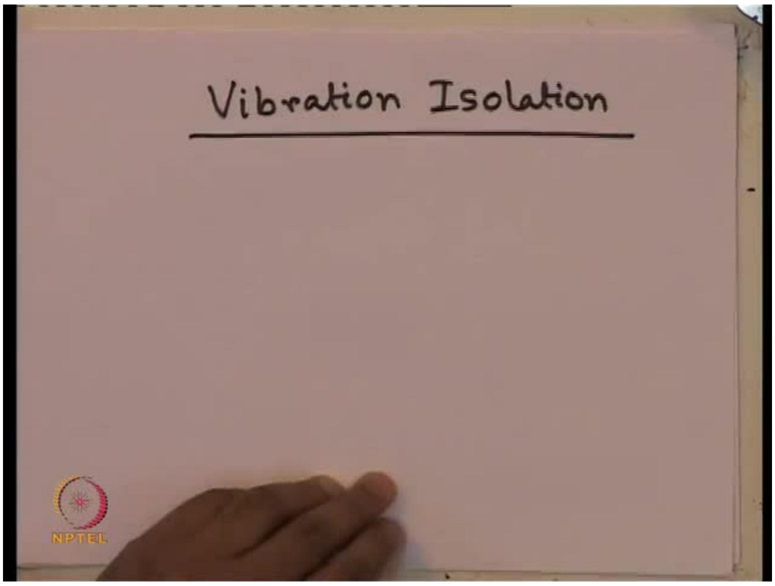
Soil Dynamics
Prof. Deepankar Choudhury
Department of Civil Engineering
Indian Institute of Technology, Bombay

Module - 2
Vibration Theory
Lecture - 12
Vibration Isolation, Vibrations Measuring Instruments

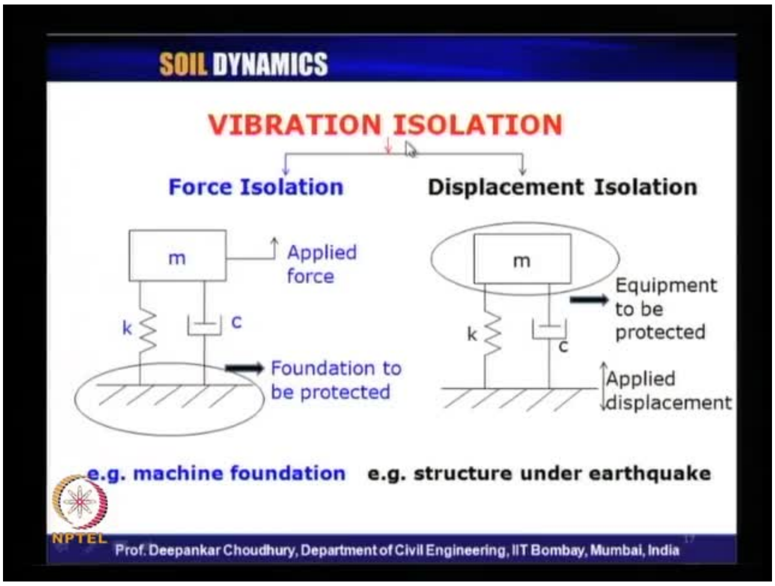
Let us start today's lecture of our soil dynamics course. So, we are continuing with the module two vibration theory.

(Refer Slide Time: 00:45)



So, with this now we will move to our next subtopic that is vibration isolation which is very important for us. How we can use the concept of vibration isolation? Let us see. So, let us look at the slide here which explains in a better way, looking at the slide.

(Refer Slide Time: 01:11)



The vibration isolation can be sub classified into two categories; one is called force isolation, the another one is called displacement isolation. So, vibration isolation can be of two categories - one is force isolation the other one is displacement isolation. And what are the basic philosophy behind these two types of vibration isolation, let us see. When we have a certain system with mass spring and dash pot and a dynamic load is applied to the system, we want that the dynamic load applied to the system should not create a huge amount of vibration or the response due to the vibration to our foundation. So, that is our concept, so what we want to, reduce the dynamic load applied to the machines coming finally its effect on the foundation. So, that is the basic concept of isolation, which is nothing but force isolation.

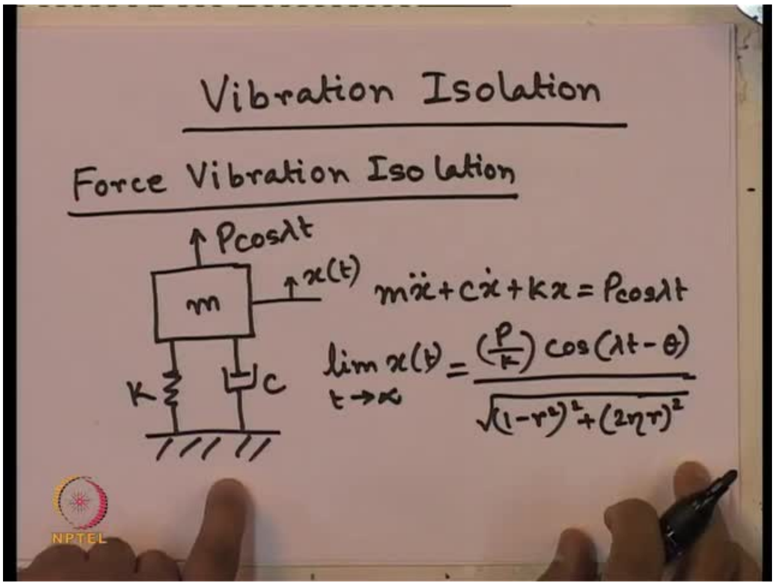
What is the example of this kind of system for machine foundation? That is when a machine is running because of that it is having an applied dynamic load, which is a large magnitude. We do not want that the same magnitude of dynamic load also transfers to the foundation of the machine because finally, that vibration of the machine should not create any problem to the surrounding buildings or the surrounding systems also to its foundation. So, what we have to do? We have to design the system in such a way that is we have to design this k and C.

We have to select the values of this damper and the spring in such a way that this foundation of the system is protected. So, to protect it from the applied force, we consider the case which is known as force isolation. And what is the concept of displacement isolation? Why it is used? In this case suppose we have a structure with mass m damper C and spring constant k, what happens when any dynamic load is acting on the ground? For example, an earth quake is acting, so the ground is vibrating and that vibration finally, affects your structure.

So, we do not want that the vibration induced with the earth quake should create a problem to our structure. So, we want to isolate our structure from that ground vibration. So, what we are trying to do, because of the earth quake there is an applied displacement at the ground level. We do not want that the same displacement should be transferred to the structure, it should be much less than that, that is our basic philosophy behind this isolation technique. So, that is why it is called displacement isolation. We want to isolate the system from the displacement, what the ground is experiencing. So, that is why the example of this kind of displacement isolation is nothing but structure subjected to earth quake loading.

So, is this clear? What are the two basic types of isolation technique we can use and depending on what is our given situation and given load. So, for the case of machine foundation we want to protect the foundation from the applied dynamic load, that is why it is force isolation. For the case of earth quake loading it is the displacement of the ground which should not be transferred to our superstructure, so that is why we want to isolate the structure from that ground displacement; that is why it is called displacement isolation. Now, let us see how we can model these things and derive this from final response, which will finally, help us to design for the system that is k and C combinations, because that is our final motto to get a better system for the design. So, let us start with the first case of isolation; that is force vibration isolation.

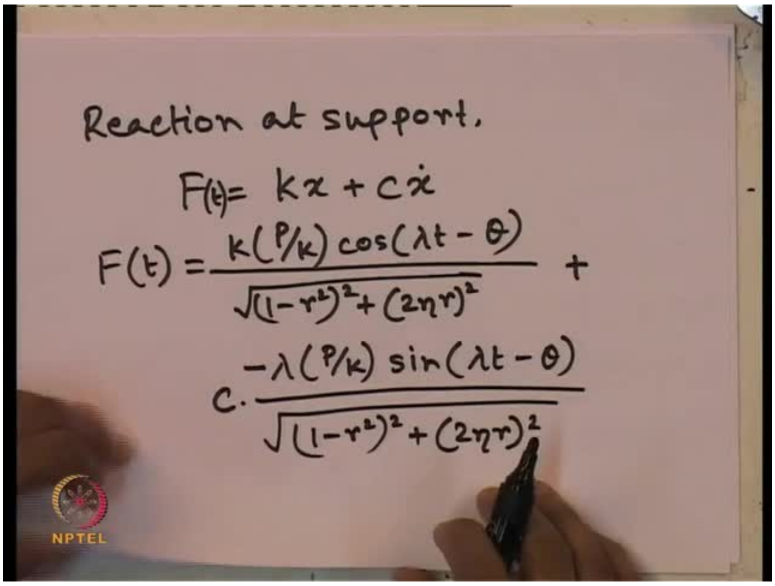
(Refer Slide Time: 06:03)



So, for force vibration isolation, what system we are considering? The single degree of freedom system k C m, it is subjected to a dynamic load P of cosine of lambda t, let us say, this is the dynamic load applied to the machine and its response as we had taken earlier x of t. Let us say so what was our basic equation of motion m x double dot plus C x dot plus k x equals to p of cos of lambda t, that we have seen earlier. Also we know what is the response of this solution limit of x of t when t tends to infinity, we have seen its response is given by P by k cosine of lambda t minus theta by root over 1 minus r square whole square plus 2 eta r whole square.

