Soil Dynamics
Prof. Deepankar Choudhury
Department of Civil Engineering
Indian Institute of Technology, Bombay

Module - 2
Vibration Theory
Lecture - 11
Response to Arbitrary, Step and Pulse Excitations,
Response to Impact Load

Let us start today's lecture of soil dynamics. We are continuing with our module 2 of vibration theory.

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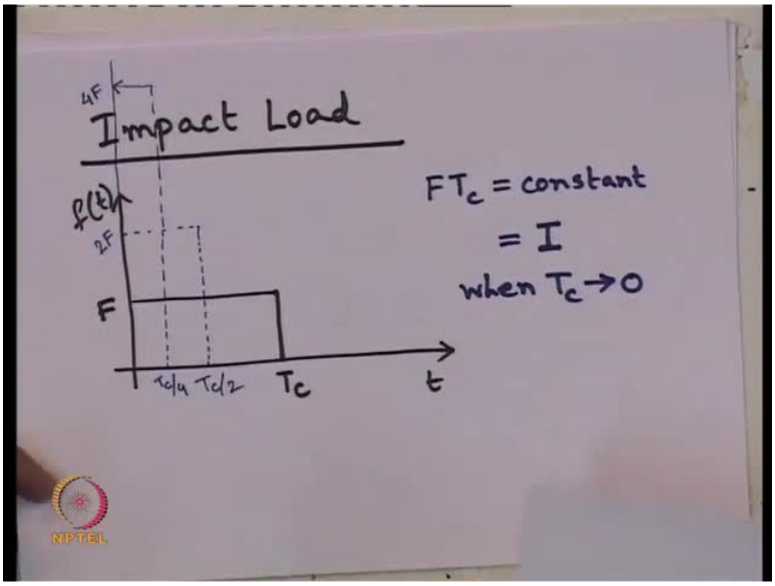

Let us see the application of this concept of heaviside step function and indicial response for case of impact load. Now, we are taking up the impact load. What is the meaning of impact load? The magnitude of the load is very high, and the duration is very small, that is the general characteristics of the impact load. What we have seen just now that a function f of t with respect to t we have solved for this case F T C. Just now we have solved for this kind of force, forced vibration what is the solution and response all this things we got. I will change it slightly now like this, I make it T C by 2, I make this 2 F. So, what I did? The area under the curve I am keeping constant, fine. Let me go further.

This is 4 F, this is T C by 4. So, in other words what I am doing F into T C is constant, I am maintaining. That can be referred as impact load I when T C tends to 0 that is when T C is very small in that Case this magnitude of F times T C we denote it as I which is nothing but the impact with T C tending to 0. So in terms of our knowledge of heaviside step function, how we can express this load let us see.

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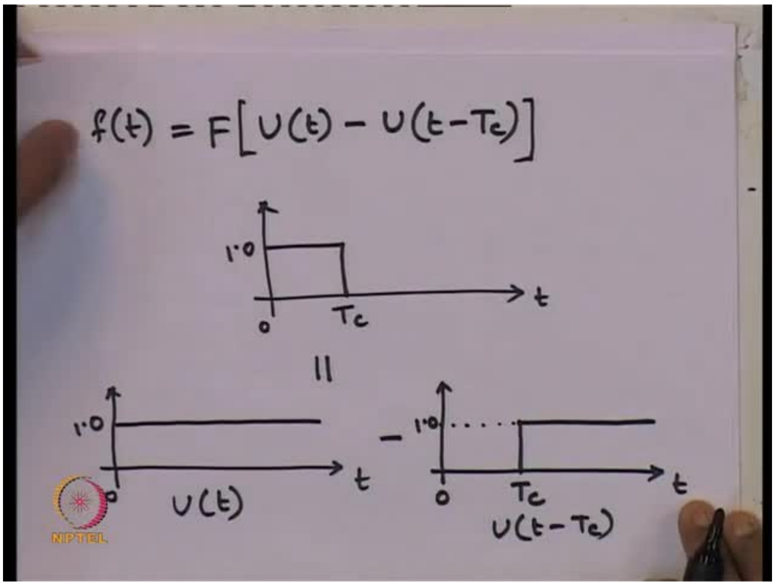

So, f of t, f of T Can be expressed as for unit load it is U of t minus U of t minus T C times F, is this clear to all of us. Let us look at here, the function f of t I am explaining in terms of heaviside step function, for unit load it is how much? One unit load U of t minus U of t minus T C, why? Let me draw it again to have more clarity. This load, if it is unit T C can be equals to this load minus this load. If you superimpose this on this we are deducting from a constant unit load which is acting from t equals to 0 to infinite time, we are deducting from that a constant load which is starting at t equals to T C and we are deducting it from here which will give us equals to this.

That is a constant load acting for a finite time and in terms of heaviside step function what is this? This is U of t and what is this one? U of t minus T C that is why I have written this function is nothing but U of t minus of U of t minus T C. So, this is for unit load because heaviside step function. If I apply F load, just multiply it with respect to F that is it. So, this is our applied force to the system.

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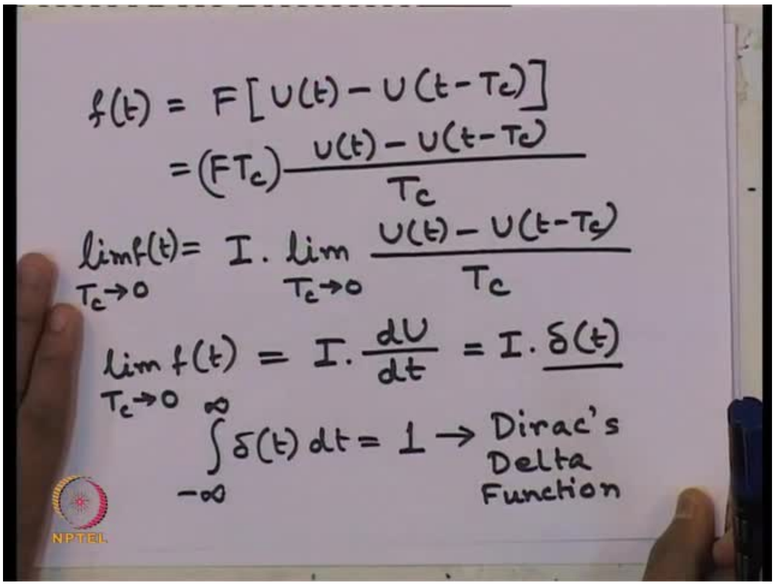

What we can do from this f of t which is F U of t minus U of t minus T C, we can write it like this F times T C U of t minus U of t minus T C by T C. We are multiplying T C here and then dividing it here. What I said? This component is known as I impact. Now, I am taking limit of this f of t when T C tends to 0 in that case this is expressed as I the impact limit T C tends to 0 U of t minus U of t minus T C by T C. What is that? So, limit f of t T C tends to 0, we have I times d U by d t. This differential which is known as I times expressed as I times delta t. delta t is nothing but first differential with respects to t of the heaviside step function which is expressed as delta t.

This delta t is called deducts delta function and how it varies, it is varies minus infinity to plus infinity delta t d t is unity, unit value. This is called deducts delta function. Now, we are introducing from heaviside step function. Its first differential with respect to time expressed as delta of t which follows this relation that if we integrate it from minus infinity to plus infinity with respect to time, it will give us a unit value that is known as deducts delta function. Now, what is the response? So, this is about the load, this part we have talked about the load. Load is expressed as for impact just I times this comes as unit. So, impact load is acting on the system, clear? f of t is nothing but we have imposed an impact load on the system. How the response should be response we can write like this.

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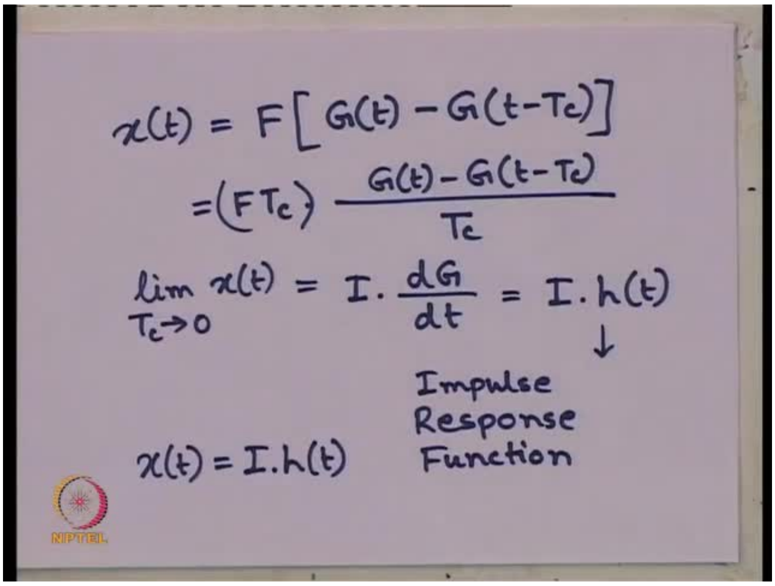

x of t response is nothing but F times G of t minus G of t minus T C, is this clear because we have mentioned earlier for heaviside step function the solution or the response is called indicial response and given by G of t. So, for unit load the same concept we are using imposing that the solution will be G of t minus G of t minus T C for unit load. So, for F load it will be just F times of the function for unit load. Let us simplify this, F times T C G of t minus G of t minus T C by T C. If I take limit this x of t when T C tends to 0, it gives us impact this is d G d t. This d G d t is expressed as h of t. h of t is the output or response for a impact load of I.

