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ADVANCED GEOTECHNICAL
ENGINEERING

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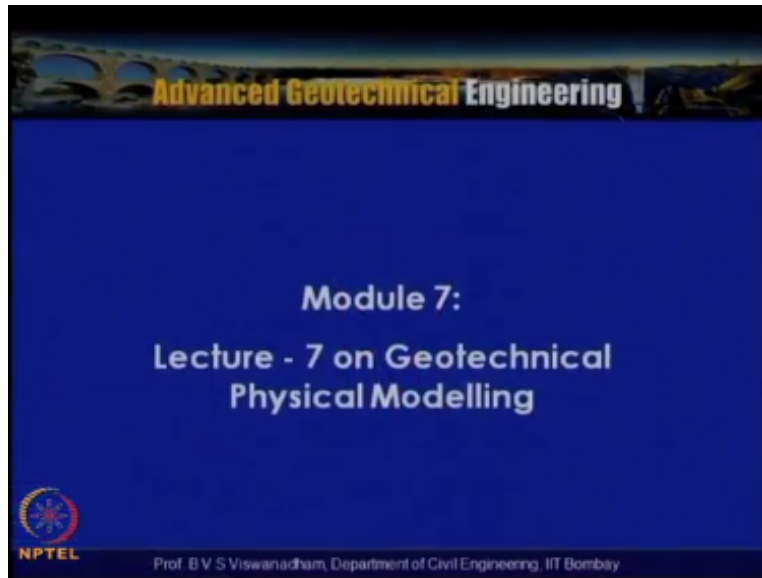
Lecture No. 56

Module – 7

Lecture – 7 on Geotechnical
Physical Modelling

Welcome to lecture series on advanced geotechnical engineering course, module-7 on geotechnical physical modelling, lecture 7.

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So module 7, lecture 7 on geotechnical physical modelling. In this particular lecture we are going to discuss about modelling of capillary rise phenomenon in a centrifuge and the respective scaling loss, and the earthquake modelling. And how these scaling loss can be verified with especially a technique called modelling of models. So modelling of models will be used for verifying the scaling loss or the scaling relationships which are actually reduced from the new phenomenon.

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Scaling law for time of consolidation

- ⇒ The distance the pore water has to travel in the centrifuge model has to be reduced by a factor of N compared to an equivalent prototype.
- ⇒ The pressure head driving the seepage flow is the same in the prototype and the centrifuge model but is applied over a distance scaled down by a factor N .
- ⇒ These two combine to result in accelerating the consolidation time in the centrifuge by N^2 .

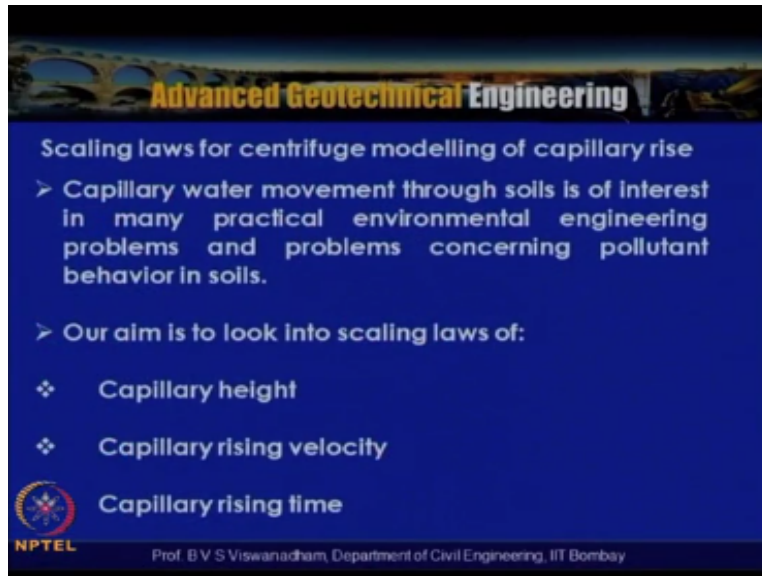
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So in the previous lecture we try to understand scaling loss for time of seepage and we also reduced based on the governing differential equation for the consolidation, we deduce also the time of consolidation. And we found that the time of consolidation is $1/N^2$ times in that of in the appropriate time. So this could be justified as the distance the pore water has to travel the centrifuge has to be reduced by a factor N compared to an equivalent prototype.

And the pressure head driving the seepage flow is the same in the prototype and the centrifuge model, but is applied over a distance down by a factor N . So these two combine to result in accelerating the consolidation time in the centrifuge by N^2 that means that this particular explanation it elucidates why the scale factor for a time in the centrifuge modelling is $1/N^2$ times that in prototype.

The distance the pore water has to travel in the centrifuge model has to be reduced by a factor N compared to equivalent prototype. So we have actually reduced this distance by $1/N$ times, but keeping the pressure head constant. So the pressure head driving the seepage flow is same in the prototype as well as in the centrifuge model, but this is applied over a short distance reduced by a factor N . So these two combine the explanation for the accelerating the consolidation time in centrifuge time by N^2 . Now let us look in to you know this scaling loss for centrifuge modelling of capillary rise.

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Scaling laws for centrifuge modelling of capillary rise

- Capillary water movement through soils is of interest in many practical environmental engineering problems and problems concerning pollutant behavior in soils.
- Our aim is to look into scaling laws of:
 - ❖ Capillary height
 - ❖ Capillary rising velocity
 - ❖ Capillary rising time

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The capillary phenomenon has been very well investigated, because of its relevance in involve mental engineering problems and problems concerning pollutant behaviour in soils. So when you have got a pollutant transport in the soil and you know relevant to some environmental engineering problems the capillary movement is very, very important.

So basically our aim is to look into the scaling loss what is the capillarity height in a centrifuge and what will be the capillary raising velocity. That means that the rate at which the capillarity height you know once it comes in contact with water and what will be the velocity of the movement of the water in the voids of soil. And what will be the capillary raising time in, what is the time which actually takes to rise is from a point at which the time contact with water to a certain levels it depends upon we all know that it depends upon the type of soil and its segregation.

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Scaling laws for centrifuge modelling of

Assumptions:

- ❖ Prototype soil and Prototype fluid as model soil and model fluid
- ❖ The characteristic microscopic length described by d_{10} of the soil, the density of the fluid and the surface tension for the fluid particle interface, which seems independent of g -level are the same in model and prototype.

i.e. $(d_{10})_m = (d_{10})_p$; $(T)_m = (T)_p$; and $(\rho)_m = (\rho)_p$

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So the assumptions basically we assume that prototype soil and prototype fluid are used in the model and in the model fluid. So that means that we actually are using the prototype soils and prototype fluids as the model soil and model fluid. And the characteristic microscopic length which is described by the particle size detail of the soil, and density of the fluid, and surface tension for the fluid particle interface which seems to be independent of G level and of same of model prototype.

That means that the D_{10} in the modelling prototype that is effective particle size which is also called an environmental engineering problems is called as characteristic microscopic length also called as characteristic length. And that is assumed to be same in model and prototype, and the surface tension which is the, for the fluid particle interfaces the fluid property and assumed to be identical in model and prototype.

And then when we use the same model pore fluid as that in the prototype, then the mass densities of the pore fluid in the model and prototype are identical. So with these assumptions and connecting to the fundamentals of capillarity, then we can actually deduce the scaling loss for the capillarity height first, and then capillarity raising velocity, and then time you know capillarity raising time.

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Scaling laws for centrifuge modelling of capillary rise

Capillary suction = $\frac{T_0 \pi d \cos \alpha}{\pi d^2 / 4}$

$u_c = \frac{4T_0 \cos \alpha}{d}$

For clean tubes, $\alpha = 0$, $u_c = \frac{4T_0}{d} = \frac{2T_0}{r}$

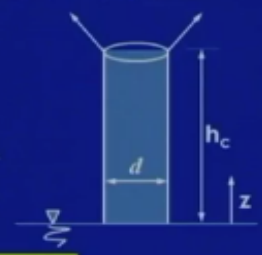
At equilibrium: $T_0 \pi d = (\pi d^2 / 4) h_c \gamma_w$

$h_c = \frac{2T_0}{r \gamma_w}$

$\frac{(h_c)_m}{(h_c)_p} = \left(\frac{2(T_0)_m}{r_m (\rho_w N) g} \right) \left(\frac{r_p \rho_w g}{2(T_0)_p} \right)$

$\Leftrightarrow \frac{(h_c)_m}{(h_c)_p} = 1/N$

This is attributed to increase in the weight of capillary fluid within the capillary tube by N



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So here in this particular slide it is assumed that all the grains are assumed that they are interconnected with thin chops you know having certain diameter D, and these diameter D is nothing, but the defines the pore size or the pore diameter. And assume that this is the water level and from which the water level rises to this level, and this hc is the completely saturated zone.

