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**ADVANCED GEOTECHNICAL**  
**ENGINEERING**

**Prof. B. V. S. Viswanadham**  
**Department of Civil Engineering**  
**IIT Bombay**

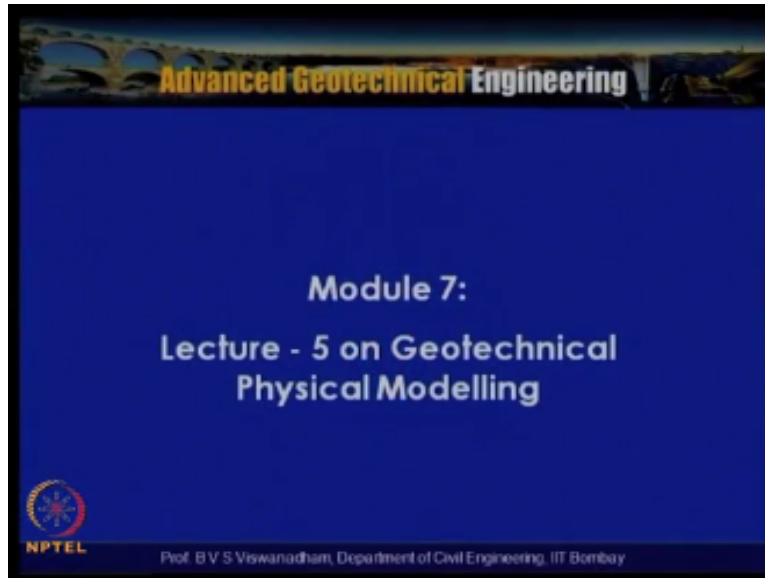
**Lecture No. 54**

**Module – 7**

**Lecture – 5 on Geotechnical**  
**Physical Modelling**

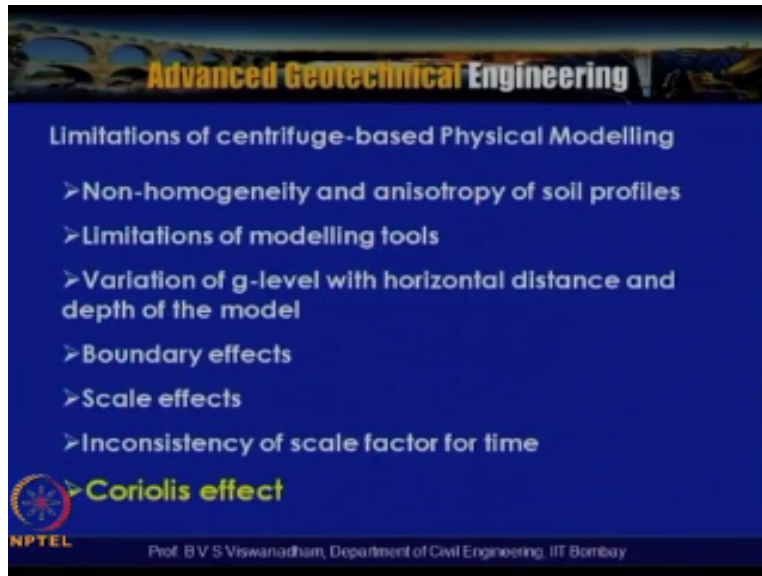
Welcome to in the course of advanced geotechnical engineering, this is module 7 lecture 5 on geotechnical physical modeling.

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So this is lecture 5 on geotechnical physical modeling and module 7, so in the previous lecture we introduce ourselves to number of centrifuges based you know the limitations which are actually involve with centrifuges based physical modeling and they are basically listed here.

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Non homogeneity and anisotropy of soil profiles which is difficult to model in centrifuge based physical modeling and limitations of the modeling tools and variation of g level with horizontal distance and depth of the model. And subsequent errors and then we also have said that how these errors can be minimized by selecting an appropriate configuration of the equipment, and the boundary effects which are between the walls of the container and soil and the scale effects which is because of the our inability to not able to scale down the particle size then one of the scale effects is called particle size effects or grain size effects.

And we discuss how this particle size effects can be avoided by doing you know especially using the principal of centrifuges modeling we call modeling of models. Then after having you know once we introduce ourselves to you know scaling laws then we realize that there are different scale factors for time for different types of forces like coriolis forces or creep forces or viscous forces or weight forces then you know the inconsistency of the scale factor for time.

Then we also discussed that whenever there is a motion of the body happens within the model then we said that you know it is going to subject to you know two types of accelerations one is Euler acceleration and second one is coriolis acceleration the as far as the centrifuge modeling is concerned Euler acceleration is not very serious because there is no much change of angular acceleration.

Because of that the Euler acceleration term is limited then but what we are having is that if you are releasing let us say you know sand particles on to the soft soil for enabling the construction

of embankment on soft soil or releasing you know certain weight at on to the acceleration gravity field. So these situations arise to cause you know coriolis effect, then also seismic perturbation which also can cause you know this you know coriolis effect.

Then we also discussed that for a coriolis effect to be you know negligible we say that the velocities have to be as more as possible like either small as possible this as small as 0.05 times  $v$ , where  $v$  is the model velocity if the event is also very fast like a blast event or a projectile event where in the ejective thrown at very, very high speed in such situations also we said that you know the coriolis effect is negligible in the scene that the zeta will be thrown with very rapid speed.

And it goes and hits the periphery of the container walls, so we need to also when the explosion events are actually modeled at high gravity there is a need for you know putting sacrificial you know sheets along the periphery this helps in reducing or tending the container boundaries otherwise what will happen is that the container boundaries are subjected to you know serious errors due to denting.


So when if the velocities of the moving particle within the model within the say greater than 0.05 time  $c$  velocity of  $v$ , with which the model is moving and less than two times  $v$ , then we said that the coriolis effect is cannot be ignore and need to be considered, and for that we have to see and how you know the coriolis effect can be minimize.

So those models which are being tested with the velocities in the range of greater than  $0.05v$  and less than  $2v$  we need to check whether the model is periphery from the coriolis effect or not. So the coriolis force arises from any movement that occurs within the centrifuges model that is what we have been discussing.  
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**Coriolis effect**

- The Coriolis force arises from any movement that occurs within the centrifuge model.
- For example, if we try to construct an embankment by raining sand in flight or if we drop a ball from the center of the centrifuge to study projectile motion and impact on the soil, or simulation of rainfall, etc.,
- Then, the moving object will have Coriolis acceleration  $2v\omega$  and Coriolis force is  $2mv\omega$ .

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And for example if you try to construct an embankment by raining sand in flight or if you drop a ball from the center of the centrifuge to study the projectile motion the impact of the soil or the stimulation of rainfall. These conditions and these are actually nothing but stimulation of this climatic events or some construction process which actually lead to this coriolis accelerations.

So then that the moving object will have the coriolis acceleration which what we said is that  $A_c=2v\omega$  and coriolis force is nothing but  $2mxv\omega$  where  $m$  is the mass of the moving object within the model, so the coriolis effect is  $2mv\omega$ . Now after having discussed the limitations we have introduced ourselves to that there are two types of machine configurations they are basically beam centrifuges and drum centrifuges.

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**Machine configurations**

- Balanced Beam Centrifuge
- Beam Centrifuge
- Drum Centrifuge

➤ The radial force acting on the central spindle should be minimized as  $|\sum mr\omega^2| \rightarrow 0$ , where  $m$  is the centre of mass of a component at a vector  $r$  from the central axis and  $\omega$  is the rotational speed of the centrifuge.

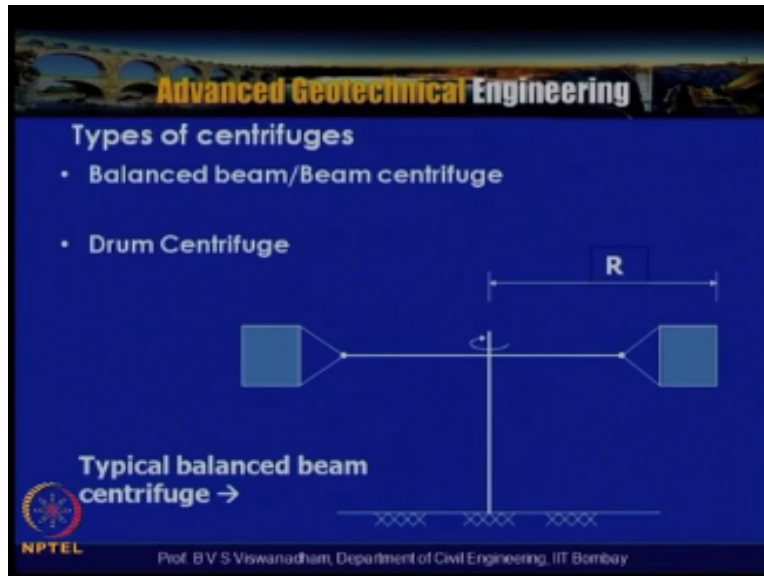
**This implies  $|\sum mr| \rightarrow 0$**

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And initially the beam centrifuges are known as balanced beam centrifuges and the drum centrifuge we said that where we have a peripheral drum and then the central tool table we try to look into the details of these machine configurations. So the radial force acting on the central spindle should be minimized as  $\sum mr\omega^2$  tend to 0 and this if you are having that means that the mass balance is actually happening on the both the sides.

