And remember this expression is valid for mu value less than 0.5, so this is once again I am telling, what is the limiting value of the Poisson's ratio for a soil, which can be used for the computation of the primary wave velocity travelling through the soil media. So, that completes us the first type of wave which we have termed as the primary wave. Now, we can do another thing to get the second type of wave, what we can do?

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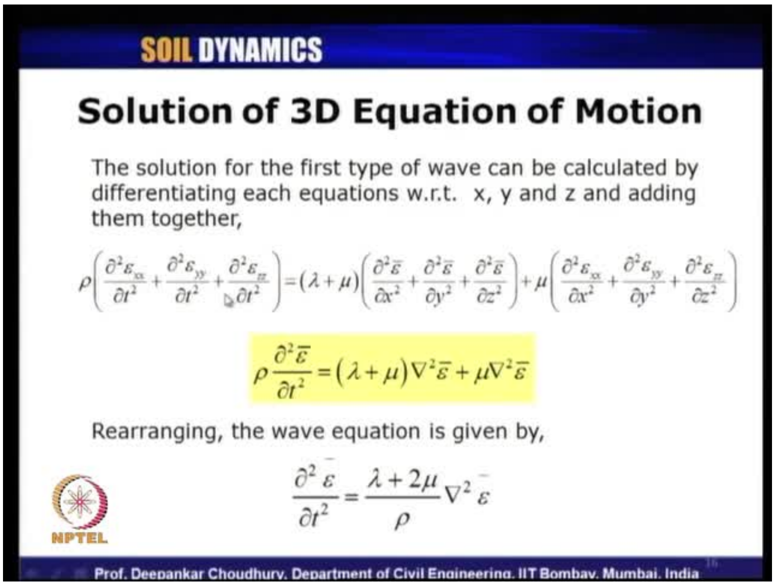

What we did earlier for the first type of wave? We differentiated all the three expressions in X, Y, Z direction and add them together because what happens when the longitudinal wave or compressional wave it passes through, that is when p wave passes through it get added because it is going in the same direction. Whereas, when the second type of wave, what we are considering as a shear wave or the torsional wave, when it travels, what we are doing here, look at here in the slide.

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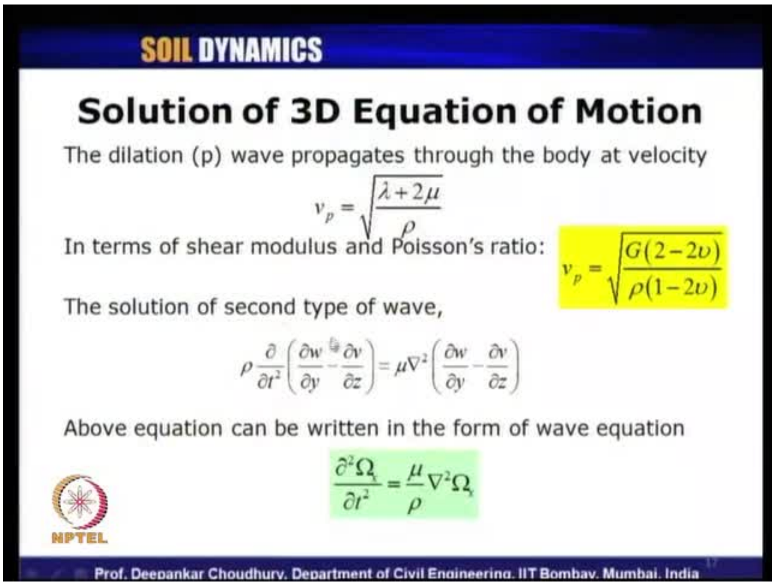

The solution of the second type will give us we differentiate it for the x axis equation what we have obtained the equation in X direction, we have to differentiate that with respect to y and z and subtract one from the other one, so that will give us the net effect of the shearing on X direction, fine? In the similar way the other two direction equation also we can obtain. So, that is the way you can see, the difference between the other two directions have been written here, to get the secondary wave propagation in X direction. So, this the generalized form of the secondary wave equation, which can be simplified and written in terms of this parameter, where this term mu by rho is expressed as v s square.