And in capillarity situation what will happen is that there is the certain zone it will completely saturated and above which there will be some fringes which actually develop, and they remain in partially saturated state up to a certain distance. And so which we actually have two heights, one is completely saturated capillarity height, other one is certain zone about that is remains in a similarly and partially saturated state.

So whatever the scale factor we deduce we assume that both these attempts which we said that completely, the capillarity height in the completely capillarity height in the completely saturated zone, and capillarity height in the partially saturated zone will follow the same you know similar zone. So we can actually write the capillary section as T0 which is nothing, but the surface tension into Πd and then this α is the angle between the made by the water film on the surface of the glass tube.

And that is nothing but the surface of the pore surfaces, but this $\alpha=0$ if you know it is a clean tube, but if are be contaminated tube then α is not equal to zero. So for clean tube $\alpha=0$, so here what we can write is that capillarity section is nothing, but t_0 into $\Pi d \cos \alpha$. So this component

what we have taken is a component and divided by the area that is nothing, but the $\Pi d^2/4$ gives the capillarity section.

So we can write U_c as $4t_0 \cos \alpha/d$ when for clean tubes or a non contaminated soil and $\alpha=0$. Then $U_c=4t_0/d$ or $2t_0/r$. So you can see that as the D pore size is actually small the U_c the capillary section will be very, very high. That means for clays the capillary section will be very high. At equilibrium, so what we do is that now at when water ranges above the water table because of the capillary phenomenon.

The equilibrium is nothing, but the equilibrium between the surfaces tension forces and the sulphate of the water column in the capillary tube what we assumed. So this tube is nothing, but you know the interconnected pore whites when we align them in a line and having approximately assuming that the diameter is D . So the t_0 (Πd)= $\Pi d^2/4 = \Pi d^2/4 hc\gamma_w$. So when you simplify this what we get is that $hc = 2t_0/r$ or γ_w . So this R is nothing, but the pore radius which is nothing, but $d/2$ and with that what we get is that the capillarity rise = $2t_0/r$ or γ_w .

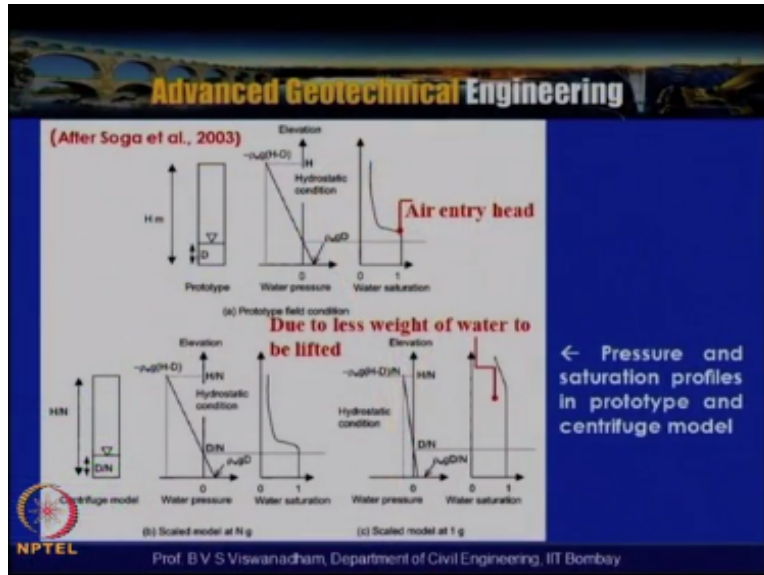
So for getting the scale factor for capillarity in model in prototype capillarity height in model and prototype and we assume that $hc(z)$, $hc(z)$ is you know the Z which is actually when is a height, you know when the water is taking place when raising with depth Z . So the Z is actually referred from here, $hc(z)$ in model and prototype is equal to when we compare the stems like $2t_0/rm$ p_wng .

Because here that mean model it is nothing, but it is N gravities and in the prototype it is which is nothing, but this is $r\rho RP$ ($\rho_w \times g/t_0 \times m$) this is in P . So now we assumed that t_0 in model is equal to t_0 and prototype and when we actually have the same soil skeleton as that in the prototype then r_m the pore radius will be identical. Then by looking into this when mass densities are identical and model into time with that what we get is that the scale factor capillarity height is $1/N$ times of that in the prototype.

That means that if you are having about 50 cm of capillarity height at for a given soil, and if you are actually testing in a centrifuge at 50G we will actually get about 1cm of capillarity height. So this particular Sc , $hc(z)_m/hc(z)$ and $P = 1/N$. So this is attributed to increase in the weight of the capillarity fluid within the capillarity tube by N . So this particular the reason what why we have go it this one is attributed to this particular fluid becomes heavier at high gravities.

So because of this the increase in the weight of the capillary fluid within the capillary tube makes the capillarity rise reduced by $1/N$ times in a centrifuge model. Now after so that all 2003.

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So here the pressure and a saturation profiles in prototype and centrifuge model are given. So in the prototype or a field situation if you look into it, so we have got the height H and this D that is the depth of water table. So below the water table what we see is that $\rho_w g(d)$ that is the hydrostatic pressure here. So above this the water column, you know this particular this column this is actually the suction which is nothing but $\rho_w g x h - d$.

So this is totally H . So this is nothing, but $H - D$. So this is, you know here minus $\rho_w g x h - d$. So this is the suction at this level. So here what you look is that up to this is the HCZ completely saturated then here you see that there is a decrease in the water saturation in the prototype. This also could be attributed to the series of the evaporation and precipitation which actually takes place. This actually makes the water table, the capillary water depleted in the portion close to the earth.

That is what we called in the odours zone. So this is the point where the air entry head takes place and then which actually depletes to this level. So if you look the similar situation in normal gravity as you look now this so called H is now h/n times, and D is d/n times. So again the hydrostatic pressure is $\rho_w \times Ng$ into d/n . So because of that here what we call is that $\rho_w d g(d)$ which is actually identical to that of in the prototype.

Similarly the suction is nothing, but $-\rho_w \times N_g \times h-d/n$. So we are again the suction is also identical. So if you see the water saturation profiles in a centrifuge model, NG model and in a prototype they are allow us, what we can see is that here you have partially saturated zone is actually commencing here also. And this is the portion were you actually have the completely saturated capillarity zone.

And above that what I said is about the fringes development or it is also called as a fingering actually takes place in this particular zone. So if you look this into this particular case simulation of prototype into $1/N$ times in $1/N$ times in normal gravity that is one gravity. So if you look into this actually has the hydrostatic pressure which is $\rho_w g \times d/n$. So this is the pressure is low and again similarly the suction is also $-\rho_w \times g \times h-d/n$, the suction is also low.

So if you look into this here the water saturation profile in a 1G model is drastically different from what we actually get in the prototype. Wherein this is actually due to as less weight of water is required to be lifted. So the water it rises to have you know very high the capillary rise it actually implies. So this is not the as realistic as that we observed in the prototype. So thus the capillary model at 1G is drastically different from the one and from the field condition.

But whereas if you are able to model this with identical soil as that in the prototype and we would buy identical fluid characteristics as that in the prototype, and at high gravities also so what it actually says that the water pressures as well as the water saturations are identical to those in the prototype. Though there is a distances are reduced by $1/N$ times. So further after having reduced and discussed about the capillarity height. Now let us look into the rate of capillary rise that is the rate of capillary rise.

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Scaling laws for centrifuge modelling of capillary rise


Rate of capillary rise

> According to Landau et al. (1967); Bikerman, 1970: A porous medium characterized by one pore size is similar to a very narrow cylindrical tube so that flowing liquid has a mean velocity.

$$u = \frac{r^2 \Delta p}{8h(t)\mu_w}$$

u = mean velocity (i.e. the flow of a viscous fluid in a cylindrical tube due to pressure difference Δp maintained at the end of the tube.

$h(t)$ = Height of liquid lifted at any instant of time t due to existing pressure difference Δp

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So according to Landau et al. (1967); Bikerman, (1970). So what actually has been assumed is the porous medium characterised by one pore size is similar to a very narrow cylindrical tube, so that the flowing liquid has a mean velocity. So according to Landau et al. (1967) and Bikerman (1970) porous medium it is characterised by one pore size, and is similar to a very narrow cylindrical tube, so that the following liquid has a mean velocity.

So based on these assumptions and further detections the mean velocity is actually obtained as $R^2(\Delta p/8h(t)\mu_w)$. So the R^2 , R is nothing but the pore radius, ΔP is the pressure head, pressure difference between the surface tension forces and weight forces of the water which is being lifted. So that U is nothing, but thing mean velocity that is the flow of a viscous fluid in a cylindrical tube due to the pressure difference Δp maintained at the end of the tube.