And whether it is with the beam centrifuge or whether it is with a balanced beam centrifuge we actually have to ensure that  $\sum mr=0$  that ensures that the horizontal the model can be rotated within the plane of rotation and second issue is that you know the bearings of the centrifuge will be you know unaffected.

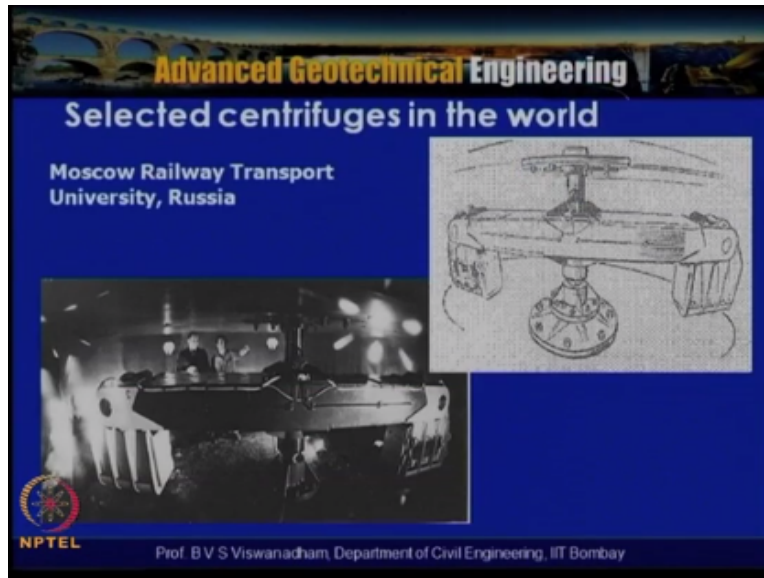
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So typical balanced beam centrifuge is actually shown in this slide where in we have the two baskets where in the models of almost equivalent weight of place and it is subjected to rotation about a vertical axis in a horizontal plane where  $r$  is the radius and if weight of the model at which this acceleration say for example if the weight is say 2 tons or 20kN and if the, if it is able to carry 20kN at 100g.

Then what we call is that as for as the capacity of the machines is balanced beam or beam centrifuges equipments are concerned it is called as capacity which is indicated as g tonnes and gkN which is nothing but pay load of the model that is 20 kN into at which the g level the 20 x 100 so it is something like called 200gkN or 200g ton capacity so the capacities of the equipment are actually indicated worldwide and then you know various ranges capacities of the equipments are there throughout the world.

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So we will try to the different centrifuges which are actually there in the world the one of the earlier centrifuges in the world which actually was you know put for the Russians that is after the Moscow railway transport university where in you have got swing baskets and then have you know the arm which is actually at the central the at the connecting to the central spindle so the pictorial or you know artistic view of the centrifugals which was know earlier is the picture here.

Where in you have got the two baskets and is called pine baskets and a central pedestal and arm or beam of the centrifuges.

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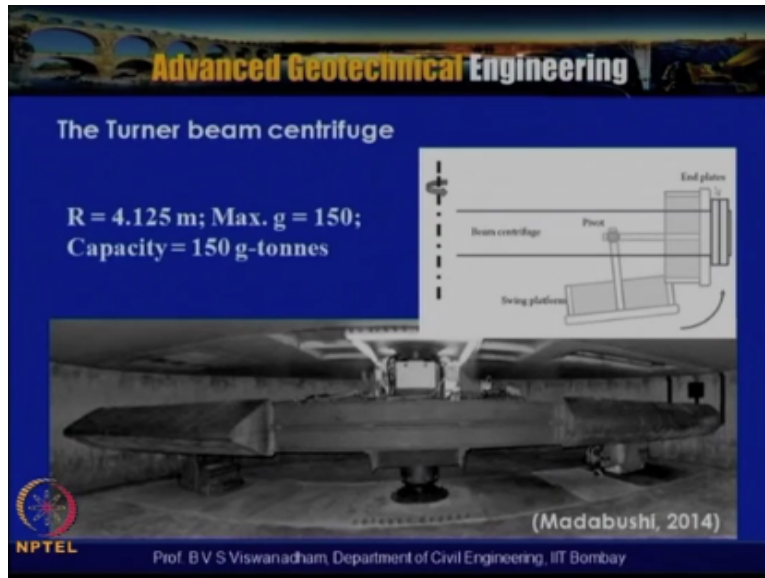




Then what we see here is a you know the rule university Boeco centrifuges in Germany where the radius of the centrifuges is about 4.15m that means that radius is measured from the center to the from here to here that is perpendicular distance so it is 4.25 when the basket swings up and the maximum g level are the capacity of this machine is 250 and the maximum pay load which can be mounted here is 2 tons.

So the capacity of the machine is said you know 400g tonnes that means that at pay load at 100g it can carry you know at 200g it can carry 2 tonnes that is the 400g tonnes capacity what we what we are actually referring so this is typical balanced beams and defused where you know as radius of 4.125m.

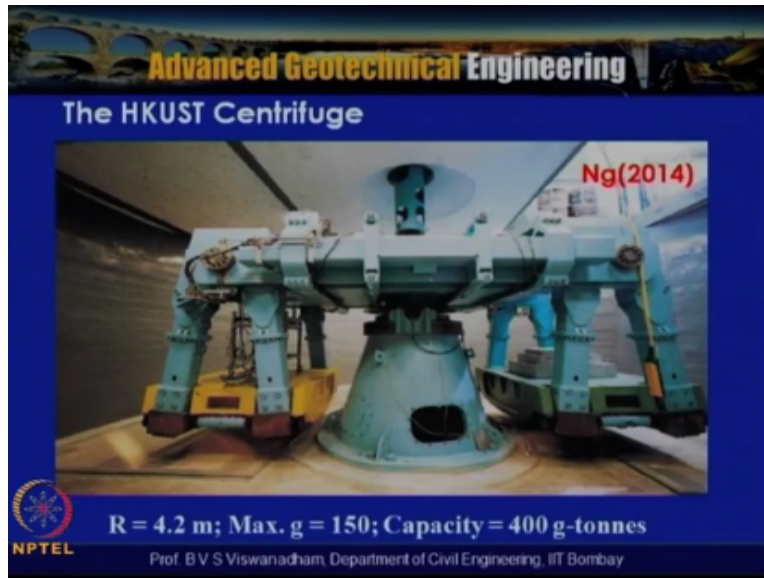
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And this is the tuner beam centrifuges this is initially it was restrain factor on centrifuges afterwards with the restrain centrifuges was converted into the a pivot mechanism which is actually shown here and the basket can be detachable and where the model is actually mounded an equivalent weight are actually placed here and so that  $\sigma_{MR} = 0$  is achieved here and this also actually has radius of  $R = 4.125\text{m}$  and Max  $g$  is 150 and a capacity of this equipment is 150g tonnes capacity.

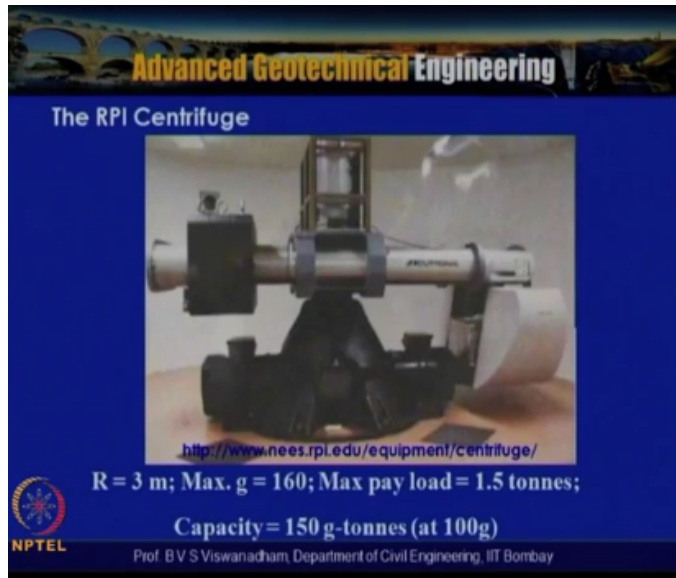
And where in you can actually see that you know the says fine basket and then this is the armor beam of the equipment so this is you know an university of Cambridge in united kingdom.

