Soil Dynamics
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Module - 2
Vibration Theory
Lecture - 13
Solutions of Quiz Questions, Multi-Degree of Freedom (MDOF) Systems

Let us start today's lecture on soil dynamics. Before we start our lecture today; let us see the possible solution for quiz one. The quiz one is already been taken. So, let us look at the slide. The questions for quiz one for our course full marks was hundred and duration was one hour and all questions has to be answered.

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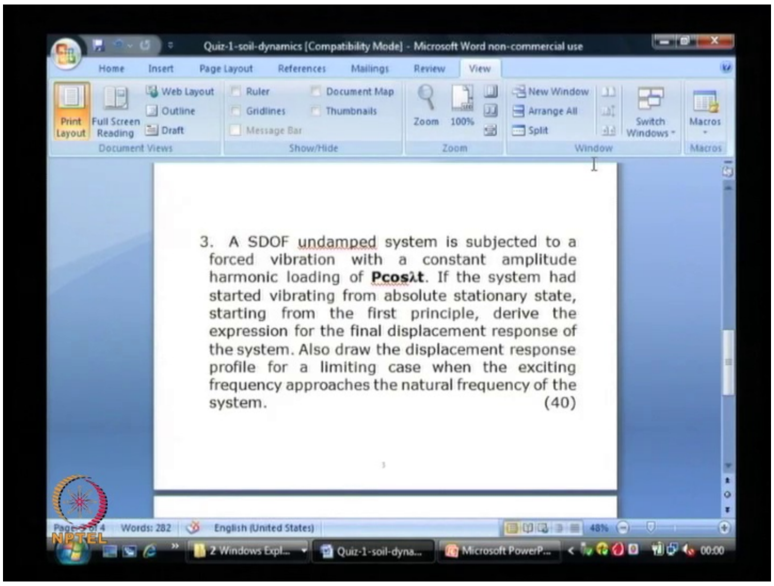

Now, let us move to the third problem. So, this was the third problem. What it said? A single degree of freedom undamped system is subjected to a forced vibration with a constant amplitude harmonic loading of P cosine lambda t. If the system had started vibrating from absolute stationary state; that is the initial displacement and initial velocity both are 0, starting from the first principle derive the expression for the final displacement response of the system. And then draw the displacement response profile for a limiting case, when the exciting frequency that is lambda approaches the natural frequency of the system. So, let us do this solution starting from our basic principle or first principle.

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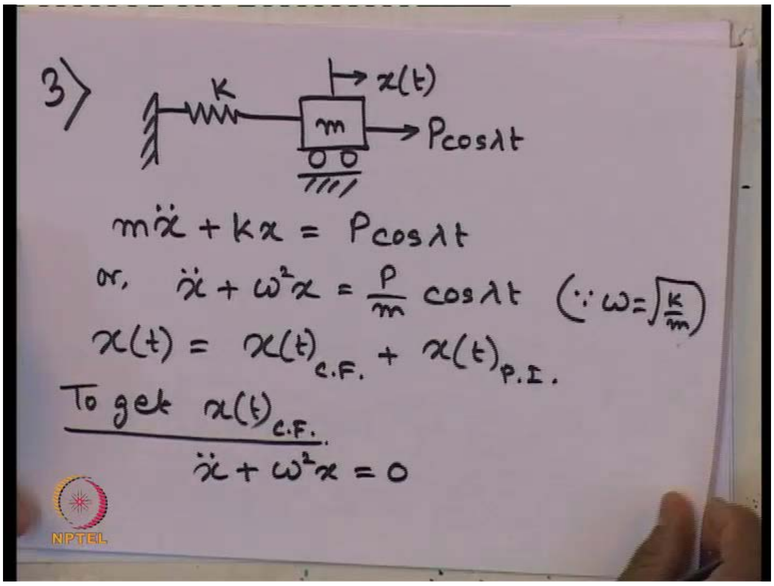

So, the single degree of freedom model undamped system we are drawing now. So, m k this is our x of t it is subjected to a forced vibration a with P cosine lambda t. So, that is the basic single degree of freedom model given to us. So, what we know? The equation of motion is given as m x double dot plus k x equals to P cosine of lambda t. That is our equation of motion or x double dot plus omega square x equals to P by m cosine of lambda t where this omega is root over k by m is natural frequency. Now, to get the solution x of t is composed of x of t complimentary function plus x of t particular integral to get x of t complimentary function. What we do? We equate it with 0. So, we can write down the equation x double dot plus omega square x equals to 0.

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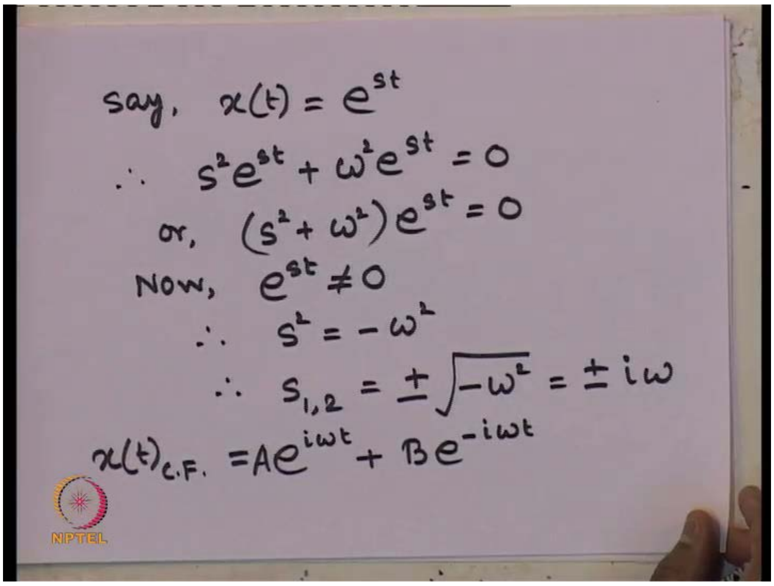

Now, let us say x t equals to e to the power s t is the form of the solution. So, we can put it in the expression s square e to the power s t plus omega square e to the power s t equals to 0 or s square plus omega square e to the power s t equals to 0. Now, e to the power s. T is non 0. Otherwise it will give trivial solution. Therefore, s square is minus omega square. Therefore, two roots of the equation. We will get s 1 s 2 as plus minus root over minus omega square which is nothing but plus minus imaginary number i omega. Therefore the complete solution, not complete solution complementary function that can be expressed as e to the power i omega t times a constant plus another constant e to the power minus i omega t because plus i omega and minus i omega; the two roots.

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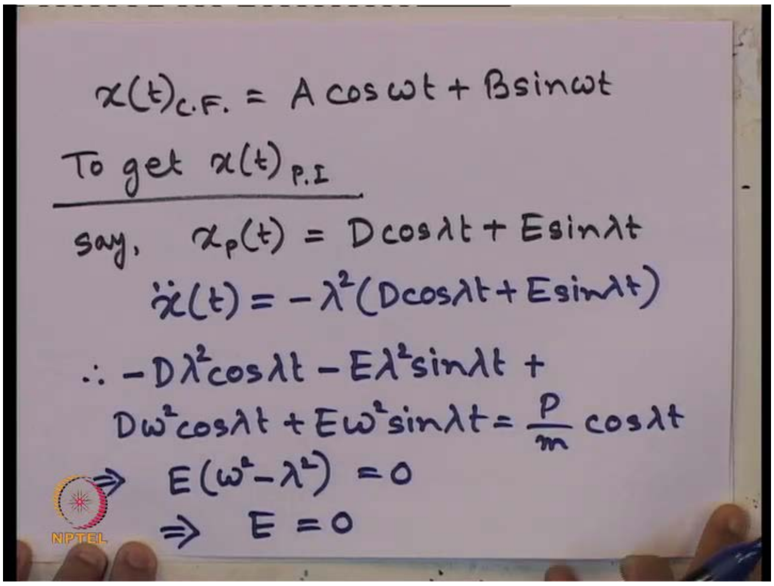

So, this equation as you know we have done earlier also in the class the solution can be expressed as A cosine of omega t plus B sin of omega t. The same thing we can express now. These constants we have to obtain later on because we have another part of the solution that is the particular integral. So, now, to get x of t particular integral what we do? First we assume some form of the solution.

So, x P of t let us say is expressed as D cosine of lambda t plus E sine of lambda t because our given function is a harmonic function. So, that is why the solution we are assuming in the form of the harmonic expression. So, it will give us x double dot t as minus lambda square D cosine of lambda t plus E sine of lambda t. If we put this in our basic equation which is nothing but this one that is x double dot plus omega square x equals to P by m cos of lambda t.