So U is the mean velocity, this is due to the flow of a viscous fluid in a cylindrical tube due to the presence of pressure difference Δp maintained at the end of the tube. $H(t)$, so $h(t)$ is the height of liquid lifted in any instant of time T due to existing pressure difference of Δp . So because of the there is a pressure difference of Δp this Δp is due to the pressure due to the surface tension forces as well as the sulphate forces of the water being lifted in the so called the narrow cylindrical tube.

If that the pressure difference is taken then that is what we get is that height of liquid lifted in any sort of time due to this particular height of liquid being lifted is due to the pressure difference of Δp and which is nothing but the pressure difference due to surface tension forces, and due to the

sulphate of column of water being lifted in the cylindrical tube, with those assumptions and discussions and we can actually further deduce this one.

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Scaling laws for centrifuge modelling of capillary rise

- When capillary forces cause a pressure difference Δp at ends of narrow tube, then Δp is the difference between the pressure due to surface tension forces and the pressure due to weight of fluid lifted at an instant of time t [i.e. $(\rho_w g h(t) \pi d^2/4)/(\pi d^2/4) = \rho_w g h(t)$]

$$u = \frac{r^2 \rho_w g [hc(z) - h(t)]}{8h(t) \mu_w}$$

By substituting $u = dh(t)/dt$ and integrating, time t required to rise of the continuous capillary zone $hc(z)$ can be obtained.

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So when capillary forces cause the pressure difference Δp at the ends of a narrow tube, then the Δp what we said is that the difference between the pressure due to surface tension forces and pressure due to the weight of fluid lifted at an instant of time. So here the pressure due to the sulphate of fluid is given like this $\rho_w g \times h(t)$ into $\Pi d^2/4$, this is nothing but the weight divided by area, area is nothing, but $\Pi d^2/4$.

So this $\rho_w g$ is nothing but the $\gamma_w h(t) \times \Pi d^2/5$. $\Pi d^2/h$ this is the area into the $h(t)$ is the height which the fluid is lifted. So this is nothing but $\rho_w g h(t)$ and from the surface tension forces with $hc(z) = t_0/R$. What we can reduce is that $U = R^2 \rho_w g (hc(z) - h(t))/8h(t) \mu_w$. So this μ_w is the

dilute viscosity of water by substituting $U = dh/dt$ and integrating time required to rise the continuous step zone SCZ can be obtained.

So by substituting for $U = dh/dt$ integrating this expression the resulting expression then the time required to rise the continuous step zone SCZ can be obtained by integrating then the time can be obtained. This time is nothing but to raise the capillary zone SCZ.

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Scaling laws for centrifuge modelling of capillary rise

$$t = \frac{8\mu_w}{r^2 \rho_w g} \left[hc(z) \ln \frac{hc(z)}{hc(z) - h(t)} - h(t) \right]$$

In a centrifuge model:

$$t_m = \frac{8\mu_w}{r^2 \rho_w N g} \left[\frac{hc(z)}{N} \ln \frac{hc(z)/N}{hc(z)/N - h(t)/N} - h(t)/N \right]$$

$$t_m = \frac{1}{N^2} \frac{8\mu_w}{r^2 \rho_w g} \left[hc(z) \ln \frac{hc(z)}{hc(z) - h(t)} - h(t) \right] = t/N^2$$

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So there after integration and simplification what we get is that $t = 8\mu_w / r^2 \rho_w g$ SCZ natural logarithm SCZ/ SCZ – ht – ht. So this is situation in the prototype. Now in the centrifuge model we know that the capillary heights and area capillarity rise into time t due to the pressure difference actually reduced by 1/n times and you know the gravity level is n times G. So we can write the time in model $t_m = 8\mu_w / r^2$ again the pore radius is identical as that in the prototype ρ_w is identical as that in the prototype.

So the term is left like that and Ng, the G term is changed as Ng and it CZ was changed as C_z/N natural logarithm $S_{EZ} / N - ht / N - ht/N$. So by simplification and rearrangement of the terms we can look into this expression like $t_m = 1/N^2$ by taking the common from the numerator and denominators what we get is that with $1/N^2 \times 8 \mu_w / r^2 \rho_w g \times S_{EZ} / ht - ht$, so with that if you looking to it this particular expression is analogy is to this particular expression.

So only thing is that now once we take when this is substituted by t then what we get is that $t_m = t/N^2$. So here also it what we get like analogue is to C phase phenomenon or they consolidation phenomenon, what we get is that the time of capillary rise is also scaled by $1/N^2$ time that in the prototype. So in line with what we reduced in the previous lecture like time for see page what we reduce is $1/10N^2$ time that of the prototype.

Similarly time of consolidation of a soil also $1/N^2$ that of the prototype similarly the time for a capillary rise also scaled by similar scale factor $1/N^2$ than that $1/N^2$ the time of the prototype. So the $t_m = 1/N^2 \times t_p$, $t_p = t$ and which is nothing but which is in the prototype. Now after having deduced this capillary time now let us look into capillary velocity.

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Scaling laws for centrifuge modelling of
Capillary Velocity

$$u = \frac{r^2 \rho_w g [hc(z) - h(t)]}{8h(t) \mu_w}$$

$$u_m = \frac{r^2 \rho_w N g \left[\frac{hc(z)}{N} - \frac{h(t)}{N} \right]}{8 \frac{h(t)}{N} \mu_w}$$

This implies that capillary velocity u_m in the centrifuge model is N times the velocity in the prototype.

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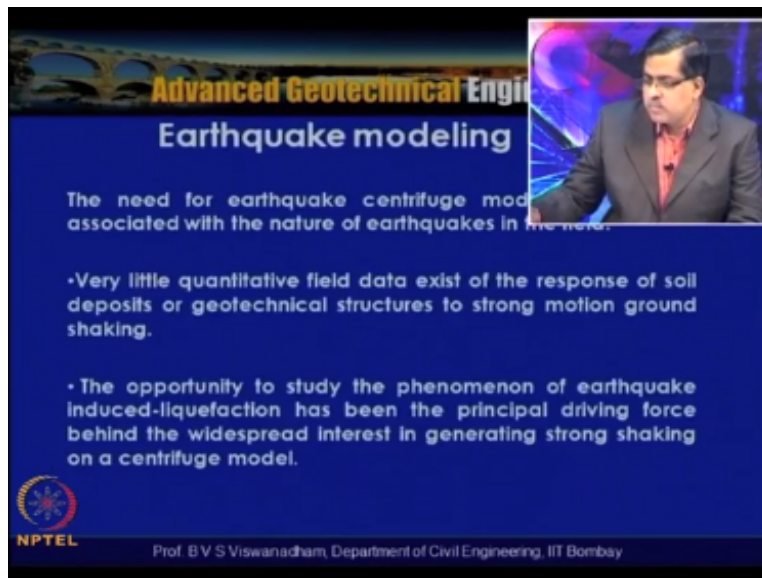
So as we have defined in the previous discussion that $u = r^2 \rho_w g \times [hc(z) - h(t)] / 8h(t) \mu_w$. So what we can do is that we can write for u_m as the velocity as $r^2 \rho_w g \times N g \times [hc(z)/N - h(t)/N] / 8 \frac{h(t)}{N} \mu_w$. So now if you compare again, if you compare with prototype and model what we get is that it implies the capillary velocity U_m in the centrifuge model is N times that of in the prototype.

So what we get is that when you take this N out what we get is that we can write it as U . U is nothing but U or U_p . So nothing but $U_m = N U_p$. So simply that the capillary velocity in the centrifuge model is N times that of the velocity in prototype. So what we have discussed is that

capillary rise which is actually which is in fall in line with like linear similitude conditions where the scale by $1/N$ times that means if the gravity level is actually increased by N times.

And the capillary height is also scaled by $1/N$ times; similarly that capillary raising time is scaled by $1/N^2$ as that of the prototype. Similarly the capillary velocity is N times that in the prototype.

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Earthquake modeling

The need for earthquake centrifuge modeling is closely associated with the nature of earthquakes in the field.

- Very little quantitative field data exist of the response of soil deposits or geotechnical structures to strong motion ground shaking.
- The opportunity to study the phenomenon of earthquake induced-liquefaction has been the principal driving force behind the widespread interest in generating strong shaking on a centrifuge model.

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So after having discussed the modelling considerations of capillary rise and consolidation and see page phenomenon. Let us look into the modelling of the earthquake in a centrifuge and what are the necessary scaling considerations are required to be considered. So the need for the earthquake centrifuge modelling of the dynamic centrifuge modelling is closely associated with the nature of the earthquakes in the field.

And very limited quantitative data exist in the response of soil deposits or geotechnical structures to strong motion of strong ground motions. So it actually says that the opportunity to study phenomenon earthquake induced liquefaction has been the principle driving force behind the wide spread interest in the generating the strong shaking on a centrifuge model.