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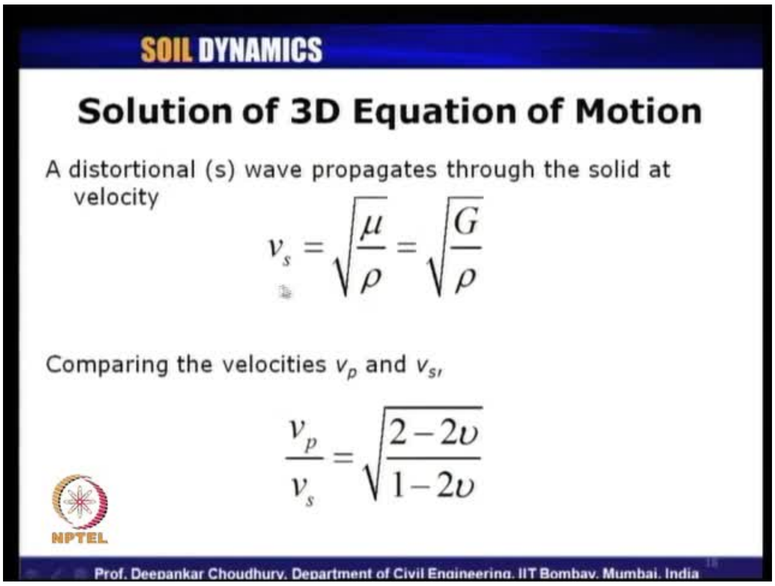

So, the distortional wave it is also called as distortional wave because it creates a torsion in the media through which the wave is travelling through. So, distortional or s wave at propagates through the solid at a velocity v s, which is given by root over mu by rho, mu is the that Lame's constant, which if we put in terms of our modulus what expressions we have obtained earlier is given by root over G by rho. Now, if we compare this two velocities that is primary wave velocity and secondary wave velocity, what we can write?

The ratio of these two velocity that is v p by v s, can be simply written in this form. That is root over 2 minus 2 mu by 1 minus 2 mu. So, this is another excellent expression, which is very frequently used in soil mechanics or soil dynamics, why? Because later on when I will discuss in our module four, about dynamic properties of soil, that time I will mention most of the times we measure this value of v s in the field or in the laboratory for a particular soil, under subjected to different earthquake or dynamic motion or dynamic loads. So, we measure the value of v s, but simultaneously we hardly measure the value of v p, what we do? We if we can identify if we can determine the value of the Poisson's ratio of the soil, then using this simplified expression we can easily obtain the value of v p in that soil media. So, that is why this equation is very much of use in practice.

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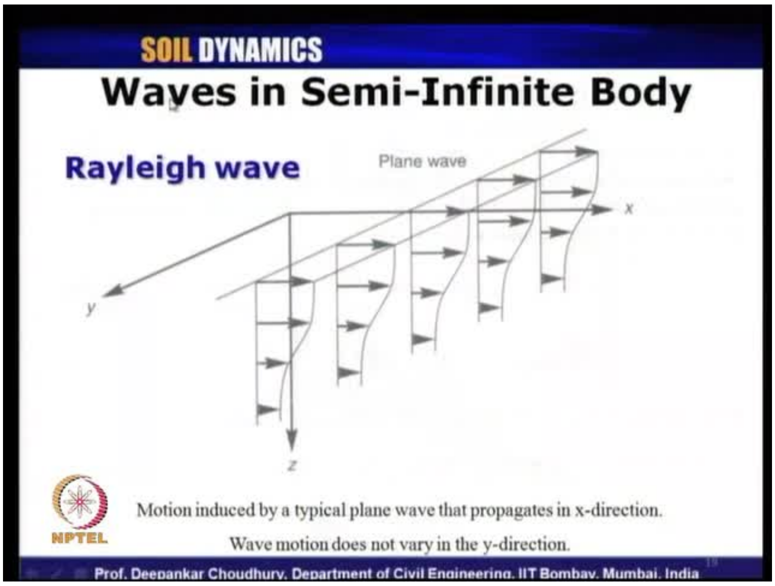

