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**ADVANCED GEOTECHNICAL**  
**ENGINEERING**

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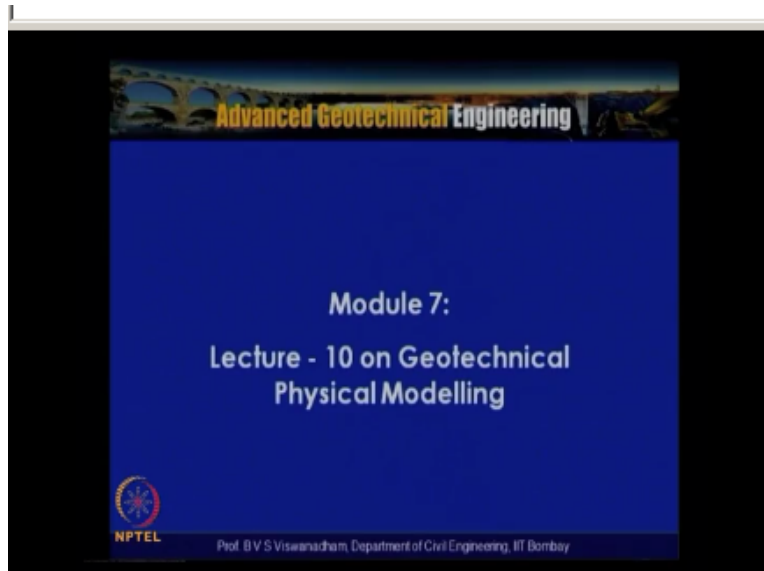
**Lecture No. 59**

**Module – 7**

**Lecture – 7 on Geotechnical**  
**Physical Modelling**

Welcome to lecture on advanced geotechnical engineering course we are in module 7 on geotechnical physical modeling lecture number 10.

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So we have introduced ourselves to the requirement of the centrifuge based physical model testing but in this particular lecture we will try to bring out relevant aspects of centrifuge based physical modeling to geotechnical problems especially when we take some selected problems in geotechnical engineering we will try to review whether the centrifuge based prescribed modeling is warranted or not warranted so we are aware now that small scale physical modeling can be performed at 1g or n g field so physical modeling at n g requires a geotechnical centrifuge to carry out model experiment.

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### Relevance of Centrifuge-based Physical Modelling to Geotechnical Problems

- Small-scale Physical modelling can be performed at 1g or Ng field. Physical modelling at Ng requires a geotechnical centrifuge to carry-out model experiments.
- Consider a few simple situations where physical modelling (small-scale) at 1g may be adequate and others where small-scale physical modeling at Ng will be required.

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So in order to carryout you know this centrifuge based physical model test require geotechnical centrifuge to induces high gravity now consider a few simple situations where physical modeling especially at small scale at 1g may be adequate and others where small scale physical modeling at ng will be required so where it is required where it is not required by taking some selected problems we can be review.

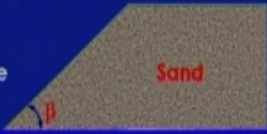
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**Slopes in Sand**

Consider a slope in sand.

$\beta > \phi$  Slope is unstable



The diagram shows a cross-section of a sand slope. The slope surface is a straight line from the top left to the bottom right. The angle between the horizontal ground line and the slope surface is labeled with the Greek letter beta ( $\beta$ ). The material of the slope is labeled 'Sand'. The friction angle  $\phi$  is indicated by a dashed line from the origin of the slope surface, representing the maximum angle for stability.

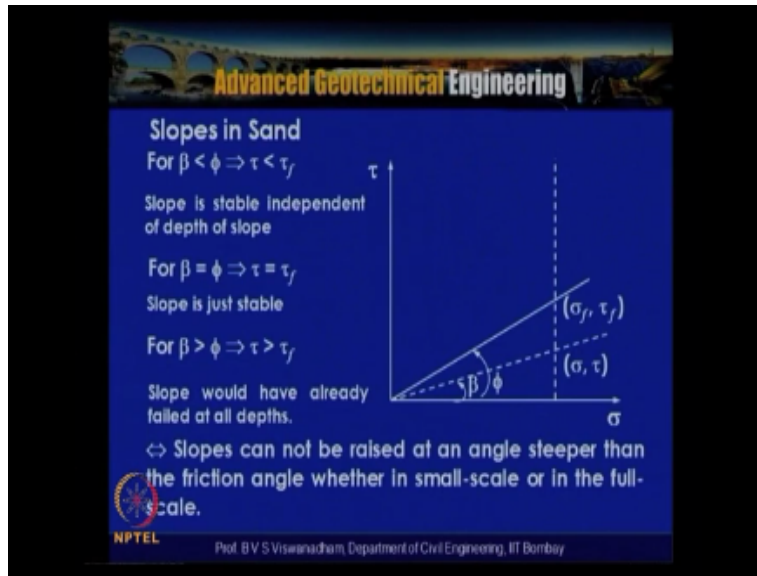
- > Slopes in sand are not stable at angles greater than the angle of repose  $\phi$  irrespective of the height of the slope.
- > For dry cohesion-less sand, the stability criterion may be stated as  $\beta < \phi$

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Now considered as a first example slope in sand so we are in this particular figure a typical slope which is formed with dry sand showed here and the slope inclination sand is and say  $\beta$  and certain height  $h$  now if we know that this  $\phi$  and is the friction angle and these the friction angle the maximum friction angle whatever it can have for stable condition is called angle of repose so for  $\beta$  greater than  $\phi$  that is for slope inclination greater than  $\phi$  the slope is non stable that means that the slope takes say you know profile which is equivalent to that shape friction angle so the slope is sand of not stable at angle is greater than angle of repose  $\phi$  irrespective height of the slope.

So where are will be the slope is sand of not stable at angle is greater than angle of repose irrespective height of slope so for dry cohesion less sand the stability criterion may be stated as  $\beta$  less than  $\phi$  for example if  $\beta$  is less than  $\phi$  we can say that stability of the slope can be ensured let us look how this can be explain by using  $\tau$   $\sigma$  plot.

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So consider you know y axis  $\tau$  and x axis  $\sigma$  and here this is the failure envelope and this is the friction angle so this is the friction angle whatever this soil can take and that  $\beta$  be this slope inclination now if you looking to this as long as  $\beta$  less than  $\phi$  the slope will be stable that has been we have been actually discussing suppose but  $\beta = \phi$  that means that all points you know in this particular line will be contact with the failure plane for  $\beta$  less than  $\phi$  the  $\tau$  is less than  $\tau_f$  that means that for  $\beta$  less than  $\phi$  you can see whatever may be the  $\sigma_1, \sigma_2, \sigma_3$  whatever may be the  $\sigma$  the  $\tau$  will be always less than  $\tau_f$   $\tau_f$  is the here stress failure and  $\tau$  is the here stress at the particular  $\sigma$ .

So for  $\beta$  equal to  $\phi$  the  $\tau = \tau_f$  for  $\beta = \phi$  moment this line is intersect with this line this line joins with this line then  $\tau = \tau_f$  so for  $\beta$  greater than  $\phi$  the  $\tau$  is greater than  $\tau_f$  so slope would have already failed at all depths that is means that slopes cannot be raised at an angle steeper than the friction angle whether in small scale or in the full scale test slopes cannot be raised at an angle steeper than the friction angle whether in small scale or in the full scale when you do not use any enforce cohesion.

So slope in sands when  $\beta$  less than  $\phi$  you can say that here for all levels of  $\sigma$   $\tau$  will be less than  $\tau_f$  similarly the slope is actually stable and independent of the depth of the slope for  $\beta = \phi$  then  $\tau = \tau_f$  the stable is just stable and for  $\beta$  greater than  $\phi$  the  $\tau$  is greater than  $\tau_f$  slope would have already failed at all depths slopes cannot be raised at an angle steeper than the friction angle whether in small scale or in the full scale.

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Un-drained analysis stability charts -

$$FS = \frac{cLR}{Wd} \quad (1) \quad \Rightarrow \text{FS safety circular arc analysis.}$$

$W = f(\gamma, H, \text{geometry of failure surface})$

$\Leftrightarrow$  Geometry of failure surface can be characterized by the three angles  $\alpha$ ,  $\beta$ , and  $\theta$

Rewriting (1):  $\frac{c}{FS} = c_r = \gamma H f(\alpha, \beta, \theta)$

$c_r$  = required cohesion to just maintain a stable slope and  $f(\alpha, \beta, \theta)$  is pure number, designated as the Stability number  $N_s$

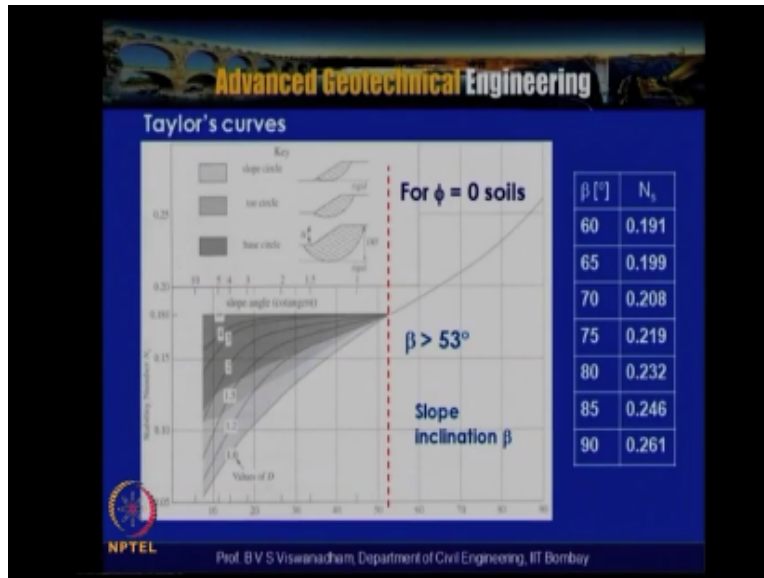
Taylor's Stability number  $N_s = \frac{c_r}{\gamma H}$

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So what we need to understand when we take in the sand is that it is irrespective of the height of the slope as long as  $\beta$  less than  $\phi$  you know the centrifuge model test are actually not warranted because even two thousand gravity or three thousand gravity whatever normal stress is actually induced this is independent of the depth so if you actually do a 1g model test with you know slope inclination less than  $\phi$  then whole group similarly let us consider you know this slopes in clay but before that let us look to the un-drained analysis stability by Taylor's method.

So from the Taylor's method actually has been reduced from the un-drained you know slope stability analysis where you know factor of safety is lowest factor safety is obtained from the circular arc analysis from this you can write weight of the you know the portion of the soil which is involved within the failure surface is the function of  $\gamma$  the unit weight and  $h$  height of the slope and geometry of the failure surface so geometry of failure surface can be characterized by the three angles which are called  $\alpha$ ,  $\beta$  and  $\theta$  and which are you know by rewriting.