So, what does it mean? If the applied force is impact load of magnitude I, then the response is I times h of t and if the impact load is unit then the response is called just h of t, for unit impact load response is h of t, for a magnitude of impact load of I the response is I times h of t. So, this is called impulse response function impulse response function. So, for heaviside step function we got indicial response, for impact force or impact load we got impulse response function. So, our x of t is expressed as in this case impact times h of t. Now, how this magnitude of h of t can be obtained? This is simple, let us take the solution of G of t, then it will be easy for us to follow.

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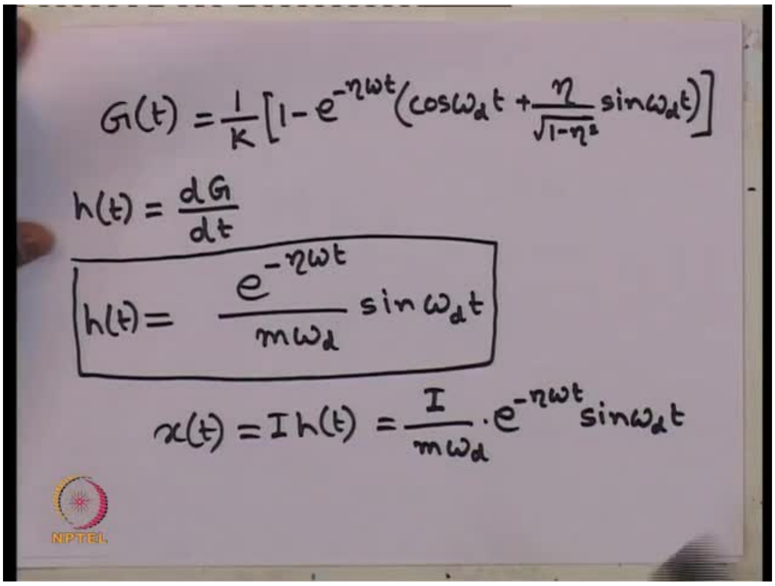

What was the solution of G of t? If we go back to our previous solution, this was the solution. Let me put it here, this was our solution for indicial response function. So, G of t means it is for unit load so it will be 1. So, 1 by K 1 minus e to the power minus eta omega t cosine of omega d t plus eta by root over 1 minus eta square sine of omega d t, that is our indicial response function. So, we need to find out h of t that is impulse response function which is nothing but simple d G d t that is we need to differentiate this expression with respect to time. We will get the impulse response function. See, if we do that the simplified form I am giving you, it will come as e to the power minus eta omega t by m omega d times sine omega d t. So, that is the impulse response function.

So, our x of t for impulse load of I will be simply I times m omega d e to the power minus eta omega t sine of omega d t. That is the total response for any impact. So, this is the solution for any impact load, what is the displacement function varies with respect to time. So, what we have derived and discussed let us look at the slides now.

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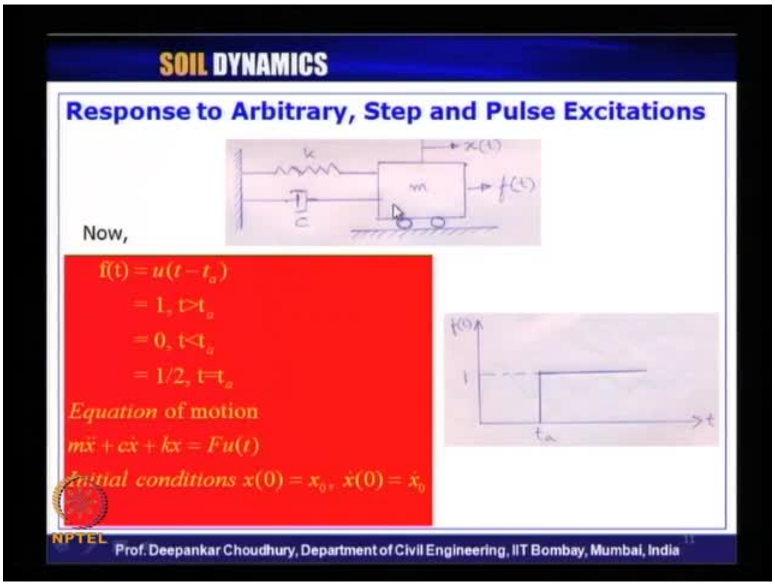

As I said the heaviside step function this one, we are expressing like this for t greater than t a, t less than t a and t equals to t a equation of motion is given like this.

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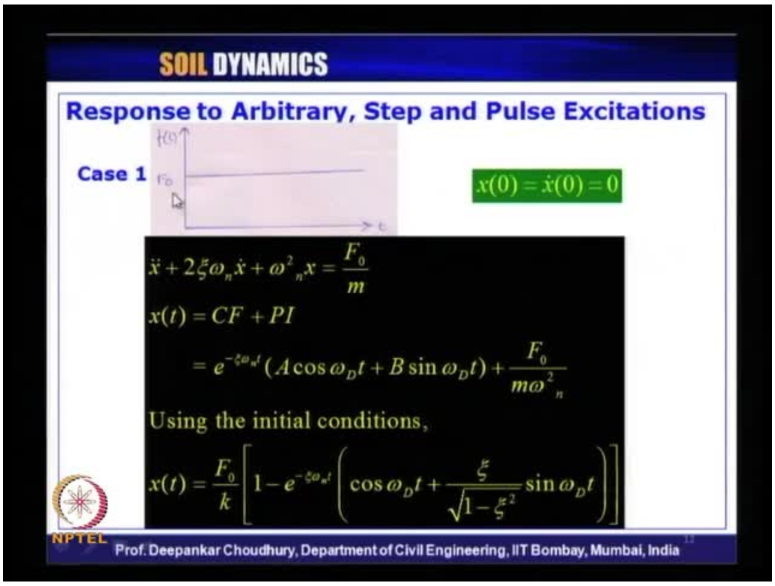

Case 1 we have considered a constant force acting on the system for infinite time, and with initial conditions of absolute rest, we got the solution like this.

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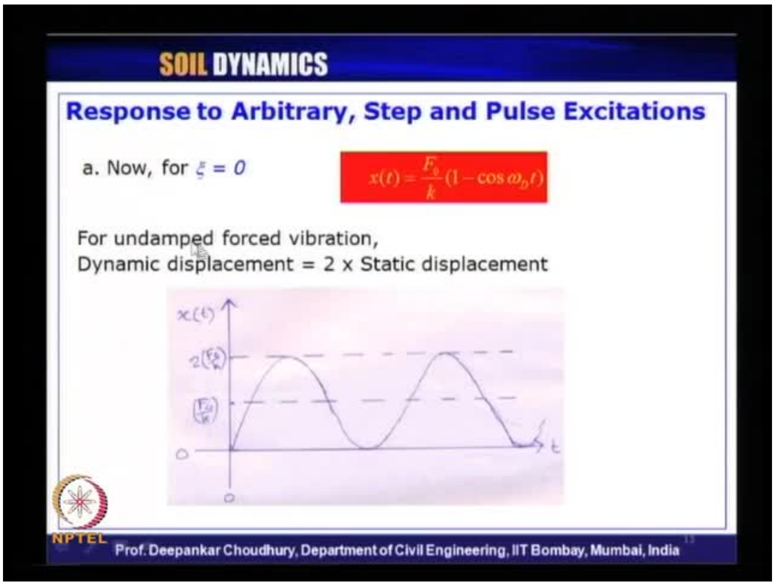

From which for a undamped system we found the vibration is oscillation about its static displacement with a dynamic magnification factor of 2.

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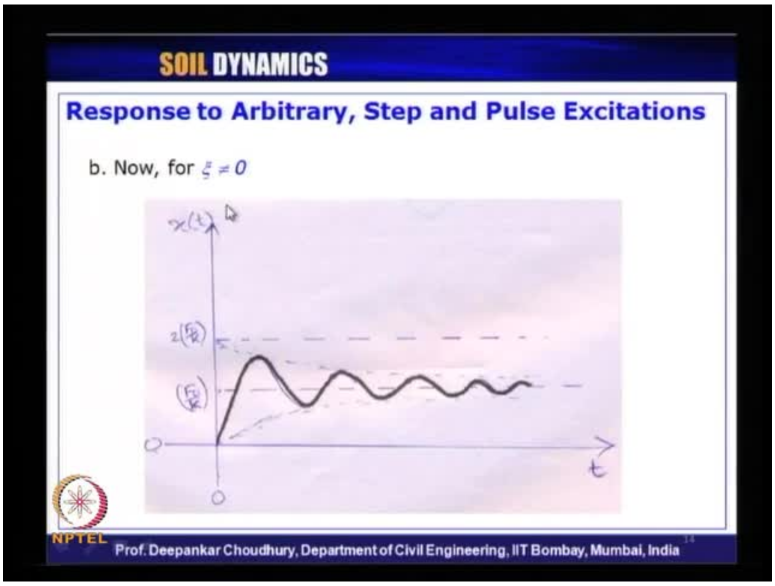

Whereas for a damped system, we found it is oscillating, but with a decay function and about this static displacement once again it was oscillating.