So, now, we are putting this that equation we will get minus D lambda square cosine of lambda t minus E lambda square sine of lambda t. That is x double dot plus omega square times x. So, D omega square cosine of lambda t plus E omega square sine of lambda t that is equals to P by m cosine of lambda t. And what we have learnt? That now we can equate the cosine components and sine components on the both sides. So, what we can get if we equate the sine components? It will give us E times omega square minus lambda square equals to 0 which will give us E equals to 0. Because this is not equals to

0 omega is not equals to lambda. And what about the other constant D? For that we will equate the cosine terms, now equating cosine terms.

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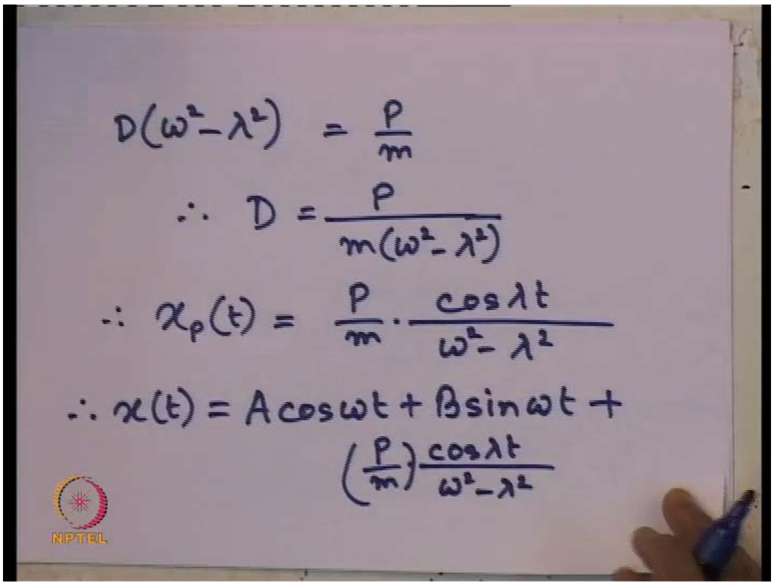

We can write d omega square minus lambda square cosine of lambda t equals to P by m. Therefore, D is P by m omega square minus lambda square. So, what is the complete solution? Therefore, x particular integral part is P by m cosine of lambda t by omega square minus lambda square because e is 0 the other component is not 0 there. Therefore, the total solution x of t is now a cosine omega t plus B sine omega t plus this particular integral P by m cosine of lambda t by omega square minus lambda square.

So, that is the complete solution. Now we have to find out these two constants using our given initial condition. So, what are the initial conditions given to us? That it starts from absolute stationary state.

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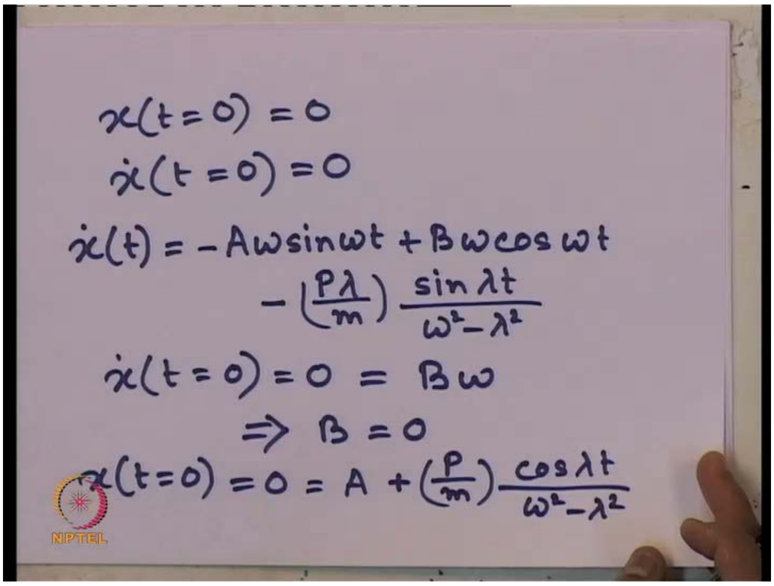

So, x at t equals to 0 is 0 and x dot at t equals to 0 is also 0 it is given to us. So, now let us differentiate this equation to find out the expression for velocity. So, this is the expression we obtain just now for the total solution of the displacement function. So, x dot t is nothing, but now we are differentiating this minus a omega sine omega t plus b omega cos omega t. Now, this one will give us minus P lambda by m sine lambda t by omega square minus lambda square.

Now, we will put these values here. So, if we do that in this equation first what we will get? If we put the velocity at t equals to 0 is 0 this term vanishes because sine 0, this term remains it is B omega this is also sine 0. So, vanishes. So, x dot at t equals to 0 equals to 0 will give us only B omega which will imply B is 0 because omega cannot be 0. So, B is 0 fine. And now if we put the first condition of displacement equals to 0 at t equals to 0 what we will get? X at t equals to 0 is 0. Let us look at the solution once again. This remains a this vanishes this remains P by m, omega square minus lambda square right. So, let us write it.

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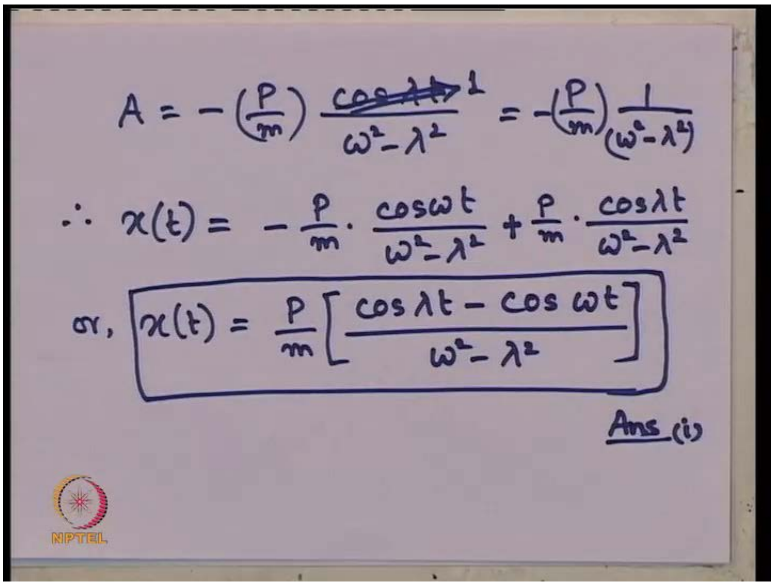

So, A plus P by m cosine of lambda t by omega square minus lambda square which will give us A as minus P by m cosine of lambda t by omega square minus lambda square. Therefore, the complete solution we can now write it as x of t equals to cosine of lambda t will not be there. Because it is one at t equals to 0 this is one minus P by m 1 by omega square minus lambda square.

So, a cosine of omega t was the solution. So, minus P by m cosine of omega t by omega square minus lambda square plus B is 0. So, that component is not present and the third component is let me take the solution once again yes. So, A cosine of omega t. So, A is this part. So, cosine of omega t B is 0. So, it vanishes and this particular integral part remains. So, plus P by m cosine of lambda t by omega square minus lambda square or x of t is equals to P by m cosine of lambda t minus cosine of omega t by omega square minus lambda square. So, this is the final solution of first part of this problem. Number three. So, this is the complete final solution of the system. That is what it is asked that derive the expression for final displacement response of the system. This is the derivation and expression of the final response displacement response of the system.

Now, what is the next part? In the next part it is asked also draw the displacement response profile, this profile not with this function. What is asked for a limiting case when this exciting frequency lambda approaches the natural frequency of the system?