One we can write  $c$  by factor of safety = it is indicated as  $c$  suffix  $r = \gamma h$  function of  $\alpha$ ,  $\beta$ ,  $\theta$  which is nothing but the so-called the geometry of surfaces indicated like this so  $c_r$  is required cohesion to just maintain a stable slope and function of  $\alpha$ ,  $\beta$ ,  $\theta$  is pure number and designated as the stability number  $n_s$  so this is actually the stability number which is put forward by the you know by Taylor in 1948 so Taylor stability number is given as  $n_s = c_r$  by  $\gamma h$ .  
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Now in order to get this you know for the different the stability number the Taylor actually has a Taylor curves where in the toes stability number actually is an y axis and this is for the different slope inclination let us see  $\beta$  is greater than 53 degrees we can see that you those then you know the independent of the depth factor and we can actually get you know different equalization like 60 degree the stability factor is above 191 so for 60 degree we can see that stability factor is above .191 and similarly for 90 degrees it is about .261 that is for the vertical cot.

So 1 by .261 into  $c$  by  $\gamma$  which is nothing by 3.81  $c$  u by  $\gamma$  which is equivalent to the critical height of s slope at which is four  $c$  by  $\gamma$  so by using Taylor's curves one can actually obtain the stability numbers and for  $\beta$  less than 53 degrees the stability number you know found.

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**Un-drained analysis stability charts – Taylor's method**

For  $\beta < 53^\circ$   $N_s = f(\beta, D/H)$

For gentle slopes, the critical failure surface goes below toe and always restricted above strong layer (hence depends on its location).

For  $\beta > 53^\circ$   $N_s = f(\beta)$  [all critical slip circles pass through toe]

This is because for such steep slopes, the critical failure surface passes through the toe of the slope and does not go below the toe.

For a vertical cut ( $\beta = 90^\circ$ )  $N_s = 0.26$  (short-term condition)

**Critical height**  $H_c = \frac{3.85 c}{\gamma}$  ← Obtained from Taylor stability number

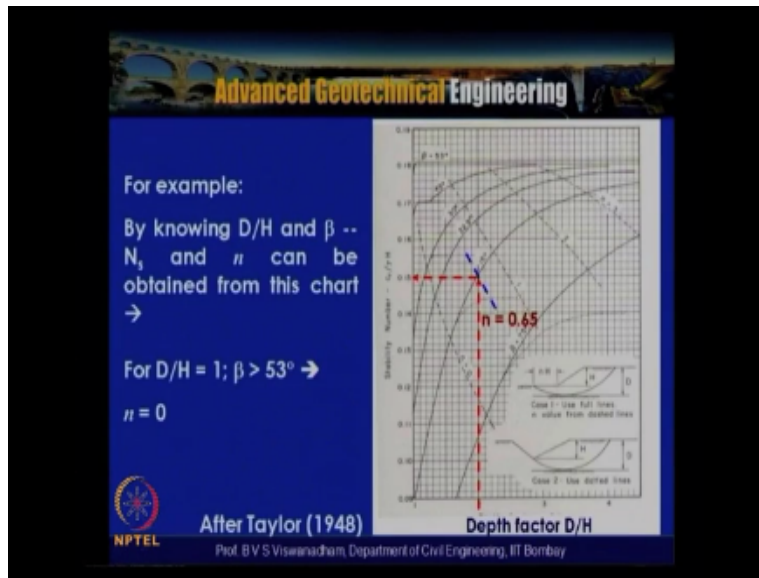
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To dependent upon you know so called D/H factor which is also indicate as small n so for gentle slopes , the critical failure surface goes below toe and always restricted above strong layer hence depends on its location so for  $\beta$  greater than 53 degrees the stability number is found to influence only on the slope inclination and failure surface is steep to past to toe of the slope so critical surface is pass to the toe so this is because for such steep slopes the critical failure surface passes through the toe at the slope and does not go below the toe so for a vertical cut  $\beta = 90$  degrees and  $n_s = 0.26$  under short term condition with factor safety =one at for critical height  $h_c$ .

We get 3.85 into  $c$  by  $\gamma$  this is actually obtain from a Taylor stability number so in case of un drained condition either with slope or with vertical cuts by using this we can actually get so why be this been shown this that this is actually used in designing some of the centrifuge model test actually particularly we know slope inclination which is actually greater than 53 degrees for you know showing the stability of slope undrained condition.

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So this is the in our example if you are actually having  $D/H$  and  $\beta$  which is less than say 53 degrees then this is the chart which required to be adapted we can see that in that case there is possibility that slope surface actually poses to the below the base so it is actually call base failure so this depth factor is given for depth factor  $D/H = 1$  it can be seen that  $\beta$  for  $\beta$  greater than 53  $n = 0$  that is the you know the this  $n = 0$  so that indicates that independent of you know the circumcircle this  $n$  value is 0 indicates that this  $n$  value is 0 indicate that slope actually.

The failure surface poses through the toe of the slope so that what is begin  $NH$  is nothing but the distance from the toe of the slope  $h$  is height of the slope and  $d$  is total height when it is given here so for by keeping  $d$  by  $h$  and  $\beta$  and  $n$  can be obtain and for  $D/H = 1$  and  $\beta$  greater than 53 degrees we get  $n = 0$  so that is indicates that you know the slope surface surfaces passes to the you know the toe of the slope.

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The slide features a blue background with white text. At the top, there is a banner with the text 'Advanced Geotechnical Engineering' in a stylized font. Below the banner, the title 'Slopes in Sand' is centered. Two bullet points are listed, each starting with a right-pointing arrow. The first bullet point discusses the relationship between the slope angle and the friction angle. The second bullet point describes the forces acting on the slope. At the bottom left, there is a circular logo for NPTEL. At the bottom right, the name and affiliation of the professor are provided.

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### Slopes in Sand

- If  $\beta < \phi$ , slope can be raised to any height even at  $1g$  or  $Ng$ , the stable slope angle is going to be the same.
- In this case, the actuating force is the body force due to gravity and the resisting force is the shearing resistance due to friction.

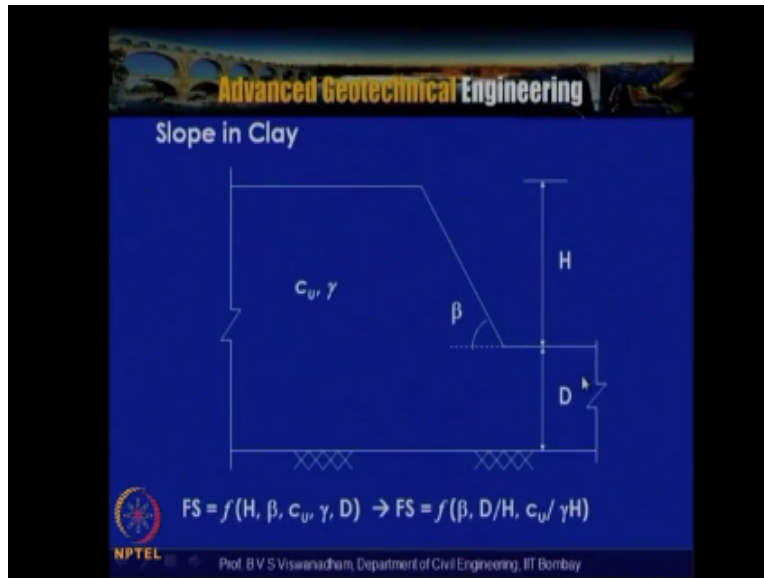
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So after having taken you know this with slope in sand we said that  $\beta$  less than  $\phi$  the slope can be raised to any height even at  $1g$  or  $Ng$  and stable slope angle is going to be the same and in this case the actuating force is the body force due to gravity and the resisting force is the shearing resistance due to friction so in case of slope in sand what we said this that if a  $\beta$  less than  $\phi$  the slope can be raised to any height even at  $1g$  or  $Ng$  and the stable slope angle is going to be the same in this case the actuating force is the body force due to gravity and the resisting force is the shearing resistance due to friction.

So let us see with the introduction from slopes in clay particularly with undrained nrc from by given Taylor stability charts lets us looking to how it can deduced by slope in clay so consider a slope in clay where we have got a  $c_u$  and  $\gamma$  as the you know soil parameters  $\gamma$  is the unit weight of the soil here in this slight the  $d$  indicates as depth below.

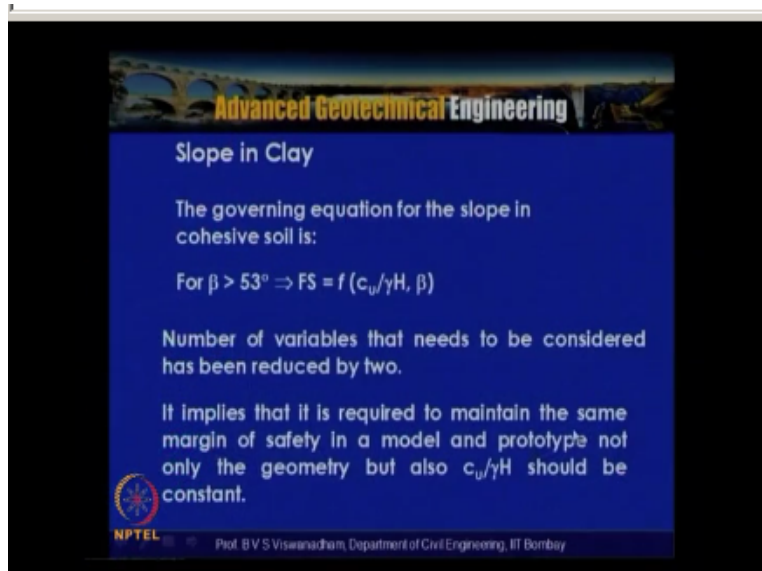
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The toe up to the film layer and  $h$  is height of the slope  $\beta$  is the slope inclination so we know that the factor of safety is nothing but a function of  $h, \beta, c_u, \gamma, d$  by using the either Buckingham theorem or rallies method we can say that the factor of safety =  $\beta$  function of  $\beta, d$  by  $h$  and  $c_u$  by  $\gamma h$  the reset that similarity between model prototype each and every  $\phi$  term has to be identical so in the process we discussed that the  $c_u$  by  $\gamma h$  has to be reduce that  $1/n$  times in order to if you actually having a small scale model reduced by  $1/n$  and times at normal gravity.

In case if you having a small scale model tested at  $\gamma_m, \gamma_p$  that is enhanced gravity then we say that  $c_u$  by  $\gamma$  need not be reduce and then automatically get deduce once  $\gamma$  increase to two  $\gamma$  in model becomes  $n \gamma$  so but it insures that to  $c_u$  by modern prototype of identical so  $c_u$  in modern prototype identical physically means that this resented be of the soil is retained so that insures that there is similarity between that model and prototype so this we have already discuss so here it implies that in order to maintain.

