

**NPTEL  
NATIONAL PROGRAMME ON  
TECHNOLOGY ENHANCED LEARNING**

**IIT BOMBAY**

**CDEEP  
IIT BOMBAY**

**ADVANCED GEOTECHNICAL  
ENGINEERING**

**Prof. B. V. S. Viswanadham  
Department of Civil Engineering  
IIT Bombay**

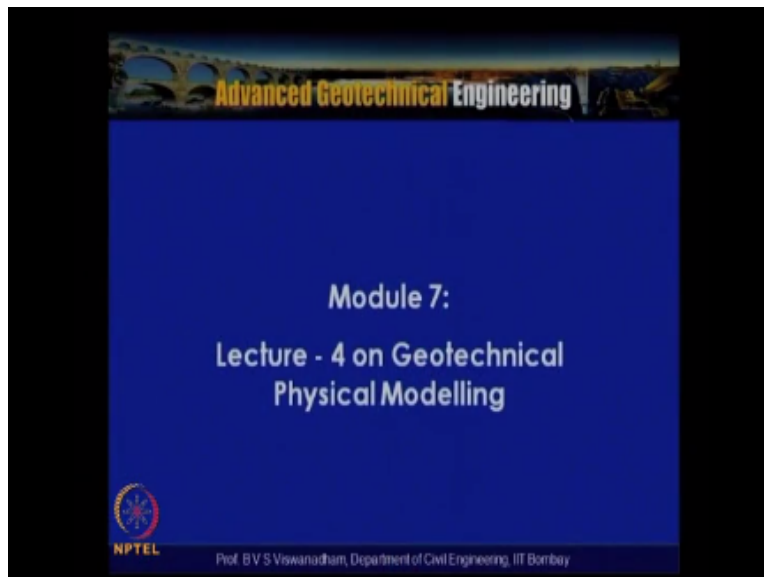
**Lecture No. 53**

**Module – 7**

**Lecture – 5 on Geotechnical  
Physical Modelling**

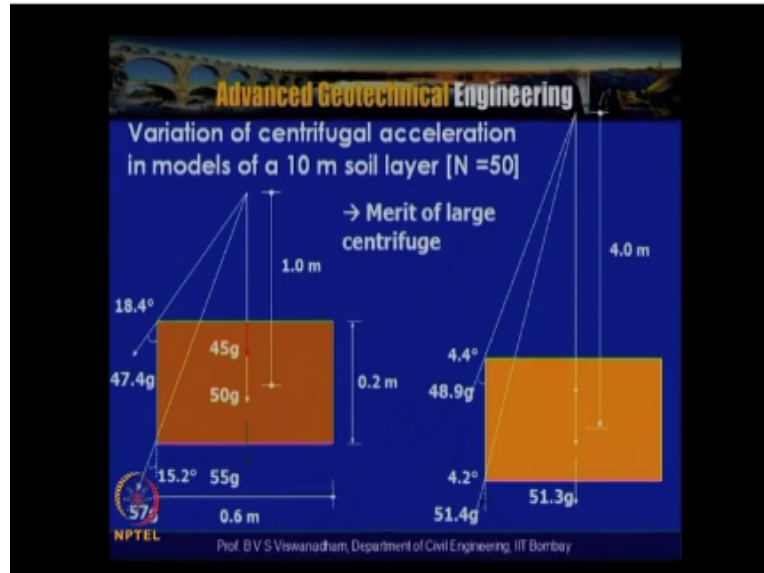
So welcome to course on advanced geotechnical engineering module 7 lecture number 4, this is lecture number 4 on geotechnical physical modelling.

(Refer Slide Time: 00:28)



So in the previous lecture we have discussed about variation of g level or gravity level with the horizontal distance as well as the depth.

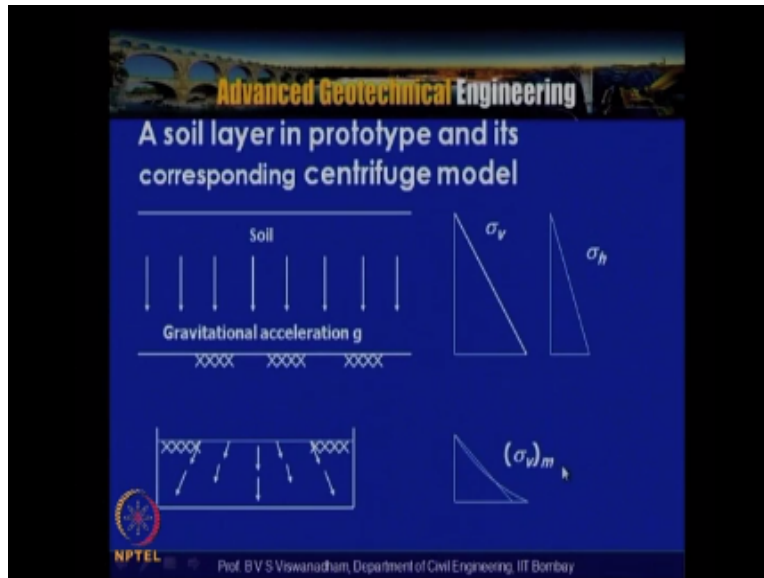
(Refer Slide Time: 00:48)



In this particular slide the merit of large centrifugal is shown, so here if you look into it there is the radius which is 1m from the centre of the shaft to certain point in the model. Similarly here this radius is about 4m, so if you look into it there is you know there is a variation horizontal distance but is limited but in the case when you take a radius of 1m here it is 45 gravity and here it is 47.4 gravity.

So there is variation of g level with the horizontal distance similarly here it is 45 gravity and at the bottom it is 55 gravities, so because here it is  $Rt \omega^2$  and here it is  $Rt + 0.2 \omega^2$  but where as if you have large mean centrifugal the variation g level is exist is prevalent but however the variation is negligible. So this actually shows the merit of large mean centrifugal. And here in this slide whatever we have discussed about you know in the normal prototype but you have got a soil state which is having certain thickness. And because of the self weight the vertical stress is  $\sigma_v$  and the horizontal stress.

(Refer Slide Time: 02:22)



If it is normally consolidated soil that is shown here, but if the model if the same prototype soil is brought and the model is constructed with a reduce thickness and subjected to the radial acceleration field then what we have discussed is that the variation of the vertical stress with the depth is not linear is non linear. Then we say that this is a going to cause certain degree of under stress in the upper half portion and the wall stress in the bottom portion.

So this variation is raised is because of the variation of the  $g$  level with the depth, so you can see that there in this particular slide and this particular diagram you know line shows the distribution of vertical stress as per the but if you look into this there is a non linear variation of the vertical stress in the model due to radial acceleration field.

(Refer Slide Time: 03:37)

**Advanced Geotechnical Engineering**

**Comparison of Hydraulic Gradient Similitude Method and Centrifuge Model tests**

	Centrifuge Tests	Hydraulic Gradient Tests
Principle	Centripetal Forces	Seepage Forces
Porous Material Soil	Self-weight Stress Increased	Self-weight Stress Increased
Solid Structure Member	Self-weight Stress Increased	Self-weight Stress Not Affected
Applications	More General Problems	Problems with Level Ground and Self-weight of Structure Member is not Important

(After Yan, 1990)

NPTEL  
Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

And before going further and we also have discussed about you know two possibilities of you know increasing the  $\gamma$  one is that variation of  $\gamma$  by increasing the  $C_p$  forces that we said that hydraulic method and in the centrifugal model test we said that this can be possible by subjecting a model to high gravities, this is done by rotating a vertical axis in the horizontal plane. So if you compare the basic principle one is that centrifugal model test, the test acts towards the centre.