So, that was our steady state response, we have seen earlier this solution how we got it. Now, how much reaction is coming to the support, because we want to minimize the reaction force coming to the foundation coming to this support. So, how much reaction force is coming to the support?

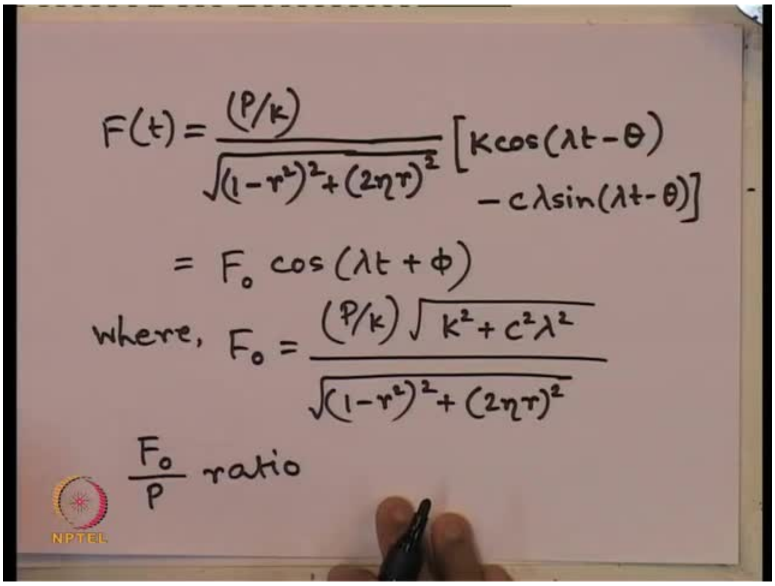
(Refer Slide Time: 07:40)



The reaction force, so reaction at support is nothing but k x plus C x dot, right? If you draw the free body diagram of each of this component that is spring and dash pot the force finally, transferred to by this spring is k x and the force finally, transferred by C is C x dot. And remember this is not just arithmetic addition, they are in 90 degree phase to each other, so remember this. This is a force addition in terms of force vector actually. Now, see if I say what should be our F of t this is actually F of t because x is the function of time so of course, it should be F of t that should be k times P by k and we are interested about the steady state response always, that is how the system is performing at time t tends to infinity.

So, P by k cosine of lambda t minus theta by root over 1 minus r square whole square plus 2 eta r whole square. This is the x of t at steady state we have seen, plus C of x dot, right? If we differentiate this what we will get? Lambda P by k sine of lambda t minus theta cos becoming sine, so we will have a minus sign here root over 1 minus r square whole square plus 2 eta r whole square. So, that is our F of t. So, if we have simplify this further, how we can write F of t?

(Refer Slide Time: 09:44)

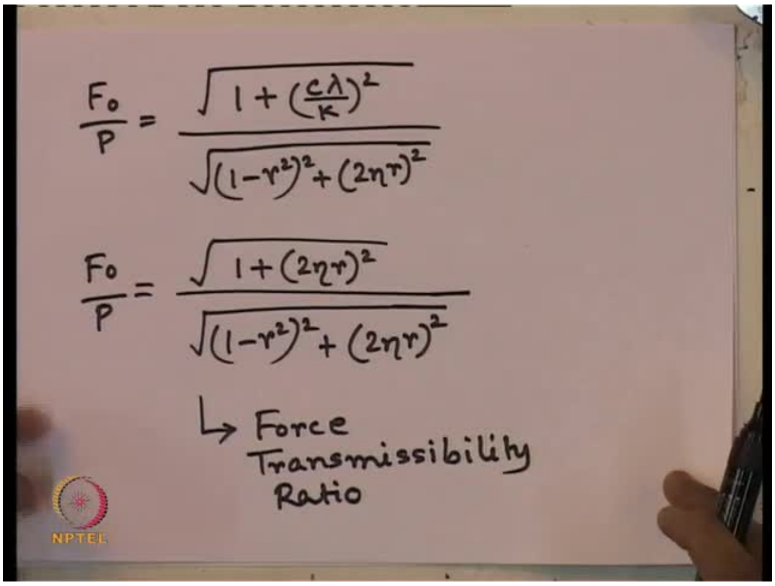


So, F of t is P by k root over 1 minus r square whole square plus 2 eta r whole square times k cosine of lambda t minus theta minus C lambda sine of lambda t minus theta. Let me put it back here again, so we can follow this. So, this common part I have taken out this is k cosine of lambda t minus theta here and this gives me minus C lambda sine of lambda t minus theta. So, this F of t, I can represent it as F naught cosine of lambda t plus say phi, where this F naught is given by P by k root over this k square plus. This square C square lambda square by root over 1 minus r square whole square plus 2 eta r whole square.

As we have done earlier also, you can express this in terms of the polar coordinates system and then simplify it in this form where F naught is given by this expression. So, what is our F naught the amplitude of this reaction force which is finally, transferred to the foundation. So, we are interested about this amplitude, as a designer. So, if we want to see how much force is getting transferred compare to how much force is applied to the system, what we should do? We should find out what is our F naught by P ratio. We are interested as a designer that this amplitude of force is transferred to the foundation and this amplitude of load was applied basically to the machine.

So, this ratio value we want to keep less than 1, then our concept of force isolation is achieved. That is we want to transfer the load to the foundation much less than what is applied to the system. So, let us see how it varies and what is the simplified form? If we take this expression once again here it will help us to follow.

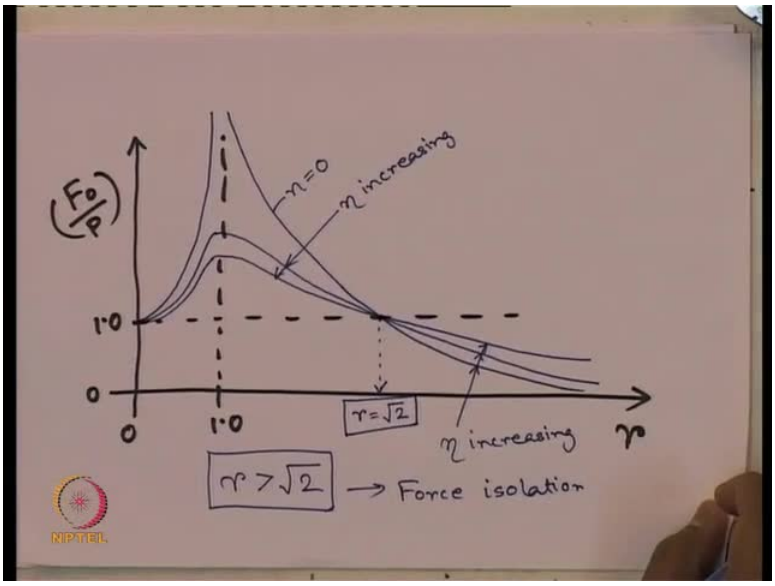
(Refer Slide Time: 12:34)



So, F naught by P is given by, this k I can take under this, so it will be root over 1 plus C lambda by k whole square by root over 1 minus r square whole square plus 2 eta r whole square. So, which if we simplify what we should get, C lambda by k, if we simplify it will give us 1 plus 2 eta r whole square because you can put k as m omega square, right? And lambda by omega is r and C you can put in terms of damping ratio. C equals to 2 m omega. So, you can put that, then from that you will get this expression omega. Omega gets cancelled and this, so F naught by P is given by this expression.

Can we find out the similarity of this function with anything, we have studied till date? Yes. It is nothing but same as transmissibility ratio. So, the force isolation is nothing but it is the force transmissibility ratio. So, how it varies now with respect to different values of r? How transmissibility ratio varies we have seen, in the same way this also varies.