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### Slope in Clay

The governing equation for the slope in cohesive soil is:

For  $\beta > 53^\circ \Rightarrow FS = f(c_u/\gamma H, \beta)$

Number of variables that needs to be considered has been reduced by two.

It implies that it is required to maintain the same margin of safety in a model and prototype not only the geometry but also  $c_u/\gamma H$  should be constant.

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The margin of safety in a model and prototype not only geometry but also  $c_u$  by  $\gamma h$  should be same.

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### Slope in Clay

If  $g$  can be increased by a scale factor  $N$ :

$$\frac{c_u}{FS\rho gH} = f\left(\beta, \frac{D}{H}\right)$$

If we increase  $g$  as we reduce  $H$  then the soil particles, strength  $c_u$  and density  $\rho$  can be kept unchanged.

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So if  $g$  can be increased by a scale factor  $n$  that was actually we have been talking  $c_u$  by factor safety  $\rho g H = \text{function of } \beta \text{ into } d / h$  if we increase  $g$  as we reduce  $h$  then the soil particles strength  $c_u$  and density  $\rho$  can be kept and change.

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**Slope in Clay**

Based on Taylor's stability chart, the maximum stable height of the slope, i.e.  $h_c$  (at  $FS = 1$ ) for  $\beta = 60^\circ$  is given by:

$$h_c = \frac{1}{0.19} \left( \frac{c_u}{\gamma} \right)$$

The acting force is the body force due to gravity and the resisting force is the constant un-drained cohesion.

➤ Hence, it is not possible to carry-out small-scale model test at 1g on slope in clay, as the slope will fail only when  $h \rightarrow h_c$ .

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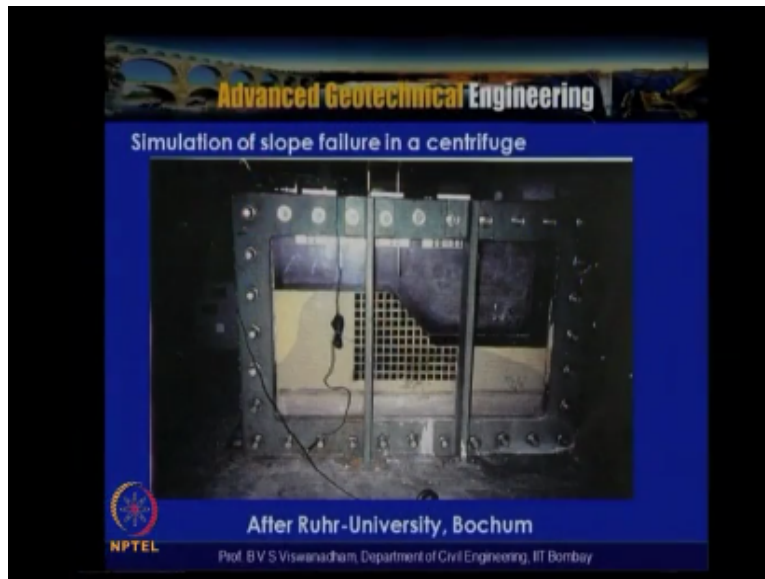
So based on Taylor's stability chart which we have discussed just now the maximum slope height of slope that is for the factor of safety = one for  $\beta = 60$  degrees is given by stability factor is .191 so we can write  $h_c = 1$  by .191 into  $c_u$  by  $\gamma$  so this indicates that in order to have in order to attain you know critical height the slope actually has to raise to this height so that you know it actually have factor of safety = one suppose if you having  $h_m$  model slope height then the ratio of  $h_c$  by  $h_m$  will be you know very high and then you which will actually ensure by factor of safety will be very high.

So the acting force is the body force due to gravity and resisting force is the constant undrained cohesion so in this case we acting in the body force due to gravity and resisting force is the constant undrained cohesion here it is not possible to carry out small scale model at 1g on slope in clay as the slope will fail only and when  $h$  vs  $h_c$  what we have understood in this case particularly with slope in clay is that based on Taylor's stability chart the maximum stable height of the slope is you know at factor safety one is for the  $\beta$  is goes to 60 degrees.

So the acting force is the body force due to gravity and the resisting forces is the constant undrained cohesion so here there is slope is only will fail if  $h \geq h_c$  if you actually maintain the slope prepared deduce by 1 by  $n$  time at kept in the elaborator it only tent to dry because it is actually has got very factor of safety in small physical dimensions so here it actually says that it centrifuge model testing is actually warranted for you understanding for stability of slope in undrained delay and also there is a requirement that the  $c_u$  has to be maintain constant and you know actually clearly says.

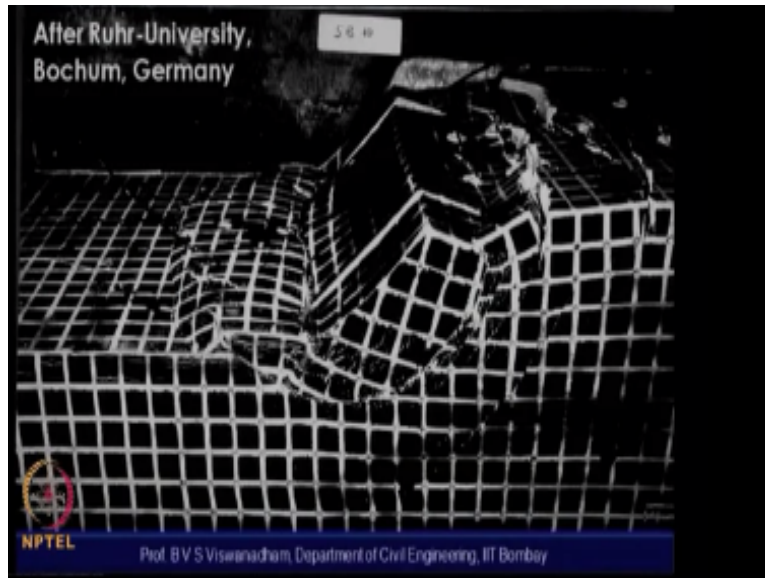
That you know it is not possible to carry out the small scale tested 1g on slope in clay as the slope actually will only fail  $h$  tents to  $h_c$  so let us see some typical you know the slope failure in a centrifuge.

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This is after Ruhr-university Bochum a front view of the slope before testing is actually shown here and this is one of the traditional fair testing of the slope which is actually with slope and clay.

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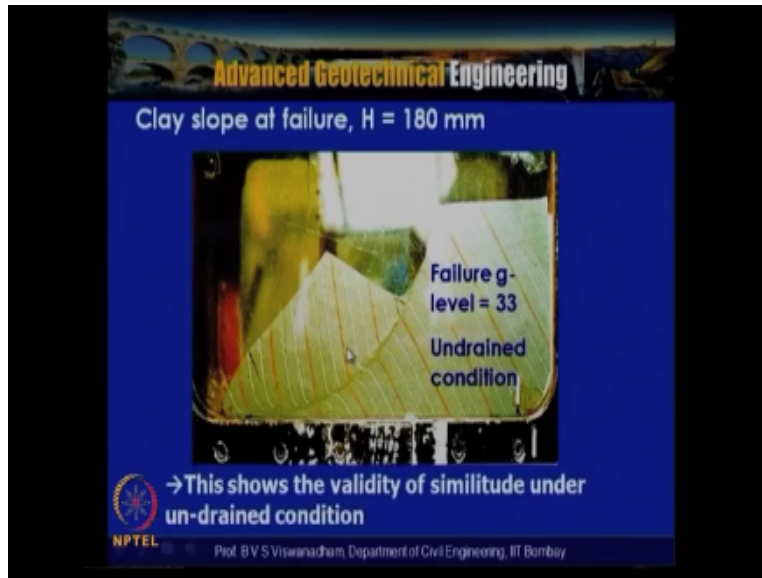


You can see that once the slope is actually you know subjected to failure you can see that development of the slip surface here so this is the formation this surface and the having parts see here the hive partion have seen here and the tension cracks formation can be seen here so this is the slope which is actually formed with saturated consoled clay and you can see that the tension cracks so these are the earlier you know imprints which actually used for the distance wishing you know the across ion of the failure face now days the advancement of techniques like particles base or digital maze correction techniques allow you to looking to even the you know formation of shear band thickness at the formation.

Of if does plain and also in front termination of model before due net failure so there is actually important information can be drive so you can see the classical you know strip circle if you new which is actually shown here from the testing done it Ruhr university Bochum Germany.

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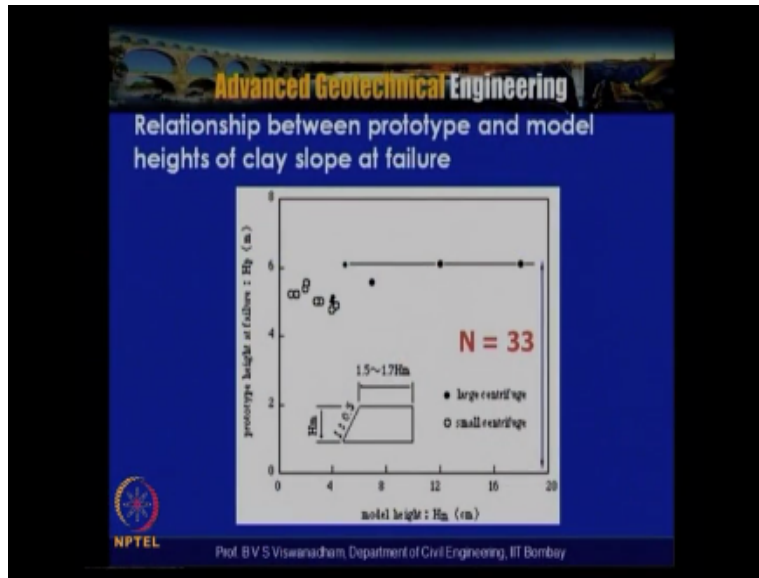




So this is a typical undrained testing which actually the slope model height of the slope is about 18cms that is  $h$  is 18cm what is actually happened is that moment the height you know the gravity level increase to 33 gravities then we can see that slope is actually undrained is strip circle failure and this shows the validity of similitude under undrained conditions.

So this shows that the requirement of the you know the centrifuge based model test to induce failure and then here it means that the critical height of the slope is nothing but  $33 \times 180 \text{ mm}$  that is equivalent height and meters is the you know critical height at of the slope at this particular soil at failure. So this relationship between prototype and model heights of clay slope of failure is shown here.