And in case of there is centrifugal model test and in the porous material soil the self weight stress increase and in the hydraulic variant method we have self weight stress increase and solid structure number it can be a pile number or it can be footing number it can be retaining wall it can be any number which is tested. The self weight stresses also increased in the case of the centrifugal model test but where as in case of hydraulic variant test and which is not done.

That means that self weight stresses are not affected they remain same and the applications in the sense in that they are more general problems composed complicated problems the construction process can be stimulated or some climatic event can be stimulated like rain fall or subjected to the earth quake or combination of the different loading forces can be stimulated in centrifugal model test which we are going to discuss in some selected applications.

But as per as hydraulic variant test is concerned we said that is goes for sandy soils and problems with level ground and the sulphate of the structure number is not important and where ever the structure number is not important and where you have got problems with the level ground like

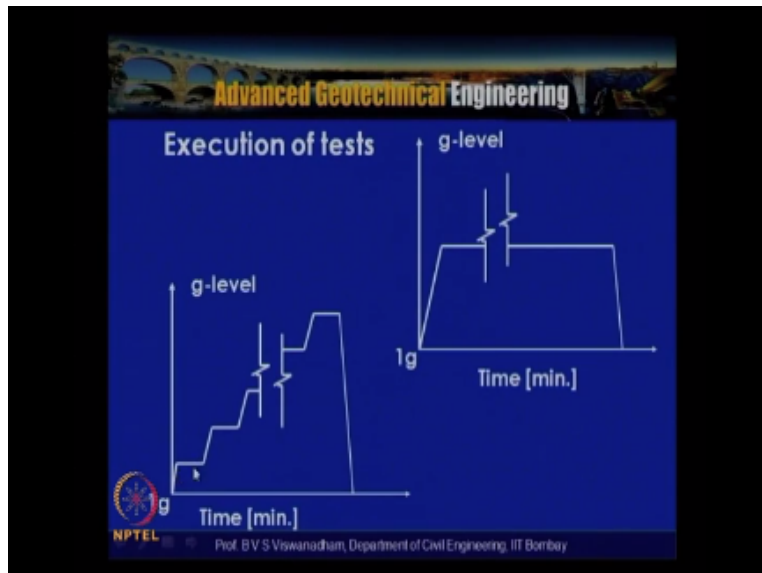
you know let us say that footing resting on horizontal surface or problems with like behavior of pile subjected to you know uplift embedded in sandy soils.

Similarly when you have got some problems like footing subjected to on the level ground these things can be studied by using the hydraulic variant stimulate method. So this particular slide gives comparison of as hydraulic variant stimulate method and centrifugal model test. And we said that as far as the centrifugal model test is concerned 2 categories of test do exist, one is called variable gravity level the other one is called constant gravity level.

The mostly the constant gravity level is adopted in the previous discussion we have discussed that when the you know gravity reaches to a constant level that means that a particular desired gravity then, when the gravity reaches to a certain level then what we can do is that you know the angle of velocity is maintained constant, so the tangential component of the acceleration will be 0. And secondly if the particular model been tested as to be safe upto the particular gravity level is been tested.

Then it can be subjected to some sort of loading or some sort of construction process or sort of effect due to some of forces like wave forces or some earth quake forces etc. Where as in case if you are interested in studying some collapse behavior of certain structure collapse behavior vertical cut or some slope which is been studied. Where in you can be studied with the variable gravity level. So here the gravity level is increased gradually at a certain speed.

(Refer Slide Time: 07:57)



And the speed at which is this is settled for example if you are increase it has to be you know keep in mind with the desire conditions like undrained conditions and if you are required to have then the waiting times also has to be small. So here this is you know a certain gravity level and this is another gravity level and then upto collapse or the failure which are actually occurred earlier.

So this is called variable gravity level but each and every gravity level you know the model actually behaves like a prototype that means that if you are actually having sandy models dry sand model the generation of pore water pressures will not be there if you are having a dry sandy soils, so upto some extent like a enforced slope constructed with the sandy soil where you are interested in the collapse behavior of the enforced slope constructed with the sandy soil.

They can be studied but each and every gravity level when they are subjected to they are equivalent to equal prototype with respect to the particular gravity level. So this is the execution of the test one is called variable gravity level and constant gravity level but nowadays with the development of different actuated systems and you know rainfall events or earthquake events, the constant gravity test are adopted widely. So the possibility of geo technical centrifugal studies include we can actually model the prototype.

(Refer Slide Time: 09:41)

The image shows a presentation slide from NPTEL. The slide title is "Advanced Geotechnical Eng". The main content is "Possibility of geotechnical centrifuge studies", which includes a bulleted list: "Modelling of prototypes", "Investigation of new phenomena", "Parametric studies", and "Validation of numerical model". At the bottom, it says "Geotechnical engineering instruction" and "NPTEL". A video inset in the top right corner shows Prof. BV S Vowanatham, Department of Civil Engineering, IIT Bombay, speaking.

That is that if you are actually interested in understanding a particular prototype structure which has been constructed whether it is safe or not or what will be its performance after 5 years or 10 years of its life can be studied by using centrifugal model technique. But however the size of the model depends upon the size of the equipment which is being used for the purpose, so if the equipment and large prototypes can be modeled very well by stimulating the majority of the characteristics of the prototype in the model.

Another possibility of the geotechnical centrifugal studies is that investigation of new phenomenon that means that a new technique or can be studied by using this geotechnical centrifugal studies where in this helps to draw guidelines and then to also assess its performance over a period of time that is before failure and it also leads to you know the improvement of the phenomenon being investigated.

So this is very interesting where new phenomenon can be studied very efficiently by using the centrifugal based modelling, then another possibility is that the parameter studies, so if you have studied for a certain dimensions then you wanted to know what will happen to the next level, so parameter studies or the parameter variation is very efficient and can be studied you know efficiently in geotechnical modelling.

Then as said earlier the numerical models are required to be validated mostly they are validated by using the full scale physical models in the case of you know non-feasibility of constructing full scale physical models let us say certain areas like nuclear installation are where there is no axis

for constructing or feasibility to construct these full scale structures. Then you know the numerical model are required to be, so one of the new is that to perform the series of the centrifugal physical model test.

So that will actually help to validated the numerical models, the another important application is that use of the geo technical centrifugal for geo technical instruction that is something like called centrifugal aided learning in geo technical engineering.

(Refer Slide Time: 12:32)

**Advanced Geotechnical Eng**

**Radial acceleration field effects**

- The earth's gravity is uniform for the practical range of soil depths encountered in Civil Engineering.
- While using a centrifuge to generate high acceleration field required for physical modelling, there is a variation in the acceleration through the model.
- This is attributed to the inertial acceleration field given by  $r\omega^2$ .
- This apparent problem turns out to be minor if care is taken to select radius at which gravity scale factor  $N$  is determined.

NPTEL Prof. BV S Viswanatham, Department of Civil Engineering, IIT Bombay

So after having discussed the possibility of the geo technical centrifugal studies that we said that by subjecting the model to about a vertical axis you know in a rotational plane there is the possibility of this radial acceleration field effects are there. So these effects actually what we said is that you know effect the vertical stress and consequently the effect also the horizontal stress distribution.

So the  $x$  gravity is uniform for the practical range of the soils depth encountered in engineering, right upto 300m or what we encounter for soil investigation in engineering  $x$  gravity is uniform for the practical range of the soils depth encountered in engineering. So while using the centrifugal to generate high axial field required for the physical modelling there is the variation of acceleration through the model.