(Refer Slide Time: 14:50)

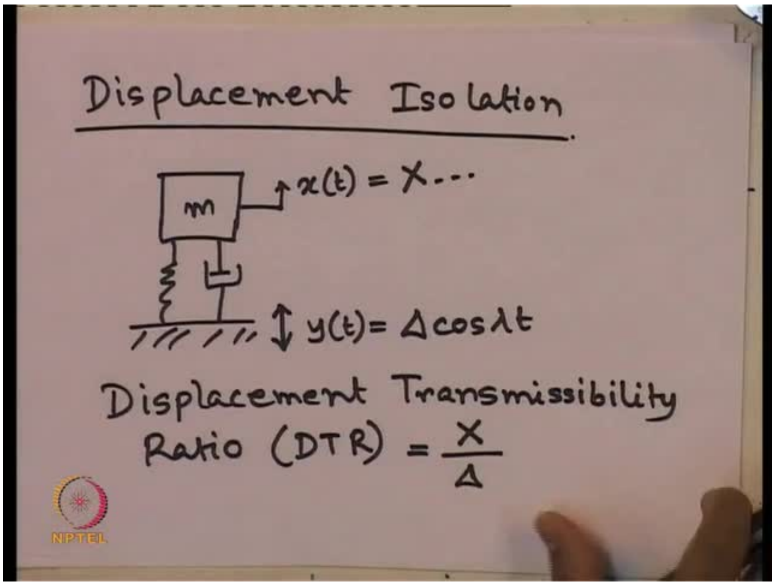


So, if we want to see the variation for different values of r, how this F naught by P ratio vary, it starts from 1 goes to infinity, one come down like this and other curves are going like this. So, it is slightly different than what we have seen for dynamic magnification factor. Remember in dynamic magnification factor for us similar case where it is subjected to harmonic loading, this was 1 in numerator, but here it is not 1. So, that is why you will get a crisscross at this point for different values of eta in this curve, so remember this.

If we actually plot this by yourself, you can see this. So, what does it mean here? It is for eta equals to 0 and in this way eta is increasing, but in this case it is a reverse way. So, here eta is increasing, so this is a important note. So, the curves are crisscrossing each other and the value where it again becomes 1 is at r equals to root 2. This is very important for us, because to apply the concept of force isolation in this region if we design we will not get the benefit, because always the force transmitted will be more than 1.

Whatever is applied we will transfer our larger force to the foundation, which is not desired. So, for design of a force isolation system, we should concentrate on this side of the graph, which means it is again a mass dominating zone. So, as a designer we should use r greater than root 2, to get the force isolation. That is the basic concept of the design for force isolation. Now, let us see the another system or another concept of vibration isolation, which is displacement isolation.

(Refer Slide Time: 17:58)

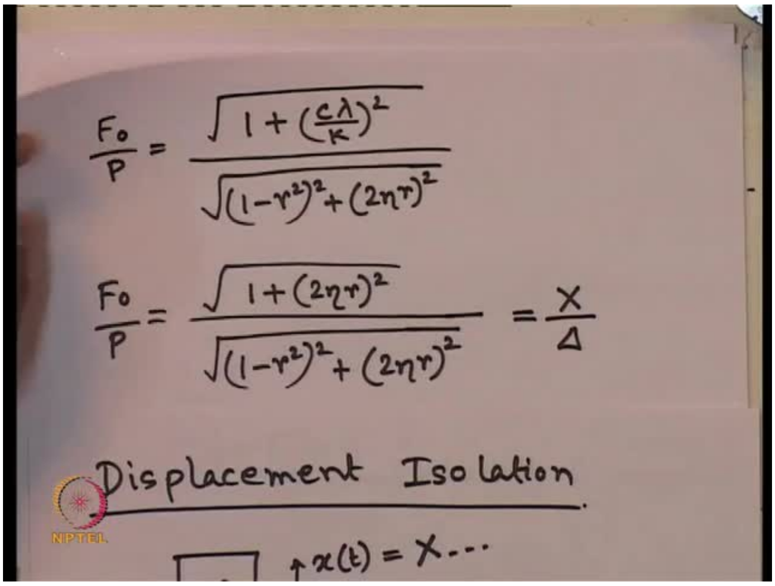


Now, we are discussing about displacement isolation. So, in this case let us draw the basic single degree of freedom system. Once again this is our X of t which is expressed as say some factor X some amplitude times some harmonic function when it is subjected to a ground displacement say y of t which is expressed as let us say delta cosine of lambda t. So, the ground displacement is given by a harmonic function with amplitude delta cos of lambda t. So, due to earthquake the displacement profile due to seismicity is like this and the response to our system structure or supper structure is like this.

See if we find out the ratio of this two, what we call this as displacement transmissibility ratio. So, earlier we have seen force transmissibility ratio, this is called displacement transmissibility ratio, DTR which is nothing but expressed as this capital X by delta that is what ever amplitude of the displacement is observed at mass level of the system or at the structure to the whatever amplitude of the displacement was applied to the system. That is similar to the concept of force isolation and the ratio was force experienced by the foundation to the force applied to the system here. Also the displacement experienced by the structure to the displacement applied to the structure and you can take it as another assignment.

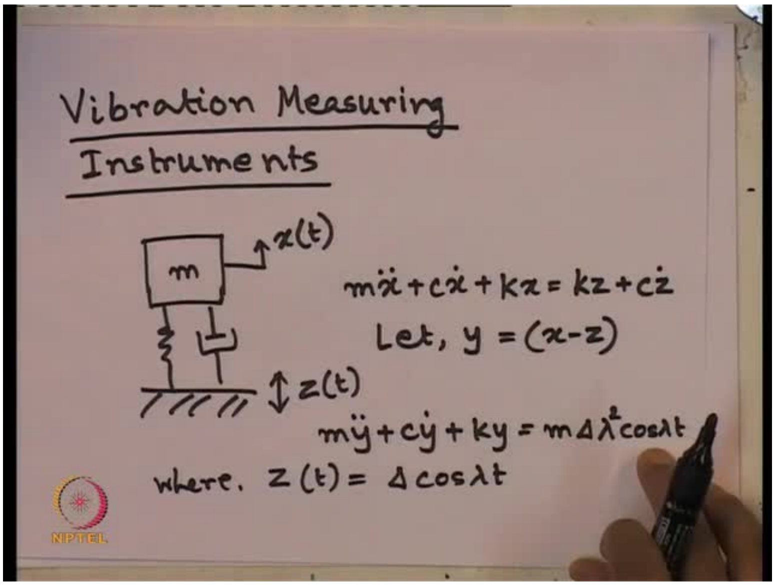
You can check whatever is the expression of this displacement transmissibility ratio, this X by delta is coming exactly same as the expression for force transmissibility ratio. So, that is why we have introduced from the transmissibility ratio as a common terminology, the expression remains same in terms of the damping ratio and the frequency ratio. So, displacement transmissibility ratio is also given by this expression.

(Refer Slide Time: 20:56)



That is X by delta, you can try to derive this as an home assignment at displacement transmissibility ratio. For design what we should get? Here also we should select the r value in such a way that, it is always greater than root 2, r is always greater than root 2. So, that this transmissibility ratio is less than 1, that means we are transferring less displacement to our structure than whatever it is subjected at the ground because of earthquake motion or some other vibration at the ground. So, with this we have come to the end of this vibration isolation.

(Refer Slide Time: 21:55)

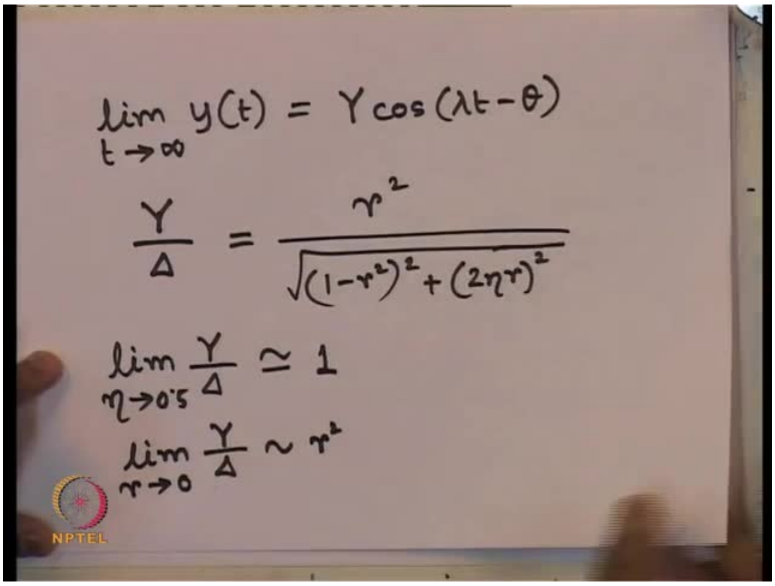


Now, let us go to another subtopic, which is vibration measuring instruments. We are now considering vibration measuring instruments. How this vibration measuring instruments are designed or how the basic concept of the design behind this vibration measuring instruments? For example, the seismogram, right? Accelerogram during the earthquake we measure the earthquake accelerations in a system seismic accelerogram. So, how those instruments are designed? What is the basic concept under the design of those seismograms? We will find out from this discussion.