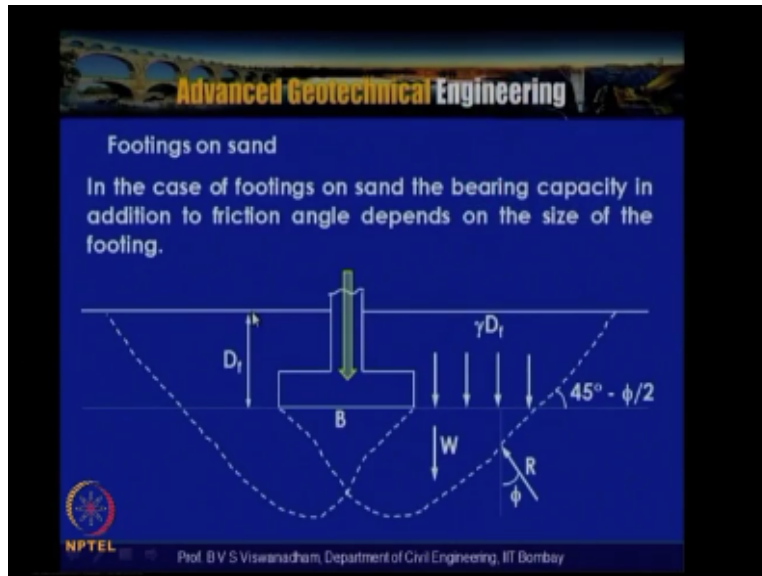
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HM= 18cm it can be seen that you know the slope is actually found to fail at 33 gravities which is actually show here is about the height of about 6m so this different height of the slope has been tested and you can see that the here this are large centrifuge equipment for use these here the small centrifuge equipment are use so the errors due to small centrifuge testing actually lead to some you know dispense but what can be seen this that when you have large being centrifuge.

We can see that the consistency result the modeling of the model found to speak well and indicate the performance of a you know critical height of if failure whether it is a 5m or whether it is at 12 cm or whether it is 18cm we can see that horizontal plate to can be obtain so this is the prototype height in the meters in y axis and model height in x axis for example here it shows the when model height is somewhere 18cms and if a prototype height cannot be equivalent to that because this is the height at failure so it actually has you know this is factor safety =one so the beret of the large centrifuge also is shown in the particular slide.

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So after having the seen two examples of slope in sand where in we set that centrifuge model testing is not required as long as the sloping relation is less than 5 that be tell us 5 and if the sloping inclination actually is greater than 5 we said that the slope are 10 to 5 but when we come to slope in clay particularly undrained condition  $\phi = 0$  we said that the centrifuge model testing is warranted and let as try to look some two different dusting bearing capacity problem one is the footing on sand let us say consider a footing having a with b is subjected to in embedded to depth of  $d_f$  a subjected to constant rick load of  $q$  in the case of footings.

On sand the bearing capacity in addition to friction angle depends on the size of the footing so let us looking into these are typical failure plains we can see that is the you know the elastic verge is actually formed and this are the radial zones and this is actually resistance generator in the embedded depth zone for above the base of the footing so this insulation is above  $45^\circ - \phi/2$  and this size is the depth of the footing and this is resistance actually derive to account of this movement.

For example when the footing load is apply and this block moves this side and this block moves this side and this is accounted by the friction actually mobilize along the periphery of this failure surface this is are failure surface is typically failure surface which are actually shown in this particular slide. So this is effect of the embedded depth of a footing that is nothing but  $\gamma d_f$  that is conferment due to embedded depth.

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Footings on sand

Net Ult. BC =  $q_d = \frac{1}{2} \gamma B N_\gamma$

$$q_d = \left[ \frac{\gamma N_q}{2} + \gamma (N_q - 1) \frac{D_f}{B} \right] B$$


> This implies that wider the footing the greater is the bearing capacity. Further, BC of a footing on sand is derived from two sources: (i) Frictional resistance due to weight of the sand below the level of the footing; and (ii) Frictional resistance due to weight of sand surrounding surcharge or the backfill.

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Now we can actually getting for the footing on sand net ultimate by capacity there is nothing but q ultimate -  $\gamma df$  is nothing but  $q_d = \frac{1}{2} \gamma b n_\gamma + \gamma df$  into  $n_q - 1$  so this is we are taken like continues footing so varying we written that ultimate capacity =  $\frac{1}{2} \gamma b n_\gamma + \gamma df$  into  $n_q - 1$  now this is simplified by writing  $q_d = \frac{\gamma N_q}{2} + \gamma (N_q - 1) \frac{D_f}{B}$  the whole bracket is multiply by b so this implies that wider the footing the greater is the bearing capacity further bc of a footing on sand is derived from two sources one is frictional resistance due.

To weight of the sand below the level of the footing and another one is frictional resistance due to weight of sand surrounding surcharge or the backfill that is you know due to this you know all around the footing particularly in this zone also so one is resistance half in this zone and resistance half in this zone.

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Footings on sand

This implies that a study conducted at 1g does not help in predicting the bearing capacity in the prototype.

This is also evident from:

$$\frac{q}{\gamma B} = f\left(e, \phi, \frac{\sigma_c}{\gamma B}, \frac{\sigma_a}{\gamma B}, \frac{E_a}{\gamma B}, \frac{d_f}{B}\right)$$

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So this implies that a study conducted at 1g does not help in predicting the bearing capacity in the prototype because the smallest the footing than you know we have got the less is the Alfred bearing capacity so B reduced by one n times the b becomes smalls so the bearing capacity also will be small so this is also you know evident we have discussed that ultimate bearing capacity of footing is found to function of several parameters and we have said that  $q$  by  $\gamma b$  =function of  $e$  and friction angle and  $\sigma c$  by  $\gamma b$  that is crushing strength of the grains and grain.

To grain cohesion that is  $\sigma g$  by  $\gamma b$  and  $eg$  that is elastic mode of the grain  $ec$  by  $\gamma b$  and the  $dg$  by  $b$  that is the  $dg$  nothing but the average part particles size raise to 2 ratio of average particles size to breadth of the footing  $b$  now we have said that footings on the sand because of the requirement whatever we have discuss in order to have the similarity centrifuge model testing is required because where the  $q$  ultimate is actually found to function of  $qd$  the ultimate to bearing capacity found to function of the size of the footing let us see in case of clay undrained condition.

Where  $\phi = 0$  and we have the only resistance from  $c_u$  that is strength of the soil so weight of wedge and the shear strength of the soil along failure plane tends to resist failure so you can see that the failure plans are distaining to difference you know in case of a you know clay in case of the you know sand you can see that the failure plain the elastic wedge is actually form and the failure plain actually this is portion is actually shifted vertically down so this is because of the roughness and friction caused by the vertical that is the surrounding soil and that is the sandy soil but incase of clay that is the option.

Because of that actually have you know the failure plain starts immediately from that elastic wedge formation will not be will be non existing so you can see that this is the typical failure surface which is actually forms here.

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**Footings on clay**

The normal forces across the surface of sliding can produce no frictional shear resistance (on account of  $\phi_u = 0^\circ$ )

Ult. BC =  $q_u = c_u N_c + \gamma D_f$

Net Ult. BC =  $q_d = c_u N_c + \gamma D_f - \gamma D_f$   
 $= c_u N_c$

For long and continuous footings,  $q_d = 5.14 c_u$

For rectangular footings (B X L) footings,

$q_d = 5.14 c_u \left(1 + 0.2 \frac{D_f}{B}\right) \left(1 + 0.2 \frac{B}{L}\right)$

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So for the footing on clay the normal forces across the surface of sliding can produce no frictional shear resistance or an account at  $\phi_u = 0$  ultimate bearing is given by  $q = c_u n_c + \gamma d_f$  then net ultimate bearing capacity can be obtain by  $q_d$   $q_d = c_u n_c + \gamma d_f - \gamma d_f$  so it is actually function of only shear strength soil and  $n_c$  for long and continues footings we can write net ultimate bearing  $= 5.14 c_u$  and where  $n_c = 5.14$  here for rectangular footings for having dimension breadth and length.

Then we can write from skeleton 1951 as  $q_d = 5.14 c_u$  into  $1 + .2 d_f$  by  $b + 1 + .2 b$  by  $l$  these are the depth factor and shear factor. Which is nothing but  $5.14 c_u$  that is nothing but you know this concentric load the inclination factor  $= 1$  so  $5.14 c_u$  into  $1 + .2$  into  $d_f$  by  $b$  into  $1 + .2$  into  $b$  by  $l$  so you can seen that incase of saturated undrained clay when we have so even if take  $\tau$  versus  $\sigma$  the more circles actually will be exhibit identical diameters that means that the irrespective of the self pressure we apply it is whatever the you know self pressure we apply you know the actually generates corresponding  $\sigma_1$  such that the  $\sigma_1 - \sigma_3$  is constant so that indicates that we actually get horizontal failure on loop and which is independent of  $\sigma$ .

So because of this particular you know using this particular logic when we have got a saturated undrained clay because as the in case of undrained condition as there is no volume change actually occurs so the  $\tau = c_u$  which is in case of complete saturated undrained condition in that situation what actually indicates is that you know in reality if actually having the similar bearing capacity problem testing on clay it says that you know the centrifuge model testing is not really required and 1g model testing the small scale model testing is required here there is no term actually the effect of you know.

The so called the breadth of the footing is independent of the size of the footing so it means that you know as long as be maintain this you know constant cohesion and for the saturated undrained conditions it actually shears in centrifuge model testing is not warranted for footings on clay so that is what actually has been described here shear strength is constant and equal to  $c_u$  hence the 1g model tests.

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Footings on clay

Here shear strength is constant and equal to  $c_u$

- Hence 1g model tests are valid in this case as the model size is not important and centrifuge experiments are warranted.

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Are actually valid in this case as the model size is not important and centrifuge experiments are warranted centrifuge model test are not warranted.

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**Advanced Geotechnical Engineering**

Cantilever sheet pile walls in sand

This wall will be stable if  $D > \alpha H$  (Where  $H$  - Height of the wall retaining the soil)

$\alpha = f(\text{friction angle } \phi)$

The actuating force is the body force due to gravity and the resisting force is the shear resistance generated due to friction. The scale of the model is not important, Whether we do experiment at 1g or  $N_g$ , the value of  $\alpha$  will be the same.

Theoretically, it appears centrifuge tests are not required.

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So let us take cantilever sheet pile walls in sand let us assume that we have got a retaining wall which is having a retaining height  $h$   $d$  is the embedded depth and this is the embedded surface now if you see that the particular ratio if  $d$  actually small and if  $d/h$  is actually say ensure you know  $d/h$  is small then is possibility that instability comes in the picture this wall will be stable if  $d$  is greater than  $\alpha \cdot h$  where alpha is function of friction angle  $\phi$  where  $h$  is height of the wall retaining the soil so here in this case actuating force is the body force due to gravity and resisting force is the shear resistance generated due to friction the scale of the model is not important you know whether.