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And this is in the hong kong HKUST centrifuges and this actually as a radius of 4.2m and maximum g level is 150 and a capacity of this machine is 400 g tonnes capacity so this is also a balanced beam centrifuges.

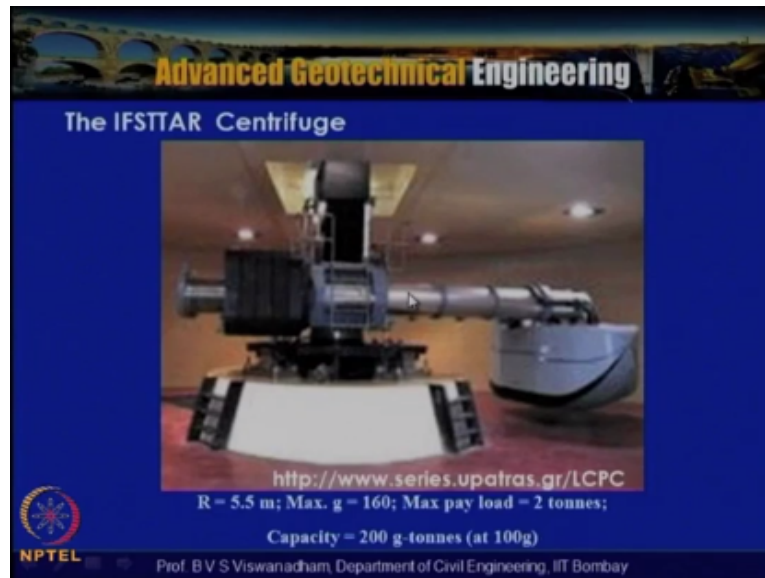
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And this is the RPI centrifuges in USA the radius of this centrifuges is 3m the maximum g level is 160 and maximum pay load is 1.5 tonnes and the capacity is 150g tonnes at 100g so you can see here it can carry 1.5 tonnes load at 10g that is the reason why capacity is 150 g tonnes and this is you know the balanced beam centrifuges where you have got a single basket or single swinging basking and this is the pedestrian and this is the adjustable counter weight.

The counter weight is adjusted depending upon the weight of the model which is actually kept so if the model is heavier then the counter weight you know it is just towards this side if the model is lighter the counter weight comes towards the center so that  $\sigma_{MR} = 0$  can be ensure.

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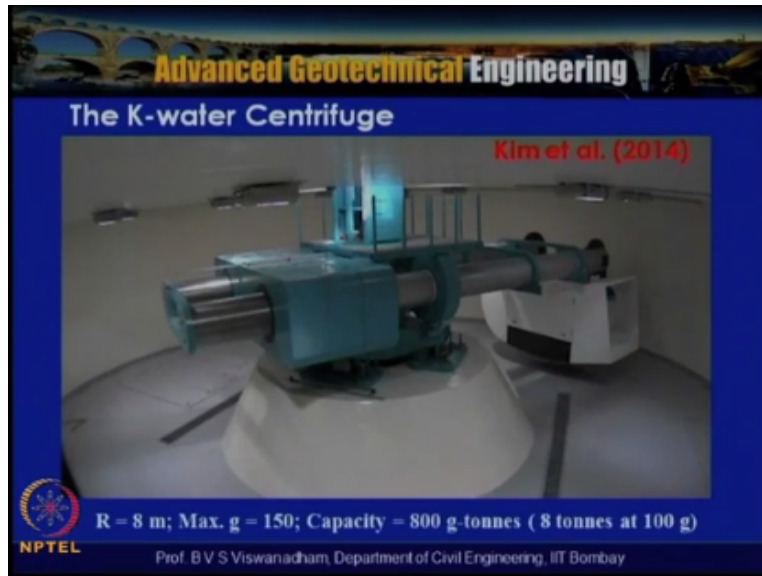
So this is IFSTTAR centrifuges in Paris in France and this is in a if we also known as LCPC centrifuges and with the radius of the centrifuges is 5.5m this is also a beam centrifuges and maximum g level is 160 and maximum pay load is 2 tonnes and the capacity is 200g tonnes capacity at 100g so that means that at 2 tonnes it can carry at 100g that is why the capacity is called 200g tonnes.

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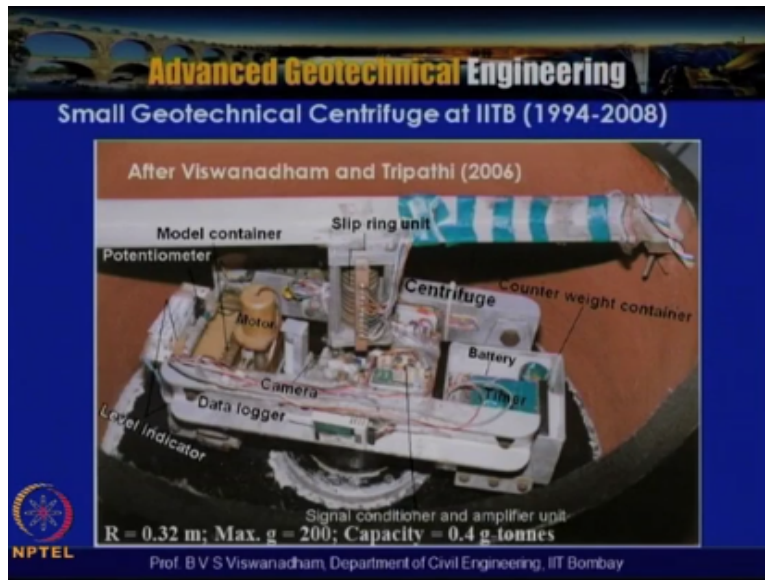
So this is the case centrifuges where in you know this is also a balanced these beam centrifuges and what we look into all these cables and these things they are like you know for activating earthquake activator that also the data acquisition components under and this also equivalent to the radius of about 5.5m and the details are actually given in Kim et al. 2013.

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And this is the k- water centrifuges and which is having the radius of about 8m and this is the again the beam centrifuges with a single basket the basket area here in this case about 2m / 2m and the maximum g level is 150 and a capacity of this centrifuges is 800g tonnes that means that it can carry a play load of about 8 tonnes at 100g 8 tonnes pay load it can carry at 100g that means that very large centrifuges models it can actually accommodate and so that the models can be tested and this k water centrifuges is called developed for studying problems like dam in stabilities and level in stabilities and other you know the water element you know the structures.

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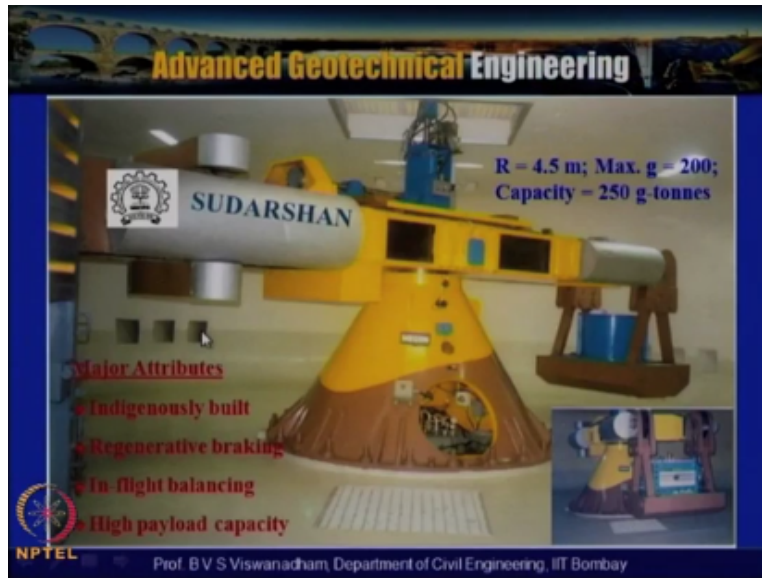


And this is you know the small geotechnical centrifuges at IIT Bombay it was actually existing between 1994 to 2008 and this is very small equipment with the bio centrifuges was actually converted into engineering geotechnical centrifuges with radius is 0.32m and maximum g is 200 the capacity is 0.4 tonnes capacity so this facility was actually used for geotechnical instruction as well as for some you know research problems which can be investigated within the you know the errors which can be allowed in this small centrifuges.