So what actually we have been discussing while using a centrifugal to generate high gravities because we wanted to have identical stresses as in the prototype. So what we said that this is

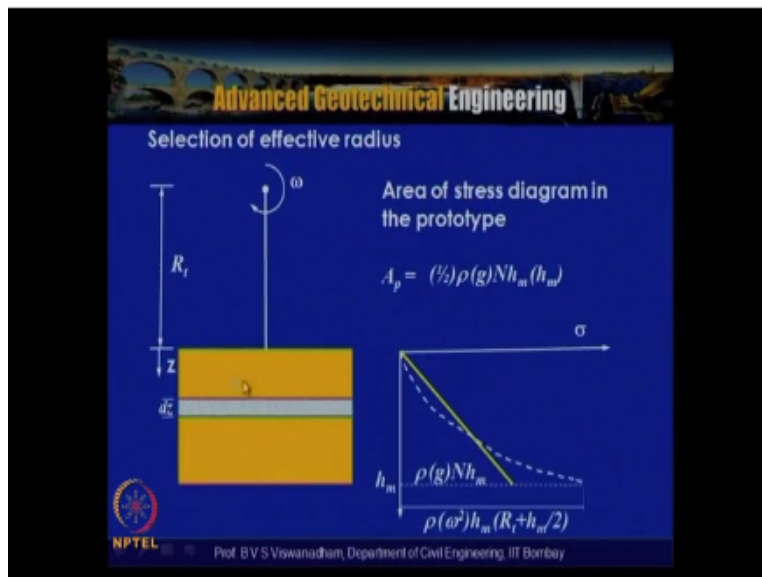


attributed to the inertial acceleration given by  $r\omega^2$  this is the initial acceleration field this is due initial acceleration field given by  $r\omega^2$  where  $r$  is the radius from the centre to a certain point in the model.

So this apparent problem turns out to be minor if you are able to take care of the you know the hardware that is the radius of the equipment at which the gravity scale factor  $n$  is determined and this also this operating radius that is called an effective radius variable to select properly then we can say that particular point this stresses in modelling prototype are identical. So this apparent problem turns out be minor if care is taken to select the radius at which gravity scale factor  $n$  is determined.

So for that what we need to do is that we have to find out you know how the variation of gravity level is there with the depth and where it is subjected to with the vertical stress distribution where it is subjected to under stress and over stress and comparing the errors then we can actually lead to you know the point at which we can actually you know maintain this gravity level can be ascertained.

(Refer Slide Time: 14:57)



So the selection of the effective radius for a beam and diffuse like you know consider a model rotating about a vertical axis which is shown here and the model is constructed with a material having low density the mass density  $\rho$  and this is the centre of the sharp the model is rotating over vertical axis and the  $V$  is the velocity in this direction and this is the small limit  $dz$  selected

at the depth  $z$  from the top surface and  $R_t$  is the radius from the centre of shaft to top surface model.

So if you look into this what we can do is that this is the you know the stress in the and this is the depth here in this direction so here this is the you know prototype system and this is the radial acceleration field, you can see that this portion there is a under stress and in this portion there is an over stress. So this could be understood because of the increasing gravity level at this depth there is in over stress possible.

So in order to determine this what we do is that again by following the same principle  $d\sigma_v$  which we can actually find out by writing like  $d\sigma_v = \rho \times z \times d\sigma_v$  can be find out and from there by integrating from 0 to  $H_m$  we can actually get  $\rho\omega^2 h_m \times R_t + H_m/2$ . So what we have is that and now here if the same point  $H_m$  if you are able to find out you know what is the stress in the prototype.

So at this level we can actually find out it is nothing but  $\rho \times$  which is the normal gravity the  $H_m$  is correspondingly the prototypes  $n$  times  $H_m$ , so  $\rho g \times nh$ . So the area of the stress diagram in the prototype is nothing but half base that is  $\frac{1}{2} \rho g \times nh \times H_m$ , so  $\frac{1}{2} \rho g \times nh \times H_m$  so this is the area of the such diagram.

(Refer Slide Time: 17:34)

**Advanced Geotechnical Eng**

Selection of effective radius

- Area of stress diagram in the cent

$$A_m = \int_0^{h_m} \rho \omega^2 z \left( R_t + \frac{z}{2} \right) dz$$

$$= \rho \omega^2 \left[ \frac{R_t h_m^2}{2} + \frac{h_m^3}{6} \right]$$

- By equating two areas of stress diagrams in centrifuge model and the prototype and by using  $N_g = R_e \omega^2$ , we get:

$$R_e = R_t + h_w/3$$

NPTEL  
Prof. B.V.S. Viswanadham, Department of Civil Engineering, IIT Bombay

So the area of the stress diagram can be obtained like this wherein this is by writing  $A_m = 0$  to  $H_m \rho \omega^2 z \times R_t + z/2 \times dz$ . So this is actually it is nothing but 0 to  $H_m \sigma_v$  so by writing  $\rho \omega^2$  that is  $R_t + z/2 + \rho \omega^2$  is nothing but the acceleration and  $dz$  is nothing but you know the thickness of the element which is under consideration. So by integrating this we get is that and then applying the  $\rho \omega^2 = R_t H_m^2/2$ . So now by equating that two areas of stress diagrams in centrifugal model or prototype.

And by using  $N_g = r_u \rho \omega^2$  this is called a effective radius which is measured from the centre of shaft to a point within the model, where we are actually going to find out the stresses are there. Now what we have done is that by equating the areas of the stress diagrams in centrifugal model in the prototype, then what we get is that  $R_e = R_t + H_m / 3$  by simplifying and the substituting we get  $R_e = R_t + H_m/3$ .

(Refer Slide Time: 19:13)

**Advanced Geotechnical Engineering**

**Selection of effective radius**

- For finding out location where maximum under stress occurs, → assume that it occurs out at a distance  $x$  from the surface.

$$d\sigma = \rho N_g(x) - \rho \omega^2(x)(R_t + x/2)$$

$$f'(x) = \rho N_g - \rho \omega^2 R_t - \rho \omega^2(x)$$

For  $d\sigma$  to be maximum  $f'(x) = 0$

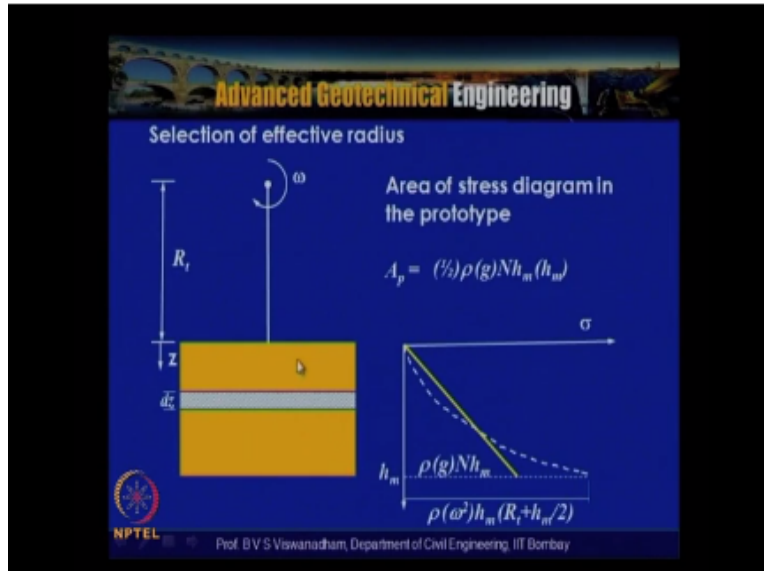
with  $R_e = R_t + h_m/3$  and simplifying, we get  $x = h_m/3$

At  $x = 2h_m/3, d\sigma \Rightarrow (\sigma)_m = (\sigma)_p$

**NPTEL** Prof. B.V.S. Viswanadham, Department of Civil Engineering, IIT Bombay

Then for finding out the location where maximum under stress occurs what we do is that, assume that occurs at a distance  $x$  from the surface.