We do experiment at 1g or ng provided you know if you able to mention  $d$  greater than 5 inch theoretically it actually appears that the centrifuge model test are not required but you know the fourth coming slides we actually looking to that you know the centrifuge model test we show the different behavior so here you know if consider cantilever shear pile wall here also there are two types of sheet pile wall one is very flexible and another one is say resist sheet pile wall in case of resist sheet pile walls what will happen is that the wall rotates about the certain.

You know the portal point above the  $\tau$  but in case if we are actually having flexible sheet pile wall there is possibility that walls undergo you know failure you know buckling failure here and undergoes you know permanent inch formation can actually occur here so where here where that is a point where the maximum shear resistance generated so theoretically for cantilever sheet pile walls in sand it says that the centrifuge model test are not warranted as long as  $d$  is greater than  $\alpha h$  where alpha is function friction angle  $\phi$ .

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**Advanced Geotechnical Engineering**

Cantilever sheet pile walls in Clay  
For stability of wall

$$\frac{4c_u}{FS} \geq \gamma H$$

In this case, the soil is saturated and is having constant cohesion.

Any experiment under 1g will have to be under prototype conditions. The actuating force is again body force due to gravity and resisting force is the shear strength due to un-drained cohesion.

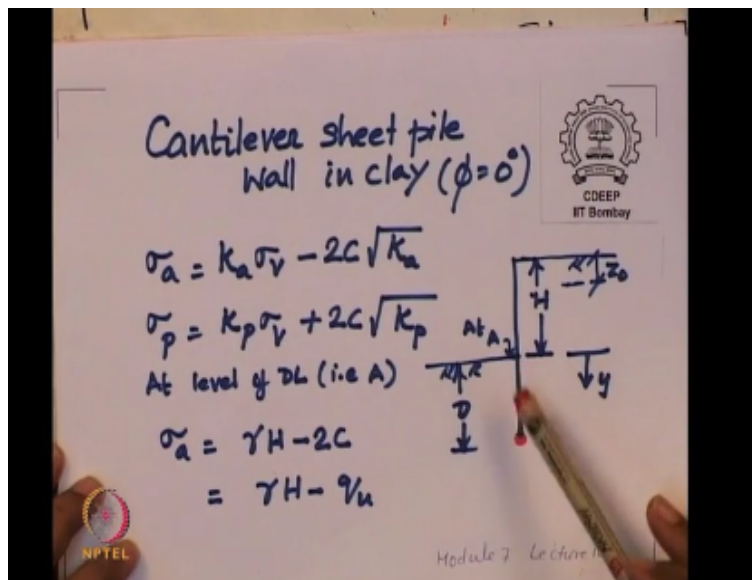
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Now let us consider for stability of wall particularly if you are actually having clay so in this case both on the wedging side that is this is the bridge level and this is let us say point A and this is the  $d$  embedded



depth and here this is the retaining height and here and here both actually location we have clay so let us look the how the stability of the wall this particular expression obtain through detailed explanation vary in we can actually look here with for the same example.

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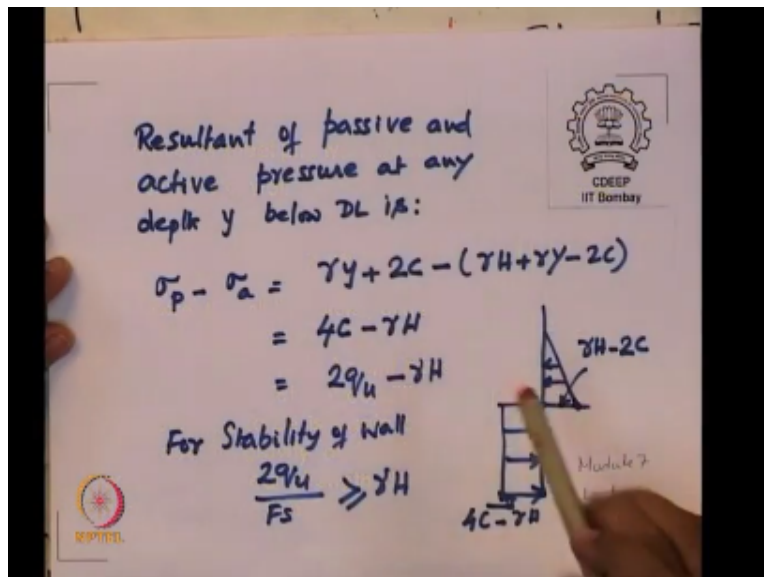


Here the cantilever sheet pile wall in clay  $\phi = 0$  and with prevalent cohesion only so here what can what actually happen is that the walls rotates about the you toe that is actually this point here and this is the dead weight surface and this the retaining height and this actually level the at reach up to the tension crackers can occur so  $z_0$  is nothing but the depth of the tension cracks and now let us see  $\sigma_a = 1$  actuate pressure  $= k_a \sigma_v - 2c \sqrt{k_a}$  but because of the  $\phi = 0$   $k = 1$  and  $k_p = 1$  and  $\sigma_p = 1$  that pressure because here the walls moves backfill and here.

It actually moves towards the backfill here the passive case actually averages so  $\sigma_p = k_p \sigma_v + 2c \sqrt{k_p}$  now let us take pressure at level of point a so here what we have is that  $\gamma h$  that is so called  $k \gamma h = 1$  with the  $\gamma k - 2c$  and which I can write it like  $\gamma h = \gamma h - q_u$  because  $q_u$  and can find comes to the strength of

the soil  $=2c$  so we can write  $\gamma h - 2c$  so this ordinate this point juts above the bridge line is  $\gamma h - 2c$  now they consider the resultant of passive and active pressure.

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At any depth  $y$  below bridge level that is this is the bridge level whatever we have said and we can actually take below the bridge level that is  $y = 0$  it is at the you know depth surface that is point a so non at pressure is actually obtain as  $\gamma y$  that is nothing but you know which is nothing but this portion  $+\gamma y + 2c$  is nothing but because you we are actually measuring from this point so  $k_p \gamma y$  so  $k_p \gamma y + 1$  so  $\gamma y + 2c - \gamma h$  that is  $+\gamma y$ .

So  $h + y$  because this entire portion is under active case ok  $\gamma h + \gamma y - 2c$  that is  $k \gamma h - 2c$  root  $k_a$  so as  $k = 1$  we can write  $\gamma h + \gamma y - 2c$  by simplification the net pressure is nothing but  $4c - \gamma h$  this is  $y = 0$  we also get at  $y = 0$  we get  $4c - \gamma h$  that means that  $2q_u - \gamma h$  now by equating this is the net pressure diagram where we have got  $4c - \gamma h$  acting over depth  $d$  and  $\gamma h - 2c$  acting over. You know the so called you know the tension crack depth once tension crack actually occurs.

Then this portion of pressure is relieved from the earth pressure so for stability of wall we can obtain you know by comparing the pressure active this side and pressure active this side a net pressure we can actually get  $2q$  by factor of safety should be equivalent to  $\gamma h$  should be equal or equal  $\gamma h$   $2q$  is nothing

but  $4c_u$  so  $4c_u$  by factor safety is should be greater than or equal to  $\gamma h$  that is what actually written here in third slide here of module 7 and lecture 10.

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Is  $4c_u$  by factor of safety greater than or equal to  $\gamma h$  so this what actually we have used in explain the whether the centrifuge model testing are required or not  $4c_u$  by factor of safety greater than or equal to  $\gamma h$  is the actually obtain from the net pressure diagram which is active side and passive side below the bridge level so in this case the soil is saturated having constant cohesion so it is actually appears that here also we can see that  $c_u$  by  $\gamma h$  actually the term is coming so like in slope in clay here also it implies that any experiment done under  $1g$  will have to be under prototype conditions.

The and actuating force is again body force is again body force due to gravity and resisting force is the shear strength due to undrained condition so any experiment under  $1g$  will have to be under prototype conditions if the actuating doing the centrifuge model test with if you are actually doing the small scale physical model test at normal gravity it implies that you know this is not realistic and may not be you know the equivalent the full scale model in the field so in conclusion the relevant of centrifuge base physical modeling what we have actually date from considering examples is that.

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**Advanced Geotechnical Engineering**

Relevance of Centrifuge-based Physical Modelling

- When the body force due to gravity is the only actuating force the shearing resistance due to friction is the only resisting force then  $1g$  model tests would be adequate for studying the phenomena (ignoring dilatancy).
- When the actuating force is an external force and not body force due to gravity and the resisting force is the constant un-drained cohesion then also  $1g$  modelling would be adequate.
- In all other cases, centrifuge based physical model tests are required.

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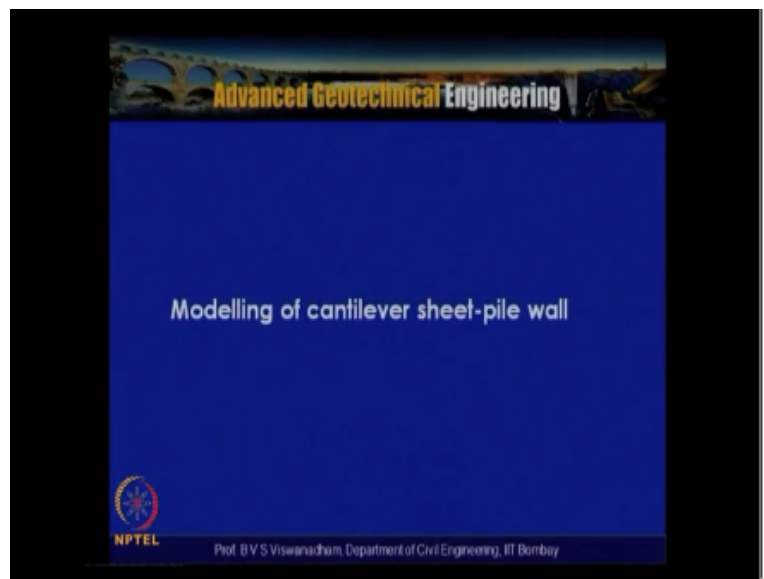
When the body force due to gravity is the only actuating force the shearing resistance due to friction is the only resisting force then  $1g$  model tests would be adequate for studying the phenomena that is means including ignoring dilatancy of soil when the body due to gravity is the only actuating force and shearing resisting due to friction is the only resisting force then  $1g$  model test would be adequate for studying

phenomena when the actuating force is an external force and not body force due to gravity and then resisting force is the constant undrained cohesion then also 1g modeling would be adequate that is actually what we are said it the bearing capacity of the footings.

On clay so in all other cases like slopes in clay, and slope retaining wall in clay what we said it that centrifuge model based physical model tests are required so in this particular discussion of the lecture what we have said it that you know for considering simple geotechnical problem we try to bring out the relevant of the centrifuge model testing then what we said is that like in agreement whatever we have been discussing if you are having footing resisting on sand it says that the centrifuge model testing is required when you are actually having footings resisting.