So one of the major attributes of the centrifuge modelling is that the climatic events like this earthquake can be modelled with you great aggress. So that the performance of the geotechnical structures to these destructive forces can be understood and it is also possible to arrive at the

remedial measures and developing a theory and then guidelines. So that a properly designed structures are constructed which can actually resist these destructive force due to earthquakes.

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Earthquake modeling

Simulation of earthquake geotechnical problems in centrifuge has grown significantly in the past decade and a variety of challenging problems are being tackled in various centrifuge establishments all over the world.

Simulation of earthquake conditions in the centrifuge requires careful consideration of:
modeling of base motion, selection of model container with non-reflecting boundaries and use of an appropriate fluid in the soil.

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So simulation of earthquake geotechnical problem in centrifuge as growth significantly in the past two three decades. And varieties of challenging problems are being tackled in various centrifuge establishments all over the world. So for this simulation of the earthquake conditions the centrifuge requires are careful consideration of modelling the base motion that means that whatever the strong motion need to be model.

And selection of the model container will be discussing that when we subject this the base motion the prototype. We actually have the sort of primary waves and shear waves. And so we need to have a special container with non reflecting boundaries. And also we actually have to use appropriate fluid in the soil if you are investigating saturated sand, and saturated silty sand soil subjected to earthquakes.

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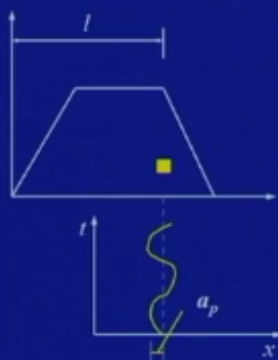
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Scaling laws for dynamic models

Considering a motion in a prototype:

$$x_p = a_p \sin(2\pi f_p t_p)$$

$$\frac{dx_p}{dt_p} = 2\pi f_p a_p \cos(2\pi f_p t_p)$$

$$\frac{d^2x_p}{dt_p^2} = -(2\pi f_p)^2 a_p \sin(2\pi f_p t_p)$$


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So these scaling laws actually deduced for a dynamic models. So here this is for a case of a prototype at one gravity. Consider similar an amendment having length L and you assume that this amendment is subjected to shaking in this direction a dynamic shaking due to turbans created because of the strong motion. So this is the time axis, this is the x axis.

Let this view the amplitude variation and as a sinusoidal variation, the AP is the amplitude which is actually indicated here in the prototype. So considering a motion in a prototype, so we can write $x_p = a_p \sin 2\pi f_p t_p$. So f_p is the frequency and t_p is the time in the prototype. Now I am differentiating this particular term what we get is that we get $dx_p/dt_p = 2\pi f_p x a_p \cos 2\pi f_p t_p$.

So here if you look into this is the velocity magnitude $f_p \times 2\pi f_p \times a_p$. So then the differentiating once again this term what we get is that $d^2x_p/dt_p^2 = (-2\pi f_p)^2 a_p \sin 2\pi f_p t_p$. So the negative sign indicates that the acceleration acting towards the centre of the axis, centre of the rotation. So here what we have done is that we have taken motion in a prototype which is $x_p = a_p \sin 2\pi f_p t_p$ and with that we have said is that by differentiating once we have got the velocity term and acceleration term by d^2x_p/dt_p^2 .
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Scaling laws for dynamic models

$$\text{Displacement} = a_p$$
$$\text{Velocity} = 2\pi f_p a_p$$
$$\text{Acceleration} = -(2\pi f_p)^2 a_p$$

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Then here what we have said is that the displacement magnitude from the expression whatever we have discussed is displacement is nothing but amplitude a_p in the prototype. And velocity is nothing but $2\pi f_p a_p$ and acceleration is nothing but $-2\pi f_p^2 a_p$. So these are the displacement velocity and acceleration terms in the prototype.

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Scaling laws for dynamic models

Considering a motion in a model:

$$x_m = a_m \sin(2\pi f_m t_m)$$

The diagram consists of two vertically aligned graphs. The top graph shows a trapezoidal pulse on a coordinate system where the horizontal axis is time t_m and the vertical axis is displacement. The pulse starts at the origin, rises linearly to a constant peak value, remains constant for a duration t_m , and then falls linearly back to zero. A downward-pointing arrow labeled N_g is positioned above the pulse. The bottom graph shows a sinusoidal wave on a coordinate system where the horizontal axis is time t_m and the vertical axis is displacement x_m . The wave starts at the origin and oscillates with a constant amplitude a_m . A vertical dashed line connects the end of the trapezoidal pulse in the top graph to the start of the sinusoidal wave in the bottom graph.

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Considering a motion in the model $x_m = a_m \sin 2\pi f_m t_m$.

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Advanced Geotechnical Engineering

Scaling laws for dynamic models

Considering a motion in a prototype:

$$x_p = a_p \sin(2\pi f_p t_p)$$

$$\frac{dx_p}{dt_p} = 2\pi f_p a_p \cos(2\pi f_p t_p)$$

$$\frac{d^2x_p}{dt_p^2} = -(2\pi f_p)^2 a_p \sin(2\pi f_p t_p)$$

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So this is actually the acceleration term which is obtained by d^2x_p/dt_p^2 from the differentiation what we have got. So what we have got this is the acceleration magnitude term.

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Scaling laws for dynamic models

Considering a motion in a model:

$$x_m = a_m \sin(2\pi f_m t_m)$$

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So now consider this similar amendment model the same configuration, but it is reduced by $1/n$ times $l_m = l_p/n$ and wherein, we consider an element which is actually subjected to let us say this is the time t_m and this is the x_m that is the amplitude is measuring this direction. So that this is actually subjected to a small amplitude now which is, a_m which is indicated here and it is that ng and $l_m = l_p/n$.

So with that what we are going to assume that the same motion is actually assumed where $x_m = a_m \times \sin 2\pi f_m t_m$. So $x_m = a_m \times \sin 2\pi f_m t_m$.

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Scaling laws for dynamic models

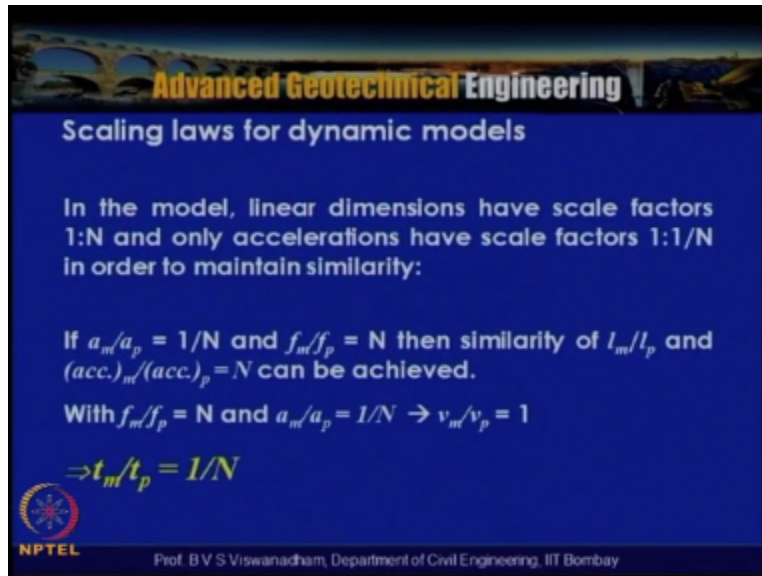
Using analogous expression for motion in the Centrifuge model, the following expressions can be derived:

$$\text{Displacement} = a_m$$
$$\text{Velocity} = 2\pi f_m a_m$$
$$\text{Acceleration} = -(2\pi f_m)^2 a_m$$

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So by using the analogous expression for motion in centrifuge model the following expression can be deduced. So again here what we have got is that displacement term a_m , and velocity term $2\pi f_m a_m$, and acceleration term the minus which is nothing but $-2\pi f_m^2 a_m$. So now what we do is that by with the analogous expression what we have done used in the centrifuge model and with that what we have got is that displacement terms as a_m , and velocity magnitude at $2\pi f_m a_m$, and acceleration term as $-2\pi f_m^2 a_m$.

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Scaling laws for dynamic models

In the model, linear dimensions have scale factors 1:N and only accelerations have scale factors 1:1/N in order to maintain similarity:

If $a_m/a_p = 1/N$ and $f_m/f_p = N$ then similarity of l_m/l_p and $(acc.)_m/(acc.)_p = N$ can be achieved.

With $f_m/f_p = N$ and $a_m/a_p = 1/N \rightarrow v_m/v_p = 1$

$\rightarrow t_m/t_p = 1/N$

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Now here in the model the linear dimensions have scale factors 1:N and an acceleration have scale factors 1:1/n that means that acceleration in model is n times the acceleration the prototype. In order to maintain similarity this is possible if the amplitude in model and prototype is 1/n and frequency is $f_m = n$ times f_p . Then the similarity of l_m/l_p and acceleration in model and acceleration in prototype = n can be a cube.