(Refer Slide Time: 19:26)



We are now assuming in at a distance  $x$  you know below the surface, the notation is different here it is just to express in terms of function  $f$  it is written in terms you know vertical distance  $x$  here. Now  $d\sigma$  which is nothing but  $\sigma n g \times$  it is nothing but the ordinate of a particular depth  $x$  below the surface of the model. That  $x$  is  $x n$  to get the prototype test and  $g$  level is  $g$  and it is the  $\rho g \times n \times - \rho \omega^2 \times x \times R_t + x^2$ .

So this is nothing but the vertical stress in the model at a depth  $x$  in the model, now that is nothing but the  $d\sigma$  the difference between the prototype test and the vertical which we are trying to find out the difference between  $\sigma$ . Now by taking differentiation of this by taking this as function  $x \rho n g \times n \times - \rho \omega^2 \times x \times R_t + x^2$  by taking  $x f'x$  we can write that it is  $\rho n g - \rho \omega^2 R_t - \rho \omega^2$  that is  $2x/2$  which reduce to  $\rho \omega^2 x$ .

So for  $d\sigma$  to be maximum  $f'x$  has to be maximum by applying the rules of the maxima and minima we can say that for  $b\sigma$  to be maximum  $f'x = 0$ , then with  $re = R_t + H_m / 3$  and simplifying we get this  $x = H_m/3$ , and again if you look into it with  $x = \rho h_m/3$  when you substitute  $x = 2h_m/3$  in this particular expression what we get is that  $d\sigma = 0$ , this term and this term = and 0.

It implies that at that particular point  $x = 2h_m/3$   $\sigma_v$  in prototype, that is where the point the centrifugal stress the distribution of a vertical stress in this centrifugal model causes the you know the vertical distribution in the prototype. So when you look back into the figure so this is that point from the surface it is actually said as  $2h/3$  and this is  $H_m$  and this is actually at 50% of the height of this point at which this is crossing.

Where the maximum under stress is occurring that is where actually what we have locate where  $x = h_m/3$ , then now we can actually find out the resulting errors after having know n the location of the you know where the under stress is maximum where the  $\sigma$  are =.

(Refer Slide Time: 22:45)

**Advanced Geotechnical Engineering**

Selection of effective radius

At  $x = h_m/3$ :

$$\% \text{ Under stress error } \frac{d\sigma}{(\sigma)_p} = \frac{\rho N g (h_m/3) - \rho \omega^2 (h_m/3)(R_p + h_m/6)}{\rho N g (h_m/3)}$$

$$= \frac{h_m}{6R_p}$$

At  $x = h_m$ :

$$\% \text{ Over stress error } \frac{d\sigma}{(\sigma)_p} = \frac{\rho \omega^2 (h_m)(R_p + \frac{h_m}{2}) - \rho N g (h_m)}{\rho N g (h_m)}$$

$$= \frac{h_m}{6R_p}$$

NPTEL  
Prof. B.V.S. Viswanadham, Department of Civil Engineering, IIT Bombay

And then what we can do is that after having so at  $x = h_m/3$  that is where the maximum under stress error is possible that is defined as  $d\sigma / \sigma_v$ . So by writing the expression with  $x = h_m/3$  we can write  $\rho h_m/3 \times n \times g - \rho \omega^2 \times \rho h_m/3 \times R_t + h_m/6 / \rho n g \times h_m/3$ . So by simplifying you know these particular expression which is written and substituting for  $n g = r \omega^2$  we get  $h_m/6r$ . so the present is under stress error is resulting  $h_m/6r$ .

And similarly at  $x = h_m$  the stress error  $\rho \omega^2 \times \rho h_m R_t + h_m/2 - \rho n g \times h_m / \rho n g \times h_m$  which is  $h_m/6r$ . So as we have equated the you know the areas of the determine the effective radius by equating the areas in the centrifugal model as well as in the prototype. We get the % under stress and % over stress as  $h_m/6r$ . So this implies that using this rule we can actually find out there is exact correspondence.

(Refer Slide Time: 24:16)

Advanced Geotechnical Engineering

Selection of effective radius

- Using this rule, there is exact correspondence in stress between model and prototype at two-thirds model depth and the effective centrifuge radius should be measured from the central axis to the one-third depth of the model (from top surface).
- Maximum error is given by  $R_u = R_o = h_m/6R_e$
- For most geotechnical centrifuges,  $h_m/R_e < 2$  and therefore the maximum error in the stress profile is minor and generally less than 3% of the prototype stress.

NPTEL  
Prof. B.V.S. Viswanatham, Department of Civil Engineering, IIT Bombay

Stress in the model type at the  $2/3^{\text{rd}}$  of the model depth and the effective centrifugal radius should be measured from the central axis to the  $1/3^{\text{rd}}$  depth of the model from the top surface. So generally they do number of interoperation do exist so this actually says that  $r_e = R_t + 1/3 \text{ of } h_m$  that  $0.33 h_m$  but there is also some other investigators have presented this  $r_e = r_t$  somewhere like  $0.4 h_m$  measured from the top surface.

So by using this rule there exact correspondence stresses between model and prototype at  $2/3$  and the effective radius should be measured from the central axis to the  $1/3$  depth of the model. So another thing is that the maximum error in the model is given by  $R_u = r$  that is  $h_m/6r_e$ , maximum error due to under stress or over stress is given by  $h_m/6r_e$ .

So for most of the  $h_m/r_e$  if it  $< 0.2$  than therefore the maximum error in this stress profile is minor if  $h_m/r_e$  is  $< 0.2$  the maximum error in this stress profile is minor and generally  $< 3\%$  of the prototypes test. So for most centrifugal  $h_m/ r_u$  is maintained as  $0.2$  then what you can see is that  $0.2 / 6 \times 100 = 3.33\%$  of stress is minor if suppose  $h_m/r_e$  let us  $= 0.5$  it is high and  $> 3\%$ . So for if you look into the centrifugal about  $1.5\text{m}$  radius onwards what we can see is that we will be able to ensure that for except for the certain models  $h_m/r_e$ .

So this actually ensures that variation the vertical stress is maintained within the limits and error can be minimized. So in the example problem whatever we have discussed just now an example problem is given here.

(Refer Slide Time: 26:55)

**Advanced Geotechnical Engineering**

**Example problem**

In a  $1/50^{\text{th}}$  centrifuge model, same material is used as that of in the prototype. Model of height  $h_m$  is subjected to rotation with a constant angular velocity  $\omega$  about a central axis. If  $R_t = 4$  m, obtain variation of vertical stress with depth at every 50 mm and compare vertical stress in model with prototype at 150 mm, 300 mm and 450 mm depths respectively. Take  $\rho = 2000$  kg/m<sup>3</sup>.

**NPTEL**  
Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

In a  $1/50^{\text{th}}$  centrifuge model same material is used as that of in the prototype that means that  $\rho$  in model prototype and the model of  $h_m$  is subjected to rotation with a constant angle of velocity  $\omega$  central axis. If  $R_t = 4$  m obtain the variation of vertical stress with depth at every 50mm and compare vertical stress in model prototype at 50mm , 300mm 450mm depths respectively.