Now, coming to some other different types of waves when waves in semi infinite body. Semi infinite body you all know, that is in one direction it is going to infinity whereas, in another direction there is a boundary like, our earth surface or ground surface. So. below ground it is extending to infinity, but at ground surface it is a ending there. So, Rayleigh wave motion induced by a typical plane wave that propagates in X direction and wave motion does not vary in the Y direction. So, the Rayleigh wave is a kind of wave which travels in this semi infinite body and its travel direction is shown here in X direction, also it is variation with respect to depth is shown, you can see here.

Whereas, in this y direction there is no variation of the wave, there is a characteristic of this Rayleigh wave. And if you notice it minutely, what you can see? The wave velocity is pretty high very close to the ground surface very close to the free surface, whereas as it goes to infinity in this direction the velocity drastically reduces. So, we will come to the specific characteristics of this type wave pretty soon in some other slides.

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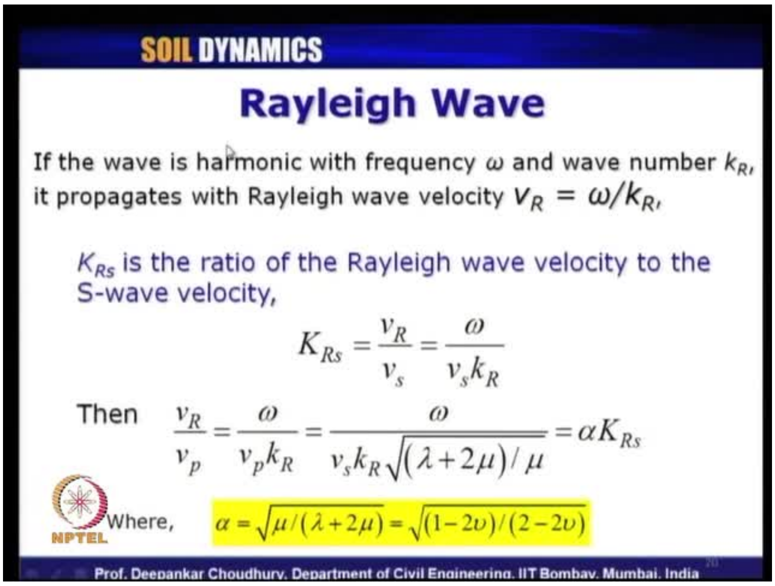

How to write down the equations of Rayleigh wave and how to compute this Rayleigh wave? Let me tell you in a simplified way, if the wave is harmonic with natural frequency, say omega what we have generally considered and wave number, let us say it is denoted by K suffix R because now we are discussing with Rayleigh waves, so that is why the wave number has been represented K suffix R. It propagates with a Rayleigh wave velocity and in that case the velocity is expressed by v suffix R. Earlier for primary wave we have used v p, for secondary wave we has used v s.

Now, for Rayleigh wave we are using v r, v r is similar way can be given by omega by K R. So, this is the relation what we have seen earlier also, it is true for any velocity related with frequency and wave number, right? Now, we are defining another term K R s. What is that K R s? It is the ratio of the Rayleigh wave velocity to the s wave velocity, so K R s is nothing but v R by v s, which is given as omega by v s times K R because v R is nothing but with respect to the wave number for Rayleigh wave equation, so omega by K R.

Now, if we take the relation or ratio between Rayleigh wave and primary wave what we can write v r by v p can be expressed like this where v p can be expressed in terms of v s and Lame's constant in this form, from our known relation between v s and v p. Now, this is denoted as alpha times K R s simplified in terms of alpha times K R s. So, what is alpha? Alpha is nothing but root over mu by lambda plus 2 mu, where these two are Lame’s constant, if we put there expressions for our known Poisson’s ratio expression and other sheer modulus expression, what the simplified form we will get? That is 1 minus 2 mu by 2 minus 2 mu, which is nothing but v p by v s, right? Just now we have seen this alpha is the ratio of v p by v s, sorry alpha is the ratio of v s by v p because this is, this has come in the denominator.