So, let me draw the single degree of freedom system again mass, spring, damper here it is subjected to single degree of freedom X of t and ground. Let us say it is subjected to z of t is the ground displacement. So, what is the basic equation of motion we can get m x double dot plus c x dot plus k x equals to k z plus c z dot. Now, let us say the difference between this ground displacement and the displacement observed at the instrument level is given by y, which is nothing but relative displacement x minus z. So, what we can write m y double dot plus c y dot plus k y is given by m delta lambda square cosine of lambda t.

If we consider this ground displacement z t is expressed as delta cosine of lambda t, right? We are considering the ground displacement as a harmonic function, due to earthquake or some other thing, the ground displacement is following a time variation as a harmonic function with amplitude like this. Then the equation of motion takes the shape of this function clear. Now, what is the measurement of the displacement?

(Refer Slide Time: 24:36)

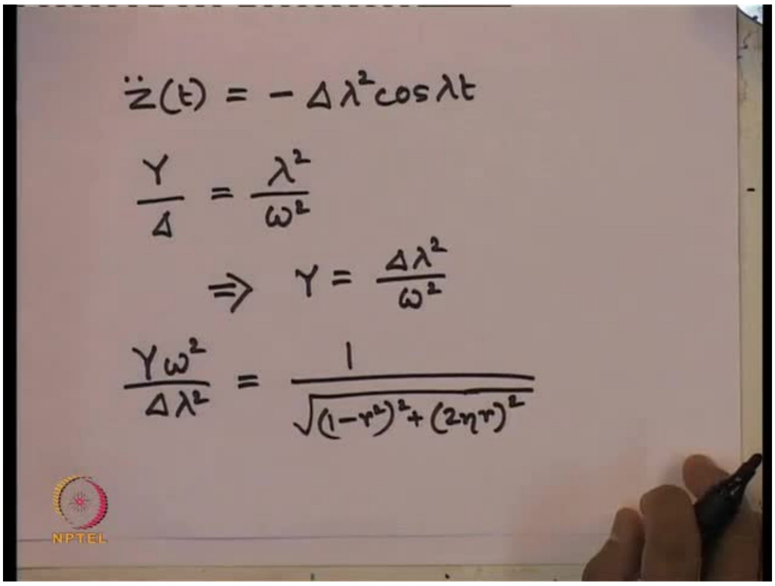


If we write down the solution, we are interested about steady state always. So, t tends to infinity y of t, how it varies? The solution of this type of equation where it is a harmonic function acting on this side, for for this type of forced vibration, what is the form of the solution? The steady state response y, of cosine of lambda t minus theta. We have seen right earlier there is a form of the steady state solutions, steady state response with y as the amplitude in steady state response amplitude.

So, what we can write, this y by delta that is response this y is relative response of course, y by delta whatever is applied is nothing but what type of force we are considering here. Similar to if we look properly to this equation basic equation, it is similar to the single degree of freedom system subjected to a rotating imbalance, right? That was the form of the equation q times of some harmonic function. So, whatever was the expression of y by delta in that case of rotating imbalance, the same expression we should get. That should be r square by root over 1 minus r square whole square plus 2 eta r whole square, so this is the measurement of the displacement.

If we take limit of this y by delta function when eta tends to 0.5, that is about half, we will get if you put in this expression it is coming almost about 1. And if you take limit of this y by delta function when r tends to 0, we get the value is about coming r square. And the other functions we can simplify and the measurement of acceleration, we can express like this.

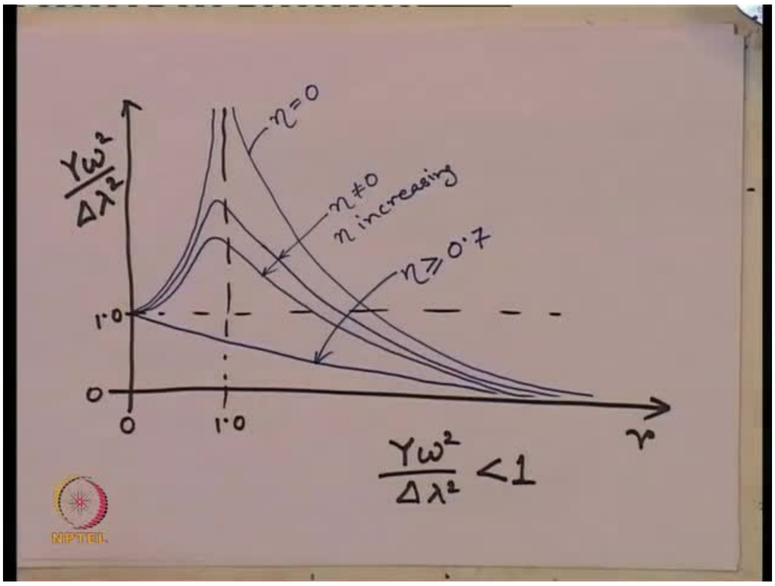
(Refer Slide Time: 26:56)



Measurement of ground acceleration z double t t is nothing but minus delta lambda square cosine of lambda t because we had assumed the displacement at the ground z of t as delta of cosine of t, so the acceleration at the ground is nothing but this, fine? So, what we can write this, y by delta is equals to lambda square by omega square, when r approaches 0. We have seen just now limit r tends to 0, y by delta approaches r square. So, what is r square nothing but lambda square by omega square.

This will give us y as a function of delta lambda square by omega square. So, if we simplify our previous expression, what we will get by putting these expressions? That y omega square by delta lambda square equals to 1 by root over 1 minus r square whole square plus 2 eta r whole square. So, this is similar to our case of forced vibrations subjected to harmonic load, so the distribution also will be similar.

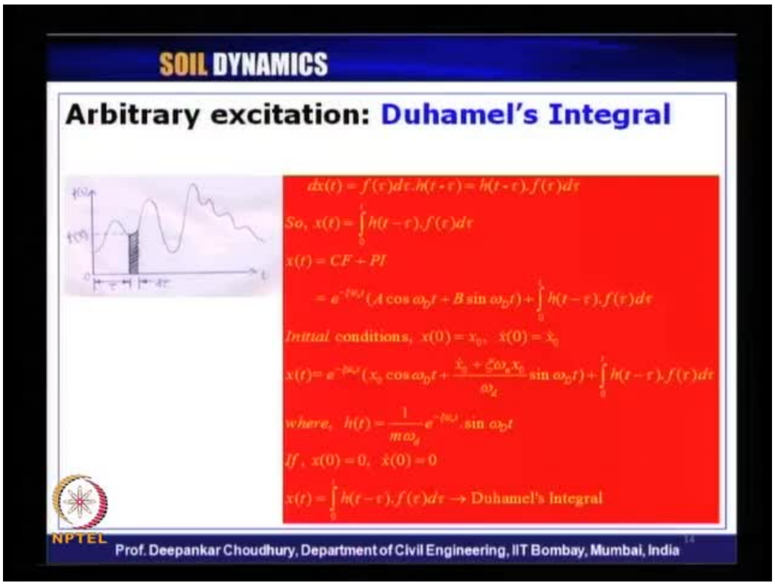
(Refer Slide Time: 28:37)



See if we want to plot the variation of this term y omega square by delta lambda square with respect to r 0, this is 1 this is 1, this is 0, we should get the variation like this for eta 0 and you can check for what value of eta. So, these are eta non zero, and eta in increasing order. So, if you check for what value of eta our this design value which we are looking at y omega square by delta lambda square, should be always less than 1. So, that we do not want any magnification at the instrument level. So, for which value of eta this comes always less than 1, you can check this and I am giving you the value, the curve will go like this.