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Scaling laws for dynamic models

Using analogous expression for motion in the Centrifuge model, the following expressions can be derived:

$$\text{Displacement} = a_m$$
$$\text{Velocity} = 2\pi f_m a_m$$
$$\text{Acceleration} = -(2\pi f_m)^2 a_m$$

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So that means that If you look into it here if you compare displacement in model am.

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Advanced Geotechnical Engineering
Scaling laws for dynamic models

Displacement = a_p
Velocity = $2\pi f_p a_p$
Acceleration = $-(2\pi f_p)^2 a_p$

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And with the displacement term in the prototype then a_m and $a_p = 1/n$.


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Scaling laws for dynamic models

Using analogous expression for motion in the Centrifuge model, the following expressions can be derived:

$$\text{Displacement} = a_m$$
$$\text{Velocity} = 2\pi f_m a_m$$
$$\text{Acceleration} = -(2\pi f_m)^2 a_m$$

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And then the in order to have the acceleration which is to be n times that in the prototype. So when you take this acceleration term in the model that is $-2\pi f_m^2 \times a_m / -2\pi f_p^2 \times a_p$. The minus, minus sign will get cancelled. Then with that what we get is that f_m/f_p has to be n times that are the prototype and when the amplitude is 1/n times that of the prototype.

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Scaling laws for dynamic models

Considering a motion in a model:

$$x_m = a_m \sin(2\pi f_m t_m)$$

The diagram consists of two vertically aligned graphs. The top graph shows a trapezoidal pulse on a coordinate system. The horizontal axis is labeled t_m and the vertical axis is labeled x_m . A downward-pointing arrow labeled Ng is positioned above the pulse. The pulse starts at the origin, rises linearly to a constant peak value, remains constant for a duration t_m , and then falls linearly back to the horizontal axis. A small yellow square is located on the horizontal axis within the duration t_m . The bottom graph shows a sinusoidal wave on a coordinate system. The horizontal axis is labeled t_m and the vertical axis is labeled x_m . The wave is a sine wave with amplitude a_m . A vertical dashed line connects the peak of the pulse in the top graph to the start of the sine wave in the bottom graph.

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With that what we get is that the acceleration will become n times that of in the prototype.

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Advanced Geotechnical Engineering

Scaling laws for dynamic models

Using analogous expression for motion in the Centrifuge model, the following expressions can be derived:

$$\text{Displacement} = a_m$$

$$\text{Velocity} = 2\pi f_m a_m$$

$$\text{Acceleration} = -(2\pi f_m)^2 a_m$$

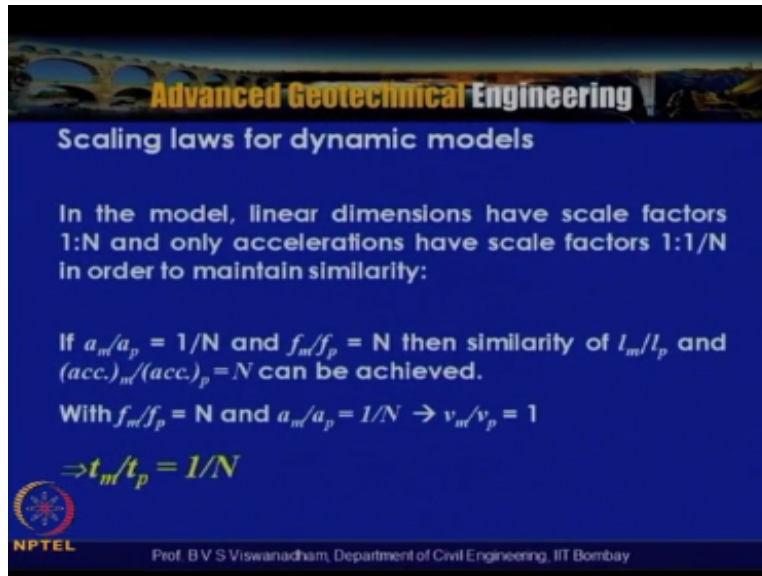
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So that means that here we have to note down that the frequency has to be n times that in the prototype that means that if you are having a earthquake with frequency of one cycle per second, then the frequency in the centrifuge model act 50G is 50 cycles per second. So when you have the frequency enhanced by n times what we get resulting time for the earthquake will be reduced by 1/n times.

Because frequency is increased by n times the time has to be reduced by 1/n times. So if you look into the when amplitude is reduced by 1/n times when the frequency is increased by n times, if you look into this velocity in model in prototype are identical that means that $a_m/a_p = 1/n$ and $f_m = n f_p$ with that what will happen is that you will get velocities identical as that in the prototype.

So if you are able to have the same motion which is subjected to this structure then what we get is that for the acceleration to be n times that in the prototype the frequency has to be n times that of the prototype and amplitude then will be 1/n times that in the prototype. And for subsequently we have the time which is nothing but $t_m/t_p = 1/n$.

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Scaling laws for dynamic models

In the model, linear dimensions have scale factors 1:N and only accelerations have scale factors 1:1/N in order to maintain similarity:

If $a_m/a_p = 1/N$ and $f_m/f_p = N$ then similarity of l_m/l_p and $(acc.)_m/(acc.)_p = N$ can be achieved.

With $f_m/f_p = N$ and $a_m/a_p = 1/N \rightarrow v_m/v_p = 1$

$\Rightarrow t_m/t_p = 1/N$

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So with $f_m / f_p = n$ and $a_m/a_p = 1/n$, what we get is that $v_m / v_p = 1$. So the velocity in model particle velocity in model in the prototype =1. So the resulting these directions implies that the time in the model is equal to time in prototype is 1/N, the time taken for the earthquake to come and send in a complete will be 1/n times that in the prototype.

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Scaling laws for dynamic models

>Time in centrifuge is compressed by a factor N and frequency increased by a factor N .

For example:

With these scale factors, it can be seen that 10 cycles of a **1 Hz** earthquake (duration 10 s) with an amplitude of 0.1 m can be represented by a centrifuge model tested at 50g subjected to 10 cycles of a 50 Hz earthquake (duration 0.2 s) having an amplitude of 2 mm.

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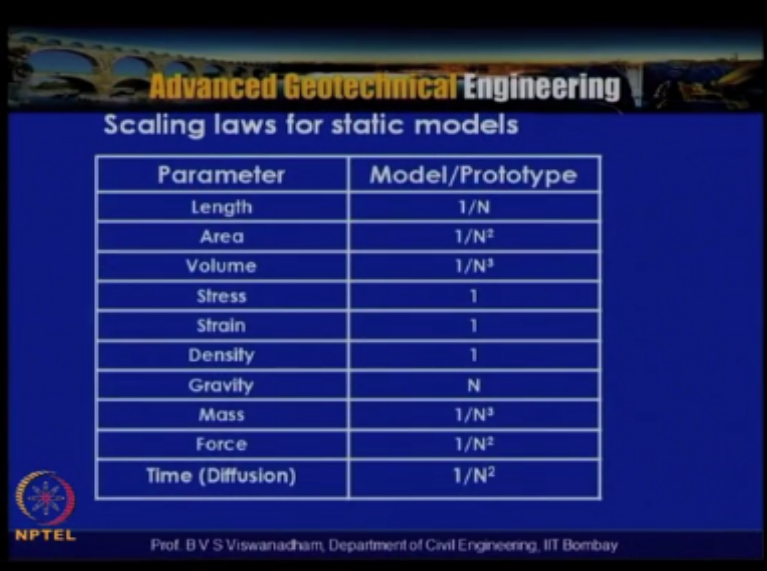
So the time in centrifuge is compressed by factor n and frequency is increased by factor n . So this is dynamic models it is required to be noted, that the time in centrifuge is actually compressed by factor n and frequency is increased by factor n . So for example, with these scale factors it can be seen that if you are having 10 cycles of 1 Hz earth quake.

The duration is say 10 seconds with an amplitude of 0.1m in the field can be represented by a centrifuge model tested at 50G subjected to say number of cycles, the 10 cycles of a 50Hz frequency. That means that the frequency which is 1Hz is no increased by 50 times 50Hz earthquake in the duration of this sees about 0.2 seconds having an amplitude of 0.1/50 that is about 2mm.

So these are the, these examples states that whatever the deductions we have for this scaling laws and with this scale factors it can be seen that the 10 cycles of 1Hz earth quake with duration of 10 seconds and with an amplitude of 0.1m can be represented by a centrifuge model tested at 50g subjected to 10 cycles of 50Hz per earthquake with the duration of 0.2 seconds. So having an amplitude of 2 mm.