So the payload which is very low is something like 2.5 kg which actually can carry so here in this particular model which is actually shown and which actually has got the you know some motor which gets activated and this you know the counter weight is actually used for supporting you know the weight of this package which is actually shown here.

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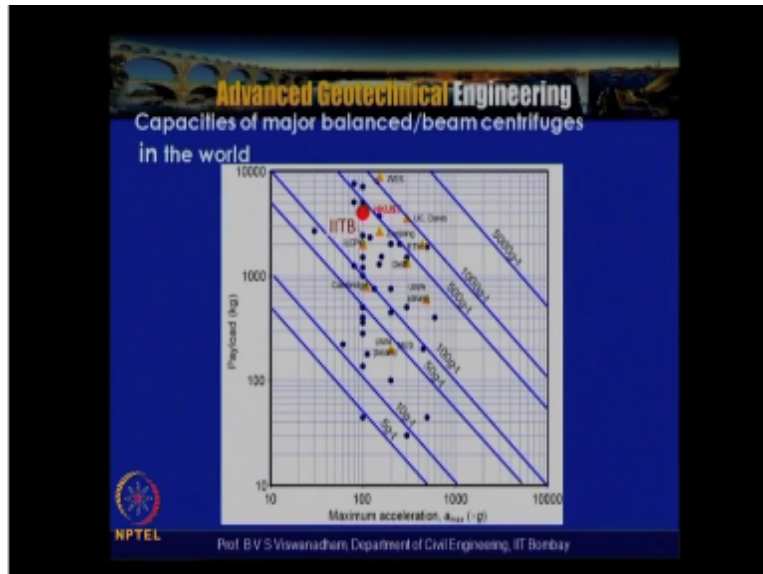




So this is the IIT Bombay's large beam centrifuges facility the radius is about 4.5m maximum g level is 200 and the capacity is 250 g tonnes and this one of the indigenously built equipment and it actually has got regenerative braking system and in flight balancing and when compared to the contemporary equipments that actually has got capacity and the g level which actually ranges from 10 gravity to 200 gravities in order to reach 200 gravities.

It can actually reach from 1g to 200 g in 6 minutes similarly from 200 g to 1 g it can actually come down are ram down in 6 minutes so it actually has got a two different rapping speeds one is 34 rpm per meter and the other one per min 34 rpm for mini the other one is 3.4 rpm per minute.

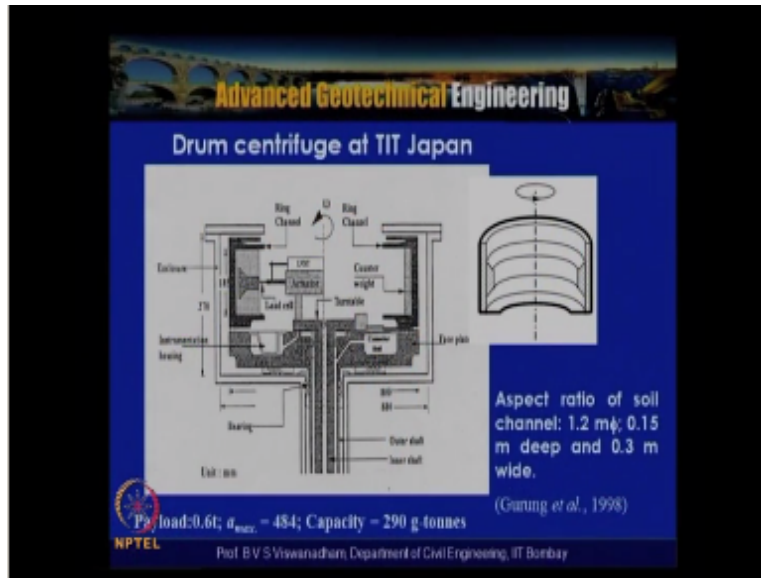
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So when you compare the capacity of the major balanced and beam centrifuges in the world where in some of the which we have discussed then IIT Bombay equipment falls somewhere here between 100 g tone capacity to 500 g tone capacity and some of the new Korean set if it actually falls weight goes to you know 800 g tone capacity and which they have the equipment which is very close to 1000g tone capacity where the r radius is about 9.5 meters.

So you know what we have is that you know this particular charge is for actually maximum acceralation and then this is the pay load so you can see that IIT Bombay say diffuse is actually can carry at 2500 kg at 100 g that is why it is actually somewhere here it is actually located so this is the center of fuse very close to it you know and then it Cambridge diffuse somewhere here so after having discussed different beam and diffuses that balanced beams and diffuses now let us look into the terms and diffuses which is this is a typical diffuse picture in blue.

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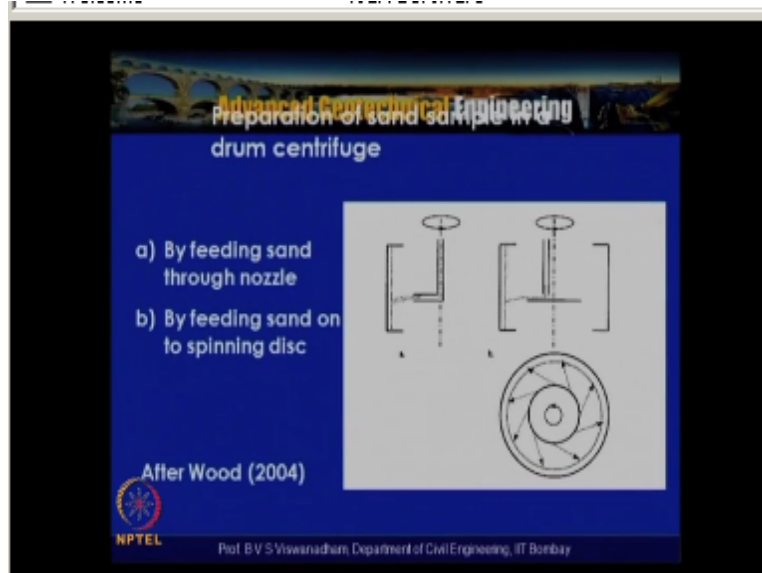


Which is at Tokyo institute of technology in Japan and here as I have been told that there is a preferred which is actually exist and where in what it is called as the aspect ratio of the channel so where in it is 1.2 meter in diameter 0.15 deep and 0.3 meter wide so where in this channel the models are actually made like this for example here an alignment model is actually made with a after having made with certain clay.

Then what is actually is done is that it is actually scrapped it is this portion is removed and this portion is removed then the shape is actually and you know then it can trusted for certain testing so what we can see is that here a particular model which is actually mounted here a uplift capacity of a certain anchor which is actually embedded in soil is been tested and this pay load capacity is actually 0.6 tones.

And Amax acieration is 484 and the capacity is 290 g tones capacity so the aspect ratio of the soil channel which actually said is that 1.25 i.2 meter diameter 0.15 d and here one advantage is that you know given stress history number of you know the test can be done then second thing is that also as the been informed before that elastic problems can be stimulated because of the large extend of the you know areas from both sides of the models been tested.

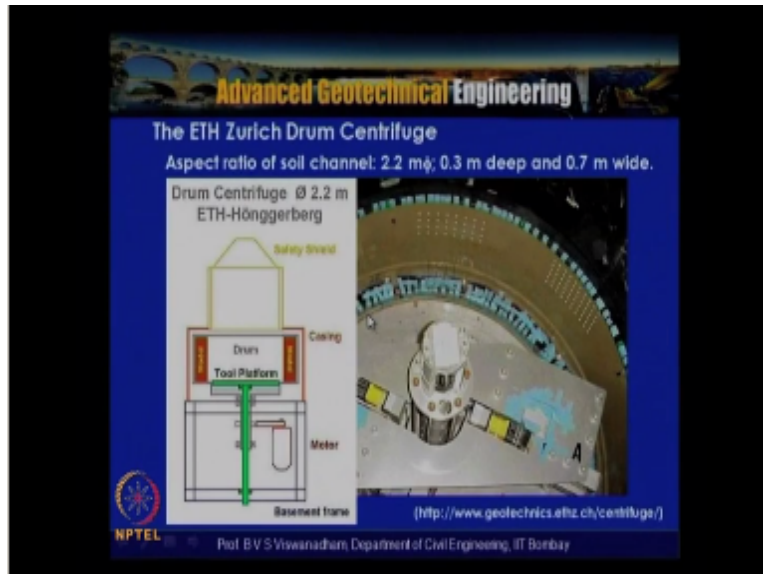
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Then the preparation of the you know model in a drums and centrifuge which is actually is a trivial and this is how the sand models are actually constructed where the sand is actually allowed to drop on to central rotating plan and both this side as well as this rotate support by vertical axis by horizontal manner in synchronize way.