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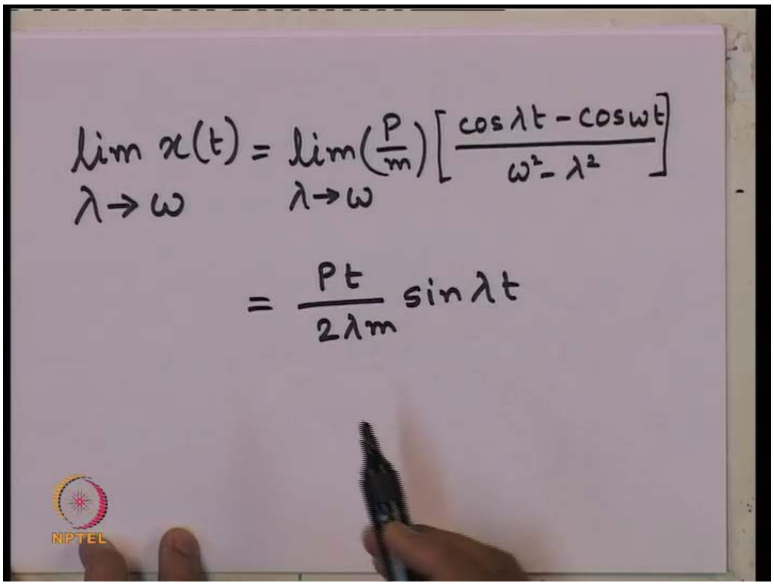

So, what we can write it is asked to find out limit of x of t when lambda approaches omega. That will be limit lambda tends to omega P by m cosine of lambda t minus cosine of omega t by omega square minus lambda square. So, when it approaches this one from our basic concept of limit what we can derive this expression gives us P t by 2 lambda M sine of lambda t. So, it becomes actually sine lambda t by 2 lambda when lambda tends to omega. This is the final response when it approaches exciting frequency approaches natural frequency. What does it mean? A kind of resonance condition is going to form. So, I asked you to draw this function with respect to time. So, I wanted to feel you that under a condition close to resonance. What will be the response of the system? So, let us draw this now. It is pretty easy to draw this function. Let me put it here.

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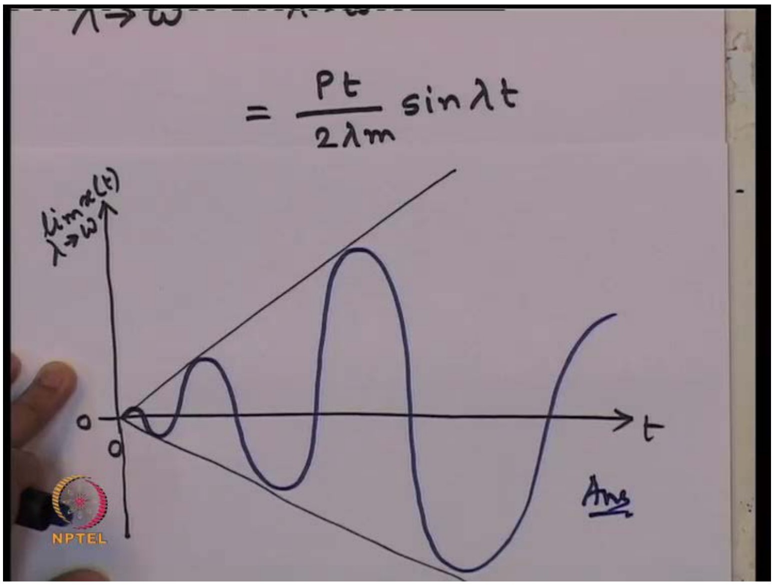

So, t I am drawing this limiting value of x of t when lambda approaches omega. What happens when t 0 it starts from 0? And how this equation we can draw easily. This will be you remember when there is the two functions and one is the multiplication of the other one sine function is harmonic function and t function is linearly increasing function with a slope of P by 2 lambda m.

So, how to draw this very easily? Let me extend this side of the axis also. So, linearly increasing lines I have drawn the slope of it is nothing, but this P by 2 lambda m right. Am just drawing the equation of y equals to c x where c is the slope of the line. So, this part I have drawn. Now there is another function attached to it which is sine of lambda t. So, the actual response is not this line, but a harmonic function like this.

So, can you see what type of disaster it is going to happen? Means when lambda approaches omega of course, in resonance conditions we are going to get maximum displacement. So, that is what it is happening. It keeps on increasing. Actually this also goes to the next line. So, it follows that envelope fine. So, this is the final answer of the problem.

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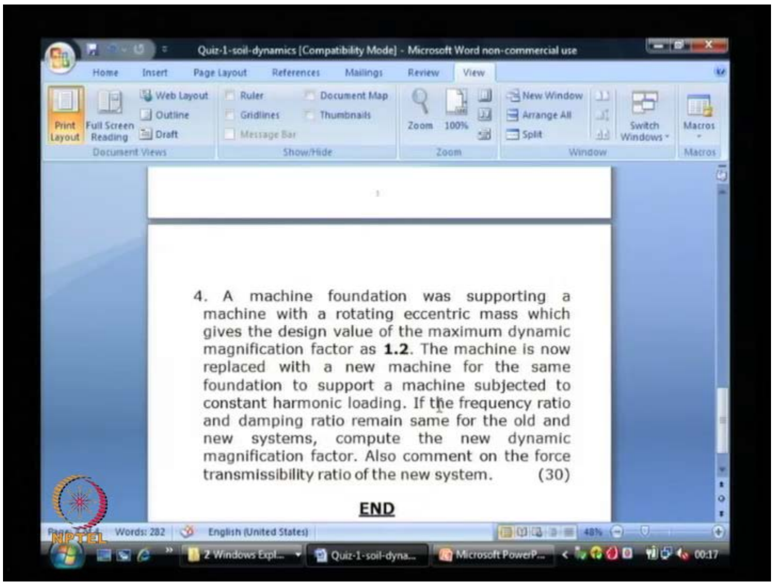

Now, let me move to the last problem of our quiz one. So, the fourth problem was a machine foundation was supporting a machine with a rotating eccentric mass which gives the design value of the maximum dynamic magnification factor as 1.2. The machine is now replaced with a new machine for the same foundation to support a machine subjected to constant harmonic loading. If the frequency ratio and the damping ratio remains same for the old and new systems compute the new dynamic magnification factor and also comment on the force transmissibility ratio of the new system. That is as a designer I want you to comment on the whether the new machine installed for the old foundation is safe or not.

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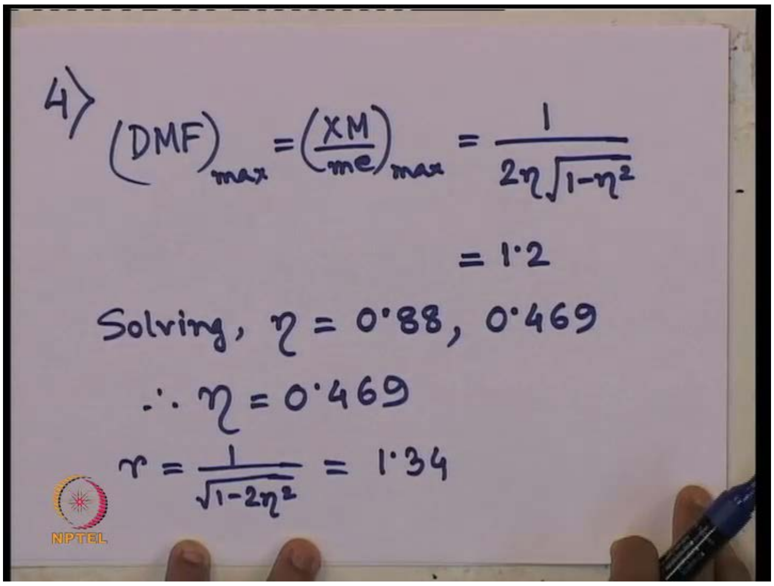

So, let us start the solution of the fourth problem what it said initially the machine foundation was of rotating eccentric mass time and for that the expression for maximum dynamic magnification factor which is expressed like X M by m e in our usual term that maximum value was 1 by 2 eta root over 1 minus eta square we have derived this things we know this things this value is given to us as 1.2. So, only unknown is eta. So, solving this we can get eta 2 values we are getting one is 0.88 another value is 0.469 and imaginary values are not considered of course, as you know the damping ratio cannot be imaginary

Now, from this again the value of eta equals to 0.88. That will not give a maximum DMF because in the previous lecture what we have understood? If eta is greater than 0.7; it always keeps on giving a no magnification assets.

So, the chosen design value of eta is 0.469. So, this is our design value given to us. Now where this maximum DMF occurs for a rotating mass type system machine? It occurs at frequency ratio r equals to 1 by root over 1 minus 2 eta square fine. Now we know this is our design value of eta. So, put it here we will get the frequency ratio as 1.34. So, this was our old system. Now, what is done? The machine is changed, but foundation remains same. Now, the new machine is subjected to constant force type harmonic loading. But for the system the damping ratio and the frequency ratio is maintained as same as the previous case of old system. So, these two values remains same. It is asked what will be the dynamic magnification factor for the new system.