So here the duration of earthquake also can be obtained by 1 cycle per second and 10 cycles we can actually get test 10 seconds, and with an amplitude of 0.1m. So here also 50 cycles per second of 10 seconds we also get here the duration of 0.2, also we can get from the time which is actually taken in the prototype like $10/50$ is also has 0.2 seconds. So this is actually example of what we actually have based on the scaling laws for the dynamic models.

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The slide features a blue background with a title 'Advanced Geotechnical Engineering' at the top. Below the title, the text 'Scaling laws for static models' is displayed. A table with two columns, 'Parameter' and 'Model/Prototype', lists various physical quantities and their corresponding scaling factors. The table is as follows:

Parameter	Model/Prototype
Length	$1/N$
Area	$1/N^2$
Volume	$1/N^3$
Stress	1
Strain	1
Density	1
Gravity	N
Mass	$1/N^3$
Force	$1/N^2$
Time (Diffusion)	$1/N^2$

The NPTEL logo is visible in the bottom left corner, and the text 'Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay' is at the bottom center.

So here based on some discussions some general summarization of scaling laws per static model was given length is actually reduced $1/n$ times area is reduced by $1/n^2$ then the prototype. And volume is $1/n^3$ time that of the prototype, and pressure is there is stress = 1 and strain =1, and density that is nothing but mass density =1, and unit wait is n times that of the prototype, mass density is identical, and gravity is n times, and mass is $1/n^3$, force is $1/n^2$, and time for diffusion is $1/n^2$.

So please note that time for diffusion is $1/n^2$, and time for dynamic activity like earthquake what we have deduced just know is $1/n$. So we will have keep in mind that we actually have two different times one for dynamic event we have got $1/n$ and for diffuser event like seepage or consolidation we have got $1/n^2$. So this is the scaling laws per dynamic models. So these are summarized here with stresses and strains are identical, velocities and model in prototype are identical, acceleration in model is n times that in the prototype, and frequency is n times that of the prototype and the time for dynamic is $1/n$ times, and mass which is actually subjected to this excitation is about $1/n^3$ times.

That means that if you are having very large chunk of mass which is subjected to strong motion in the field, we can that particular structure can be reduced by $1/n^3$, and the same structure can be subjected to equivalent to that in the field. So that we are able to get the identical response of a

field structure under consolidation in the centrifuge model. So if you look into now there is conflict which actually has been addressed as for as the diffusion time as well as the dynamic events time.

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Conflict of scale factors for diffusion and dynamic events

- > In modeling stability of clay embankments, the dynamic time scaling factor of 1:N should apply (since no seepage flow or diffusion of water)
- Subsequent to earthquake, any dissipation of excess PWP would be modeled using a time scale factor of 1:N².

However, a problem arises in the study of liquefaction of saturated fine sands where excess PWP dissipation will occur during the earthquake event.

In that case, it is necessary to ensure that the time scale factor for motion is same as that for the fluid flow.

$$t_m/t_p = 1/N^2 \text{ or } t_m/t_p = 1/N$$

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So in modelling the stability of clay amendment that dynamic scale types, scale factor of 1:n should apply, since no seepage of flow or diffusion of water occurs. Because if you are having a dynamic stability of clay amendments there is a possibility that we are able to bifurcate this timings with 1:n that you can say that during dynamic event we can actually take 1/n and subsequently if a long term see page occurs, then long term deficient occurs we can actually take 1/n².

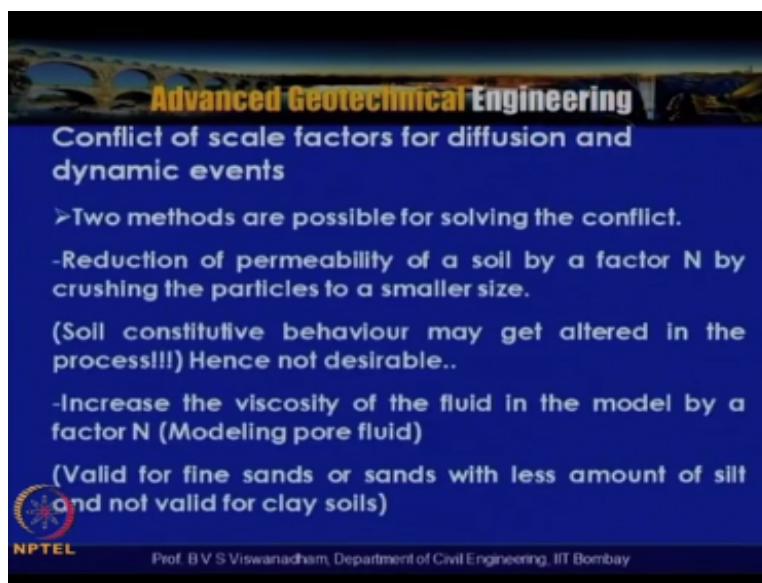
But subsequent to earthquake any dissipation of excess pore water will be model using a time scale factor of 1/n². So however the problem arises in the study of liquefaction of saturated fine sands with excess pore water pressure dissipation will occur during the earthquake event. So problem here actually occurs is that whatever when you looking to this there is deviation from the field event when before tenuous of the magnitude of the excess pore water pressure if the diffusion actually commences.

And this is possible for certain type of soils like sands and silty sands having non-plastic fines. There is a possibility that the situations they happen in simultaneously in that case we have a conflict of scale factor for diffusion and dynamic events. So in that case it is necessary to ensure

that the time scale factor for motion is same as that for fluid flow. That means that we actually have to say and go with one scale factor.

So t_m/t_p is $1/n$ and t_m/t_p in principle it actually indicates that it is $1/n^2$ times prototype. So if you looking to this out of the two time factors scale factors the $1/n^2$ is relatively faster compared to even the dynamic scale in factor for time. So we actually have to select such a way that one time is actually adopted, one time scale factor is adopted, if the both dynamic events and future events occur simultaneously. So that we can actually operate on the particular model, so that the results of the centrifuge model represent that in the prototype.

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Conflict of scale factors for diffusion and dynamic events

- > Two methods are possible for solving the conflict.
- Reduction of permeability of a soil by a factor N by crushing the particles to a smaller size.
(Soil constitutive behaviour may get altered in the process!!!) Hence not desirable..
- Increase the viscosity of the fluid in the model by a factor N (Modeling pore fluid)
(Valid for fine sands or sands with less amount of silt and not valid for clay soils)

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So for that actually two methods are actually possible for solving this conflict. One is that in a reduction of the permeability of soil by a factor n by crushing the particle to a smaller size. So this was actually tried by many investigators wherein, let us say that $k = cd \cdot 10^2$ the Hagen formula when you reduced D_{10} model in prototype by $1/n^2$ times there is a possibility that we can actually reduce the permeability $1/n^2$ times.

And then increase the gravity by n times so that the permeability of soil is identical to that in the model prototype. But the soil constitute due to behaviour they get altered in the process hence not desirable. So the scaling down of the duration is not acceptable solution, because when you scale down the sand turns out to be silt, and the silt and sand behaviour is actually drastically different as far as the stress strain relationship between the soil is concerned.

So the reduction of the permeability soil by a factor n by crushing the particles to a smaller size and though this avenue is there this is not possible, because the soil constitute to behaviour make it altered in the process. Another possibility is that increasing the viscosity of the fluid in the model by a factor n, that means that replacing the conventional fluid that is water by a fluid which is actually having system of ideal characteristic with that what you can actually see that the increasing in the viscosity of fluid in the model by a factor n will actually in to solve for this conflict.

But this particular modelling changing of model for feed actually value for fine sands or sands with less amount of silt and not valid for clay soils, because it is not possible to saturate clay soils with highly viscous fluid pore feed.

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Conflict of scale factors for diffusion and dynamic events

$$k = \frac{K\gamma}{\nu}$$

$$k_m = \frac{K_m \gamma_m}{\nu_m}$$

$$k_p = \frac{K_p \gamma_p}{\nu_p}$$

$$\frac{k_m}{k_p} = \frac{K_m (N \gamma_p)}{(N \nu_p)} \left(\frac{\nu_p}{K_p \gamma_p} \right)$$

$$\frac{k_m}{k_p} = 1$$

Using Darcy's law:

$$v_m = i_m k_m = i_p k_p = v_p$$

$$\frac{v_m}{v_p} = 1$$

$$\frac{i_m}{i_p} = \frac{1}{N}$$

For eg. 100 C. St Silicone fluid is 100 times more viscous than water but has virtually same density.

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Now consider here in this particular slide $k = k\gamma/\nu$, the ν is nothing but kinematic viscosity of the pore feed, which is in the model. So now we look into this k_m is equal to, the k is nothing but the

absolute permeability k_m/k_p , and k_p is nothing but $k_p \gamma_p$ and v_p . So k_m/k_p is equal to, now if you look into this now $k_m = n\gamma_p$, so γ_m changes to $n\gamma_p$. And then if you are able to have in for v_m as Nv_p where the fluid is actually selected is n times viscous than.