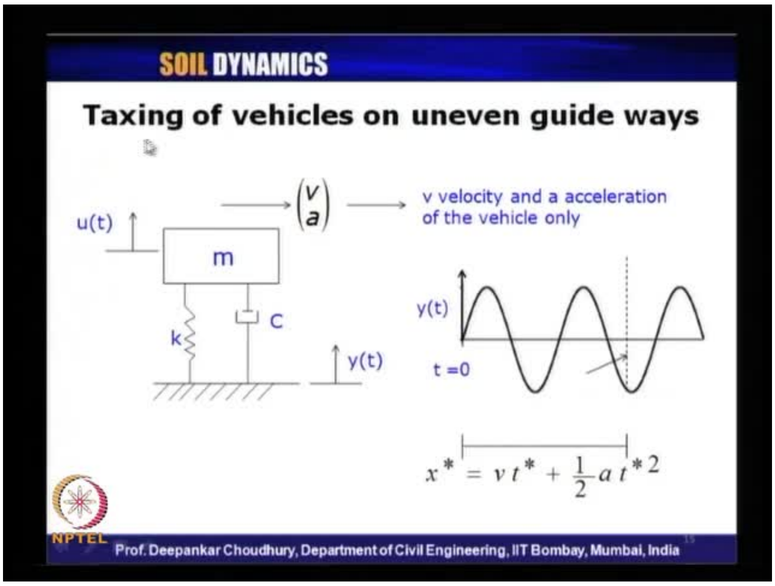
So, what is that value of eta? This will be when eta greater than or equals to 0.7, in that case the value of this parameter y omega square by delta lambda square is always less than 1. So, for vibration measuring instruments, we select the damper in such a way that the damping ratio of the system is more than or equals to 0.7. So, you can verify this using a simple calculation, whenever it is, always you equate this expression to one and you will get the value of eta for which it is always 1, irrespective the value of the r. So, that is the basic principle used for designing vibration measuring instruments. So, whatever we have learnt today, let us look at the slides here.

(Refer Slide Time: 31:06)



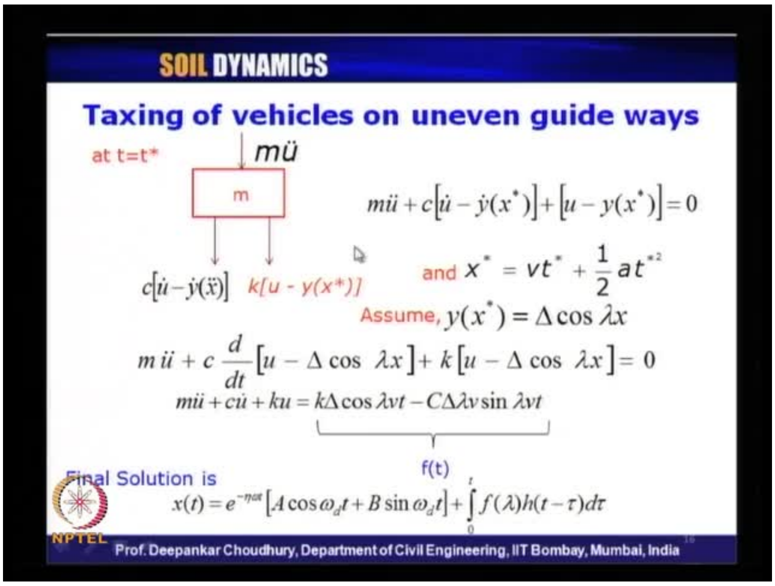
So, we talked about arbitrary excitation like this, using the concept of Duhamel integral taking an infinitimal strip and then integrating it we get the particular integral similar to the concept of the impact load. And the final solution we can get using Duhamel's integral when the system is starting from absolute rest condition.

(Refer Slide Time: 31:33)

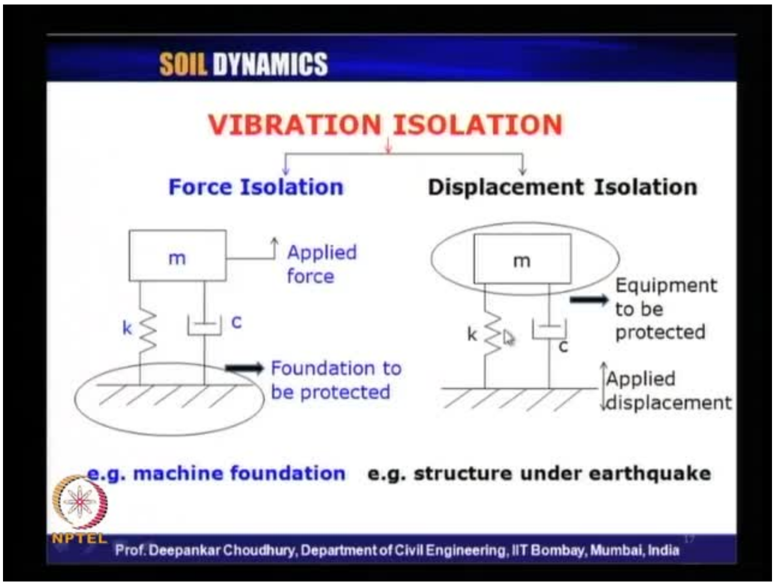


Then we have seen the taxing of vehicles on uneven guide ways the system is moving and the dynamic response at the system level is u of t, on ground it is y of t and we assume the ground undulations as a harmonic function like this at a particular elapse of time t star. What is the displacement on the ground or the movement on the ground that will be a function of y, which will finally, give us the solution of the system like this.

(Refer Slide Time: 32:02)

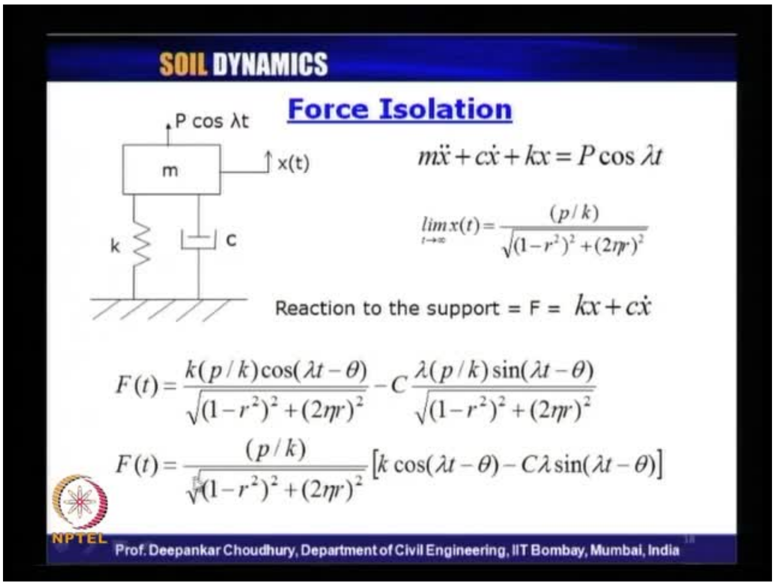


(Refer Slide Time: 32:07)



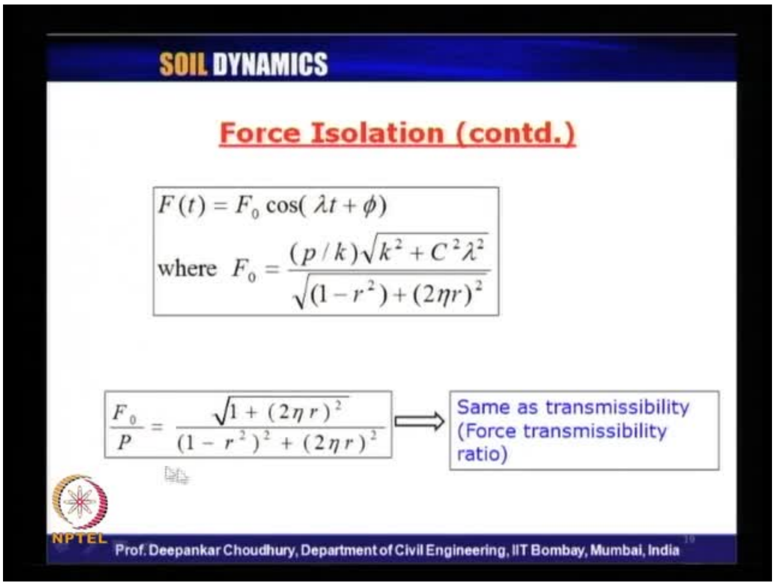
Then we have used the concept of vibration isolation, force isolation and displacement isolation.