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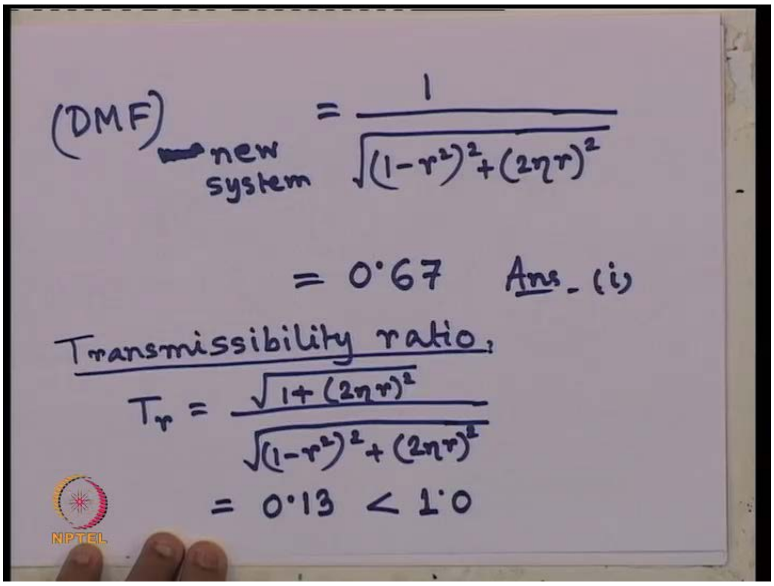

So, DMF for the new system for new system which is subjected to harmonic loading with constant amplitude that is given by 1 by root over 1 minus r square whole square plus 2 eta r whole square. Now we have already computed r is 1.34 and eta is 0.469 because they remain same.

So, just put this values we will get the DMF of the new system. It is of course, not the maximum because in earlier case where the maximum occurs for this case. It is not the point where the maximum occurs. So, this is the solution of first part. That is what it is asked that what is the new dynamic magnification factor and the last part says the transmissibility ratio; force transmissibility ratio that t r is given by root over 1 1 plus 2 eta r whole square by root over 1 minus r square whole square plus 2 eta r whole square. That is the expression for transmissibility ratio.

Now, eta is known, r is known. You put these values. We will get it is coming as 0.13 which is much lower than 1. So, as a designer I asked you to comment on this. So, the new system will be very safe because it is transmitting much lower force than what it is subjected to the foundation. So, our foundation will be very much safe in the new condition also. So, that the comment you should provide as a designer at the end of this problem fine. So, with this we have come to the end of the solutions of forth is quiz one.

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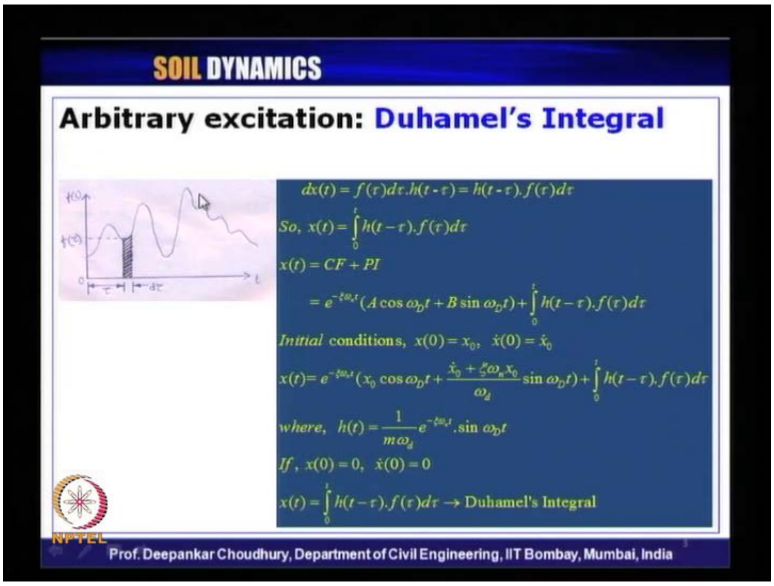

Now, we will start our today's lecture with module 2 on vibration theory a quick recap of what we have learnt in the previous lecture. We have seen for any arbitrary excitation how we can compute the solution using the concept of Duhamel's integral if it starts from absolute stationary state. It is similar to the concept of derivation for the impact loading. But it is considered in small infinitesimal strips and then integrating over the entire time for which the excitation is working we can get the complete response of the system.

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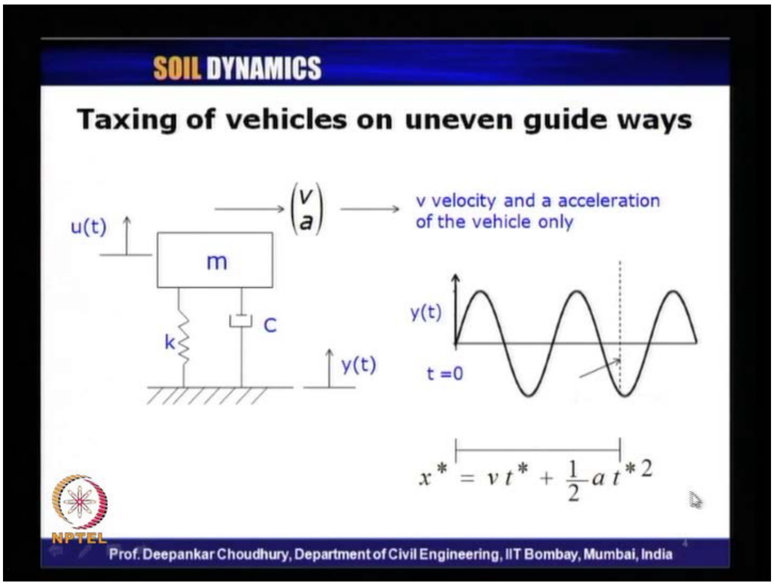

Then we have seen the case of dynamic loading coming due to the taxing of vehicle on uneven guide ways. When a vehicle is moving at a constant velocity V and there is undulations on the ground. Then the distance travelled at a time t.

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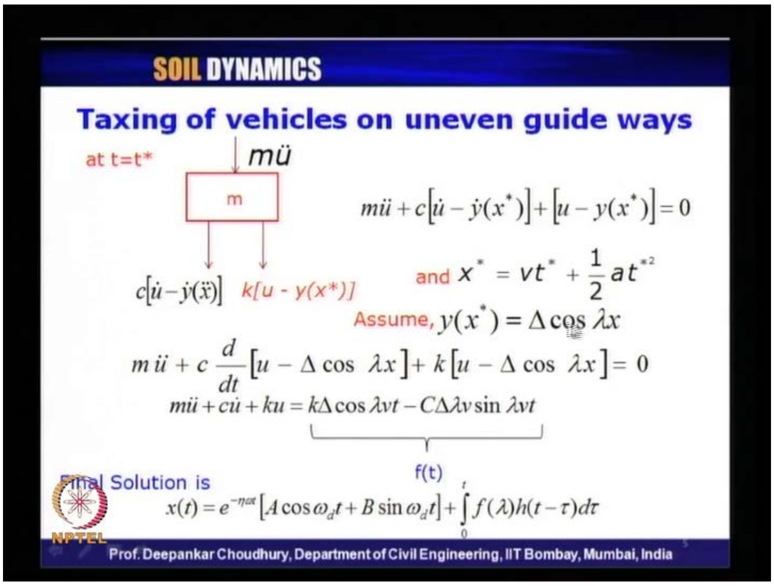

We have computed and then the final solution we have seen in this form which can be easily obtained depending on the profile of the ground which we have assumed as a harmonic profile.

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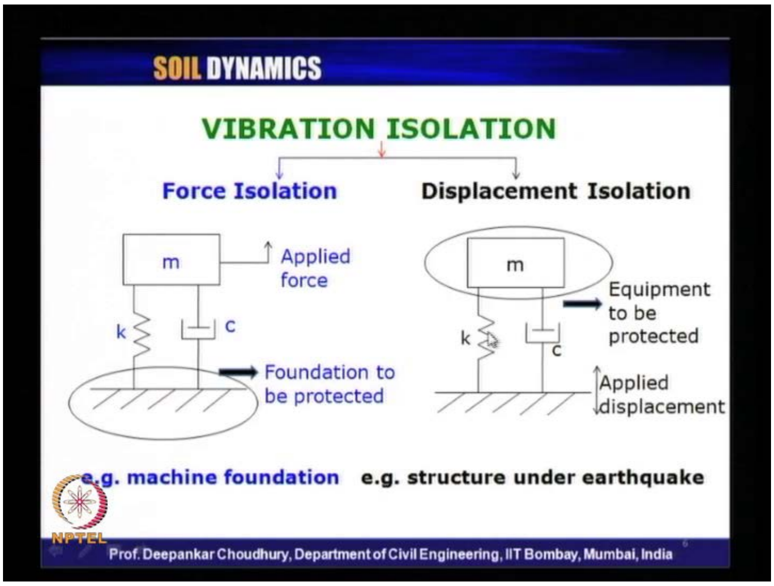

Then we have seen vibration isolation. There are two types of vibration isolation force isolation and displacement isolation. And where are the applications, practical application on force isolation? In case of machine foundation we want to make our foundation safe with respect to the applied load to the machine. And the displacement isolation the practical application in terms of structure subjected to say under earthquake loading where the earthquakes come to the ground level. And applied displacement due to the earthquake should not be transferred or magnify at our super structural level. So, that is why we want to protect our super structure. So, that is the concept of using displacement isolation.