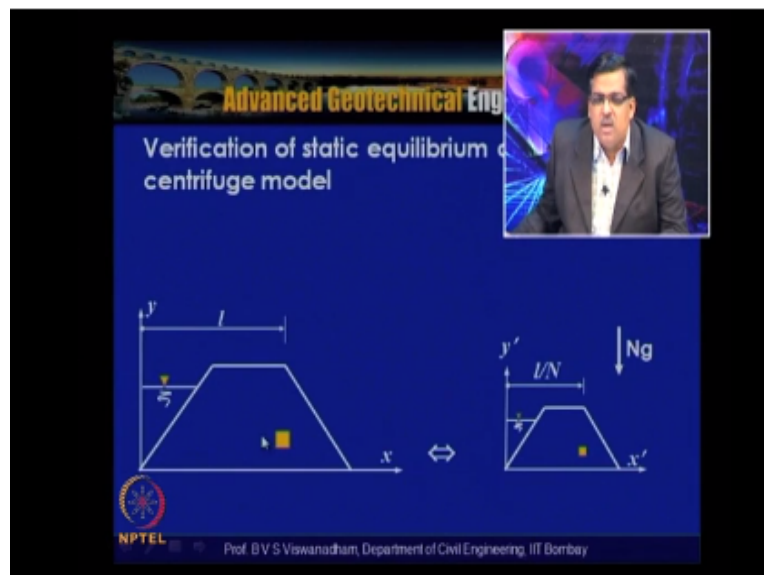
So here what we need to do is that the  $R_t$  is given that is 4 m then the height of the model is about 450mm, so what we can do is that by the height of the model which is about 450mm and that  $r_e = R_t = 4 + h_m/3 = 0.453$  so  $r_e = 1.15 + 4$  and  $R_t = 4.15$ m and by knowing  $n = 50 \times 9.81 = 4.105\omega^2$ , you can actually find out  $\omega$  you will get it in radius per sec and if you would like to convert this  $\omega$  from to rpm  $2\pi/60$ .

And for converting from rpm to radian per sec  $2\pi/60$ , so for by knowing  $\omega$  and writing the stress in the model and prototype at different depths we can find out what is the variation of the vertical stress in the model as well as in the prototype. So if you look into it by at the depth exactly at  $2h_m/3$  that is  $2 \times 450/3 = 300$ mm you will see that the stress in model prototype are identical that is where the  $d\sigma = 0$ .

And a depth of 150mm you will see that the under stress error is maximum and depth  $h_m$  that is the base of the model  $h_m = 450\text{mm}$  you will see that, so we can also find out by going 450 that is  $0.45 h_m / 6 \times 4.15$  and you can also calculate what is the percentage error due to. So this is how the solution for the problems and then for calculating the stresses by need to use while substituting  $\rho = 2000\text{kg}$ .

So verification of the static equilibrium of a centrifuge model so here what we have said that now in vertical stress variations they do exist but let us say that we have you know a prototype and if you are having certain element under equilibrium. So we need to satisfy static equilibrium are satisfy are not that we can say that you know the model response will be equal to as that as the prototype. So consider a alignment which is shown here and this dimension.

(Refer Slide Time: 30:39)



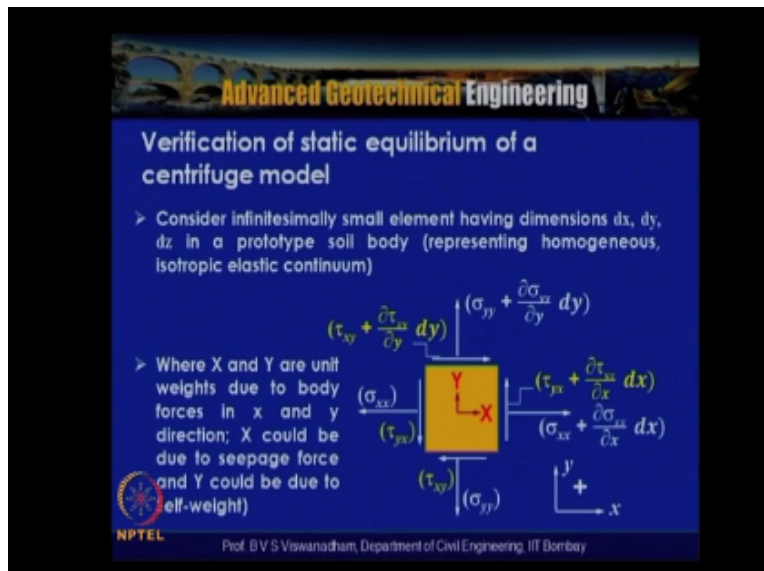
These are the x axis and y axis and it is actually as c page as well as subjected to some forces due to c page as well as the sulphate forces and consider the element within the model as it is shown here. So assume that the same model is reduced by 1 n times by maintain this length as  $L/n$  and



maintaining this gravity as  $ng$  that means and this  $l$  dimensions as been reduced and then here this element is actually having  $dx\ dy/\ dz$ .

Similarly here with the same density both the models are constructed with the mass density and here the volume of element which is consider as  $dx\ dy/\ dz$  and here it is by reducing by  $1/n$  times that is  $dx\ dy/\ dz$ . So this model represents the conditions to the prototype.

(Refer Slide Time: 31:48)



Now what we do is that considering two dimensional equilibrium let us see what will happen when you wanted to derive static equilibrium of a centrifuge model. So consider small element having dimensions  $dx, dy\ dz$  that is what we have been discussing.  $X$  and  $y$  which are actually shown here they are the units weights due to body forces in  $x$  and  $y$  direction and  $x$  could be due to  $c$  page and  $y$  could be self weight of the forces.

So the here we have the two dimensional element when it is subjected to the horizontal stresses and vertical stresses so here this is the rate of the change of the stress because of that increase in stress. So which is nothing but  $\sigma_{xx} + \rho\ xx\ x\ dx$  and this we have consider the elements in

tension, similarly the stresses in y direction here it is  $\sigma_{yy} + \rho_{yy} x dy$ . So this x body force acting in the x direction this y body in the y direction.

And this is the x is positive y is positive in this direction, now we have shear stresses we cannot shear stresses  $t / x +$  and here  $t_{yx}$  and here  $t_{xy} dy / t_{xy}$  that is here so when we consider now the equilibrium of force that effects  $f_y$  and so you will get that what we get is that by for equilibrium is that we have to ensure because it is two dimensional  $\sigma_{fx} = 0$ ,  $\sigma_{fy} = 0$  and  $m_x = m_y = 0$  now taking moment at the centre of the element.

Now with  $\sigma_{fx} = 0$  what we can do is that  $\sigma_{fx} = 0$  so here  $\sigma_{fx}$  is nothing but  $\sigma_{xx} + \tau_{xx} + \tau_x x dx$  this you know acting on the area which is nothing but  $dy/dz$ . Similarly here  $\sigma_{xx}$  stress acting on the area  $dy/dz$  so the net force is nothing but  $\sigma_{xx} x dx dy dz$ , so if you subtract this one similarly when you take forces in the shear stress in this direction and body force in this direction and simplifying the.

(Refer Slide Time: 35:00)

**Advanced Geotechnical Engineering**

Verification of static equilibrium of a centrifuge model

- > For equilibrium:  
 $\sum F_x = \sum F_y = 0$  and  $\sum M_x = \sum M_y = 0$
- > Taking moments about center of element gives:  
 $\tau_{xy} = \tau_{yx}$

With  $\sum F_x = 0$  and simplifying, we get:

$$dx dy dz \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X \right) = 0$$

**$\left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X \right) = 0$  (As  $dx dy dz \neq 0$ )**

NPTEL Prof. BV S Viswanadham, Department of Civil Engineering, IIT Bombay

After simplification what we get is that  $dx / dz x r + x = 0$ , so this particular you know expression here we write is as  $dx dy/dz$  by volume of the element cannot be equal  $= 0$  where it is read as  $\tau_{xy} / xy = 0$ .