Then what will happen is that this particulars are actually thrown tangetionally on the peripheral like how these the sand models are done and the similarly some clay models which are actually done by feeding either sand this is either feeding the sand through nozzle and so that the sand is actuality dropped on to the channel which is been constructed which is been used for constructing a model.

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So this is the one of the largest you know drum centrifuge in the world and this is the ETH Zurich Drum centrifuge and the pictorial view is actually shown here where in this is the model in which this is been constructed and this is the of the drum which is actually shown here and this is the central true table.

And which the test is on progress where you will actually see that both the central true table as well as the drum rotates in other so this is actually about 2.2meters in diameter in radius and when in you can see that you know it is a particular case you know the investigation which is actually for stone Colum reinforced clay been investigated what you see the dots and the module steel columns so this is the during drum centrifuge.

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Relationship between ramp angle,  $\theta$  with RPM,  $\omega$  for beam centrifuges

$$\sum F_r = mr\omega^2 = T$$

$$T = m\omega^2 (R + l \sin\theta)$$

$$\sum F_y = N - mg = 0$$

(neglecting vertical acceleration)

If there is no angular acceleration of the model about its centre of mass (i.e. P) then there can be no moments about P.  $\Rightarrow$  Resultant of T and N pass through P

$NT \sin\theta = T$

$g \tan\theta = r\omega^2 \left(1 + \frac{l}{r} \sin\theta\right)$

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Now after having discussed about the relationship between the ramp angle I mean the different machine configuration and what we said is that we have got beam centrifuge and balance centrifuge are we have you know the drum centrifuges in the case of that drum centrifuge the model is actually prepared within the channel which is attached to the local centrifuge of the drum and now a days you know some of the drums centrifuge are also equipped with some baskets.

Where in the baskets are actually attached to the you know the peripheral of the drum so with that you know some confine models can be you know tested and now as far as the beam centrifuge and balance centrifuge is consent we need to understand what is the relationship between ramp angle that is the angle which is supplied by the basket by when it is actually in the 1 g condition that is

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Relationship between ramp angle,  $\theta$  with RPM,  $\omega$  for beam centrifuges

$$\sum F_r = mr\omega^2 = T$$

$$T = m\omega^2 (R + l \sin\theta)$$

$$\sum F_y = N - mg = 0$$

(neglecting vertical acceleration)

If there is no angular acceleration of the model about its centre of mass (i.e. P) then there can be no moments about P.  $\Rightarrow$  Resultant of T and N pass through P

$$N \tan\theta = T$$

$$g \tan\theta = r\omega^2 \left(1 + \frac{l}{r} \sin\theta\right)$$

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When the basket is vertical and the weight will be acting downwards and when it you know the rotation starts about vertical axis let us say that this is  $\rho$  which is actually rotating at vertical axis and there is spring of the basket takes place so at any point of time  $t$  once it starts rotation of a you know the vertical axis that angle be  $\theta$  then you know assume that these are the radial axis and this is the  $y$  axis vertical.

And this is the frictionless for you have got a tangential force acting that is on the normal force acting upwards and this is the model package and then model package and the basket weight assumed that this is the center of mass that is the point  $p$  which is actually the weight acting downwards including.

The basket weight as well this is actually acting downwards now by taking you know this angle is  $\theta$  and this is  $mg$  now at this point from the distance from the edge to the point  $p$  which is you know the distance is  $l$  so this distance is  $x \sin \theta$  so if this is radius from central of centre of shape to the centre point of the inch it is  $r_h + l \sin \theta$  where  $r = r_h + l \sin \theta$  now by taking you know  $\sigma_{fr} = 0$  and  $\sigma_{fi} = 0$  we can write  $\sigma_{fr} = 0$  if you write you know what actually happens is that when the model rotates above the vertical axis in horizontal planked there is the radial acceleration which actually develop and the radial force which actually develop the centrifuge force which is nothing but  $mr\omega^2$  as to be balanced by  $T$ .

So  $T = mr\omega^2$  so we can write  $T = m\omega^2 r$  for substituting  $r = r_h + l \sin \theta$  so the  $r$  is nothing but  $r_h + l \sin \theta$  which is indicated as  $R$  here  $r_h + l \sin \theta$  and by taking  $\sigma_{fi} = 0$  we can write  $n - mg$  where is  $mg$  and

neglecting vertical acceleration so if there is no angle acceleration of the model above the centre of the mass  $p$  and so that can be no movements and we can actually assumed that the resulted of  $T$  and  $N$  passes through this centre of the gravity.

That is the centre of the mass point  $P$  is the resulted of  $T$  and  $N$  that means that this angle  $N \tan \theta$ ,  $\tan \theta = T/N$  and that angle  $\tan \theta = T/N$  so we can write that  $T = N \tan \theta$  now by taking  $n_g = mg$  and then substituting from  $\sigma_{fr} = 0$  whatever the expression we got  $T = mr \omega^2$  for  $n$  we substituting  $m$  so it is  $Mg \tan \theta = mr \omega^2 * r_h + L \sin \theta$  now what will happen is that  $m$  and  $m$  will get canceled so this indicates that the ramp angle is independent of the you know the mass so what we get is the expression after simplification.

We get the  $g \tan \theta = r \omega^2 / (1 + 1/r) \sin \theta$  so were  $r$  is small  $r$  is nothing but  $r_h + L \sin \theta$  so what we can actually what it means is that you know the this equation once you know this configurations we can actually calculate what is the you know the ramp angle that is you know this is  $\theta$  were in both left hand side as well as right hand side so we actually need to use a iteration process so that you able to get the relationship between  $\theta$  and  $\omega$ .

So as the  $\omega$  you know reaches to certain value what we tend to see is that the  $\theta$  becomes close to  $90^\circ$  and then it remains constant then that means that there is the point what we actually say that you know the designed gravity level is actually achieved once the result level is achieved as that we do a test a constant gravity level or at the variable gravity level.

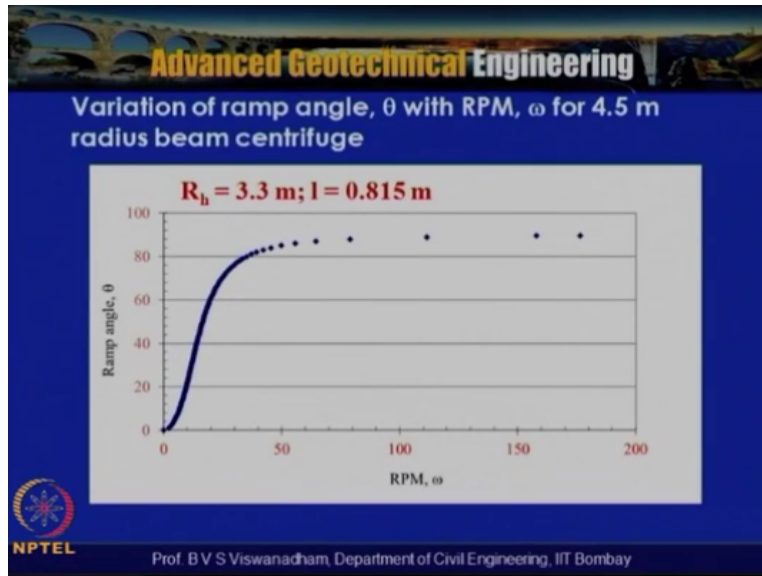
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So this is you know the swing in action of IIT Bombay large centrifuge is actually shown and this view is actually obtained by Camera which is actually located in the centre and vary in the camera gives the swinging up of the angle so what we can see is that the model actually is swinging up and it remains the horizontal position s long s you know the machine is actually you know under rotation so this is you know large centrifuge in action at IIT Bombay so once again this is shown okay.