So if you got having water actually has one centistock of kinematic viscosity, and if you are able to replace with that is n centistokes, that is $n v_p / k_p \times v_p$. So with $k_m = k_p$ by simplification what we get is that $k_m/k_p = 1$ the permeability's are identical. So by using the Darcy's law $v_m = i_m = i_m k_m = i_p k_p$ with that what we get is that when you have got $k_m = k_p$ then we got $v_m = v_p$.

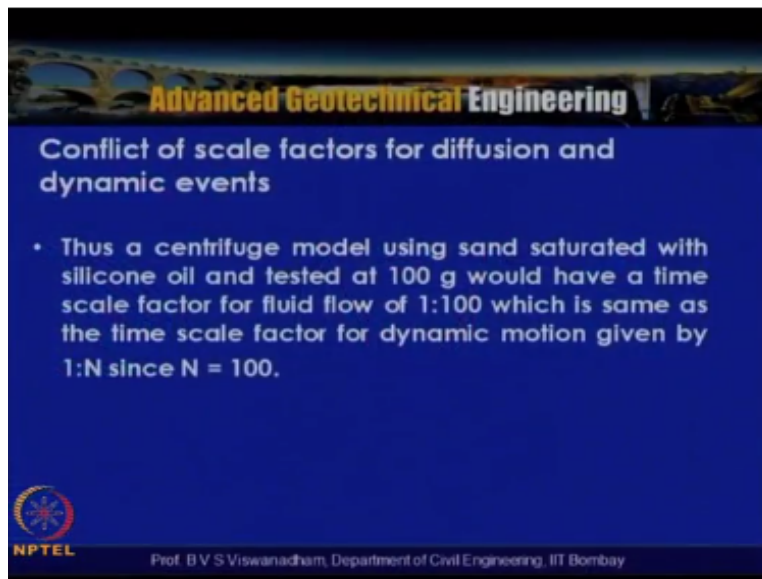
So here in this particular case what we have used is that $i_m = i_p$ definition that is with h/n . So with $v_m = v_p$ and $t_m/t_p = 1/n$ times. That means now this particular scale factor what we were actually getting for diffusion event because we are actually maintaining $k_m = k_p$ and with $v_m = v_p$. What we are getting is that $t_m/t_p = 1/n$.

So that means that the replacement of a pore fluid with higher viscosity is a viable option for investigating their problems when we have a conflict of scale factor for diffusion and dynamic events. Say for example, 100 centistokes silicon fluid is 100 times more viscous than water but has virtually has same density.

So here very, very important is that whatever the pore fluid which we actually considered model pore fluid which you consider to replace the conventional pore fluid they can have, it can have a higher viscosity, but the mass density of the pore fluid model pore fluid which is actually selected to replace the conventional pore fluid has to be identical. So this is basically required to ensure that effective stresses does not alter.

That is if you are actually having a pore fluid which is actually heavier then there is a possibility that the effective stresses will get actually altered. So one principal requirement is that either the mass densities of this pore fluid are moderate pore fluid and conditional pore fluid have to be almost identical. So with that what you can see that we may be able to have this particular option is valid.

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So thus a centrifuge model using sand saturated with silicon oil and tested at 100 g would have a time scale factor for fluid flow of 1:100, which is same as the time scale factor for dynamic motion given by 1: N since $N=100$. So if you are actually having this replacement of 100 centistokes silicon oil, then there is a possibility that we will actually have identical scale factor for the diffusion as well as these things.

So with this, what is physically happening is that, by altering the pore fluid by placing the conventional pore fluid with model pore fluid having the viscosity, we are actually making the diffusion event slower. So with that what is actually happening is that we are able to match with the dynamic event, and then we are actually going close to the fluid conditions when the saturated sandy deposits or deposit with silts is actually subjective to some sought of a strong motion.

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Requirements of ideal pore fluid

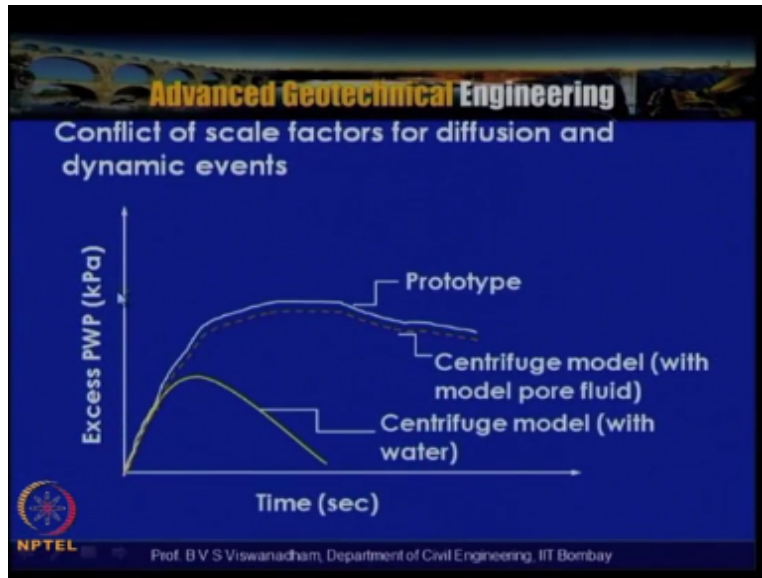
- The fluid shall be like water, a Newtonian fluid.
- It must have the same compressibility as that of water.
- It must be chemically polar to use along with silts and sands.
- If the fluid has the same density and surface tension as water so that capillary effects will be properly scaled.
- The presence of different fluid should not alter (i) strength properties and (ii) damping characteristics.

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So what are the requirements of the ideal pore flow in the sense that the fluid shall be like water and Newtonian fluid and it must have the same compressibility as that of water and must be chemically polar to use along with the silts and sands. And if the fluid has same density as surface tension as water, so that capillary effects will be properly scaled, and the presence of different fluid should not alter the strength properties and damping characteristics.

So that it does not mean that the presence of different fluids should not increase or decrease the strength properties. They have to be same and also the damping characteristics have to be same. These are some of the selected item pore fluid characteristics.

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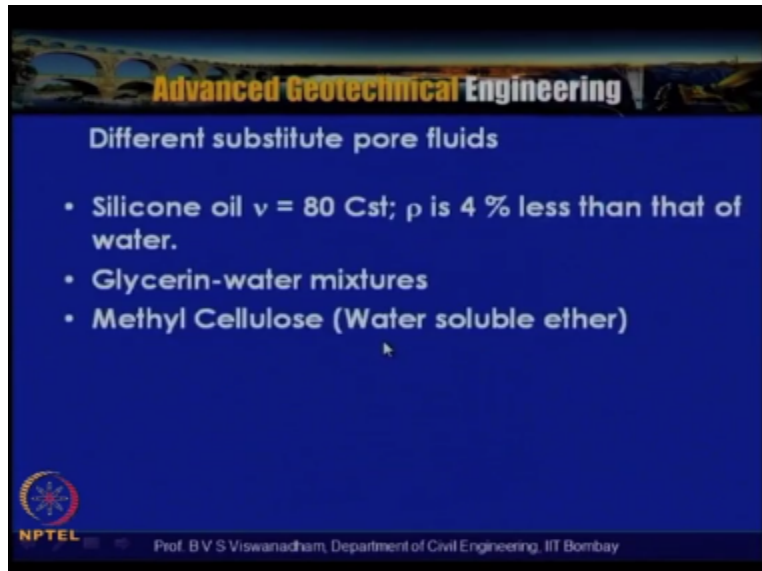


Now the physical explanation is actually given here in this particular chart, for example, when there is a saturated sandy deposit, subjected to this earthquake in the prototype. So you can see that how the excess pore water pressure with time takes place in the prototype. So, the variation is, which is like this which is shown here. But when we have a centrifuge model, with water as the pore fluid, what actually happen is that the excess pore water will rise with excess because of the size with disturbance.

But the dissipation actually commences very rapidly. So, this is because of the nature of this $t_m/t_p=1/N^2$, the diffusion event is actually faster than which actually happens in the field. So by replacing with conventional pore fluid, the schematic flow of the excess pore water pressure variation with the time that is actually given here, this is the time, which is in the earthquake shaking time.

You can see that this is almost identical to that in the prototype. So by doing with replacing with conventional pore fluid with a model pore fluid having high viscosity, a centrifuge model results represent closely as that in the prototype, so which is actually explained and shown here in this particular figure.

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So some of the different substitute pore fluids which are actually called as silicon oil. Silicon oil actually has 80 centistokes of kinematic viscosity and low densities almost equal to that of water, 4% less than that of the water, and it is also some investigators actually use Glycerine water mixtures, and in some laboratories they use model pore fluids.