(Refer Slide Time: 32:12)



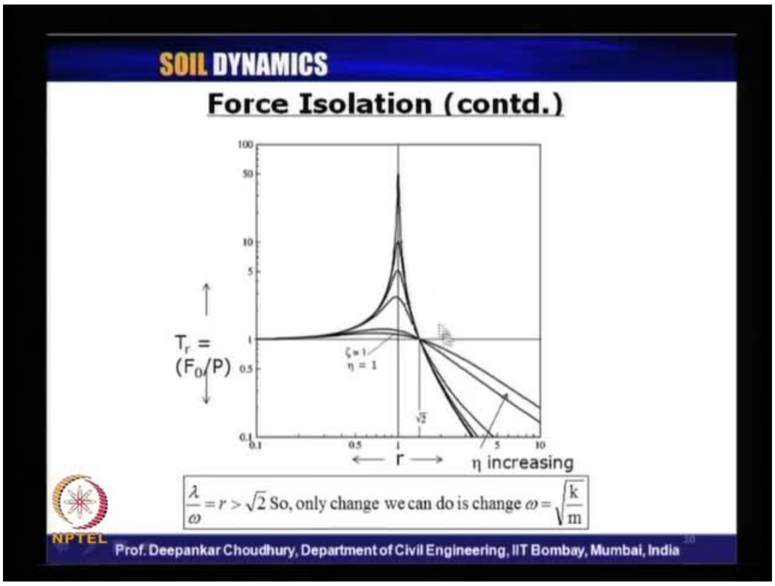
For force isolation we want to isolate this foundation from the applied load.

(Refer Slide Time: 32:19)



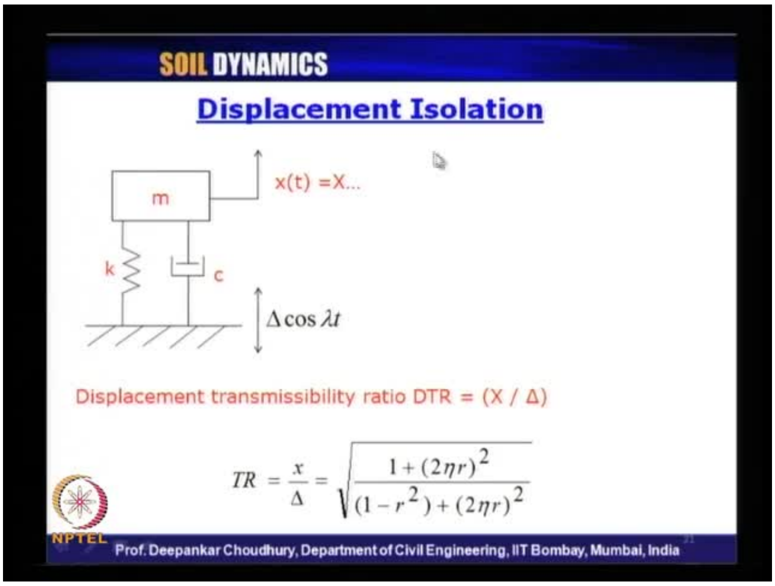
Finally, we got the expression same as transmissibility ratio, so this case we call it as force transmissibility ratio.

(Refer Slide Time: 32:27)



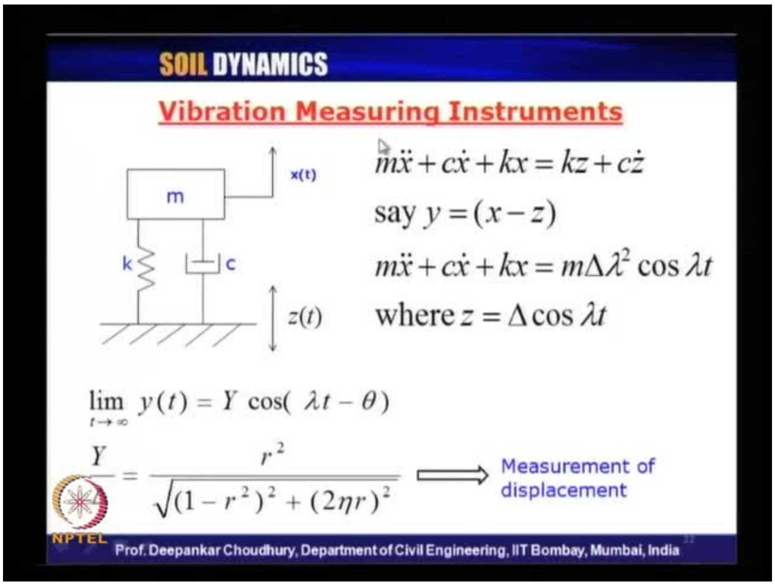
And the variation is like this with r greater than root 2. We can design the system, so that the force transmitted to the foundation is always less than whatever is applied to the system.

(Refer Slide Time: 32:42)



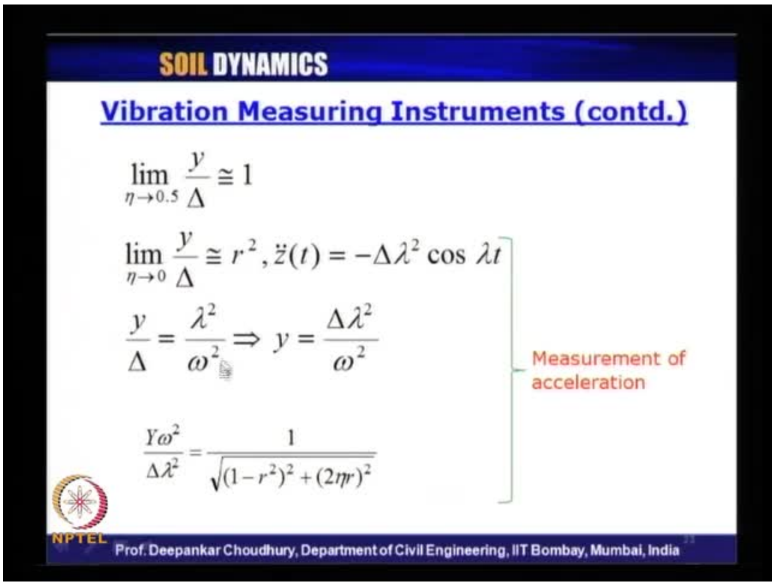
In case of displacement isolation, the displacement transmissibility ratio is given by the same expression of transmissibility ratio. So, here also we want to reduce the displacement coming to the structure whatever is applied at the ground.

(Refer Slide Time: 32:57)



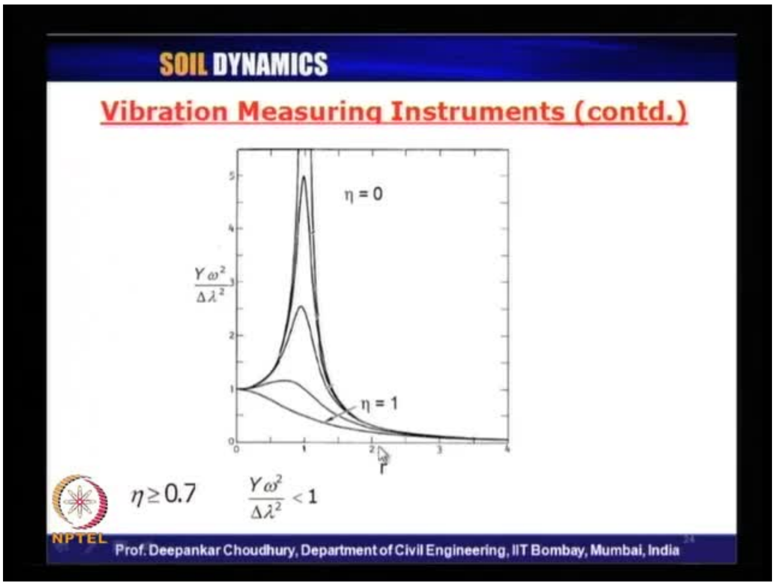
And for vibration measuring instruments, the basic concept we have seen how to find out the equation of motion.