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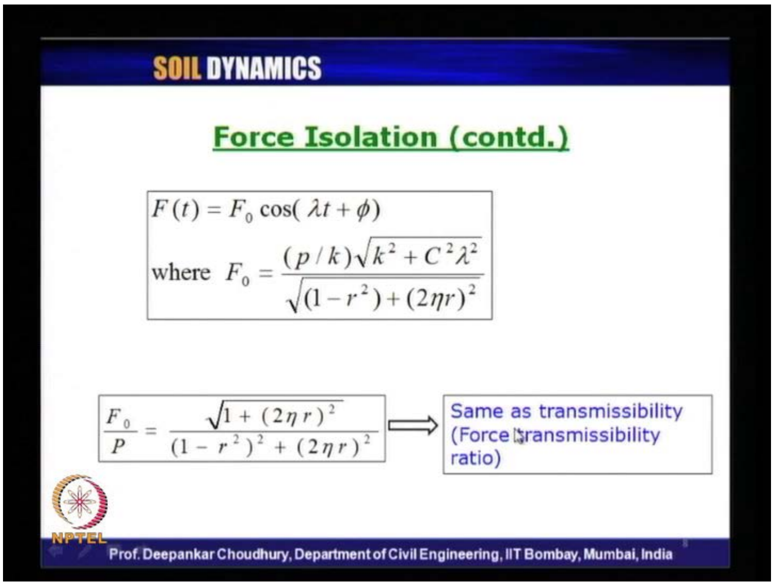

Then we have derived the expression for force isolation from which the expression of forced transmitted to the foundation to the force amplitude of force applied to the machine was obtained. And the expression is nothing, but exactly as transmissibility ratio.

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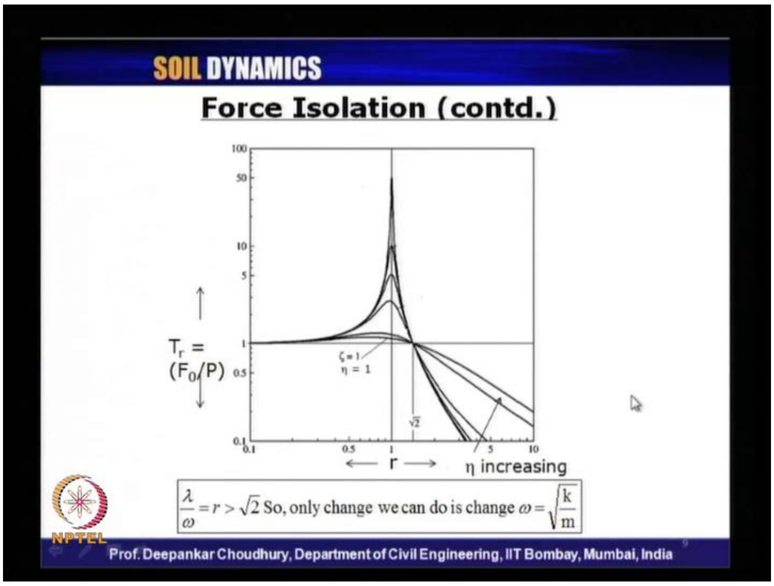

So, we termed it as force transmissibility ratio which varies with respect to different frequency ratio and different damping ratio like this. And we have seen that the value of r greater than root two will always give us the transmissibility ratio, force transmissibility ratio less than one which is desirable for the system as a designer.

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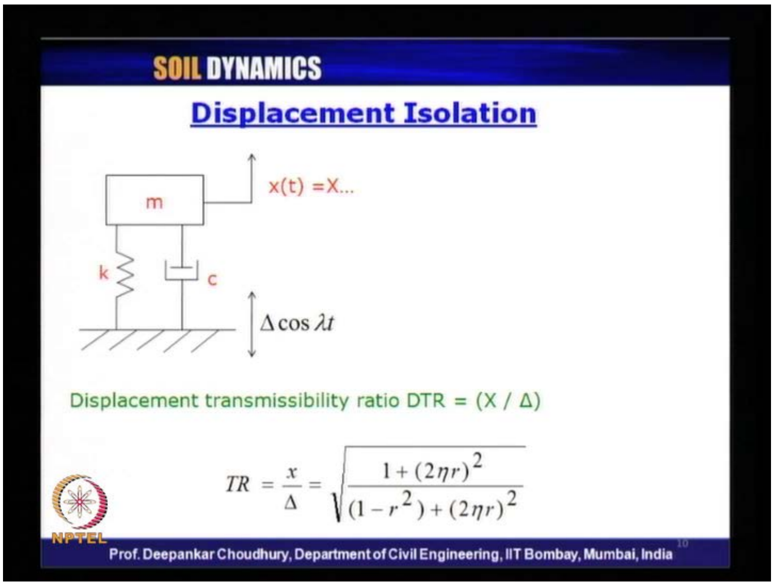

Similarly, for the displacement isolation we have seen if the displacement function for the ground displacement is considered as harmonic. Like this the transmissibility ratio can be expressed in the same form of the force transmissibility ratio. The expression remains same and that is why the distribution also remains same with respect to the frequency ratio and for different damping ratio values.

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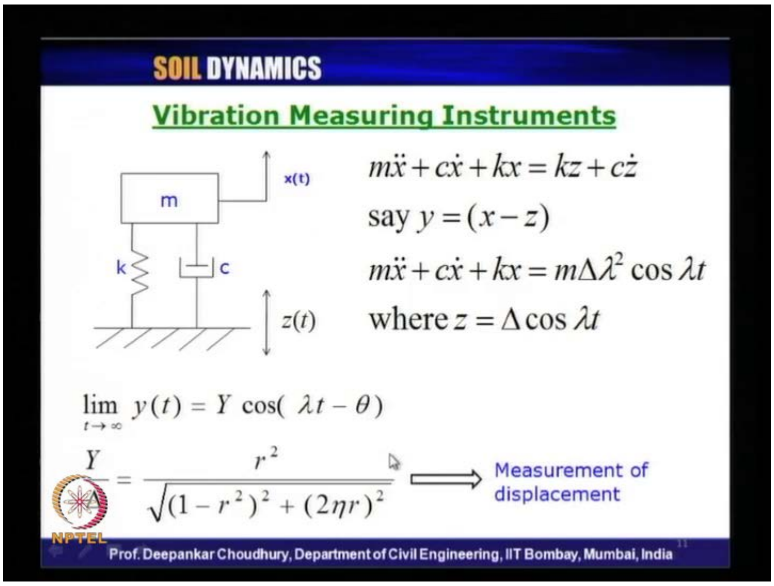

Then we have seen how the vibration measuring instruments are designed to measure the vibrations of the ground.

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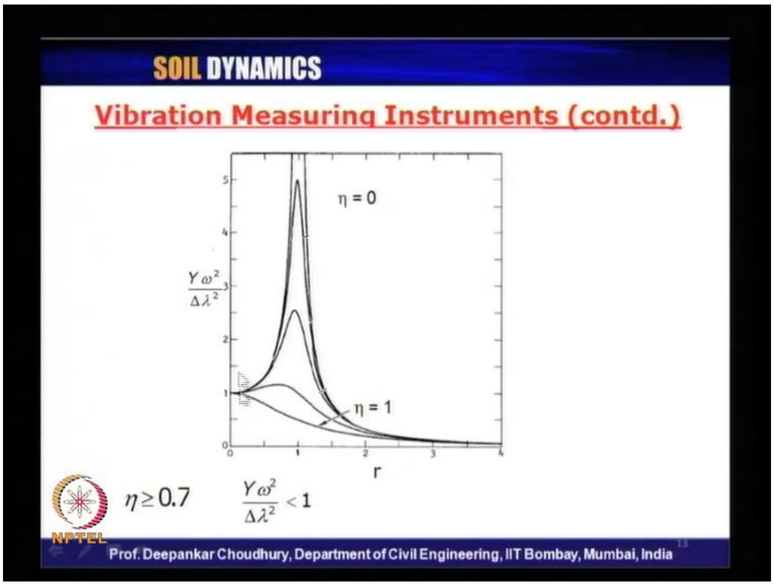

And the measurement of displacement can be carried out and this also varies a profile with respect to different frequency ratio. And for value eta greater than 0.7 whatever be the value of r, we have seen the this magnitude will be always less than 1.