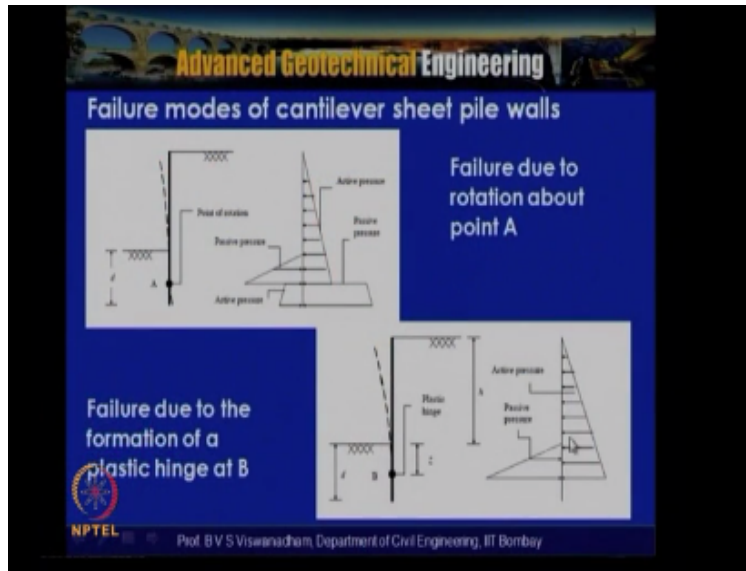
On clay it says that the 1g model test will stand good and if you are actually having you know the cantilever sheet pile walls in clay it says that centrifuge model testing is warranted and when you are actually having cantilever sheet pile walls in sand it says that theoretically it centrifuge model test are not warranted you know because as long as  $d$  is greater than  $\alpha h$  so now you know this particular discussion about whether centrifuge test.

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Are warranted particularly for modeling of cantilever sheet pile wall let us looking to this with the test actually has been done at IIT Bombay so here the failure modes of cantilever sheets pile walls are given so we have two typically failure modes.

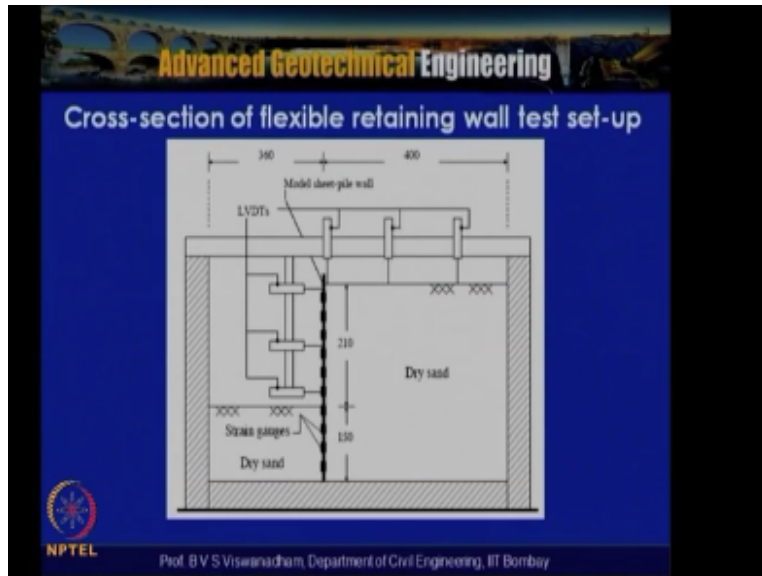
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One is the wall failure in other one is material failure so in this case of the upper figure where what actually happens is that you know material failure occurs the failure due to rotation about point a so we can see that the wall reverse rotation and about point a there is a point of rotation then we have got active pressure and passive pressure and here this wall comes towards you know the passive zone and this is the active zone so this is the net pressure where you know where you have active and then passive and here again passive and active but in case when you are actually having failure due to formation of a plastic hinge that is the material failure material of the sheet pile wall and h is the you know height of the soil above the dead surface d then you know.

We can see this is the active pressure and this is the passive pressure and at this point this wall tries to undergo for rotation and formation of plastic hinge actually takes place here so this particular issue what actually has model by using variable gravity method and then also verified by using by numerical modeling.

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So here consider cross section of flexible retaining wall test set up where the walls is actually instrumented with the strain gauges basically to measure the bending movement so the bending movement are actually obtained by pasting strain gauges and calibrating by applying by non loads and once we get to be actually get the for each application of the load we will get the pending movement the theoretically it can be calculated and for each application of the load for response of the strain gauges to be applied load can be obtain so based on each that strain gauge calibration factors of pending movement with volts can be with output volts can be obtain.

So the in linear range of if you are actually taking and that is actually gives the calibration factor for the individual strain gauges so once during the test once it is actually subjected to let us say in this particular method what we have done is that taken you place take container we are having 76cm in length ad 20cms in breadth and having you know retaining height of 21cms and 15cms is the embedded depth and wall is 2cm above the you know the base of the above the base of the container and what are actually has been done is that you can see that boast measure.

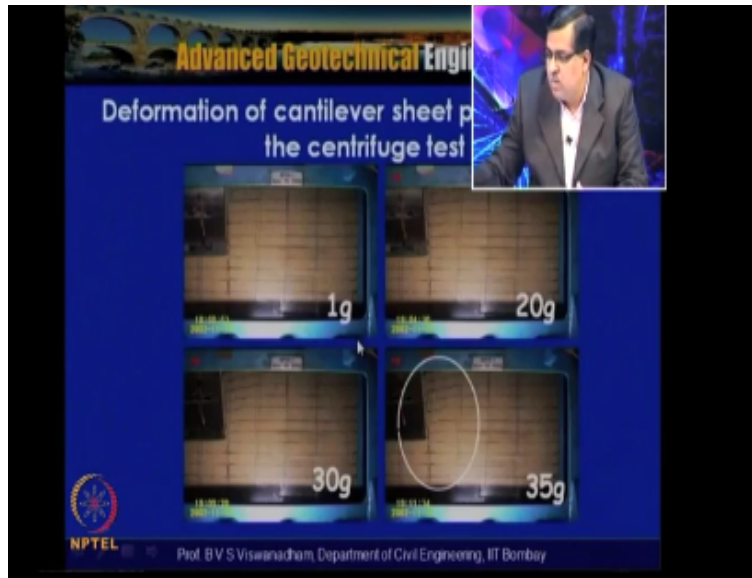
The surface settlements and eliminative are used measure the lateral movement and this gas are used to measure the bending movement during flight.

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So here the perspective view of the model prepared for mounted on the swinging basket of the shown here. The wall is modeled using in thin aluminum plate having 3mm thickness and the sand was actually find sand and which is having average particle size of about 0.15mm and is a purely greeted sand and which is placed at 55% density purposefully to induce large lateralize pressures and here what has been said is that in order to observe the formation of the rupture planes colored thin color sand lines were actually drawn both on the active side as well as towards the passive side also.

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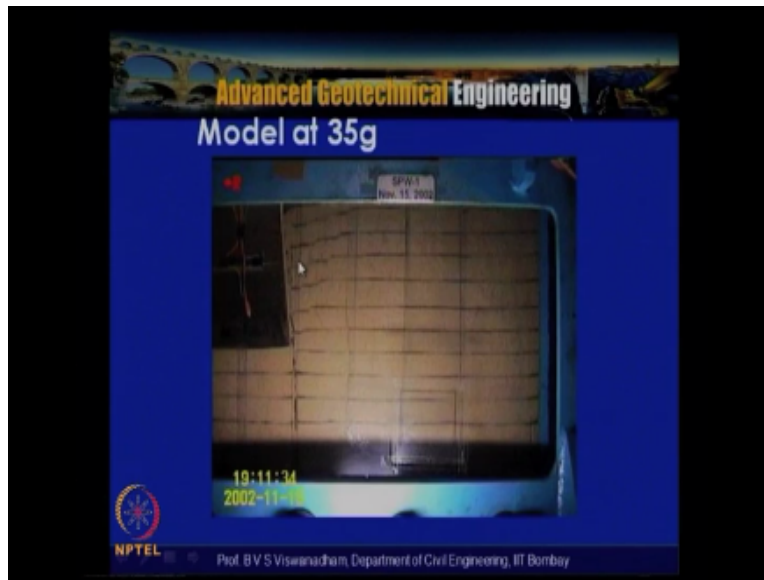


So this is as it has been told that this particular method the testing was actually adopted is the variable gravity level testing. So the wall the model has been subjected to increased gravities and this picture actually obtained from a camera mounted in front of the model, so that it captures picture of the desired area 1g this is the 20g, 30g, 35g, so you can see that as the gravity level is increased though we are actually though it each theoretically it actually says that the  $d/h$  ratio at 1g it is stable, but it does not mean that as 20g the same stability is actually ensured.

So that is what actually in the previous discussion when we have discussed about the relevance of centrifuge based physical modeling, we said that as long as  $d$  is greater than  $\alpha$  times  $h$  we can ensure that factor safety that is only valid theoretically but in priority it also depends upon the stiffness of the shear pile wall and incase of when you have got some flexible shear pile walls which actually very weak in nature then you can see that you have got situation like the formation of a the plastic hinge and then the development of the rupture lines can be seen very clearly here. So the close view of the picture at 35 gravities is actually shown here.

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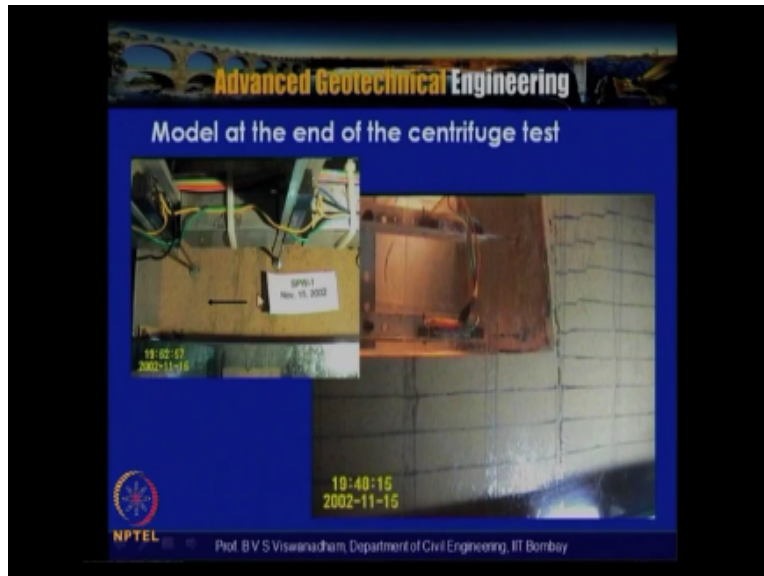




So this actually shows that the wall actually has undergone a permanent deformations and the bottom portion is not subjected to any moment and it can be seen that at this portion actually has undergone the multiple cut like slip plains and this indicates the so called when we take different lines and at the top also this has been observed over the length the breadth of the container because we have taken a plain strain container so because of that the entire portion get shifted down wards like k step type deformation.