(Refer Slide Time: 33:06)



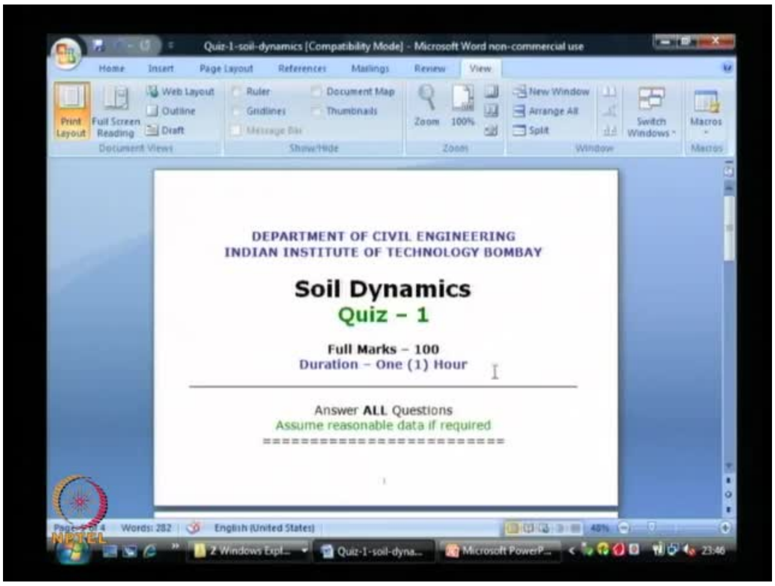
And then the solution, how it varies the measurement of accelerations.

(Refer Slide Time: 33:15)



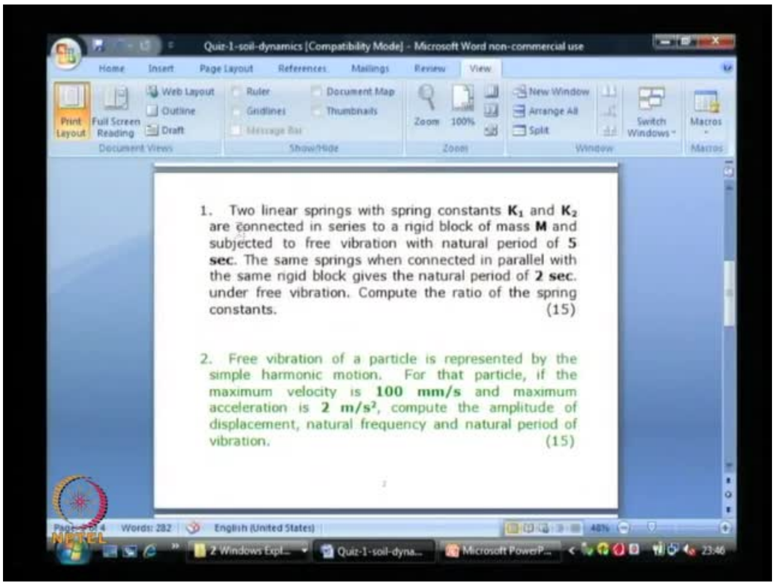
And this is the design technique to make this parameter always less than 1, our damping ratio of the system should be always greater than or equals to 0.7, so that will give us the concept behind the design of the vibration measuring instruments. So, with this we will stop. Let us see the possible solution for quiz one. The quiz one is already be taken, so let us look at the slide, the questions for quiz one for our course.

(Refer Slide Time: 34:02)



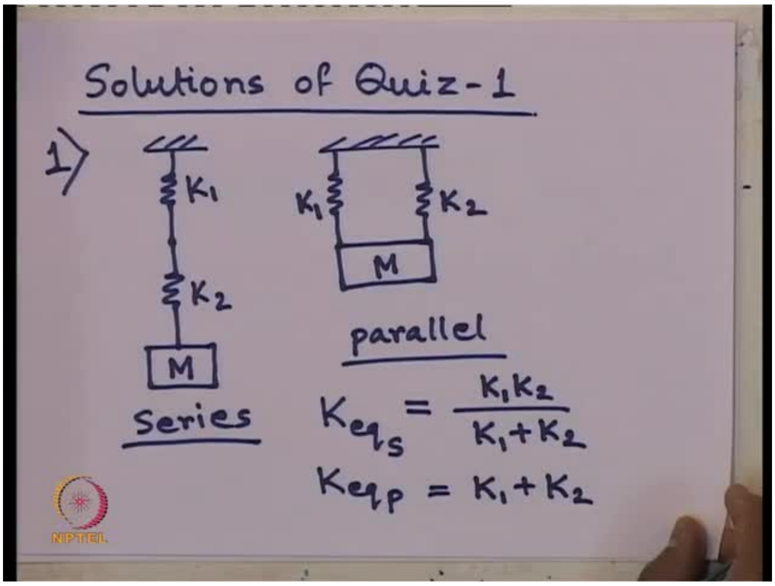
Full marks was 100 and duration was 1 hour and all questions has to be answered.

(Refer Slide Time: 34:13)



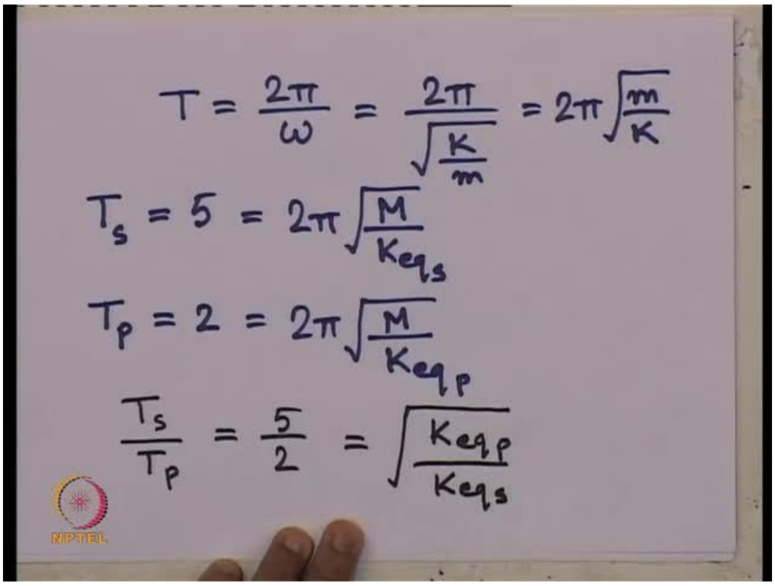
The first one was two linear springs with spring constant K 1 and K 2 are connected in series to a rigid block of mass M and subjected to a free vibration with natural period of 5 seconds, the same springs K 1 and K 2 when connected in parallel with the same rigid block gives a natural period of 2 second under free vibration. Compute the ratio of the spring constant and this problem carries 15 points or 15 marks. So, let us see the possible solution for this problem which is pretty direct and easy.

(Refer Slide Time: 34:59)



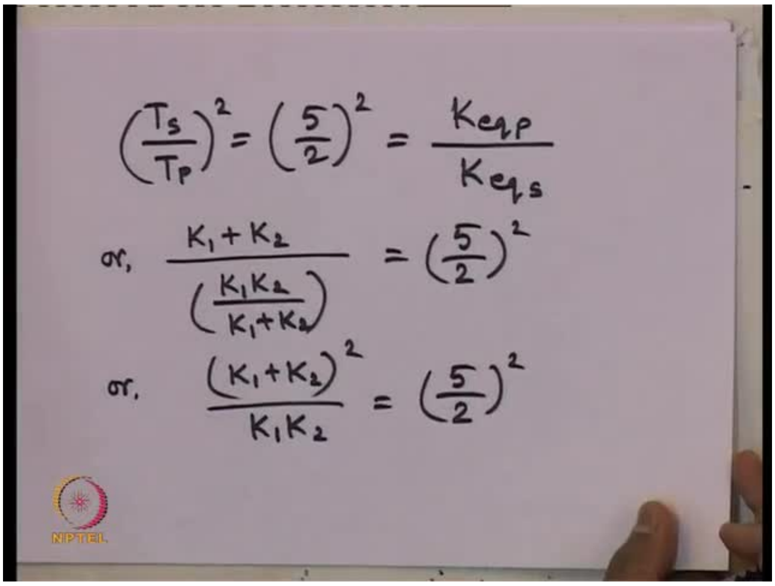
So, solutions of quiz one. The first problem what it says, two linear springs with spring constant K 1 and K 2 connected in series. So, we have two linear springs connected in series attached to the rigid block of the mass M and K 1 and K 2 are the spring constants. And the second case it says they are connected in parallel and connected to the same rigid block of mass M K 1 and K 2. This is in parallel and this one is in series. Now, we know the expressions for equivalence spring constant, for these two cases. K equivalent for the case series is how much? It is K 1 K 2 by K 1 plus K 2. We have derived this in our lecture and for parallel K equivalent parallel is K 1 plus K 2, these are the equivalent spring constant. Now, how we can find out the natural period when it is subjected to free vibration?