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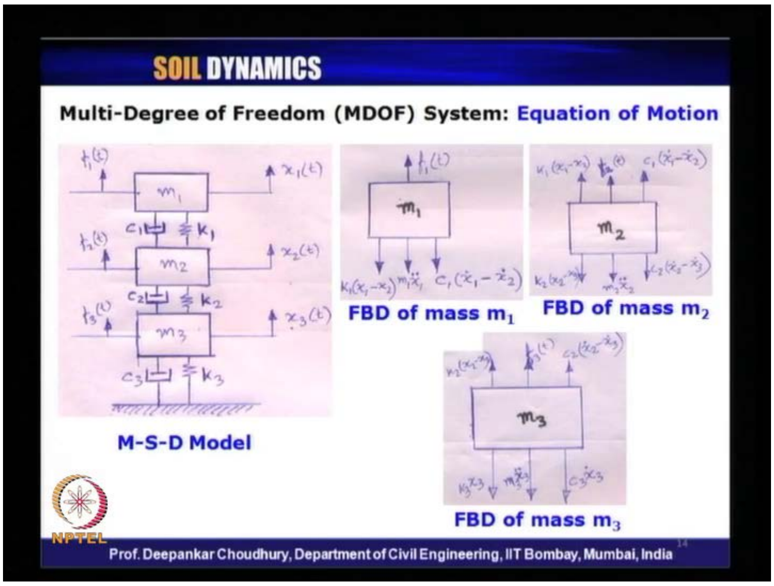

Now, coming to today's lecture we are starting with new subtopic multi degree of freedom system basic equation of motion. We want to formulate for multi degree of freedom system. So, let us do the derivation for the basic multi degree of freedom system. How we can address this problem?

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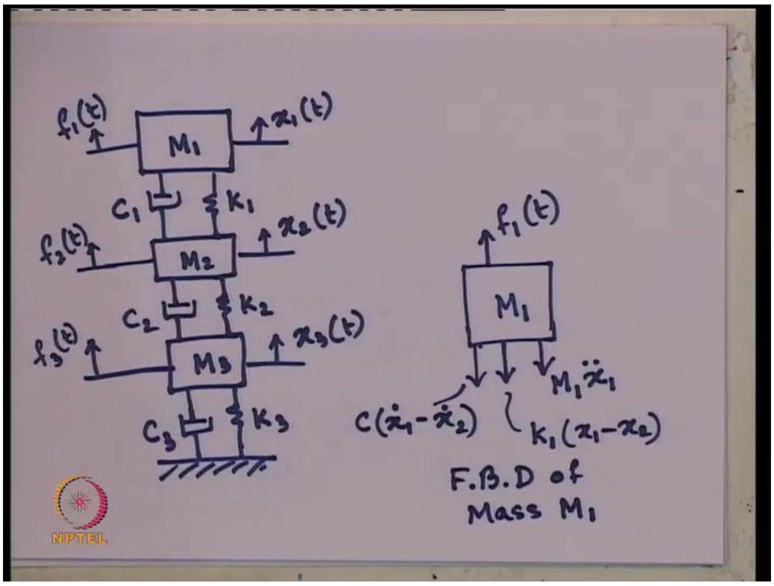

So, let us draw the multi degree of freedom system. Here first this is mass M 1 damper and spring spring K 1 damper C 1 then mass M 2 mass M 2 damper C 2 spring K 2. Then mass M 3 mass M 3 damper C 3 spring K 3 and this is the fixed end. Now, what are the degrees of freedom of the system? This is x 1 t this is x 2 t this is x 3 t. So, I have drawn a system with 3 degrees of freedom x 1 x 2 and x 3 and to this the forces applied are external. Dynamic load applied are f 1 t on this first mass f 2 t on this second mass M 2 and f 3 t on the third mass M 3.

So, these are all the externally applied dynamic load on the system. So, this multi degree of freedom system, now if we want to derive the equation of motion for this basic system what we need to do? First thing, to draw the free body diagram of each component; that is for each mass; now we will draw the free body diagram. So, let us do that free body diagram for the first mass will be this is M 1 it is subjected to f 1 t as externally applied dynamic load. Now, x 1 t I have given at one particular instant of time in this direction. So, the forces of resistance will come and it will act like this f I will be M I x 1 double dot. This is inertia force. Then there will a spring force and a damper force. Now, how much will be the spring force spring force should be? K 1 times the relative displacement between this mass and this mass. So, x 1 minus x 2 and the damper force should be C times relative velocity between these two mass. So, x 1 dot minus x 2 dot. So, these are the forces acting for the first mass. So, this is FBD of mass M 1.

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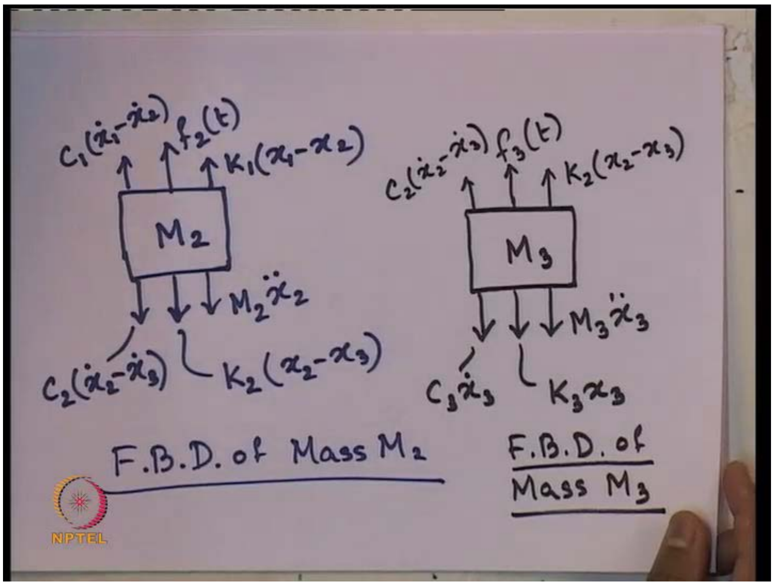

Now if we want to draw the free body diagram of the second mass M 2 what it should be for M 2? It is subjected to the externally dynamic load f 2 t. Now, x 2 is in this direction at one particular instant of time vertically upward we have taken inertia force will be M 2 x 2 double dot in this direction. And what are the spring forces? This spring force and this damper force this spring force should be K 2 times. Let us look at here. Once again this spring force K 2 times relative displacement between this two masses and the damper forces will be C 2 the relative velocity between these two masses.

So, K 2 times x 2 minus x 3 and these damper forces C times x 2 dot minus x 3 dot is there. Any other force yes because to maintain the internal equilibrium of this two system. Whatever the forces are acting from this mass M 1 to mass M 2 connected through this damper and spring that equilibrium internal equilibrium has to be maintained. So, whatever this damper force and spring forces are acting in this direction downward direction; the same magnitude forces must acting upward direction to maintain the internal equilibrium of the system fine. So, what will act? Here we will have this K 1 times x 1 minus x 2 and this is C 1. So, C 1 times x 1 dot minus x 2 dot. So, this completes the free body diagram of mass M 2.

Now, the third mass. That is mass M 3 we want to draw the free body diagram. What are the forces acting on it? The externally applied dynamic load is f three t on this side we will have M 3 x 3 double dot as a inertia force because again we have taken at one instant the direction of moment like this. So, inertia force is in this direction and spring force damper force connected because of this K 3 and C 3. Now, they are connected to a fixed support.

So, there is no relative displacement between x 3 and nothing is there. So, it should be K 3 times x 3 and this should be C 3 times x 3 dot. Any other force yes we have to now maintain again the internal equilibrium between this mass M 2 and M 3. So, whatever these forces C 2 x 2 dot minus x 3 dot and K 2 x 2 dot minus x 3 dot where acting on mass two in downward direction. The same force in opposite direction must act to maintain the internal equilibrium because of the connectivity through this C 2 and K 2 clear. So, we will have here K 2 x 2 minus x 3 and here C 2 x 2 dot minus x 3 dot.