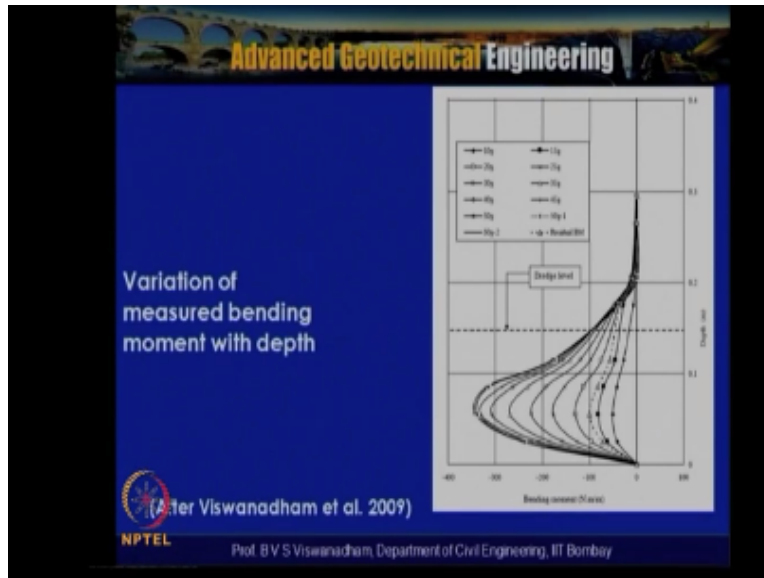
And so this one of the classical failure which actually observed for the deformation of a sheet pile wall and putted in sand with very low stiff wall. So this is variation on returns soil settlements distance from the wall then you can see that 20g 30 g 35, 45, 50 then you can see the settlements increase so these are the settlements either from the test of the wall.

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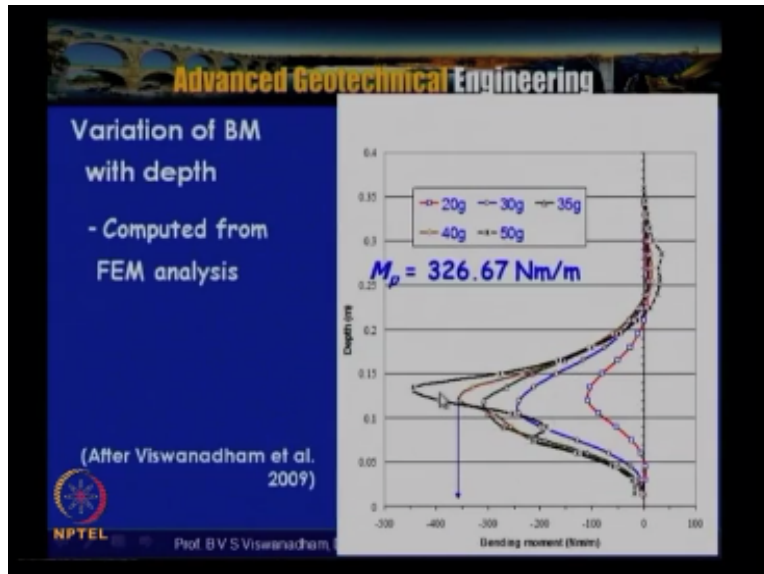
So again so has been told here so this actually from the front evolution can be seen that this is the wall so these are the step type deformations actually are shown and the corers view of the wall with the ranker shown here so he considered that this one plain passing here and one plain passing here and plain actually finally this regular point.

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So from the data which is actually obtained from strain and the bending moment actually plotted, that you can see bending moment are found to increase with the given so this is at you know 20 g 30 g like the and once we have blocking the gravity to you know 1g so there is a net bending moment like shown, so you can see that with during centrifuges test actually reach beyond up to gravity level 35g so you see that you know the sharpness of the cores tend to increase and with actually happen because the wall are actually has been subjected to a bending moment which is actually more than the plastic moment capacity of the wall. If that times actually there a attention of plastic takes place that is what actually happen in this particular case.

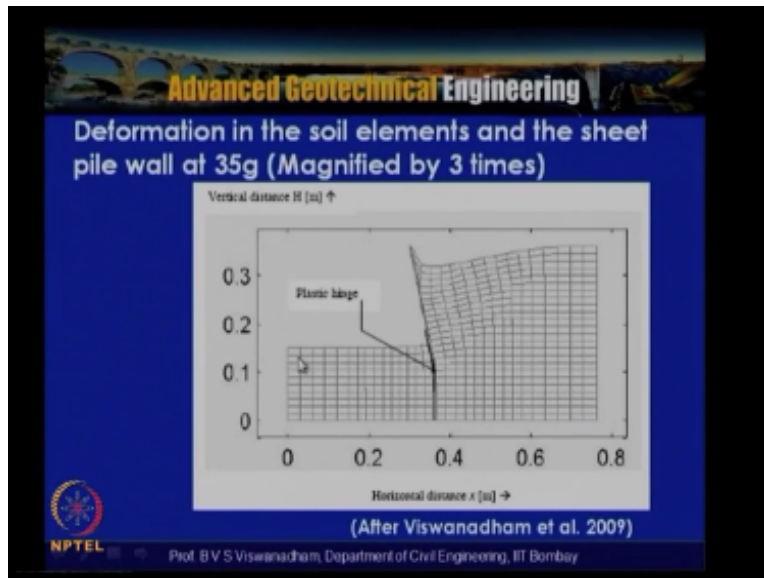
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So here are the same issues actually you know compared with no FEM analysis of the same problem with increasing gravity by using sand and software were in a when you compare here you see that at this particular point are the plastic moment capacity of a 3mm thick wall made of aluminum having you know eg that is the  $\sigma_y$  of aluminum about 72 MPa which is actually obtained has you know the plastic moment capacity about 326.67 Nm/m.

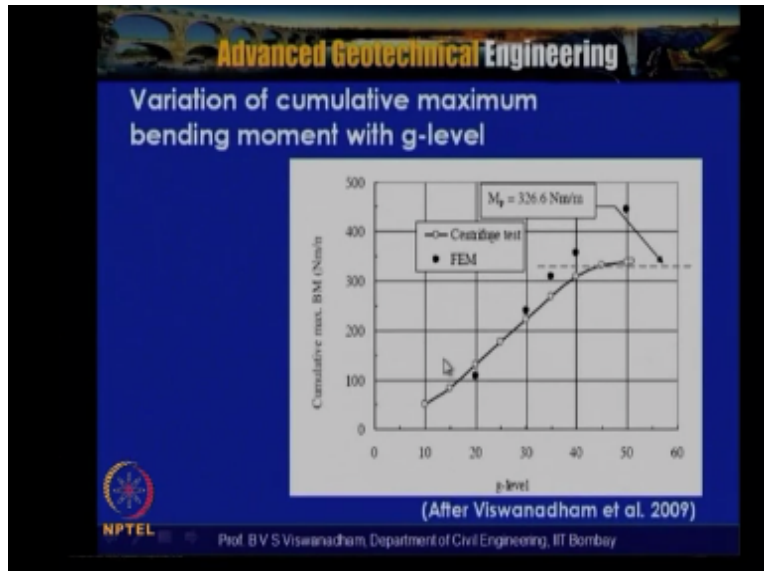
So it can be purely seen that this particular point cross at the plastic moment capacity and went into our formation of the plastic in case of random method also you can see that the sharpening bending moment can be observed.

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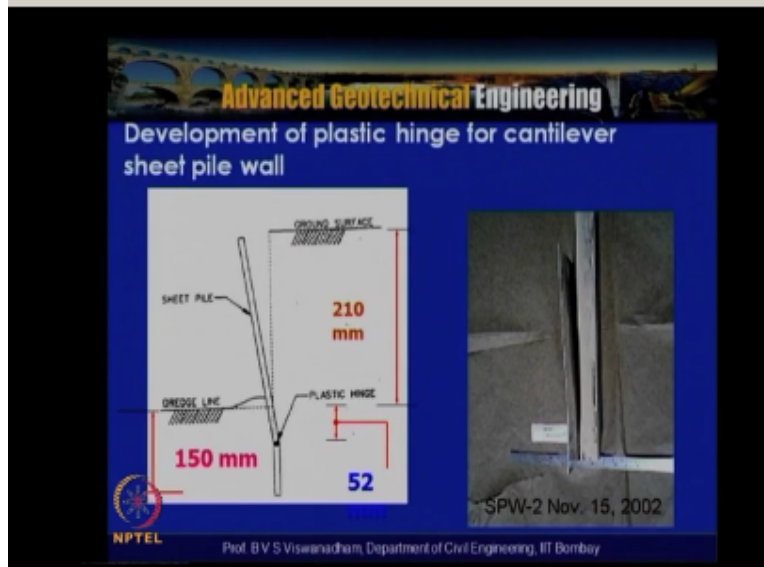
So this is the deformed, deformation of the soil elements on the sheet pile walls at 35g basically you can see here also the plastic inch formation can be seen very, very clearly.

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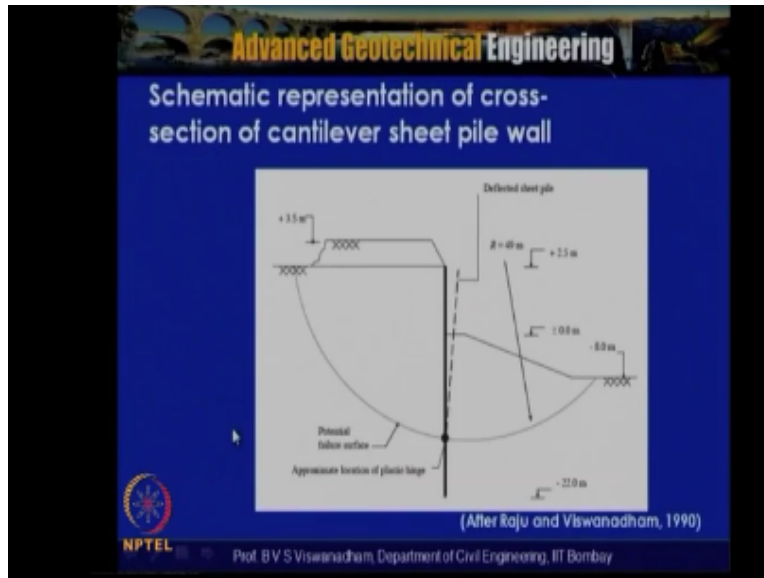
So variations of the cumulative maximum bending moment with g-level is actually plotted and when you see said increasing g-level bending moment is actually increasing the same situation also measured in the you know FEM and this is actually level where you know the so called 326.6 meter that is the plastic moment capacity so you can differently say that somewhere between 35 to 40 ,40 to 45 the wall actually attained you know the plastic formation actually has taken place in sub action has taken place and developed the further so this is what actually you know we have thrower by using this.