(Refer Slide Time: 37:03)



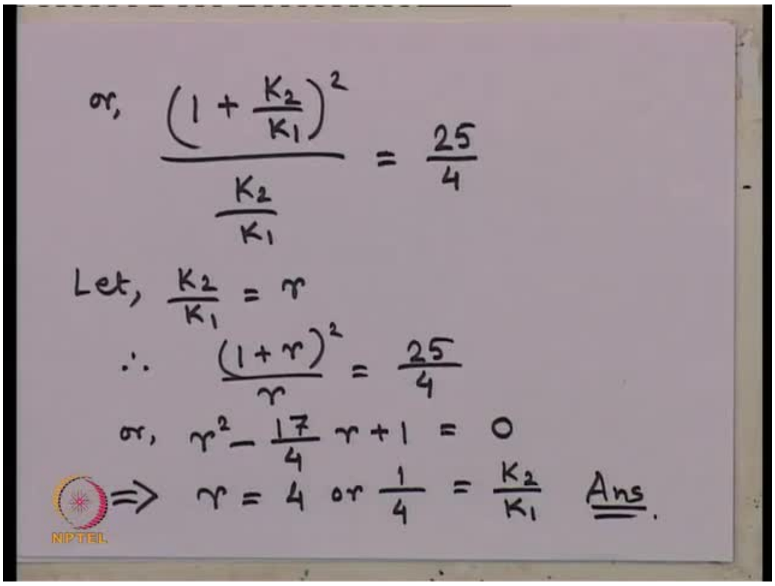
The expression for natural period T is nothing but 2 pi by omega. Now, omega is circular natural frequency, which is expressed as root over K by M. So, T is expressed as 2 pi root over M by K, so what we can see the time for the series connection is given to us, how much? 5 seconds. So, 5 seconds, so it is expressed as 2 pi root over M by K is K is equivalent, under that series connection. And T in the case of parallel connection is given to us as 2 seconds, which is 2 pi by 2 pi into root over M by K equivalent in parallel. So, the next part we can write like this T S by T P is equals to 5 by 2 equals to how much? We are dividing this by this, so we will get root over K equivalent for parallel by K equivalent for series.

(Refer Slide Time: 38:45)



Now, on simplification, what we can write, T S by T P whole square which is 5 by 2 whole square is K equivalent in parallel by K equivalent in series or what is K equivalent in parallel? That is nothing but K 1 plus K 2 and what is K equivalent in series? That is K 1 k 2 by k 1 plus k 2, which is equals to 5 by 2 whole square or K 1 plus K 2 whole square by K 1 K 2 equals to 5 by 2 whole square, so further if we simplify this we can get, 1 plus K 2 by K 1 whole square by K 2 by K 1 equals to 25 by 4. Now, let us say this K 2 by K 1 is r.

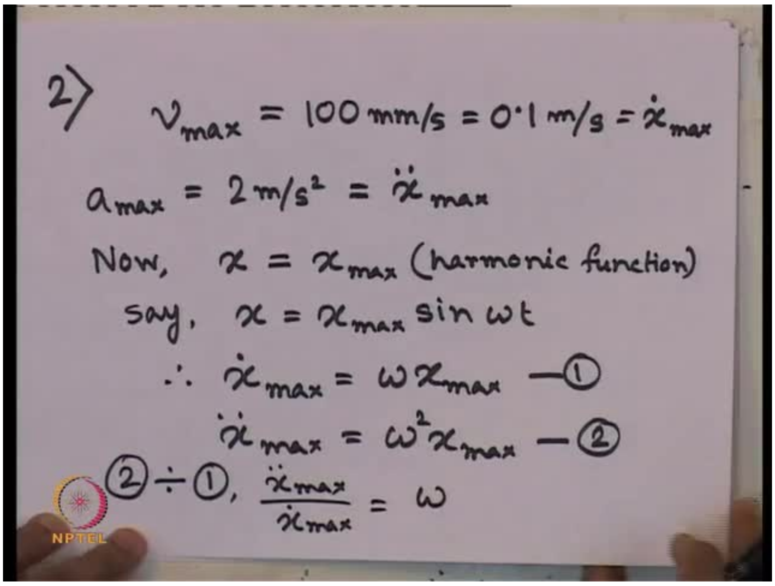
(Refer Slide Time: 39:43)



Therefore, 1 plus r whole square by r equals to 25 by 4 or r square minus 17 by 4 r plus 1 equals to 0. If we solve this, it gives us r as equals to 4 or 1 by 4, which is nothing but the ratio of that two spring constants K 2 by K 1. So, this is the answer. So, the ratio of the spring constants what is asked in this question here on the slide, you can see compute the ratio of the spring constants we have obtained, it is either 4 or 1 by 4.

So, that was the solution for the first question, very simple. The second question was free vibration of a particle is represented by the simple harmonic motion, for that particle if the maximum velocity is 100 millimeter per second and maximum acceleration is 2 meter per second square, compute the amplitude of displacement, natural frequency and natural period of vibration. It also carries 15 points or 15 marks.

(Refer Slide Time: 41:35)

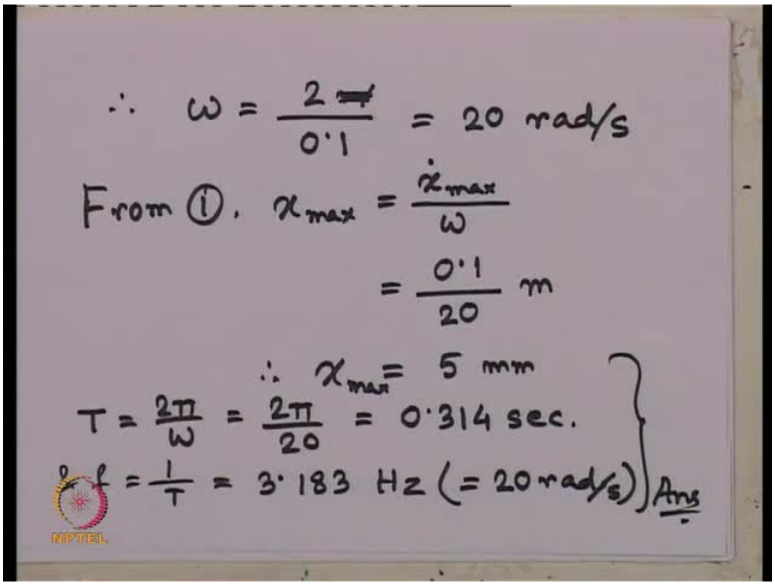


This is also pretty simple and easy problem which probably you must have solved in your high school physics. So, what is given to us? The maximum velocity for that particle subjected to simple harmonic motion V max is 100 millimeter per second. So, let us convert it to out SI unit, 0.1 meter per second, which is as our usual notation x dot max. And maximum acceleration is given as 2 meter per second square which is already in SI units, no need to convert and in our usual notation is x double dot max. Now, what is mentioned that it follows a harmonic motion, so if the particle follows a harmonic motion the displacement profile we can write in terms of x equals to x max, that is amplitude times the harmonic function.

That is what the equation we can write. So, let us say it follows like this, x equals to x max say sine of omega t. Where omega is natural frequency and t is the time. So, that is the form of the particles subjected to harmonic motion. So, what we can write therefore, the velocity x dot max, if we try to find out x dot max that is nothing but omega times x max, right? And what is the maximum acceleration? That is nothing but omega square x max.

We are talking about only the magnitude, so we do not concern about the plus sign or the minus sign, it is just the magnitude, maximum magnitude. So, if now we operate this, that is equation two divided by equation one, what we can get? This x double dot max by x dot max is nothing but omega, right? So, now both the values of x double dot max and x dot max are given to us.

(Refer Slide Time: 44:07)



Therefore omega is x double dot max is 2 meter per second square. Let us not put the unit, just the value magnitude we are putting. And x dot max is 0.1, so 20, this much radian per second that is a unit for omega, right? Now, from equation one, from equation one, let us put it here once again, what we can write? x max, that is maximum amplitude of displacement is nothing but x max dot by omega. So, the value of x max dot is given to us, which is 0.1 meter per second and this is 20, so so much of meter. So, this gives us 5 millimeter that is the one answer that is compute the amplitude of displacement, this is the amplitude of displacement.

x max is 5 millimeter and what is the natural period? T equals to 2 pi by omega so 2 pi by 20 gives us 0.314 second as the natural period of vibration. And the natural frequency f is 1 by T which is 3.183 in terms of hertz unit, which is nothing but if you put it in the form of circular frequency, also does not matter. So, which is 20 radian per second the same thing. So, these are the three answers which has been asked in this problem, pretty simple and direct, right? So, let us stop here today. We will continue our lecture in the next class.