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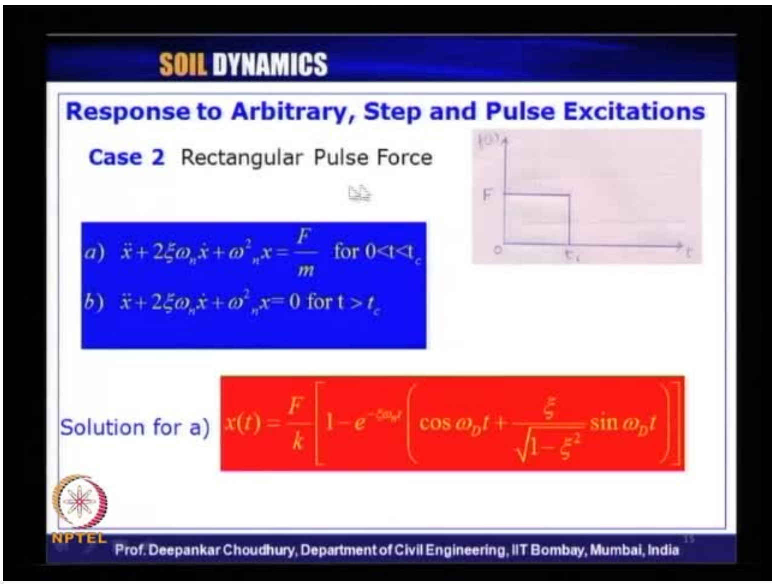

For rectangular pulse force like this we got two cases, one for a finite time when the force was acting and beyond that time, it is a free vibration.

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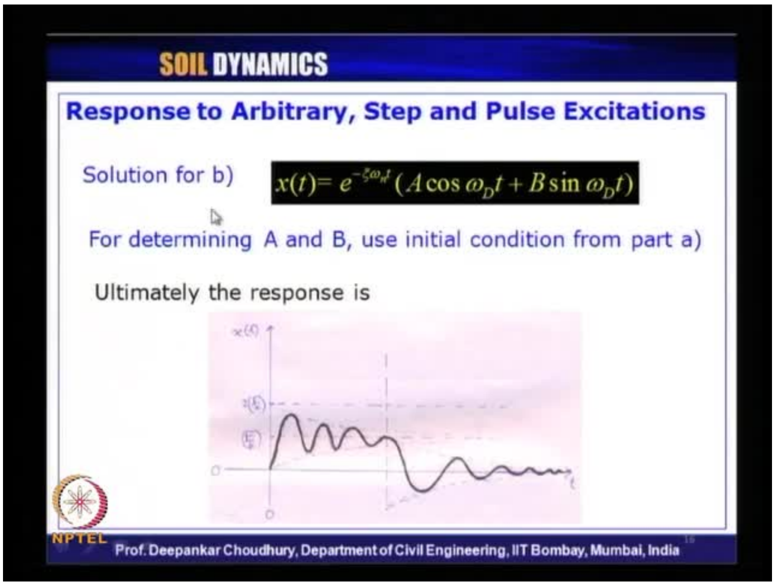

The complete solution for both part A and part B gives the response like this, initially it was oscillating when the load was acting till time T C about the static displacement beyond that it is oscillating about the base 0 line for the response.

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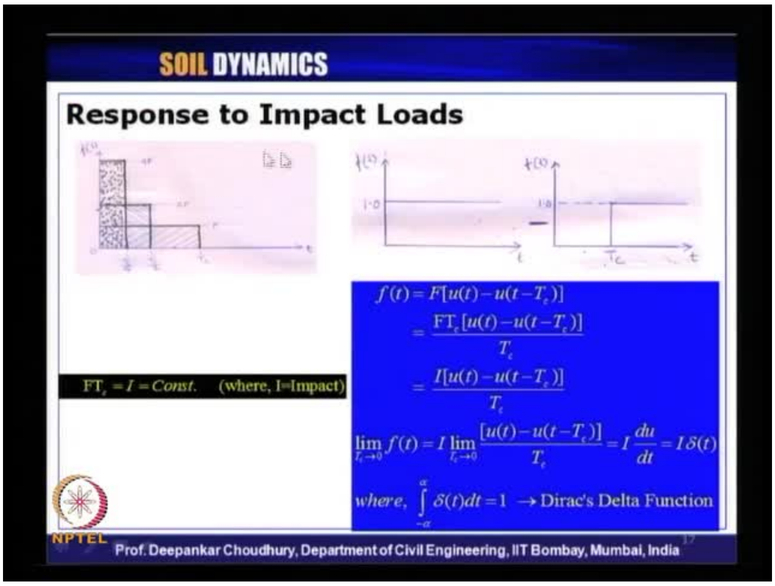

And then we have seen how to consider the response of impact load, how to mention the force function, the exciting force function for a impact in terms of Dirac's delta function.

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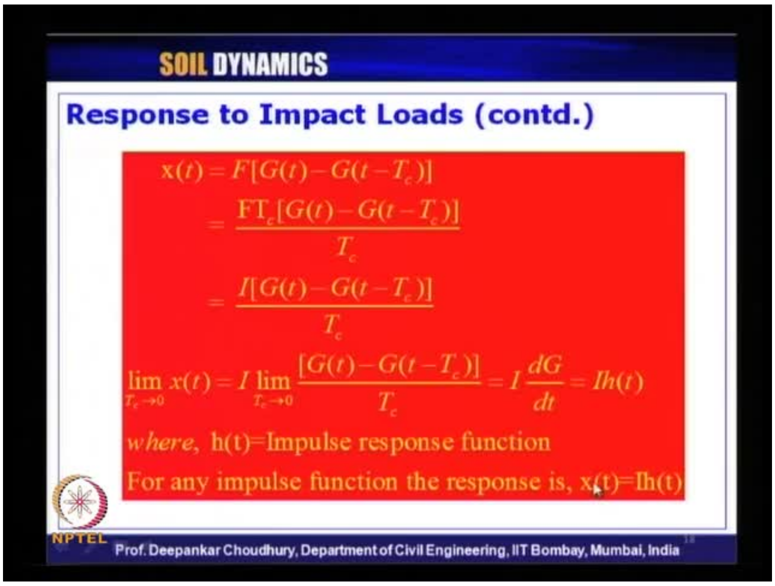

And finally, the response what we get it is called impulse response function h of t which is nothing but the differential of the indicial response function.

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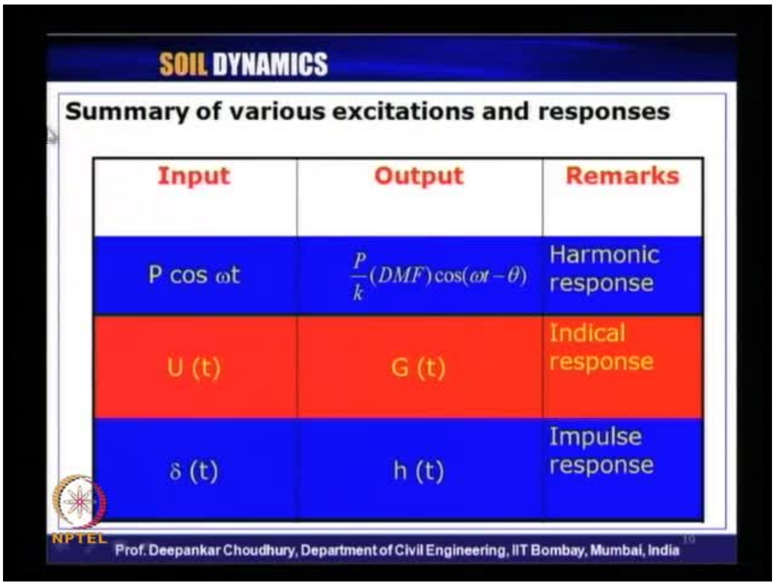

So, in summary what we have learnt till date, if we have input force vibration of harmonic in nature the output is the static displacement times the dynamic magnification factor and a harmonic function is the steady state displacement and response we call as harmonic response. Then we have seen if our input is heaviside step function then output is called indicial function or indicial response. It is mentioned that it is known as indicial response function. Also we have seen if the input is Dirac's delta function then the output is h of t which is known as impulse response function.

So, these are the three major categories of excitations for the forced vibration which we have come across for different input, what are the different output? And how and what way they are called. So, from next class onwards we will see for any arbitrary load. Till now we have not considered any arbitrary load, from next class onwards we will take any arbitrary load and how to get a solution for that we will see for a forced vibration.

Then for given any type of force to our single degree of freedom system, we can solve the problem and get the, its response so final solution of displacement with respect to time. With this we will end. So, we are continuing with the module 2 vibration theory. Let us look at the slides.