So, 1 minus 2 mu by 2 minus 2 mu, this alpha is the ratio of v s by v p, which also can be seen if we put this expression. s which will now help us to further simplify the equation in this form that the dilation of wave propagates through the body at a velocity v p which is nothing but primary wave velocity which is expressed as root over lambda plus 2 mu by row in terms of lame’s constant, So, this will be v R by v s. So, alpha has to be v s by v p, so that it equates with respect to this one, fine?

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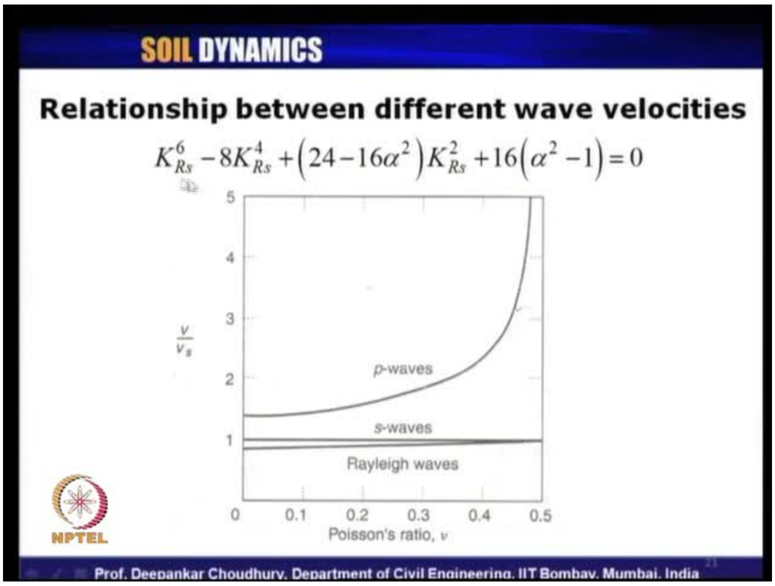

Finally, what is the final equation used to solve for Rayleigh wave velocity? To compute the Rayleigh wave velocity this is the expression, that is K R s to the power 6 minus eight K R s to the power 4 plus 24 minus 16 alpha square times K R s square plus 16 times alpha square minus 1 equals to 0. So, in this K R s, we have already defined ratio of v R by v s alpha is nothing but ratio of v s by v p. So, in that way if for different values of Poisson’s ratio, if we want to see how these three velocities, about which we have studied just now, that is p wave s wave and Rayleigh wave, how they vary with respect to v s.

Because see this K R s and alpha both are expressed in terms of v s, right? because alpha is ratio of v s by v p and K R s is also the ratio of v r by v s. So, everything is expressed in terms of shear wave velocity, why? again to iterate a, reiterate this one, that in laboratory and field we generally measure the shear wave velocity. So, if we can compute the shear wave velocity of the soil, then easily we can compute the primary wave velocity and the Rayleigh wave velocity of the soil, using these two expressions. And how the variation behaves for different ranges of Poisson’s ratio? Let us look at this chart, So, with variation of Poisson’s ratio you can see this v can be v p, v s or v r depending on which line you are selecting, okay? So, that is why when we are talking about s wave, this is a line which is nothing but at constant value of one because this ratio obviously will be one, v s by v s, irrespective of Poisson’s ratio. However, you can see the p wave values starting here like this and here it will become infinity because at 0.5, it is not giving any particular value.

So, that is why the equation is not valid at value of mu equals to 0.5 but just lower than that it is valid. Also the Rayleigh wave, if you see the variation of Rayleigh wave with respect to s wave, it is pretty close. The velocity of Rayleigh wave is very, very close to s wave velocity, it depends on mu value but for all practical purposes we can assume Rayleigh wave velocities about 90 percent of shear wave velocity. So, that is the practical design consideration most of the time we geotechnical engineers we use. So, practical value used by geotechnical engineers for Rayleigh wave velocity is about 0.9 times the shear wave velocity, though the exact value can be easily computed using this expression by solving this expression and depending on the values of Poisson’s ratio.