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Now after having a seen the you know how the you know the machine were it is subjected to rotation above the vertical axis it swings up and at times some constant you know the ramp angle so now in this particular slide the variation of the ramp angle  $\theta$  with Rpm  $\omega$  for radius of 4.5 meters radius is centrally shown here.

So what can be see is the by using that equation this has been found put by  $R_h=3.3$  meters and  $l=.815$  meters so the from the centre of the inch to top surface and basket is the it is you know above 1.2 meters so this is actually assumed as .815 meters so once this is calculated what actually takes place is about 34 to 35 Rpm.

You know when it actually attains so called the horizontal T and you know the angle will be very close to  $90^\circ$  but it cannot be  $90^\circ$  by because there is one g gravity which is actually acting downwards us even the what we have discussed is the radial acceleration filed when it is actually happening the  $Ng$  which is acting towards the model and then 1 g is actually acting downwards what we have is that the resultant gravity level which is 1 which we need to considered.

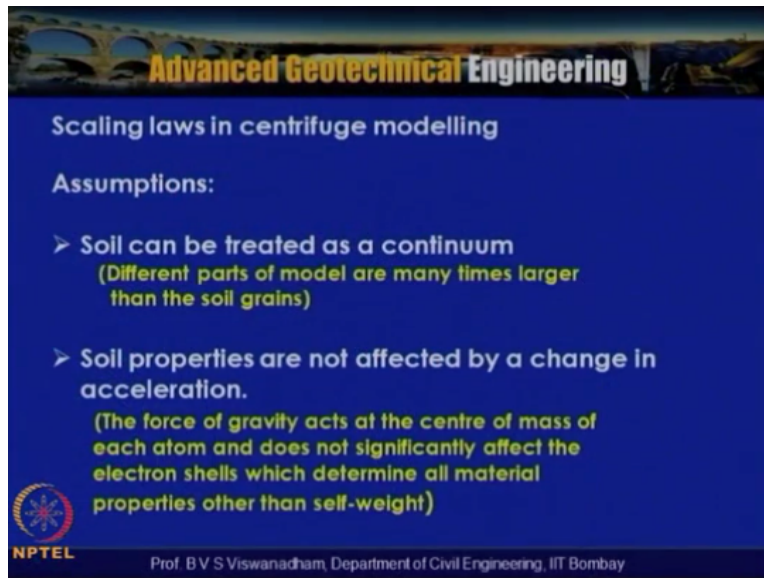
This is nothing but root over  $n^2+1*g$  so if you say that  $N=100$  that  $100^2 +1$  root over which is almost equivalent to 100. you know some decimals in to times g so which is regarded as 100g and this is for you know typical 1.1 meter radius centrifuge were in  $R_h=.74$  meters that is from the centre for the shaft to you know the centre of the inch and l is actually assumed as .22 meter here.

So this implies that this small as radius it actually takes longer Rpm to become flat form to be horizontal to 210 to 220 Rpm so it actually acquire angle of above  $89^\circ$  also so it actually implies that the smaller the radius the beam centrifuges around be used above a certain  $\omega$  value up to that time you know we cannot actually claim that gravity conditions have been achieved and this are actually called as sudo gravity conditions.

So 1. we need to note down is that the mass is actually independent of that when we are calculating the ramp angle the mass is independent from the you know the discussion whatever we had and second issue is that the beam centrifuges around be used above a certain  $\omega$  so were in we can actually you know get the desired you know Rpm then beyond which we can actually can be for example if radius of 1.1 meter is used.

So you know then know these scaling laws which are actually going too discussed cannot be applied appropriately so before discussing about scaling laws.

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**Advanced Geotechnical Engineering**

Scaling laws in centrifuge modelling

Assumptions:

- > Soil can be treated as a continuum  
(Different parts of model are many times larger than the soil grains)
- > Soil properties are not affected by a change in acceleration.  
(The force of gravity acts at the centre of mass of each atom and does not significantly affect the electron shells which determine all material properties other than self-weight)

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Let us look into the you know this simple assumptions which are actually the earlier the scaling laws were reduced by 1998 and this also by start based on the you know definitions the fundamentals definitions the you know scaling laws are been reduced some of the fundamental assumptions that we have been put forward is that the soil can be treated as a continue that means that different parts of model are many times larger than the soil grains.

That means that if I have a length of the container, the length of the containers should be many times larger than the soil grains similarly if you are actually having a footing the footing should have reasonably large when you compare with these size of the grain, so that actually will you know the soil can be treated as a continue.

And this is one of the conventional assumption in soil mechanics and the soil properties are not affected by the change in acceleration because we said that in centrifuge there is physical modeling the we in order to actuate the identical stresses in model to protect we say that  $\gamma_m = n \gamma_p$  so wherein  $\gamma = \rho$  into  $g$  with that you know with  $gm/gd = n$  if you are able to achieve with identical soil as that in the prototype what we said is that the  $\gamma_m$  becomes  $n \gamma_p$ , so that means that you know the soil properties are not affected by change in acceleration.

Except the you know unit weight which actually determining the you know the result of the sulphate so the force of this you know the for the explanation for this you know this is assumption that actually given by square field 1980 the it is said that the force of the gravity act at the center of the mass of each atom and does not significant affect the Alton shells which

determine the all material properties other than the sulphate, so only accept this in weight the rest of the properties are assumed to be not affected so this assumption you know ensures that you know identical soil properties are Hc and 5 and that also we actually reduced from the you know the dimensional analysis are in then theory of the models.

That they should be identical in modeling prototype, so these are the some fundamental assumptions as far as the scaring laws in centrifuge modeling is concerned, now coming to the similitude in geotechnical engineering wherein like in conventional fluid mechanics wherein we actually have the similitude definitions like linear similitude kinematic similitude.

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**Similitude in Geotechnical Engineering**

- Linear similitude  

$$\frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{H_m}{H_p} = \frac{1}{N} \quad A_m/A_p = 1/N^2; V_m/V_p = 1/N^3$$
- Kinematic similitude  

$$v_m/v_p = N_v; a_m/a_p = N_a$$
 For an identical soil in the model and prototype,
- Dynamic similitude  

$$F_m/F_p = N_F$$

$$\rho_m = \rho_p \rightarrow N_M = N_V = 1/N^3.$$

For identical effective stresses in the model and prototype:  

$$N_{\sigma'} = N_{\sigma} = N_u = \sigma_m'/\sigma_p' = \sigma_m/\sigma_p = u_m/u_p = 1$$

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And dynamic similitude so where in the linear similitude in the sense that whatever the dimensions which are actually there in prototype they have to be scaled by 1/N times and the factor is constant for all the dimensions that means that if I have a length and breadth and height then it is length and model prototype which is also called as length scale factor and then and that scale factor BM/Bp and height scale factor is HM/HP has to be reduced 1/N and if the length and breadth are actually reduced by 1/N.

Then area which is nothing but  $A_m/A_p = 1/N^2$  and similarly the volumes are reduced by  $1/N^3$  that is that  $V_m/V_p = 1/N^3$  and so this is as far as the linear similitude is concerned and it similarly kinematics similitude is concerned wherein we have  $V_m/V_p = N_v$  that is nothing but the scale factor for velocity and similarly the accelerations  $A_m/A_p$  has to be =  $N_a$  that is the as far as the

scary factor for the accelerations concerned, so velocities and acceleration where also have to be scaled.

And then the similarity need to be maintained similarly you know when for all for an identical soil in model type with the  $\rho_m = \rho_p$  the scale factor for the you know mass and volume are equal that is nothing but  $m = N$  suffix  $m$  is nothing but the scale factor or mass =  $N$  suffix here  $B = 1/N^3$  the dynamic similitude is nothing but  $F_m / F_p = N^f$  wherein you know we can actually say that this in the case of dynamic similitude even if you have different forces like  $C^+$  forces or let us say.