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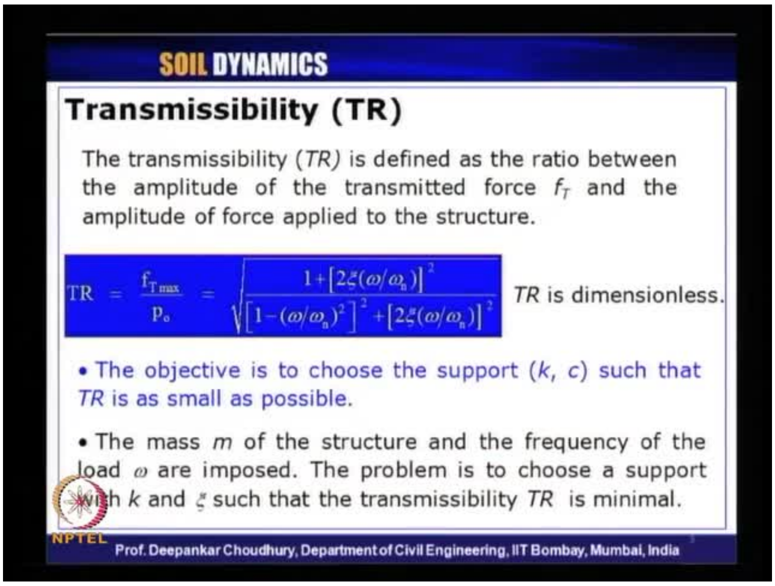

So, a quick recap of what we have learnt in the previous lecture. We have learnt about a new terminology called transmissibility ratio and how it is defined we have seen. It is nothing but the amplitude of transmitted force to the amplitude of force applied to the structure and the equation is given by this which is a dimensionless parameter.

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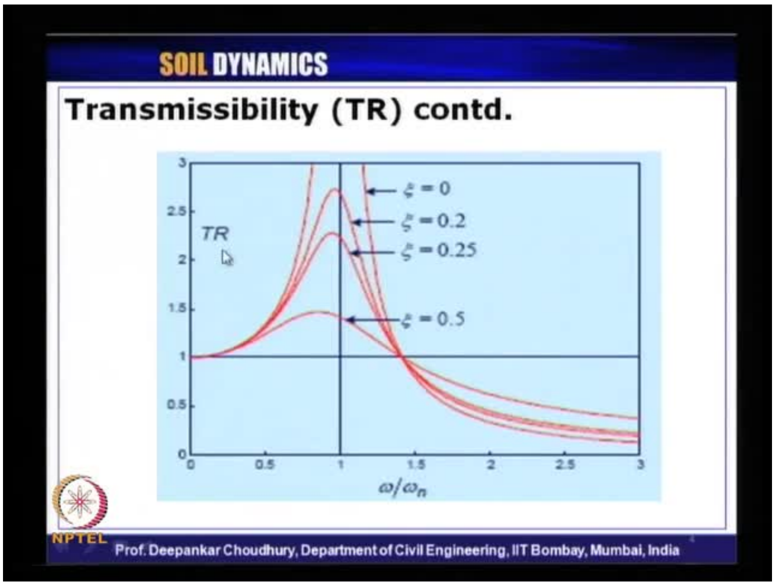

And the distribution of transmissibility ratio with frequency ratio for different values of damping ratio can be obtained like this and we have seen it starts from 1, transmissibility ratio is 1 when frequency ratio is 0, it again reaches the value of transmissibility ratio equals to 1 at a particular value of frequency ratio and we have obtained from derivation this value is root 2 and beyond that value of r equals to root 2 we get transmissibility ratio less than 1.

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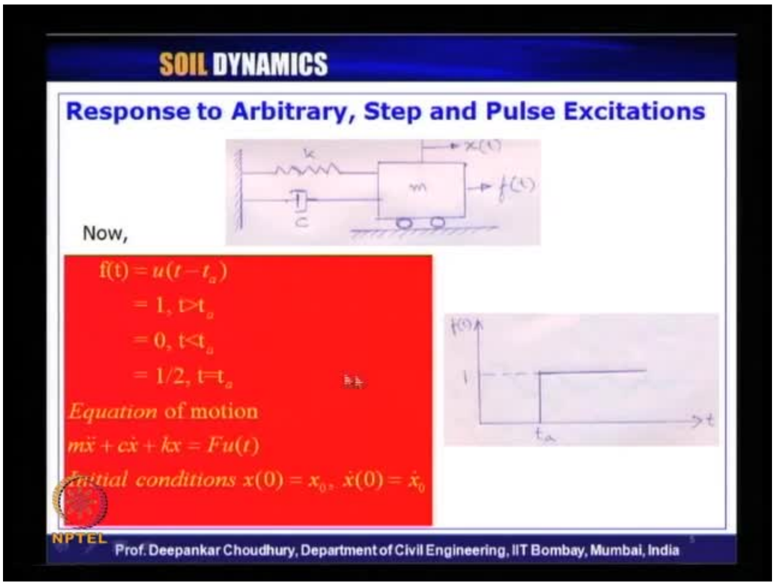

Then we have considered single degree of freedom system subjected to any kind of arbitrary excitation or step excitation and pulse excitation. So, this f of t we have expressed in terms of heaviside step function like this u of t minus t a, this is equals to considering this figure f of t varies with respect to time. It starts at time t equals to t a and then keeps continuing and the magnitude of it is unit value, then the heaviside step function is given by this expression mathematically and its value is unit when time is greater than t a, but when time is less than t a then it is 0 and at t equals to t a we take the average of these two and we get this value.

So, our equation of motion for any force f of t in terms of heaviside step function becomes F times that heaviside step function because this is for unit load so for any magnitude of load F, amplitude F we can express it like this.

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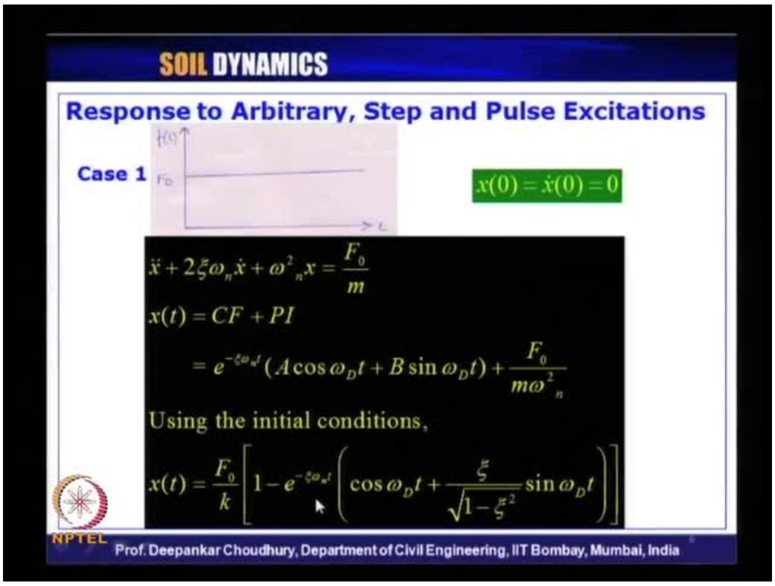

And now applying our initial conditions we have considered different cases like when there is a constant force applied to the system for infinite time with magnitude F naught and it is starting from absolute rest condition then we have seen what are the complimentary function and particular integral, the complete solution and then using the initial conditions we got the complete solution of the system like this.

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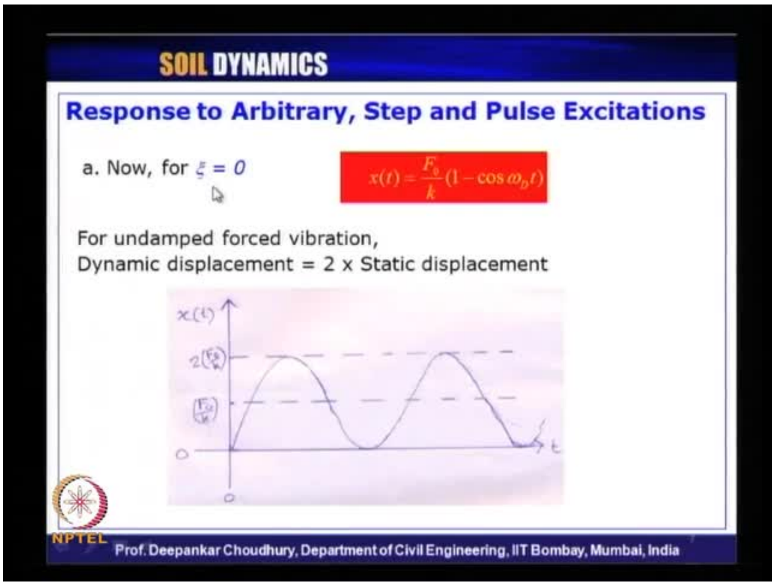

And if we considered the undamped system that is with damping ratio equals to 0 the solution is a harmonic function with the amplitude, it is oscillating about the base axis F naught by k which is nothing but our static displacement and the dynamic displacement maximum can occur two times of that. So, our dynamic magnification factor for this problem is nothing but two and the response we got like this.

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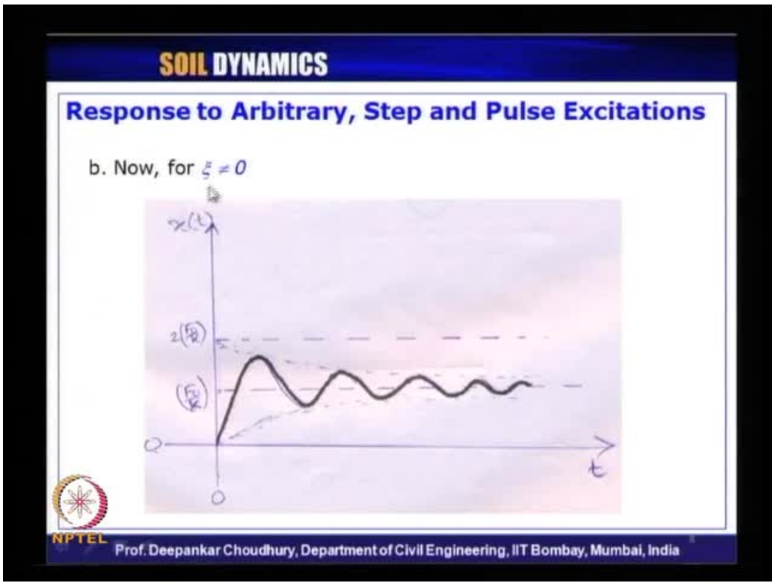

Whereas, for a damped system when the damping ratio is not 0 we have seen that it again oscillates about this base line of F naught by k, but there is a decay function so with time it decays and finally, at time t equals to infinity it reaches the static displacement.