(Refer Slide Time: 35:43)

**Advanced Geotechnical Engineering**

**Verification of static equilibrium of a centrifuge model**

With  $\sum F_y = 0$  and simplifying, we get:  $\rightarrow$

$$\left(\frac{\partial \sigma_{xx}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + Y\right) = 0$$

(As  $dx dy dz \neq 0$ )

In case of  $N_g$  model, considering a small element having volume  $(dx dy dz/N^3)$ , it yields

$$\left(\frac{\partial \sigma_{xx}}{\partial x/N} + \frac{\partial \tau_{yx}}{\partial y/N} + NX\right) = 0$$

$$\left(\frac{\partial \sigma_{yy}}{\partial y/N} + \frac{\partial \tau_{xy}}{\partial x/N} + NY\right) = 0$$

NPTEL Prof. B.V.S. Viswanadham, Department of Civil Engineering, IIT Bombay

Similarly we are looking into  $\sigma_y = 0$  in the y direction and simplifying the different forces what we write, so what we get is that  $\partial \sigma_{yy} / \partial y + \sigma_y / \partial y + \partial \tau_{yx} / \partial x + y = 0$  so this is again as  $dx dy dz$  we 0 what we write is that this equation, nothing but the 2 equation one is here in the y direction as it is shown. So in case of  $N_g$  model considering that the small element having a volume  $dx dy/dz$  now what we get is that  $\partial \sigma_{xx} / \partial x / n + \partial x / \partial y / n + nx = 0$ .

And then in the y axis  $\partial \sigma_{yy} / \partial y / n + \partial \tau_x + \partial n / n = 0$  so this x and y are nothing but the body forces or equal unit weights in x direction and y direction material  $y/n$ .

(Refer Slide Time: 37:01)

**Advanced Geotechnical Engineering**

**Verification of static equilibrium of a centrifuge model**

With  $\sum F_y = 0$  and simplifying, we get:

$$\left(\frac{\partial \sigma_{xx}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + Y\right) = 0 \quad (\text{As } dx dy dz \neq 0)$$

In case of  $N_g$  model, considering a small element having volume  $(dx dy dz/N^3)$ , it yields

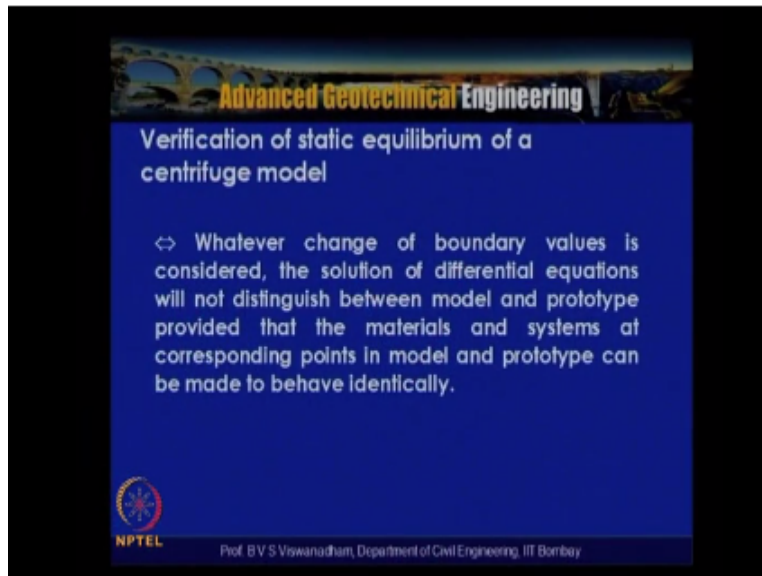
$$\left(\frac{\partial \sigma_{xx}}{\partial x/N} + \frac{\partial \tau_{yx}}{\partial y/N} + NX\right) = 0 \quad \left(\frac{\partial \sigma_{yy}}{\partial y/N} + \frac{\partial \tau_{xy}}{\partial x/N} + NY\right) = 0$$

NPTEL Prof. BVS Vasanadham, Department of Civil Engineering, IIT Bombay

Now with that what it actually if you look into this previous expression by simplifying this n will get cancelled and cannot be = 0 so  $\partial \sigma_x / \partial x + \tau_x \partial + x = 0$ ,  $\partial \sigma_y / \partial y + \partial \tau_x / \partial + y = 0$ . So these equation whatever we got for prototype are analog is to the 1 what we have got in the z phase model, so this indicates that the centrifuge based physical model satisfies the static equilibrium conditions what they are actually existing in the equal prototype.

So what we have actually reduced from this discussion that for convince we have actually have taken 2 dimensional and we have written the forces in the x and y direction only and we have taken  $\sigma_f = 0$ ,  $\sigma_{fy} = 0$  and by simplifying what we did use is that the equilibrium equation in the x and y direction and afterwards we converted them into the ng model and considering a small element having volume dx dy dz it is equation which are identical to those in the prototype. So we can say that the static equilibrium conditions of this centrifuges model are satisfied.

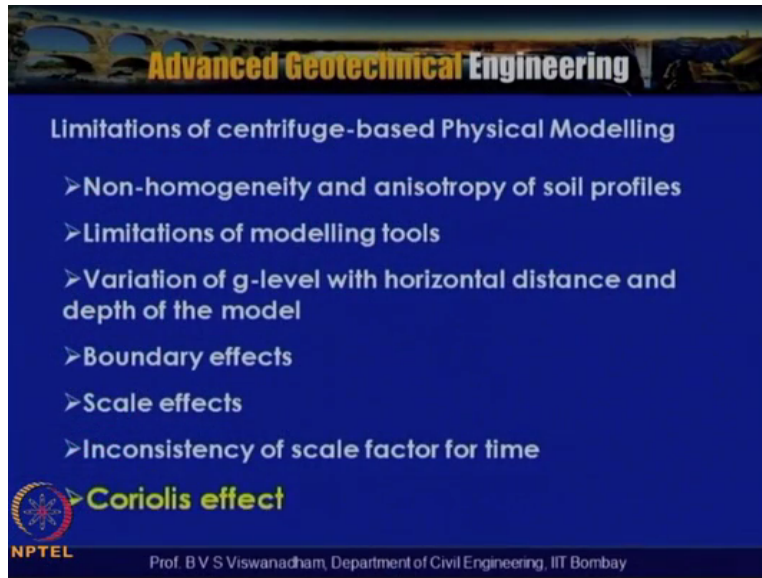
(Refer Slide Time: 38:25)



So what are the changes of the boundary value is concerned the solution of differential equation will not distinguish between model in prototype, provided that the materials and the system at corresponding points can be made behave identically. So what we have noted is that whatever the change of the boundary values is consider the solution of differential equations will not distinguish between model and prototype.

Provided that the materials and the system at corresponding points in model prototype can be made behave identical. Now in the limitations of you know after having discussed the like how the vertical stress can be minimized due to radial acceleration field and also how we can actually satisfy static equilibrium equations of the prototype in a model. Now let us look into limitations like any modelling techniques suffers from some limitations.

(Refer Slide Time: 39:26)



**Advanced Geotechnical Engineering**

**Limitations of centrifuge-based Physical Modelling**

- Non-homogeneity and anisotropy of soil profiles
- Limitations of modelling tools
- Variation of g-level with horizontal distance and depth of the model
- Boundary effects
- Scale effects
- Inconsistency of scale factor for time

➤ **Coriolis effect**

**NPTEL** Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

And limitations of some modelling tools the sizes and also things required to be considered and variation of g level with the horizontal distance at the depth of the model, so this we said that whatever whether it is a beam centrifuge the variation of the g level with the depth cannot be eliminated but however in case of a we can actually say that the variation of  $g$  with the horizontal distance especially for a small centrifuge is limited because with nature of the model.

But similarly for slope model for example some investigators like in 1991 they have given the curvature for the top surface of the slopes to suits the curvature of the centigrade. This is basically done to eliminate the variation the g level with the horizontal distance and the boundary effects these models particularly especially the sandy soils they exhibit you know very severe boundary effects that is nothing but the friction due to soil and the containers of the inner side.