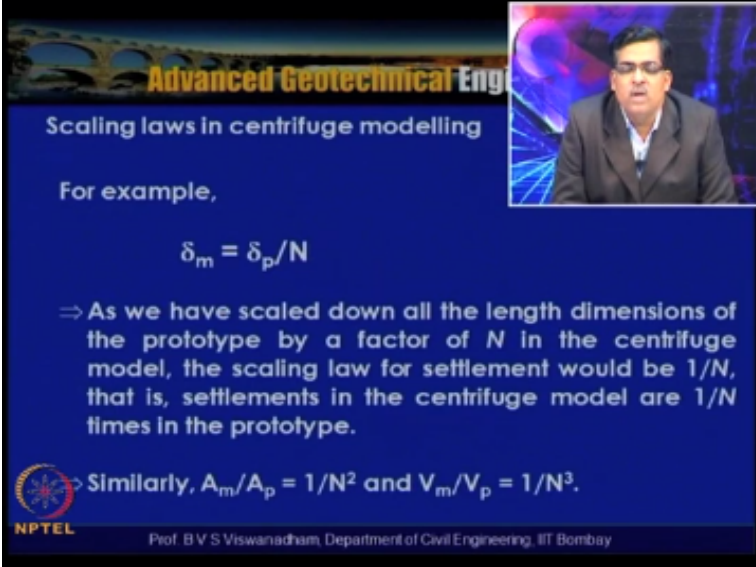
You know creep forces or some discuss forces or we are having some dynamic forces or if you are actually having some weight forces what will be the type of the force what dynamic is similitude says that the dynamics similitude is says that it actually should have a constant scale factor for the force then only we can say that the dynamic similitude is satisfied, so for identical effective stresses in model in prototype and what we says is that the  $N \sigma$ - that is the scale for a effective stress and  $\sigma$  scale factor for.

Total stress and  $U$  the scale factor for pore water pressure where it we have  $\sigma - / \sigma_p - = \sigma / \sigma_m / \sigma_p = U_m / U_p$  where  $U$  is nothing but the pore water pressure in model and prototype so this pore water pressure is nothing but defined as  $\gamma_w$  into the height let us say  $h$ , so you know when we have the  $\gamma_w$  in model is  $n$  times  $\gamma_w$  in prototype the even the with reduced column of water height of water what we actually get is that identical pore water pressure as that in the prototype when you have the identical pore water pressure as that in the prototype and when you have the identical stresses.

Total stress is in model prototype or identical then the effective stresses are identical when we have the effective stresses identical is achieved then the soil stress strain behavior or the soil shear strength is actually simulated accordingly, so now here the scale in factor for the last in set first modeling as we have said that from the linear similitude point of view length and the breadth and height or reduced by  $1/N$  so in that case the displacements they can be due to settlements or pile deflections or they can be you know lateral deflection of a wall and these displacements have to be.

1/N times as that in the prototype that means that if you are having a 500mm displacement and that 50g that means that it is about only 10mm displacement it should regard, so that means that the  $\Delta p/N$  so we have scale down the length dimensions of the prototype by a factor N in the centrifuge model the scaling for the settlement mean you know 1/N.

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Scaling laws in centrifuge modelling

For example,

$$\delta_m = \delta_p / N$$

⇒ As we have scaled down all the length dimensions of the prototype by a factor of N in the centrifuge model, the scaling law for settlement would be 1/N, that is, settlements in the centrifuge model are 1/N times in the prototype.

Similarly,  $A_m/A_p = 1/N^2$  and  $V_m/V_p = 1/N^3$ .

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That means that as we have scaled the length scale factor by 1/N times the displacements have to be also 1/N, so the settlements in the centrifuge model are 1/N times that in the prototype, so similarly the areas which are all actually involved let us say  $A_n/A_p = 1/N^2$  so let us if you look into you know something like 0.72m / 0.36m area into 40 gravities, so this will actual represents about 450<sup>2</sup>m of area in the field conditions, so this actually represents events the small areas when they are actually provide tested at higher gravity level.

so there is a possibility that they represent the large areas under considerations similarly the volumes which are actually involved  $V_m/V_p = 1/N^3$  then the we have already proved that the stresses are identical in centrifuge.

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Scaling laws in centrifuge modelling

➤ We have already proved that the stresses are identical in the centrifuge model and the prototype.

$$\frac{\sigma_m}{\sigma_p} = 1$$

As  $\delta_m = \delta_p / N$ ,       $\epsilon = (d\delta) / \delta$

With  $(d\delta)_m = (d\delta)_p / N$        $\rightarrow \epsilon_m / \epsilon_p = 1$

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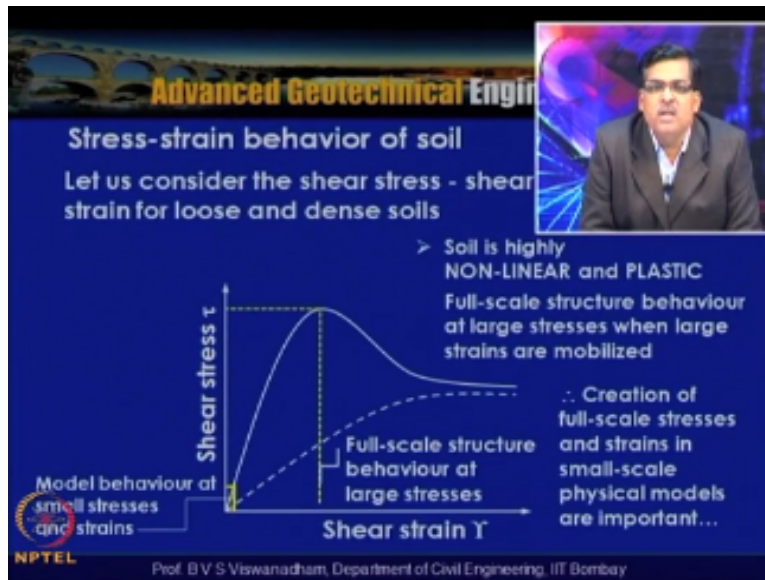
In model and prototype we said that the stress in the model and prototype are identical because of the enhancement of the unit weight, so with that  $\sigma_m / \sigma_p = 1$  and we also said that because as we came down the maintain the linear simulated of you know the length dimensions then the displacement settlements have to be  $1/n$  times of the prototype. So this strain you know which is caused due to you know that displacements of the particles are the total displacement of a body.

It can be within the soil mass means the crushing of the soil particles and you know the bending of the soils particles in case of clays. So these strains which are actually nothing but it can be due some distortion or due to crushing of the grains are due to movement of a rigid body. So if you define the strain as  $d\sigma / \sigma$  with as  $\sigma_m / \sigma_p$  is  $1/n$  so that small change in the displacement that also as to be small, that is  $d\sigma$  in model =  $d\sigma_p / n$ .

So with this what we can say that you know with  $\sigma_m / \sigma_p = 1/n$  and  $d\sigma$  in model =  $d\sigma_p / n$  so  $\epsilon_p / \epsilon_m = 1$ , so this ensures that identical stress and strains in model prototype and this actually has got you know the relevance as far as the stress strain period of the soil in concerned.

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So this we have discussed that if you actually having you know I g model test conducted at small stresses so initially the model which actually will have you know one stress because you know when we have the low stresses the soil will actually have higher stiffness.

And though the scale factor from the dimensional analysis says that the  $\Delta m / \Delta p / L$  by the virtue of the simulating modelling, but what I actually you know we measure is that in the model test is that you know the less is that the less settlements in the, for the structure being constructed. But when we look into the real prototypes conditions where the stresses are high, and as the strains are actually high the stiffness of the soil actually falls.

So with that the real prototype conditions when it is actually there with the real stresses. And then what is actually happen is that the stiffness is low. So the settlements are large. So this actually shows that the soil behavior is actually highly non-linear and plastic. So in order to capture the identical behavior the creation of the foot scale stresses and strains in the small scale models is very, very important.

This slide we have already discussed, but to bring the relevance of the, as we are discussing about the stress and strain modeling, we actually trying to bring this discussion once again. Now let us look into different aspects of the scaling loss and centrifuges modeling. So here like we have force work and energy.

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Scaling laws in centrifuge modelling

**Force, work, and energy**

- Considering the basic definition of Newton's second law of motion, a force  $F$  acting on a body of mass  $m$  will cause an acceleration  $a$ , such that:  $F = ma$

$$\frac{F_m}{F_p} = \left(\frac{m_m}{m_p}\right) \left(\frac{a_m}{a_p}\right) \quad \text{with } m_m/m_p = 1/N^3 \text{ and } a_m/a_p = N$$

$$\Rightarrow F_m/F_p = 1/N^2$$

For example, a 3000 kN force in a 50 g centrifuge test scales down to be only 1200 N. This is another advantage of a centrifuge test, as we can easily make actuators to load piles, retaining walls, slopes, etc., and forces that need to be applied by these actuators are relatively small.