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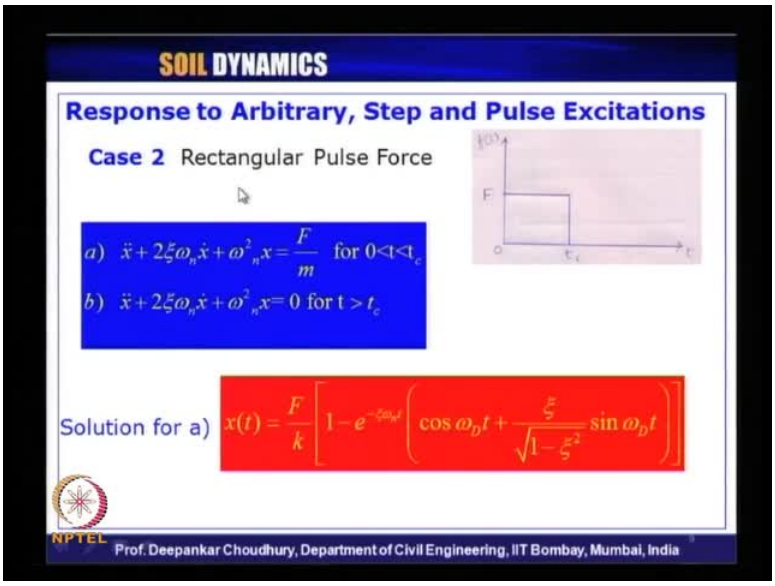

Then we have seen a rectangular pulse force which is acting for a finite amount of time T C with a constant magnitude F and that we can express in two equations. The first one when it is a forced vibration F by m for a time up to T C. Beyond time T C it is nothing but a case of free vibration and the complete solution for part a just now we have obtained like this.

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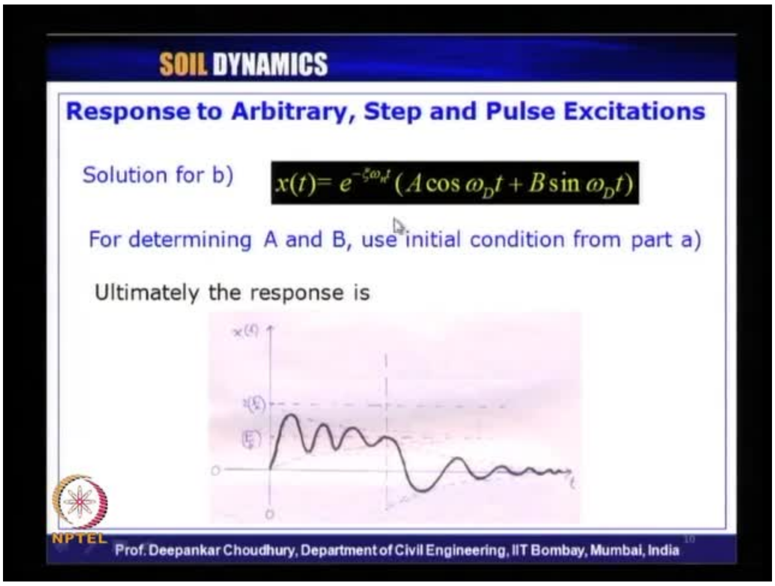

And for part b to obtain this two constant A and B what we have used? The initial condition was coming from the solution of this part A that is at t equals to t c whatever at the values of x of t and x dot of t those are the initial condition for our second part of the equation where for free vibration. So, that is why the ultimate or complete response we got like this that initially it is oscillating about this line F by k that is static displacement and beyond time t c it just drops and oscillates about this base line of 0. So, this was the complete response we have seen.

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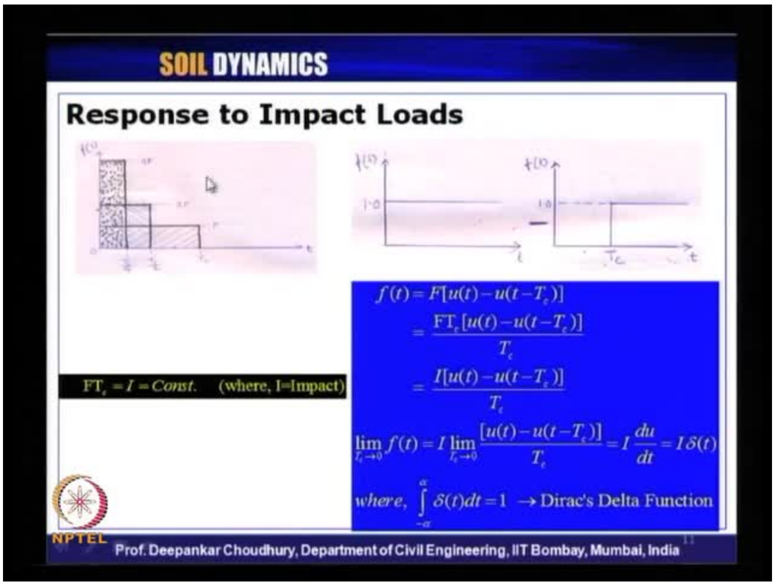

After that we have seen the response to impact loads. Impact load we can compute by keeping the area constant F of t, what we have seen? F times T C curved under this line we are keeping constant. So, that is what the concept of impact load is the time for which it is acting is very, very small however the magnitude of this load is pretty high. So, that is the concept of impact load and this type of loading we can express in terms of a unit load constantly applying, then we can deduct the remaining portion of this load from starting from time T C.

So, a mathematically we can express this function of impact load F of t like this which in terms of impact that is F times T C remains constant considering this as a impact value, we have expressed this in terms of this expression which is nothing but we have seen is expressed as I times delta t where delta t is nothing but the differential, first differential with respect to the time of heaviside step function and that delta t when we integrate it from minus infinity to plus infinity we can get the unit value, that is called Dirac’s delta function.

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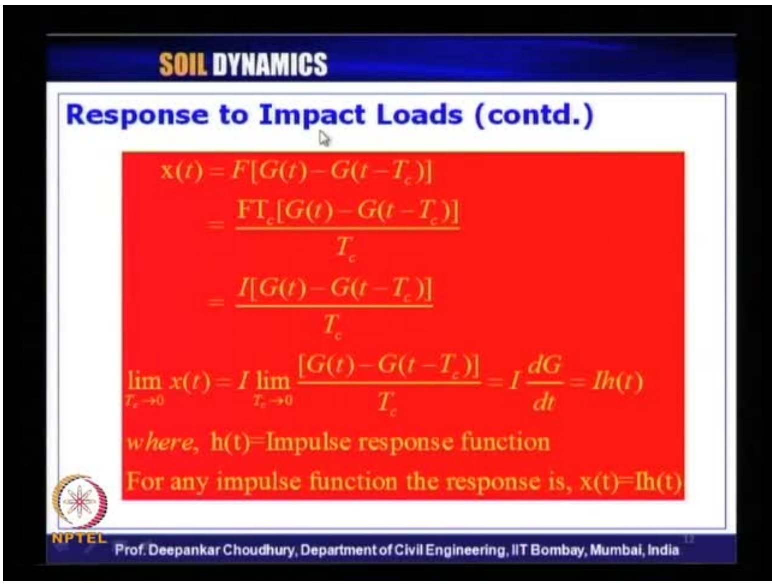

And the response to this impact load we have seen x of t which is nothing but for a force F magnitude it is nothing but the response of that heaviside step function which we have mentioned as indicial response. So, indicial response G of t for unit load and the remaining portion we have deducted as we have seen it expressed mathematically in terms of I times d G d t which is equals to I of h of t. h of t is called impulse response function which is nothing but the first derivative with respect to time the indicial response function.

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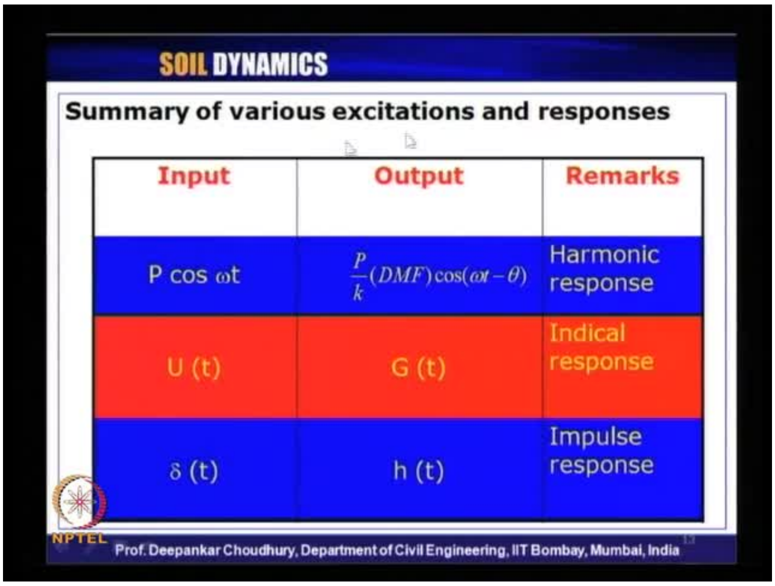

So, this was our summary of whatever we have studied for different cases of forced vibration, when the input is harmonic loading the output is also harmonic and these are remember, these output I am talking about only the steady state response. The complimentary function is always attached to it. So, these response of output means these are the steady state or long term response. So, this was our final output of steady state response for a harmonic loading and the response we call as harmonic response. When we have heaviside step function the response is indicial response, when we have Dirac's delta function then the output is h of t that is impulse response.