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And you can see that the post investigation have actually invent that so called you know the plastic inject actually formed 52mm below the you know below the bridge line and this is the 150mm which is the about depth so you can see that this is actually point physically you know here adjust point so you can see that you know this is a point where the sharpening of the bending moment also occur.

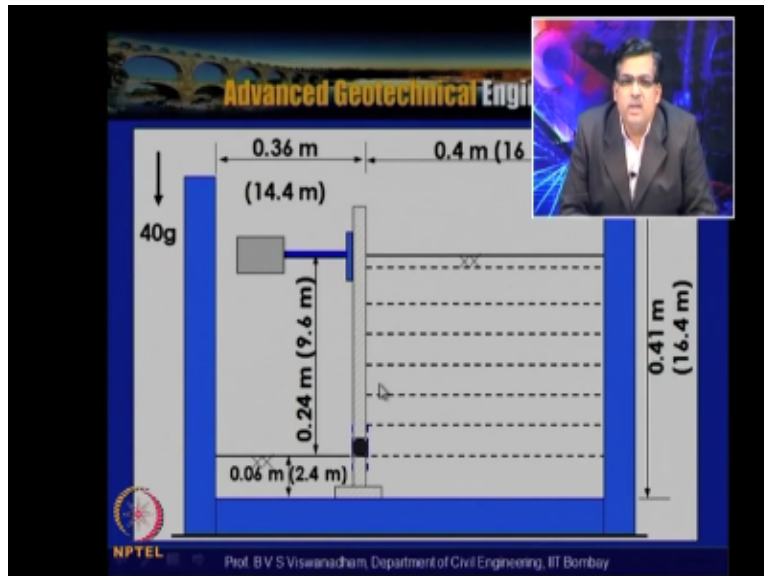
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So this is co-ordinate with one of the you know problem which actually happened in one of the project in one of the sides wherein in order to some decipher war contraction centimeter you know beyond that particular surface of the wall what actually deformed at this level the wall is actual observed to the defect point 5meter also when the post investigation also actually carried out found out you know the wall which is loose having schematic modules that let to the formation of the plastic so you know this is actually level of the what particular practical problem what actually has been discussed if only differences is that in case of in this problem that we say edge level. And then occurrence also there.

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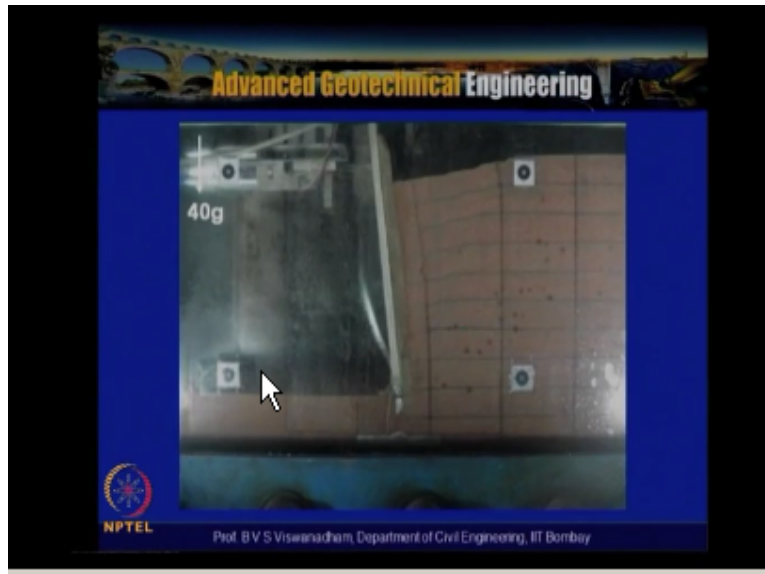




So let us consider you know a typical you know failure study for a tested 40g so here with same contained actually used wherein we actually have got resort returning wall aluminum and hint is actual place this point so this is point 24 meter 40g = 9.6 meter and this will be 0.4m at 0.4m(6) and this is .36 meter this is =14.4 m until depth of the contain is about .41 m that is 16.4 m so baseline which we having the 0.06 m into 40g and here this position in order to prevent the sand particles entry with the revenge thin actually placed here.

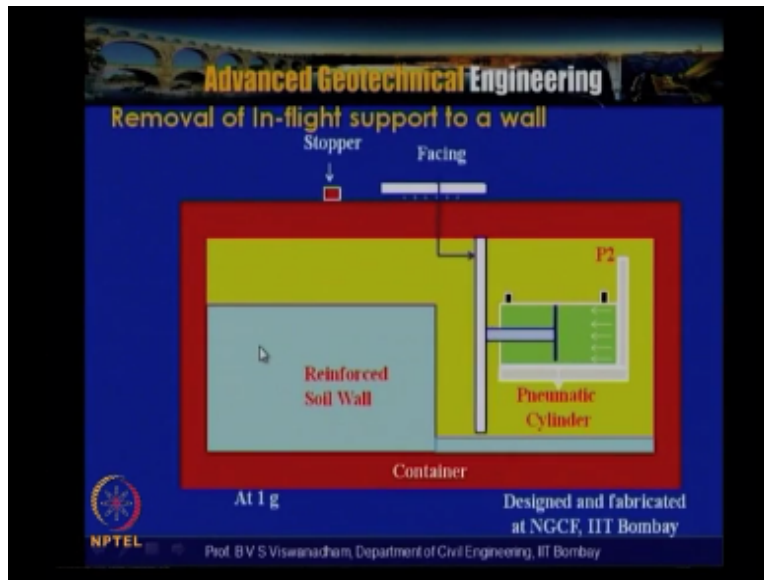
As shown this figure so actually have been done the wall is actually cropped usual with very high-pressure and this centrifuge gravity with is 19 to 40g you want to prevent this passive you know the passive mode of failure the mechanical crop of the provided in this side so that you know the volume not move forward the you know the practical so here particular we want to interested in module in the active mode of failure once we want to module to active failure what actually done is that once we reached to the particular gravity so the pointy is 0 then what will happen is that the wall, tells to move that mean, so this is really shown.

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In this next slide with give brief animation which we actually shows that this actually picture taken by a camera 40gravity so you can see that the pressure which is actually released 40g so you can see that formation of cluster clays so this is the test which is actually we have done at 40 gravities and where you can see that the failure surface actually most others like this so this is other typical examples of you know the returning wall mode of particular active state of failure can be remember.

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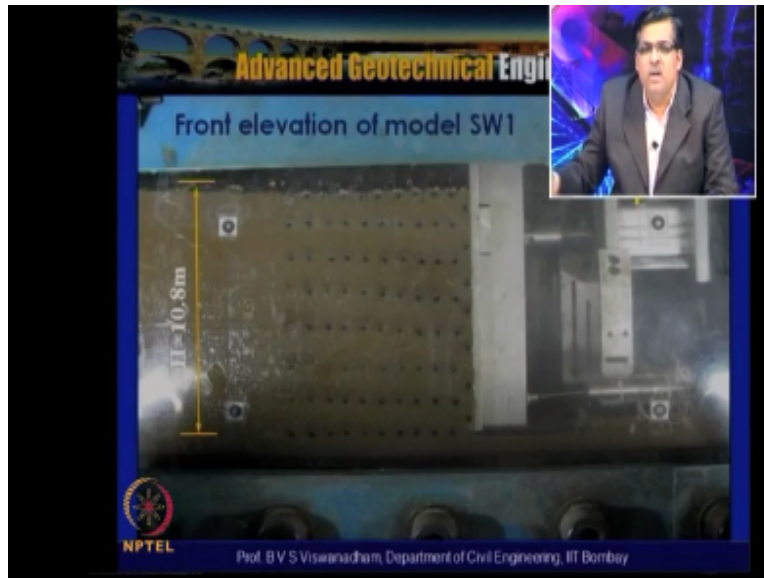


So in this case consider you know let us say we wanted currently very recently in 2013 in-flight system was actually driven, Where in what we have is that let us say we are going to in forces soil wall or a soil nail wall or you got a particular wall, so what we actually have is that we have got a wall support system which is attached to a bearings and there is a mechanical stopper then numerical cylinder is actually placed here let us say that we actually have got two ports and one port allow pressure here other port allows to pressure put in the out pressure put in the pressure in the reverse direction.

So let us assume that initially we have got a  $p_2$  it applies and the wall is actually held from coming toward the side by restraining with a liquid force. And here also the care has been taken that the wall will not move by putting a mechanical stopper towards the soil wall being tested. So let us look into this how this happen once this is at  $1g$  states so once we go we actually to go to  $ng$  that means we have got an equilibrium or  $k$  condition is achieved establishing your forces equilibrium and forces because of this pressure applied and because this wall is actually supported on this Barings.

So the wall height wall weight and all those things will not come into the picture here. The next level what we do is that we try to apply pressure  $p_4$  and remove the support. So that now this particular wall at  $ng$  if  $h/m =$  wall height. And when this actually happens at  $ng$  so it is equal and height is  $h$  meters in the prototype.

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So let us look this in the real demonstration where in we can see that 10.8m of wall at 40g you can see that how the wall is undergoing deformation because this particular technique allows one to test actually the movement of the center wall support system is also monitored by using you can see that which are actually placed here and they measure how much.

So if this actually moves about five cm in a short duration. So with that what actually happens is that we can actually see that how the wall undergoes movement and done this actually can have an impact on the deformation behaviors and other aspects can be studied so this is type of test which actually has been done by at IIT Bombay by developed in-flight wall support system at 40g. So this is a typical some reinforced soil wall constructed with marginal back field material and where it is compacted at well settle of the back field.

So you can see that there is the tension cracks are actually formed multiple number of tension cracks are formed and which actually led to the excessive deformation of a wall at the top most zone and this is similar situation if the wall is actually there in the field this one then they know you can see this the step of deformation can actually happen at for 10.8m wall.

So in this particular lecture what we try to understand is that the relevance of the centrifuge based physical model testing we are try to bring and then based on that we actually also try to see some selected examples so in this module the geotechnical physical modeling where we have seen that how small scale physical model testing at particularly carried out at high gravity is relevant to many of the geotechnical problems and we also have brought out is that if you are actually having certain conditions like footing resting on clay.

There is a possibility that the 1g model testing the small scale 1g model testing also falls good. So this is how the centrifuge based physical model testing is also applied for number of geotechnical problems for studying for subjected two different types of forces which actually gives the behavior which is actually close to the real food scale structure.

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