So, that completes the FBD of mass M 3. So, is it clear now how the free body diagram of each of the masses. We have drawn now from this free body diagram what we have to do we have to use D’almbert’s principle. So, now, let us use the D’almbert’s principle for each of this free body diagram for equilibrium conditions. We will consider for each of this masses and what we can write from FBD of mass M 1 which is expressed here are shown here.

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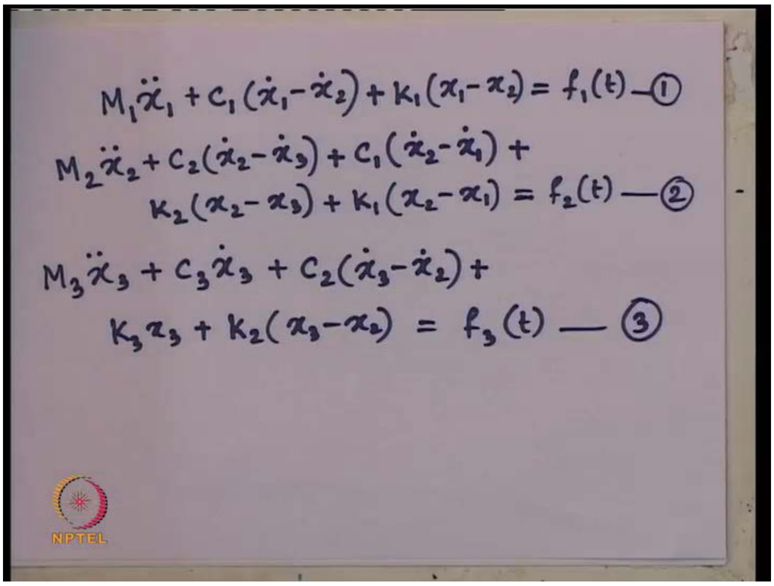

So, using D’almbert’s principle we can write M 1 x 1 double dot in this direction plus C 1 x 1 dot minus x 2 dot plus K 1 x 1 minus x 2. This is equals to f 1 of t. This is first equation we got using D’almbert’s principle for this free body diagram of mass M 1.

Now, let us use the free body diagram of mass M 2. So, free body diagram of mass M 2 is here. Now, with us what we can write M 2 x 2 double dot plus C 2 x 2 dot minus x 3 dot plus C 1 x 2 dot minus x 1 dot plus K 2 x 2 dot minus x 3 dot K 2 into x 2 minus x 3 plus K 1 x 2 minus x 1 equals to f 2 t. So, this is the second equation. Do you agree with me? What I have done this one is in upward direction this is in downward. So, I made it downward by using x 2 dot minus x 1 dot. Similarly, this also I made it downward K 1 x 2 minus x 1. So, that is why all these terms I have taken in one side and the externally applied force I have taken on the other side.

Now, from the free body diagram of the third mass we can use the D’almbert’s principle and write the equation like M 3 x 3 double dot plus C 3 x 3 dot plus C 2 x 3 dot minus x 2 dot plus K 3 x 3 plus K 2 x 3 minus x 2 equals to f 3 minus t. That is the third equation. Here also what we have done? These two forces were acting upward. We make it downward by just changing the sign x 3 dot minus x 2 dot x 3 minus x 2. So, that is why we have taken everything on this side and these are the three governing equations we are getting using D’almbert’s principle for each of the three masses. Now, if this entire state of equation we want to put in a simplified form. What we can use? We can use the matrix form. Then we can represent these equations in a better way or in a easier way.

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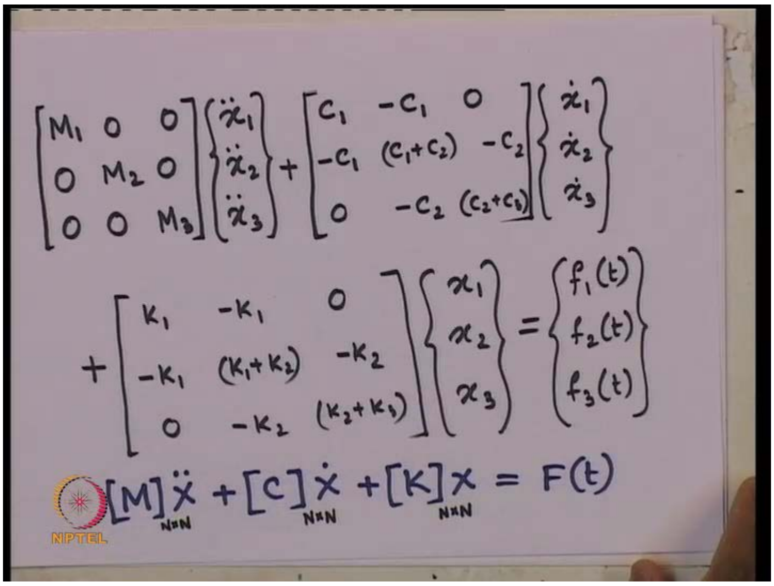

So, let us put this equation in a matrix form and let us see how it will look like in the matrix form. I have M 1 M 2 M three 0 0 0 0 0 0 with this the vector is acceleration vector x 1 double dot x 2 double dot x 3 double dot. Then I have the next matrix C 1 minus C 1 0 minus C 1 C 1 plus C 2 minus C 2 0 minus C 2 C 2 plus C 3 and associated vector is velocity vector x 1 dot x 2 dot x 3 dot plus K 1 minus K 1 0 minus K 1 K 1 plus K 2 minus K 2 0 minus K 2 K 2 plus K 3 and associated vector is displacement vector x 1 x 2 x 3 which is equals to the externally applied dynamical load vector f 1 t f 2 t and f 3 t.

So, you can see these are nothing, but the same equations one two and three we have written in the matrix form. So, in other words we can simply write it as M x double dot plus C x dot plus K x equals to f of t. What does it mean? Even for multi degree of freedom system the basic governing equation of motion the form of basic governing equation of motion remains same. That is mass times acceleration damper times velocity plus spring constant times displacement equals to applied dynamic load. Whatever we have seen in the single degree of freedom system, the same equation is valid for multi degree of freedom system. Only change is for single degree of freedom system. These were just a single quantity or values. Now, for multi degree of freedom system these are the matrices. That is this is the mass matrix this is damper matrix. This is stiffness matrix and for single degree of freedom system this acceleration velocity and displacement were just again a single parameter. In this case these are the vectors, acceleration vector velocity vector and displacement vector. Also this right hand side earlier was a single force single value force. In this case it is a forced vector for number of degrees of freedom.

So, if suppose if you have a two degree of system what should be the size of mass matrix 2 by 2? Size of this damper matrix 2 by 2, size of this stiffness matrix 2 by 2 and size of this acceleration vector 2 by 1. Size of this velocity vector 2 by 1 size of this displacement vector 2 by 1, size of this forced vector 2 by 1. So, for n numbers of degrees of freedom system we will have mass matrix with N by N damper matrix N by N stiffness matrix also n by n, whereas the acceleration vector will be vector of n by 1. This velocity vector will be of n by 1 and displacement vector will be n by 1. Similarly, the applied dynamic load vector will be of size n by 1. So, this is the general form of the equation governing equation of motion for a multi degree of freedom system?

Now, in this case what you have seen all the matrices. This matrix what you have seen it is perfectly diagonal. In this case these are also symmetric matrix about diagonal. They are symmetric. Both damper matrix stiffness matrix mass matrix and this is absolute diagonal other elements are 0 other entries are 0. So, let us see whether this is the always case or not. Let us take one example through which we will show the effect of coupling of different parameters and if the different parameters are coupled. What we need to do to get the solution?