So, we had stopped up to that a portion. Let us start with any arbitrary load. So, when any arbitrary load is acting on a single degree of freedom system, how we can find out the response of that system? Let us see.

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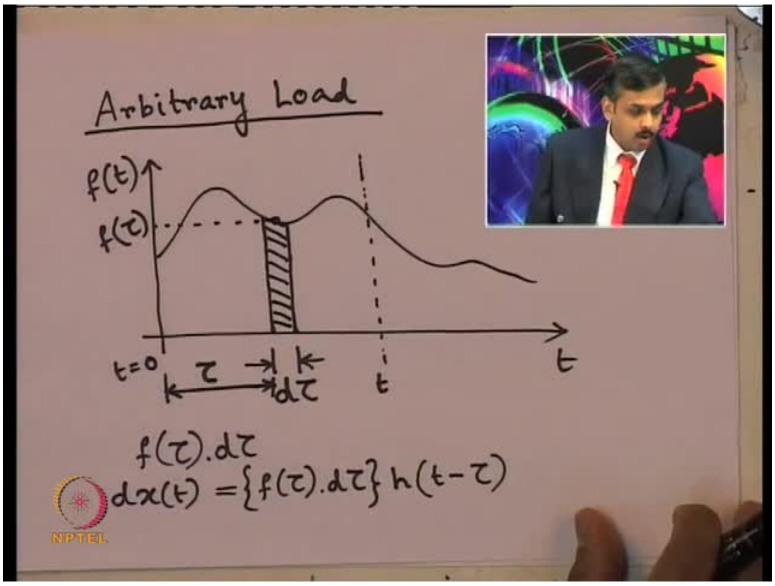

So, for arbitrary load we are expressing suppose the load acting on our system, the dynamic load of course, we are talking about something like this, some arbitrary load with respect to time. Now, this f of t varies arbitrarily with respect to time. What we can do? We can take a very small infinitesimally small slice. So, this slice we are considering under this curve. So, this small duration of time let us say d of tau. So, for us very small duration of time d of tau we are considering how the force is varying under this curve and suppose this occurs after a elapse of time tau.

So, from the beginning of the force that is at t equals to 0. From t equals to 0 after a time elapsed tau let us say we are considering an infinitesimally small time segment d tau and we want to find out how it varies and we want the final response let us say at any time t. So, at any time t we are interested to know about the response. At this point, infinitesimally small magnitude let us say the value of f of t is f of tau. The magnitude of f of t at the center point of this infinitesimally small time is f of tau, let us say. So, what we can say the area under the curve is how much f of tau times d tau, that is the area under the curve.

So, what we can write the response of any system which is subjected to this kind of loading what can be said? The d of x of t suppose, the response we are indicating for this infinitesimally small time duration for which that load is acting is d of x of t. That response is nothing but what we have seen, it is similar to our impact load. Why? The duration is very small and magnitude is having pretty high value. So, what is the response for a impulse load? The response for a impulse load is nothing but that impulse force I times h of t, impulse response function. So, how much is the value of I here? I is the nothing but area under the curve.

So, in this case it is f of tau times d tau that is our I times h of t. Now, h of t is the impulse response function when it starts from the beginning, but in this case the impulse load is acting after the elapse of time t equals to tau. So, it should be h of t minus tau. Do you agree with me? The impulse has started after an elapse of time tau. So, at any time t at any time t we want to know what is the response of the total system. So, the response should be I times h of t, in this case our t is t minus tau because it is started beyond time t equals to tau. So, the response of the system is given by this when it is subjected to this infinitesimally small load on the system, at any time t the response will be like this.

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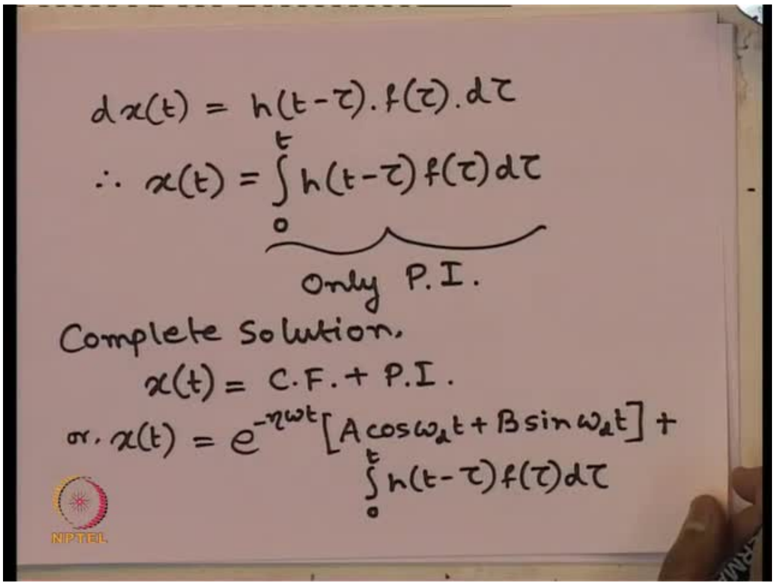

So, if it is so what we can rewrite this like d of x of t is given by h of t minus tau times f of tau times d tau. Just we have rearranged it. Therefore, the complete response x of t for the entire system which is subjected to that kind of several infinitesimally small impact load over a time say t what should be the total response nothing but the integration time t is 0 to t h of t minus tau f tau d tau. So, it is quite simple mathematics, what we do? We just integrate this infinitesimally small area over the time for which the load is acting, to say it is acting for a time up to t, then integration under the curve will give us the complete response.

So, the complete response should look like this x of t integral 0 to t h of t minus tau f tau d tau. Now, remember this part is the only particular integral part of the solution. So, the complete solution so complete solution for this problem that is any system subjected to arbitrary loading will be complimentary function plus particular integral which is nothing but x of t complimentary function is similar to the solution for free vibration. So, e to the power minus eta omega t A cosine of omega d t plus B sine of omega d t plus this component of particular integral that is integration 0 to t h of t minus tau f tau d tau, this is the complete solution for any system subjected to this kind of any arbitrary loading.

Suppose, for earthquake loading we can use this solution as the response of the system subjected to an earthquake loading. Now, we simplify this what we will get? Now, we have to use the initial conditions.

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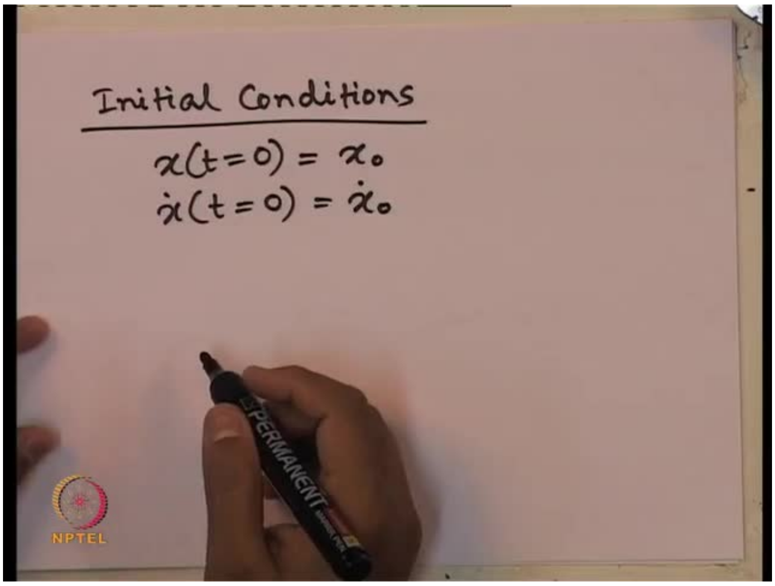

So, our initial conditions let us say a given as x at t equals to 0 is x naught and the velocity x dot at t equals to 0 is x naught dot as we consider for the other cases.