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Now considering the basic definitions of Newton's second law of motion, and we can say that a force  $F$  acting on the body, we can actually define it as  $F=ma$ , where  $M$  is the mass, and then  $A$  is the acceleration. So force acting on the body in the direction of that acceleration. So in order to get the scale factor for the force,  $f_m/f_p=m_m/m_p/a_p/a_m$ , and with  $m_m/m_p=1/m_q$ , because we have used the same soil as that in the prototype with maybe retaining the  $\rho_m/\rho_p=1$ .

And reducing the volume  $v_m/a_p=v_m/v_p=m_m/m_q$ , the  $m_m/m_p=1/m_q$  and enhancing the acceleration  $N$  times, then it is  $a_m=na_p$ , so what we get is that  $f_m/f_p=1/n^2$ . So this actually has the practical relevance of this is that, if you are having say, 3000 kilo Newton of force in 50 gravities centrifuge test, the scales down to be only 1200 Newtons. That means that a 3000 kilo Newton force in a 50g centrifuge test scales down to only 1.2 Kilo Newtons.

So for example this is a case for example, if you wanted to do the lateral load capacity of a large case and foundation, then you need to develop a 3000 Kilo Newtons in the sense that 6000 Kilo Newton this training capacity cantilever many times under these type of situations are difficult to achieve in the field. And this is another advantage of a centrifuge test that as we can actually make actuators to load piles, retaining walls or slopes, or amendments, or dams.

And forces that need to be applied by these actuators are relatively small. So another advantage of the merit of the centrifuge will actually if you look into it, we can make the actuators to apply very, very small forces, but these forces correspond to the forces which are actually correspond

in applying the appropriate scaling conditions as per the gravity level being considered. So this is this core factor for the force is concerned.

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Scaling laws in centrifuge modelling

Force, work, and energy

The basic definition of work done  $W$  is the product of a force  $F$  moving through a distance  $d$ .

$$\frac{W_m}{W_p} = \left( \frac{F_m}{F_p} \right) \left( \frac{d_m}{d_p} \right) \quad \text{with } F_m/F_p = 1/N^2 \text{ and } d_m/d_p = 1/N$$

$$\Rightarrow W_m/W_p = 1/N^3$$

$\Rightarrow$  This scaling law for work done suggests that the work done in a centrifuge model is relatively small compared to that in a prototype. This is also advantageous for centrifuge modellers.

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Similarly, consider the work, the basic definition of the work done  $W$  is nothing but the product of the force moving through a distance  $D$ , that means that  $W=Fx D$ . So the work energy, the work done in moving the body by a force  $M$  through a distance  $D$  is given by  $W=Fx D$ , where  $W$  suffix  $M$  suffix  $P=FM/FP \times DM/DP$ .

Now having known with  $FM/FP=1/N^2$  and  $DM/DP=1/N$ ,  $DM/DP$  is nothing but the distance from the linear similitude point of view, it is scaled by  $1/N$  times. So with that when you substitute back what we get is that  $WM/WP=1/N^3$ . So the scaling law for work done suggests that the work done in a centrifuges model is relatively small compared to that in the prototype.

The work done is nothing but  $WP/N^3$  that means that the work done in this centrifuges model for the same energy is actually is compared to be very small so this is also another advantage as for as the centrifuge is concerned, so the scaling law for the work done suggest that the work done centrifuge model is relatively small compare to that in the prototype.

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Scaling laws in centrifuge modelling  
Force, work, and energy

Consider the definition of potential energy PE normally expressed as energy lost by a falling mass  $m$  through a height  $h$ ,  $\rightarrow PE = mgh$

$$\frac{PE_m}{PE_p} = \left(\frac{m_m}{m_p}\right) \left(\frac{g_m}{g_p}\right) \left(\frac{h_m}{h_p}\right) \quad \text{with } m_m/m_p = 1/N^3, g_m/g_p = N$$

$$\text{and } h_m/h_p = 1/N$$

$$\rightarrow PE_m/PE_p = 1/N^3$$

Thus, centrifuge modelling can offer a very effective way of investigating the effects of explosions on buildings, earthen dams, dams, retaining structures, etc., without the need to conduct these studies at full scale, which can be both expensive and damaging to the environment.

NPTEL Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

Now after having discussed about the work as for as the definition of the units and is concerned with from the physics definitions concern work and energy are equal but consider the definition of the potential energy the potential energy  $pe$  which is equal to normally expressed as energy lost by falling mass  $m$  through a height  $h$  so wherein  $pe$  which is nothing but we can actually say that  $p = n gh$  so we can say that  $pe$  in model  $pe$  in prototype which is nothing but we can actually write it like the product of  $m_m / m_p$  and  $g_m / g_p$  and  $h_m / h_p$ .

So with  $m_m / m_p = 1 / n^3$  and the acceleration which is  $n$  times that is  $g_m / g_p = n$  and  $h_m / h_p = 1 / n$  so we can write that the potential energy in model in prototype is  $1 / n$  times. So here which actually says that you know the centrifuge model can offer a very effective way of investigating say for example when you are trying to see the energy which is actually realize due to some explosion or a blast load effect on certain geo technical structures or buildings or earth and dams or dams or a training structures without the need to conduct these at full scales many times they are expensive and they damaging to the environment.

So in such situations see a centrifuge modeling offers in effective way of investigating the effects of the explosions on this buildings wherein the energies are actually modeled which is actually you know which is nothing but  $1/n^3$  times that of the energy of the prototype say  $pe_p$  which is nothing but  $n^3$  times potential energy suppose this concept is something like if you are dealing a type of ground improvement technique like you know dynamic compaction wherein a known weight is actually dropped over a height  $h$  with a potential energy you know  $h$ .

So wherein what we actually get is that you know the  $1/n^3$  so this concept of you know the energy modeling can be used for understanding some problems like the liquefaction mitigation measures are a ground improvement methods like dynamic compaction, wherein we will be able to simulate with identical stresses and strain fields we will be able to see the response of a particular model applied with the different energies where the different parameter studies can be done.

Similarly the kinetic energy which is also again said as  $\frac{1}{2}mv^2$  and with that kinetic energy also is modeled as  $1/n^3$  wherein  $\frac{1}{2}mv^2$  when this mass is actually is  $m/n^3$  and wherein velocity at present as for as the dynamic velocity or the motion is concerned which is  $v_m = v_p$ , and where in with that what it actually tells is that the kinetic energy in model prototype is  $1/n^3$ . So with that what actually happens is that now the work energy and you know potential energy and kinetic energy which is actually modeled as  $1/n^3$ .

So let us say that we are actually interested in studying a impact load on the you know on particular foundation suppose a ship impact load on the pile foundation wherein the certain weight of a model is actually the moving weight is released and to make it to hit the you know the foundation. So that actually mass the you know the modeling of the impact on the particular foundation.

So in this particular lecture what we try to understand is that you know different you know the scaling lose the basic scaling lose which are actually require for centrifuge modeling and then you know further we have you know different forces like cps forces or we have some dynamic forces or we have some weight forces or the weigh forces are basically they are you know due to body forces or due to the structurally externally applied loadings that is nothing but you know application of a load to your footing or application of load to a lateral load to a foundation.

Then in that situations this actually scales down you know require to understand the scaling lose then you know in this particular lecture we are actually try to look in to the basic you know parameter which actually as for eleven to centrifuge modeling this is a scaling lose then we also have seen different machine configurations and we said that different type machines which are balanced in beam and beam set if you just some select machines are actually are you know shown then we also have deduced relationship between ramp angle and  $\omega$  that is the angular velocity of the model.

Then we said that the beam centrifuges can be used depending upon the type of the configuration on the equipment the smaller the you know radius then they can be used only up to the rpm is will be large to attain this you know the so- called 89.5 or a 90<sup>o</sup> horizontal plane. So you know that means that they can be used only beyond that certain rpm, otherwise you know the up to that stage actually what it actually said is that the sudo gravity conditions provide. Then we also have discussed that you know the typical the drum centrifuge which are actually available and how you know the model preparation can be done by using sand in the drum centrifuge actually was introduce.

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**NPTEL  
Principal Investigator  
IIT Bombay  
Prof. R. K. Shevgaonkar  
Prof. A. N. Chandorkar**

**Head CDEEP  
Prof. V.M. Gadre**

**Producer  
Arun Kalwankar**

**Project Manager  
Sangeeta Shrivastava**

**Online Editor/Digital Video Editor  
Tushwar Deshpande**

**Digital Video Cameraman**  
Amin Shaikh

**Jr. Technical Assistant**  
Vijay Kedare

**Project Attendant**  
Ravi Paswan  
Vinayak Raut

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