So this need to be eliminated by special techniques particularly like applying a white petroleum grease or lubricating assent without effecting the transparency of the you know the front view transparent glass or wall and end by applying sheets in a special way we can you know eliminate this friction angle upto reduce this upto in a big way. So the boundary effects are important and need to be control by especially for sandy soils models.

In case of a we are having a high plastic soils that is you know this need to be arrested by applying by lubricant the inner sides of the containers, so that the boundary effects are minimized, in case of sandy soils due to friction between the wall and the sand grains. Then the other thing is that these models they subject to scale effects particularly one of the predominant

effects which we actually have discussed is that because of our inability to not able to model the grain distribution.

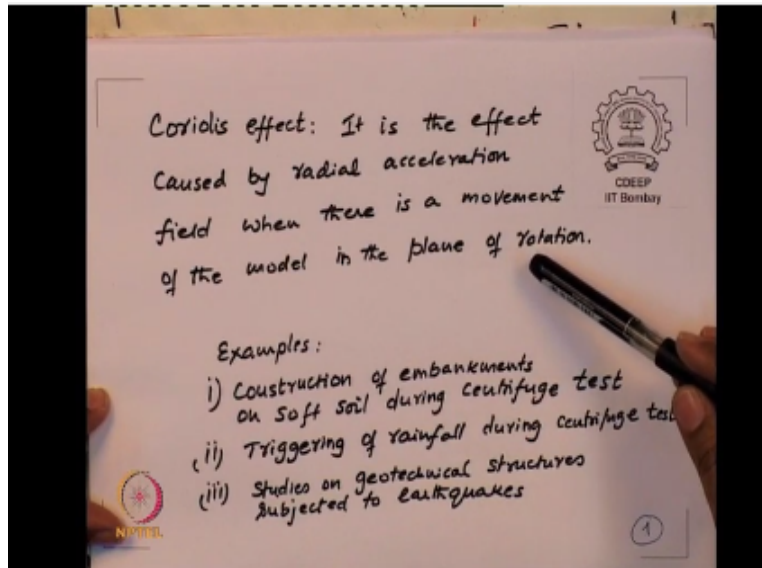
These scale effects are can be minimized and also need to be found out how they are effecting the model behavior, so this we have the one of the possible scale effects in the centrifuge based physical model is due to the grains effect and which will be discussing and how this can be solved or attended by using the same centrifuge based physical model, it is nothing but what we call modelling of model.

Then we are actually deducing you know times scales factors for the time we will find that you know each and every phenomenon will changes for example if it is something like dynamic time or if it is for some viscose force you actually have a different times. So this inconstancy of the scale factors for the time is also said as the one of the limitations. Some times when we have got scale factors for the time for some dynamic cases as well as some diffusion cases when they are different.

And I the phenomenon occur simultaneously then we actually have to resolve the some sort of you know a alternative so that this consideration are satisfied. Then one of the major affects particularly the effect which is called as Coriolis Effect which is actually caused due to movement of you knows the particle within the model. If this supposes the elasticity is  $v$  and when you are comparing with the velocity then we have to see that how this Coriolis Effect can be you know considered.

So let us look into this particular Coriolis Effect in depth and what is that theory behind this Coriolis Effect.

(Refer Slide Time: 44:03)



So what we said is that the Coriolis Effect which is nothing but it is the effect caused by the radial acceleration this is again the effect caused by the radial acceleration field when there is the moment of the model in the plane of the rotation, so this moment of the model can actually occur because of certain like construction process like assume that we are having a soft soil bed and with certain stresses and there we want to construct embankments because in the construction of embankments on the soft soil happens in different stages.

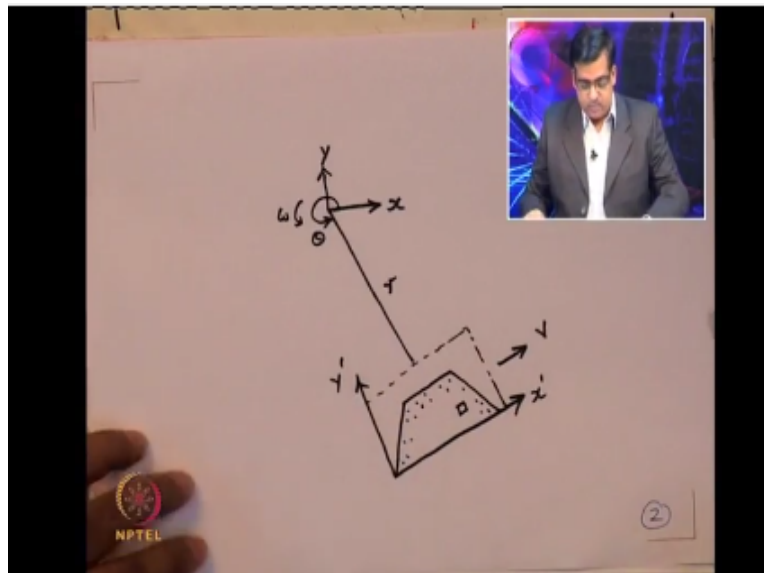
So let us say that we are actually stimulating that, for that what we do is that we use the hopper technique, so when the particles is released to construct sand embankments during flight. So the construction of the embankments of soft soil during centrifuge test and to triggering of rainfall during centrifuge test for example. If you wanted study the effect of the rainfall on constructions where we actually modelling this rain drop let us when they are actually you know stimulated in the form of a mist or in the models.

When these drop let they are subjected to when they are released in the high acceleration field, they can be subjected to you know this Coriolis Effect acceleration, that is basically the Coriolis Effect is the effect caused by the radial acceleration field when there is the moment of the model within the plane of rotation. The another thing is that subject studies on the earth quakes for example when the model is subjected to earth quakes factor due to with the help of some actuator which can actually induce this phenomenon.



And what it does is that this is subjected to the particle velocities are such that they can actually get experienced with you know this Coriolis Effect, so for some model which is actually required like construction embankments on soft soils are let us say when we are studying rain falls effects on structures it is mandatory to check whether the model is actually free from effect or not. So in order to understand the theory of the Coriolis Effect consider a model rotating.

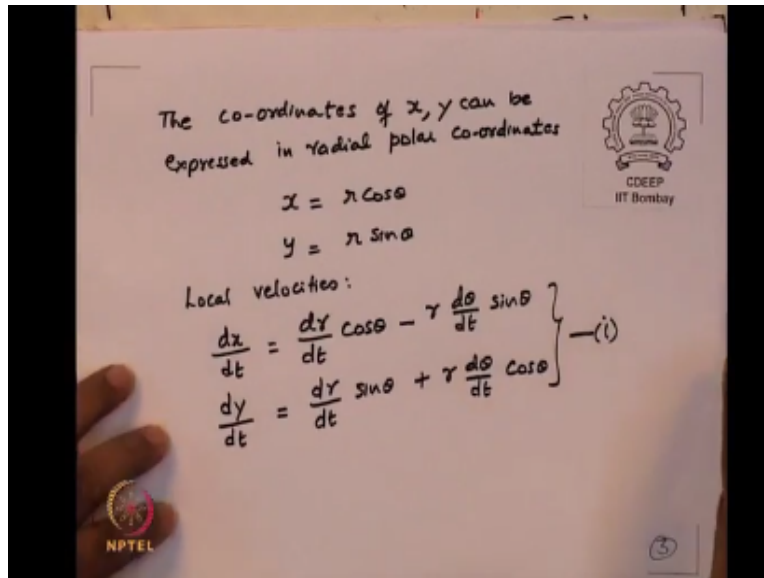
(Refer Slide Time: 46:48)



About vertical axis in the horizontal plane what we are seeing is the plan view and here is this is the centre of the shaft and x and y are the global coordinates at the centre of the shaft and the model is rotating in this direction and this is the velocity  $v = r\omega$  this is the radius which is actually upto the element which is considered here this is an embankment and constructed with a material having  $\rho$ .