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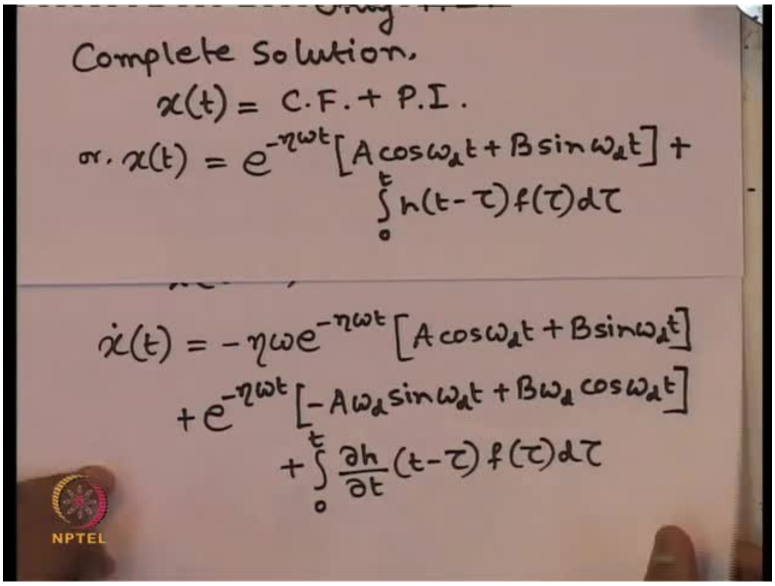

What we will get if we differentiate the solution with respect to time, the velocity we should get as so let me take the equation once again here. So, it will be easy for us to follow. Now, I am differentiating this displacement, I am differentiating this displacement function with respect to time to get the velocity. So, I should get minus eta omega e to the power minus eta omega t times A cosine omega d t plus B sine omega d t plus e to the power minus eta omega t minus A omega d sine of omega d t plus B omega d cosine of omega d t.

And then this particular integral part I am differentiating with respect to time, what we should get integration 0 to t del h over del t t minus tau f of tau d tau. Now, with this if we apply the boundary conditions what we should get? At t equals to 0 it is x naught. So, from this expression if we put t equals to 0 what we should get? We are only getting A, this is one A this vanishes, this also vanishes because time t equals to 0. So, we are remaining with only A equals to our x naught and what about if we put t equals to 0 for x dot we will get minus eta omega A plus B omega d plus this vanishes. So, this part vanishes with this, we will get the constants. Now, the constants are given by similarly, as we have seen for earlier cases.

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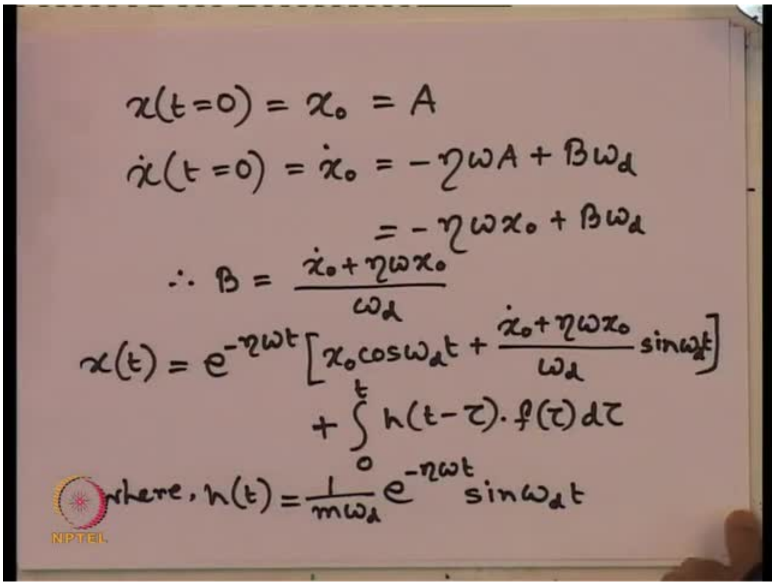

This is our A and x dot at t equals to 0 equals to x naught dot is nothing but minus eta omega A plus B omega d which is minus eta omega x naught plus B omega d. Therefore, B is x naught dot plus eta omega x naught by omega d. Therefore, the complete solution using the initial conditions takes the shape of e to the power minus eta omega t x naught cosine of omega d t plus x naught dot plus eta omega x naught by omega d sine of omega d t plus this particular solution 0 to t h of t minus tau f of tau d tau, and what we know? In this solution where we know h of t is given by 1 by m omega d e to the power minus eta omega t sine of omega d t.

So, this is the solution we know about h of t. So, for h of t minus tau we can write down the expression here and then we can integrate to get the total result. We can solve for this, for any arbitrary response, arbitrary loading for any arbitrary loading what is the response of the single degree of freedom system. Now, from this solution I can mention a special case, I can just put a remark for a special case when the system is starting from absolute rest condition, that is the initial displacement and initial velocity is exactly equals to 0. What will happen? So, this is the special case now I am discussing.

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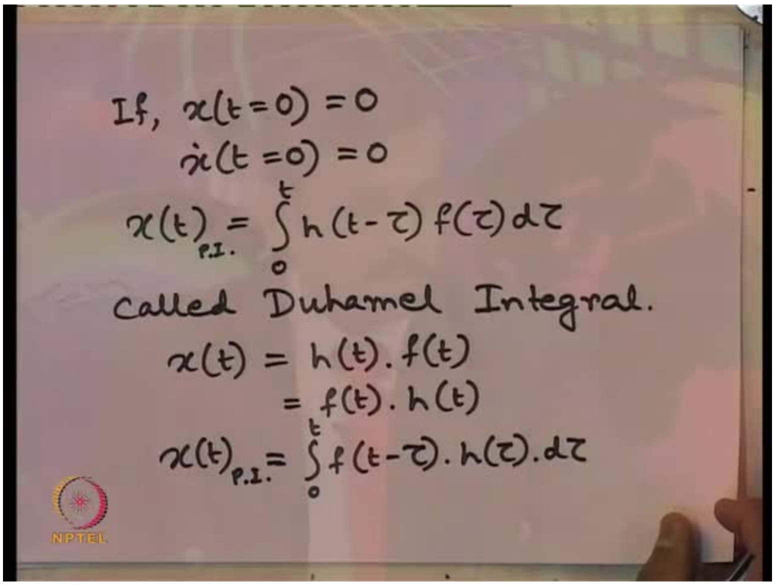

If, x at t is equals to 0 is 0 and x dot at t is equals to 0 is 0 in that case x of t which the particular integral part h of t minus tau f of tau d tau for t tends to infinity. So, I am talking about only the particular integral part in that case when the body starts from rest the steady state response, this gives us the steady state response, particular integral part always give us the response at time t tends to infinity. So, this is nothing but the steady state response that is called Duhamel integral. So, we can solve it the concept of Duhamel integral when it starts from rest.

What does it say? That if we have a function which is a product of two functions like this. Suppose, a function x of t is given by h of t multiplied by f of t then we can rewrite it in this form also. That is what the Duhamel integral says which means for our case x of t that P I we can rearrange and put it in this form f of t minus tau h tau d tau. So, they are interexchange able that is what the Duhamel integral says, if its starts from rest. So, it will be much easier for us to solve this expression, why? h of tau is known to us, h of t we know so in that expression you put h of tau then d tau with this you can integrate over the time t from varying from 0 to t.

So, in that way the complete solution for any system subjected to arbitrary dynamic load can be obtained using the concept of Duhamel integral if it starts from absolute rest. Now, let us take another special case of dynamic load.

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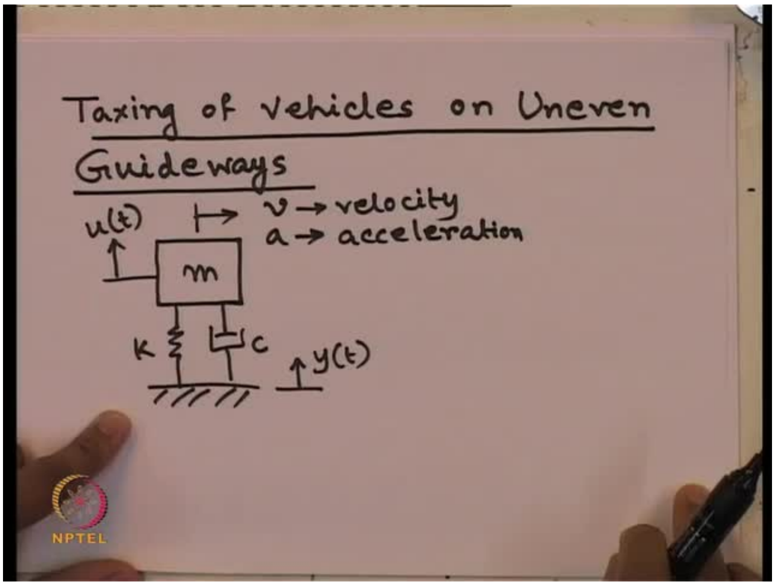

I have already mentioned earlier that is called taxing of vehicles on uneven guideways. Uneven guideways it can be pavement, it can be runway. So, in the first lecture I have mentioned that the origin of dynamic loads, one of the cause is also the undulations of the pavement, runway etcetera. So, when any vehicles is taxing means either starting or landing suppose in case of aircraft whether at the time of taking off or at the time of landing due to the unevenness of the runway there will be additional dynamic load to the system and why it happens? How it happens and how to get the solution of that kind of dynamic load which is acting on a single degree of freedom system?

Let us see how we can consider. So, I am drawing a single degree of freedom model of the system with mass m, I have say a displacement function single degree of freedom u of t, the system is like this, the stiffness damper. This is nothing but the vehicle; m is the mass of the vehicle or lumped mass of the vehicle which is in contact with the ground means at the time of taking off or landing. So, that is why taxing of vehicles and on the ground the displacement function, let us say it is given by y of t because of the undulations of the ground. And this vehicle is moving on the ground with say velocity v, v is the velocity of the vehicle and a is the acceleration of the vehicle.