And recently for the past one and half decades people are actually using Methyl ether or methyl cellulose, which actually used in the food dyeing industry as water mixable substance which is used for producing high viscous fluids. So this is possible here with Methyl ether or it is also called as a metal house, and with that the desired viscosity range can be actually obtained. The more we will be discussing with the examples from the works carried out by our investigators.

So, these are the slide actually given them different substitute pore fluids. Now after having discussed about different modelling scaling loss, but we need to have again means of verifying this. So the modelling of models are also called as modelling of prototype.

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Modelling of Models

- Modelling of models is a technique used in centrifuge modelling to ensure that the scaling laws derived earlier are valid.
 - ❖ Modelling of prototypes
 - ❖ Modelling of models

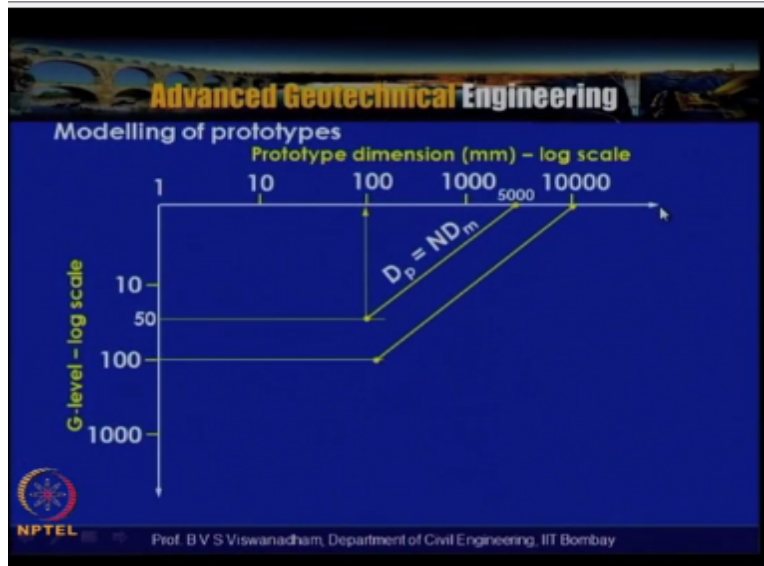
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Basically, it is a technique used in centrifuge modelling to ensure that the scaling losses derived earlier are valid. Like we have reduced the time scale for capillary rise is $1/N^2$ time the prototype. So that is actually done by using modelling of models. So this modelling of models is nothing but when you say that $GL = 1$, that is $G_1, L_1 = G_2, L_2 = G_3, L_3 = G_4, L_4 = 1$. That means that here what you are having is that we actually have different models, which can be done.

So similarly in the hydraulic gradients method, so then we actually have different pressure differences with different thickness of soil layers. There also we can actually have different models, which actually represents the identical result of the, let us say, a particular footing resting on sand subjective to, so called pressure difference between top and bottom.

Similarly, here the modelling of models the two subcategories is modelling of prototypes and modelling of models.

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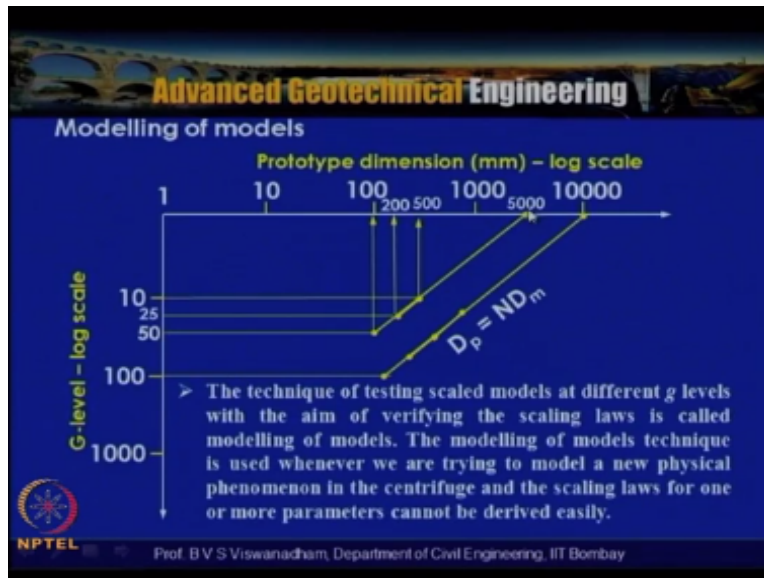


So in this, what we have seen is that, where we have what the prototype dimensions are given on the log scale and gravity level are given in the log scale in the vertical axis. So, here when you have a 5 metre prototype, let us say a footing size of 5 metre is scaled down by 100 mm at 50 g. So, this is nothing but the $D_p = 50 \times 100 = 5000$ mm. So this is directly a modelling of a prototype into a centrifuge model.

So when you have you a prototype of 10,000, let us say retaining value of 10,000 mm that is 10 metres, and then this is modelled by at 100 g it is a height of 100 mm. So 100×100 is about 10,000 mm. So, this is a here we are corresponding this is at 1 g and this is at the prototype dimensions 5000 and 10,000.

Similarly, when you try to look into when a prototype exists, we are trying to look into the corresponding model.

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And when other case is that, when the prototype does not exist, what we actually do is that, we actually have three different models. Each model represents the model of modelling with each other, and then they also represent overall a corresponding identical prototype. For example, here when you have 50g at 110cm prototype dimensions, it is one model. Similarly, a 25g with 20cm dimensions, it is one model and at 10g with 50cm dimensions, it is one model.

But model one, model two, and model three, they actually, when they represent a prototype having 5m dimensions, so the example here is 5 metres. So, this $D_p = ND_m$, which is nothing but $N1D_m$, $N2D_m$, and $N3D_m$, but were all in the level, which is actually falling down this line, represents the corresponding prototype in this. For example, here also what we have is that we have got at height of 100mm and at 100g 10,000, then we actually have got another model.

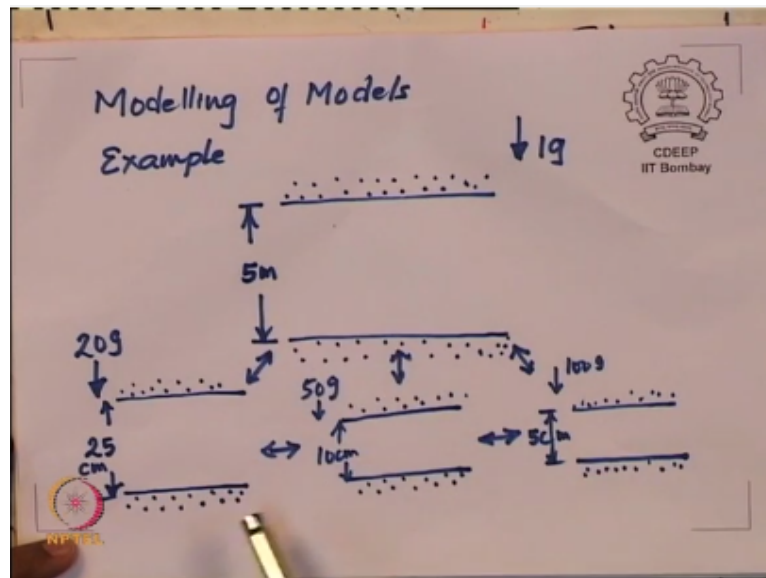
We reduced the dimension with reduced the g level and increased the dimension. When you have decrease in the g level, when we actually have increase in the dimensions of the model. So here the technique which is actually, the technique of testing scaled model at very different g levels with the aim of verifying the same scaling laws, this particular technique is called modelling of models.

So the modelling of models is actually used in for verifying the scaling laws, which are actually reduced for existing phenomenon, like are for the new phenomenon or anything, which is actually being investigated. And the modelling of modelling technique is also used, whenever we

are trying to model a new physical phenomenon in the centrifuge, or in the scaling laws for one or more parameters cannot be derived easily.

And one of these modelling of model technique is also used for checking whether the so and so model is actually subjective to the scale factors or not.

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So as an example, let us look into this particular figure, where in we have a thick layer of consolidation takes place in 5m thickness in both top and bottom, we have open layer, that is sand layers. Then this at 1g, but what it actually speak about the modelling of models is that with the thickness of 25 mg in the model at 20 g this again this is a model of this particular prototype case.

And when we reduce this by another 15cm that is the model dimension 10cm at 50g, it corresponds to again this particular prototype, which is 5m. And then when they reduce further to 5cm increase the gravity load to 100g. So here see model 1, model 2, model 3. They are actually models of each other, but they are also in individual when they say, they are actually models of a particular prototype.

So, the idea is that all these three models should have identical time settlement behaviour as that of in the time settlement behaviour of the prototype. So in this lecture, we actually have

discussed about the capillary of scaling laws and dynamic model scaling laws, and then we also discussed about the technique called modelling of models.

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