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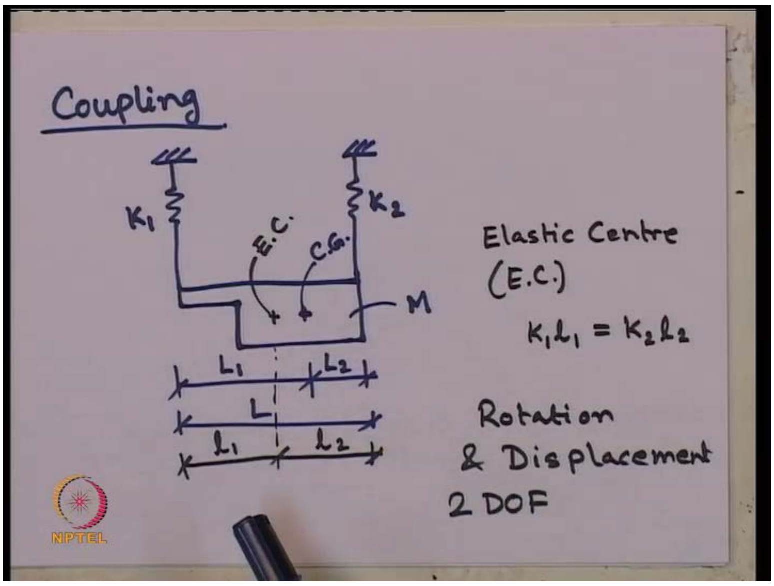

So, effect of coupling we are considering for different parameters. Let us take a simple problem of a rigid beam with varied cross sectional area which is having a mass of capital M attached to two spring say K 1 and K 2. And considering initially just an un damped system to show the effect of coupling and say this is the C G of the beam. So, this is center of gravity and the distance of the centre of gravity from this two ends are say capital L 1 capital L 2 and the total length of this bean is capital L.

Now, these two springs are connected in such a way that the elastic center of the system is here. So, this point is E C. What is E C? Elastic center. How elastic center is defined? Elastic center is such that let us say from two ends the distances are this is small l 1. This is small l 2 elastic center is such that K 1 l 1 is equals to K 2 l 2. At that point that is the definition of the elastic center. Now, we will see that for this system two degree of freedom system. We are considering both rotation and displacement of the system. So, both rotation and displacement of the system, we are considering for this which is giving us two degree of system. Now, selection of the axis for the rotation is extremely important. Why? Let us see. What I said the selection about which this beam will rotate the axis will rotate extremely important for this problem to know the effect of coupling. Why it is? So, let us see.

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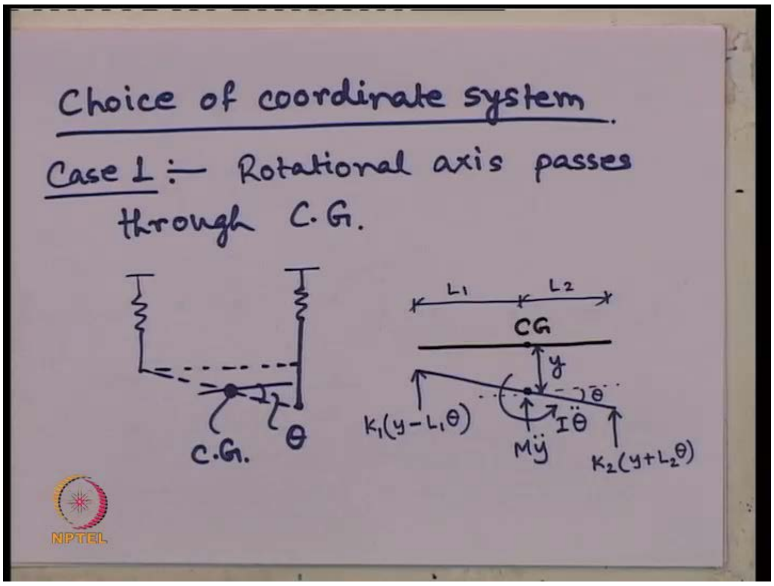

So, we can assume the choice of co-ordinate system. What do you say this as choice of co-ordinate system with respect to the rotation of the beam? We are considering the different cases. The first case I am considering when the rotational axis passes through the center of gravity of the system. What does it mean? If I just draw it like this what I want to say the line diagram, I am drawing it has rotated about its C G.

So, about C G its rotation is angle is theta. So, this is the center of the gravity. So, what has happened? The two degrees of freedom for this beam I am considering it has displaced vertically down at a particular instant. So, this is the direction of motion I am considering. It has displaced as well as it has rotated and for that rotation we are considering theta it has rotated above its center of gravity. So, this is the case I am considering here. So, what will be the free body diagram for this system? If I draw the line diagram this was the location of our C G. Earlier, now the beam has moved to a new position like this and it has rotated above this C G and that angle of rotation we are considering as theta.

So, it has moved down as well as it has rotated about the center of gravity. What are the forces should come on the system at C G? We will get suppose if the vertical moment up to this point C G is y, then there will be a vertical inertia force M y double dot acting at the C G because of its vertical movement also it has rotated. So, there will be rotational inertia force also which will try to put it back to its original position; so i theta double dot in this direction.

So, that it goes back to its position. Now, two springs are connected at the two ends spring forces will act. What will be the spring force at this end? It will be vertically upward because it goes back to its original position try to. So, K 2 times what is the displacement for this spring that is y plus L 2 times theta. Am I right? Because see the distances L 2 is from C G to this end; so L 2 times theta fine and this side it is L 1 times theta. So, this distance is L 2 and this distance is L 1 and another spring force that also will act vertically upward because see final displacement of this point is downwards. So, it will again try to put it up. So, it will be K 1 times y minus L 1 theta. This free body diagram is clear. So, if these are the forces, now what we can do? We can write down the moment equilibrium about the C G. So, if we write the moment equilibrium about C G what we should get?

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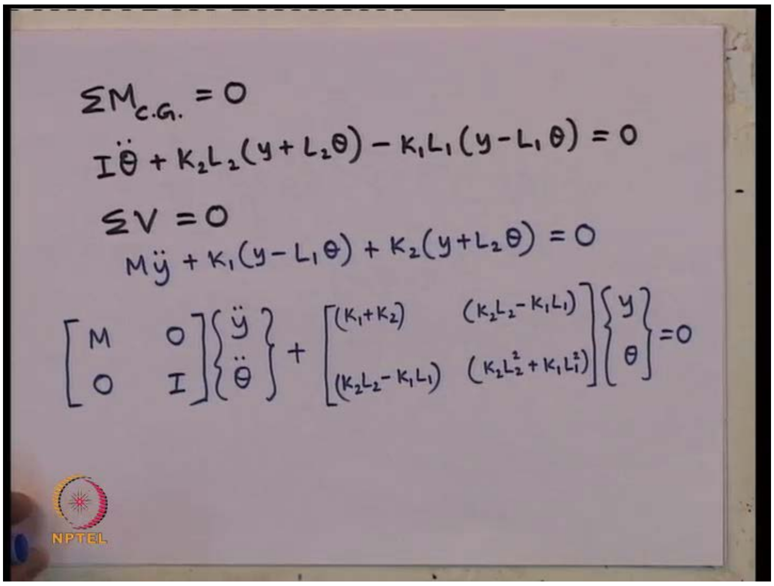

So, sum of moment about that C G should be equals to 0. So, let us put it here. So, that we can see all the forces anti clock wise I theta double dot; so, I theta double dot. This M y double dot is not creating any moment about the C G. What else it is creating? This is also creating anti clockwise. So, K 2 L 2 y plus L 2 theta and this is creating clockwise moment. So, minus K 1 L 1 y minus L 1 theta equals to 0 and another equation we can use which is sum of vertical forces is equals to 0 for the system which will give us M y double dot vertically upward. This is also vertically upward vertically upward. So, plus K 1 y minus L 1 theta plus K 2 y plus L 2 theta equals to 0.

So, this two equation we got, which we can help us to get the two degrees of freedom solution which is in terms of y and theta those are the two degrees of theta right vertical displacement and rotation. So, this one we can represent it in matrix form very easily M 0 0 I. Then the acceleration vector y double dot theta double dot plus K 1 plus K 2. Then K 2 L 2 minus K 1 L 1 then K 2 L 2 minus K 1 L 1 then K 2 L 2 square plus K 1 L 1 square times y and theta equals to 0. We considered the free vibrations case. So, this is our mass matrix. This is stiffness matrix. You can see mass matrix is diagonal. Symmetric stiffness matrix is symmetric about this. These two are same elements, but it is coupled can you follow these two gives us the two equations M y double dot then with theta nothing is there K 1 y plus K 2 y then this 1 K 1 minus K 1 L 1 theta k plus K 2 L 2 theta equals to 0. This is this equation and the second one is 0 with this I theta double dot and this one is K 2 L 2 y minus K 1 L 1 y plus K 2 L 2 square K 2 L 2 square theta. This gives K 1 L 1 square theta equals to 0.

So, with this we have seen for this case mass matrix is uncoupled, whereas, the stiffness matrix is coupled. Now, we can consider the other cases with different center for the rotation. That is for the first case we have considered the C G as the point of rotation. Now, if we take other any other point say elastic center as point of rotation what should be our equation of motion? And then we will see any other point say any corner of the beam we have taken point of rotation. Then what should be the equation of motion? So, this depends on the choice of the coordinate system. That we will see in the next lecture.