So, with this velocity and acceleration for this vehicle what we can mention how this y of t varies with respect to a distance which is travelled by the vehicle. Let us see how we can express it? Suppose, the undulations or unevenness of the pavement or runway that is the guideways we express it in terms of a harmonic function. That is we are idealizing the undulations of the ground in terms of an harmonic function. You can consider other functions as well.

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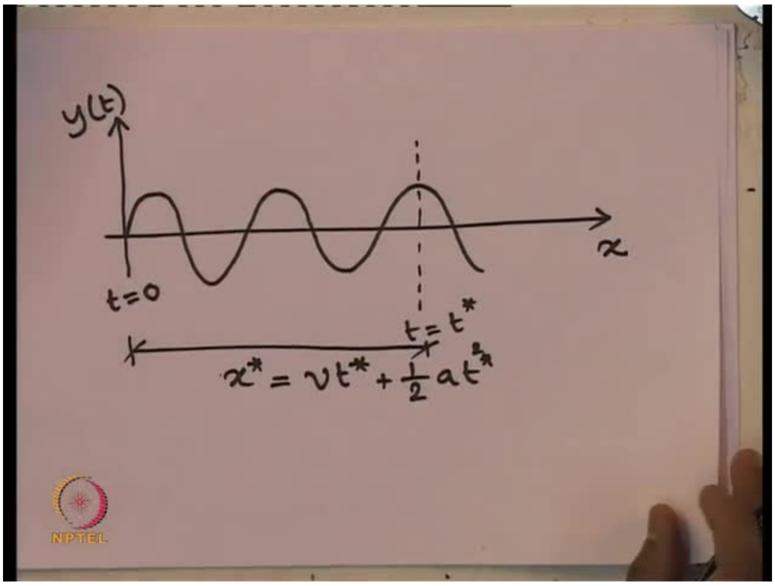

But, to start with let us say I am considering with respect to ground distance x this y of t the vertical this undulations say it is falling some harmonic function like this. That means ground undulations we have like this and this is at say t equals to 0 and the vehicle has reached this point at say t equals to t star that is after sometime the vehicle has to move horizontally right in the x direction. So, it has reached after time t equals to t star some distance of x.

So, that distance which is travelled by the vehicle x star after a time of t star how much it should be? It should be v of t star plus half of a t star square. The distance travelled by the vehicle is nothing but the velocity times time elapsed plus half of acceleration times time square. We know this equation. So, what happens when the vehicle reaches this time t equals to t star, what is the free body diagram of the system? Let us see.

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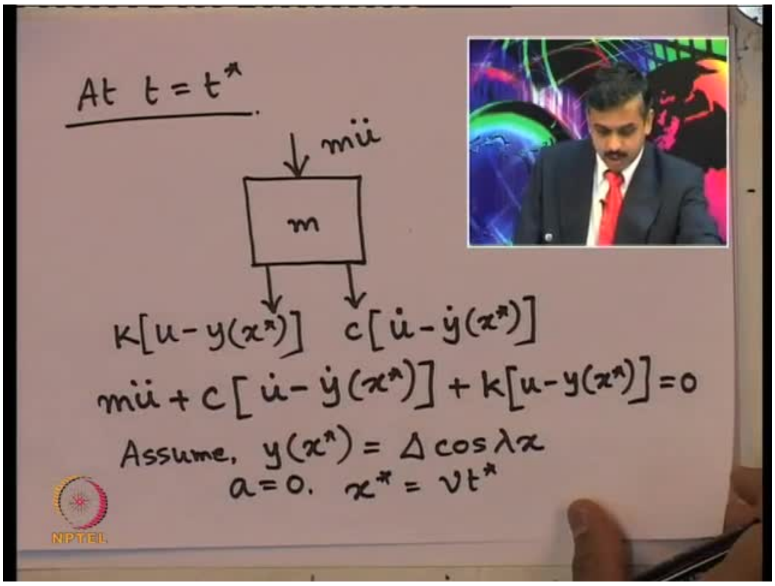

So, at t equals to t star, we are now interested to see the free body diagram of the system mass m, what should be the inertia of the system? Inertia will be m u double dot because u was the displacement at mass level. So, inertia of the system will be m u double dot and the spring force and damper force, how much it should be? The spring force should be k times the relative displacement between pavement and the mass level or the vehicle level whatever is the relative displacement. So, what was the displacement function we have considered? Let us look back in the basic first picture.

So, here this was the function for the displacement profile on the ground and this was the response at the mass level. So, the relative displacement between these two will give us the magnitude for the spring force as well as the damper force. So, the spring force should be K times u minus y of x star because y is the function of x star. We have seen, we have assumed that it is the harmonic function. So, at t equals to t star the displacement it has travelled is x star.

So, y at that time whatever is the displacement function, the relative displacement times the spring constant will give us the spring force and whatever the damper force, that should be C times the velocity u dot here minus the velocity at the pavement level y dot x star. So, relative velocity times C will give us the damper force. So, for this system now we apply the D Alembert's principle what we can write m u double dot plus C u dot minus y dot x star plus K times u minus y of x star equals to 0.

Now, let us assume this y function as a function of x in terms of just now we have considered as a harmonic function. So, assuming the undulations of the ground y of x star as a harmonic function say delta cos of lambda x where delta is the amplitude of the displacement at the ground and it is following a harmonic pattern. So, let us say cos of lambda x lambda being the frequency. That we can easily obtain from the wavelength conditions of the, because from the length and the velocity of the vehicle we can get the lambda value for the system.

So, with these assumption let us say another condition that the acceleration is 0. So, whatever is our x star that is given by v t star. So, it is moving at a constant velocity. So, in that case what we will get if we simplify and put this expression here?

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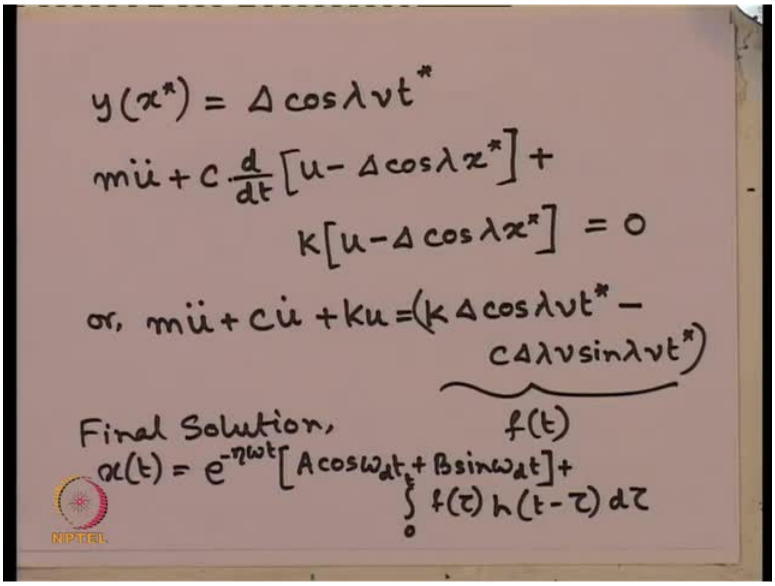

So, our y of x star is given by delta cos of lambda v t star, when it is moving at a constant velocity. Then y of x star is given by this. So, how the equation of motion is given m u double dot plus C d of d t u minus the displacement we are talking about. So, delta cos of lambda x star plus K times u minus delta cos of lambda x star, this equals to 0 or m u double dot plus C u dot plus K u equals to now let us take this K delta cos of lambda v t star minus, this is minus when it goes this side plus, but that we are differentiating with respect to time, so cos become minus sign so that is why minus minus C delta lambda v sine of lambda v t star.

So, this is nothing but our f of t. So, what we have done? Due to the guideway unevenness on that single degree of freedom system that is on the vehicle which is moving at a constant velocity on the pavement which is undulated in the form of a harmonic function. The basic governing equation of motion is given by this, where the dynamic load is coming because of the undulations and at an elapse of time t with lambda as frequency and delta as the amplitude of the ground movement. We can get this as the function.

So, the final solution of this we will get as similar to the other cases. The final solution we can represent as x of t equals to e to the power minus eta omega t A cosine of omega d t plus B sine of omega d t plus integral 0 to t f of tau h times t minus tau d tau because that was our solution for any arbitrary load.

So, this we are considering of f of t so for any arbitrary load this is the final solution. You can put this expression of f of t and get this integration done. Suppose, it is starting from absolute rest condition you use the Duhamel's integral and get the solution. So, that is the way we can find out what is the response of a moving vehicle on ground and this will give you an idea, if you take any example problem later on we will see. Suppose, there is an undulation because of that reason the dynamic load experienced at the vehicle when it is moving at a particular velocity we will see how it is related to the mass of the vehicle.

So, that is why I said earlier and at the first lecture suppose there is a heavy vehicle moving at a very high speed even though the undulations of the road are present we feel very less amount of jerking. So, this x of t nothing but it will give us the feeling of how much jerking we are getting, this u of t at the vehicle level. Whereas, if we have a very light weight vehicle, a small car that is mass is pretty small, if it tries to run or if it tries to move at a very high velocity we can experience a huge amount of u of t. Even it may give us the maximum DMF that is the resonance condition also can arise.

So, you can check all this aspects when you are designing a particular car that is what should be its limiting velocity, what should be the depending on your road conditions and of course, the undulations of the road etcetera. So, that can be designed very easily, fine. So, with this we will stop today's lecture here. We will continue our lecture in the next class.