So what we have to see is that when this is subjected to the so called what will happen to the acceleration components in this, so what we need to do is that we have to first express this in terms of and try to get the acceleration at the centre of the shaft and then by using the method of transformation the transform the local acceleration within the local axis within is  $x'$  and  $y'$ .

(Refer Slide Time: 47:57)



So now what we do is that the coordinates of  $x$  and  $y$  can be expressed in radial polar coordinates as  $x = r \cos \theta$  where  $r$  is the radius from the centre of the shaft where  $x$  and  $y$  origin is there to element which is actually considered in the previous slide. So  $y = r \sin \theta$  so we have  $x = r \cos \theta$  and  $y = r \sin \theta$  so local velocity can be obtained by differentiating this  $dx/dt = dr/dt \cos \theta - r d\theta/dt \sin \theta$  similarly  $dy/dt = dr/dt \sin \theta + r d\theta/dt \cos \theta$ .

So let this be as 1 so these are local velocities and then by differentiating once again what we get is that local but these are reference to global axis at the centre of the shaft.

(Refer Slide Time: 48:57)

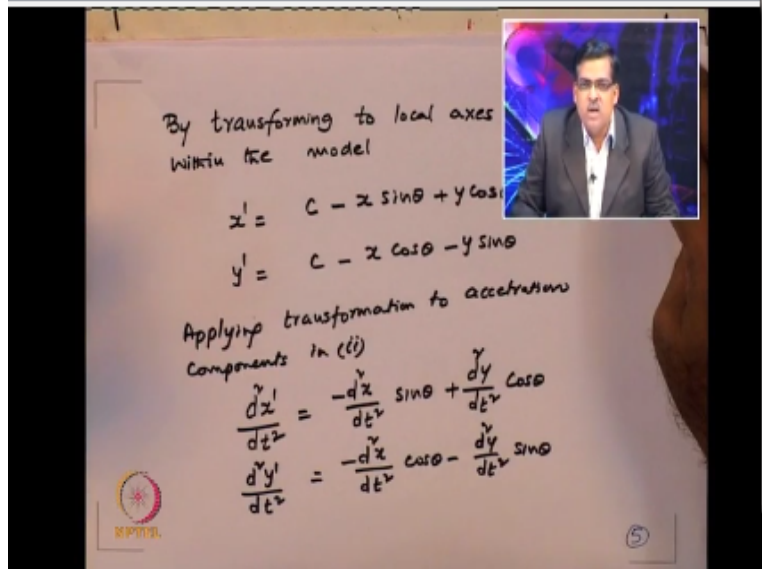
Local accelerations:

$$\frac{d^2x}{dt^2} = \frac{d^2r}{dt^2} \cos\theta - 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) \sin\theta - r\left(\frac{d^2\theta}{dt^2}\right) \sin\theta - r\left(\frac{d\theta}{dt}\right)^2 \cos\theta$$

$$\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2} \sin\theta + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) \cos\theta + r\left(\frac{d^2\theta}{dt^2}\right) \cos\theta - r\left(\frac{d\theta}{dt}\right)^2 \sin\theta \quad \text{---(ii)}$$

So the local acceleration as given as  $\frac{d^2x}{dt^2} = \frac{d^2r}{dt^2} \cos\theta - 2 \frac{dr}{dt} \frac{d\theta}{dt} \sin\theta - r \frac{d^2\theta}{dt^2} \sin\theta - r \left(\frac{d\theta}{dt}\right)^2 \cos\theta$ , so similarly in the y axis  $\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2} \sin\theta + 2 \frac{dr}{dt} \frac{d\theta}{dt} \cos\theta + r \frac{d^2\theta}{dt^2} \cos\theta - r \left(\frac{d\theta}{dt}\right)^2 \sin\theta$  this is 2. Now what we do by applying the transformation of from the global axis to local axis by the method of what we can write is that.

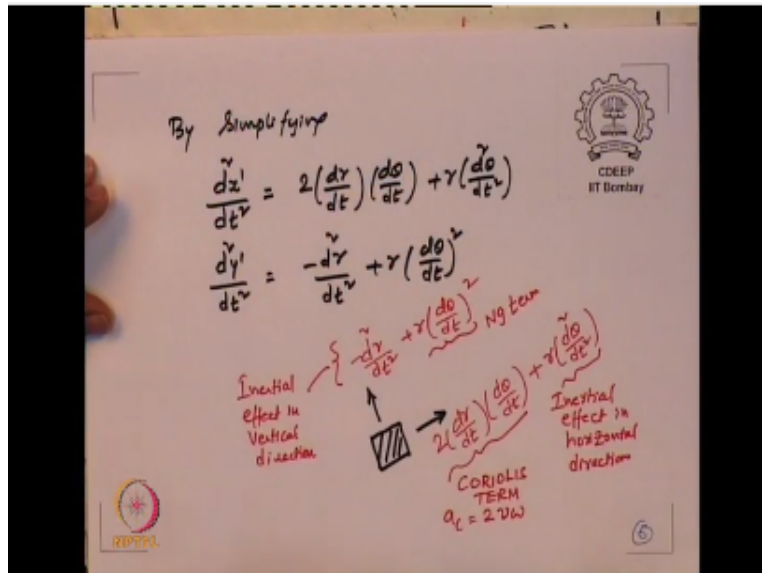
(Refer Slide Time: 49:46)



The basic equation is that  $x'$  so here you can see that one axis is getting transform from the centre of the shaft to the axis within the model which is actually rotating, so  $x'$  along  $x$  axis of the model that is horizontal axis of the model  $y'$  is now be vertical axis of the model so we can write  $x' = c - x \sin \theta + y \cos \theta$ ,  $c$  is the constant  $y' = \text{constant} - x \cos \theta - y \sin \theta$ , so in order to get this things what we do is that to get the local acceleration with reference to axis within the model first you do the differentiation get the  $dx'$  and  $dt$  and then after further doing successive differentiation to this what we get is that  $d^2 x' / dt^2$ .

So the  $d^2 x' / dt^2$  ton works out to be  $-d^2 x / dt^2 \sin \theta + d^2 y / dt^2 \cos \theta$ , similarly in the  $y'$  direction  $d^2 y' / dt^2 = -d^2 x / dt^2 \cos \theta - d^2 y / dt^2 \sin \theta$ . So by we know that  $d^2 x / dt^2$   $d^2 y / dt^2$  with reference to grovel coordinates at the center of the shaft. Similarly we also know here and here by substituting and simplifying what we get is that the following tucks which is nothing but.

(Refer Slide Time: 51:17)



By simplifying the substituting for  $d^2 x / dt^2$  and  $d^2 y / dt^2$  what we get is that  $d^2 x' / dt^2 = 2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$ , so if you look in to it this particular element when it is you know actually has two components of acceleration one is this  $2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$  and similarly we have  $d^2 y' / dt^2 = -\frac{d^2 r}{dt^2} + r \frac{d\theta}{dt}^2$ . So if you look the you know this particular element which is consider in the centrifuges model what we can write is that this particular term in the horizontal direction that is  $2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$  that is this particular term.

So this particular term is called as the coriolis acceleration term so when the model when the element is subjected to certain amount of acceleration so what we are the acceleration field what we see is that there is some lacteal acceleration which is actually acting on the element which is actually called as coriolis terms is also called as a coriolis acceleration term which is nothing but  $2 v \omega$ , ac a suffix c is indicated as coriolis acceleration which is nothing but  $2 v \omega$  + this term is due to inertial effect in horizontal direction.

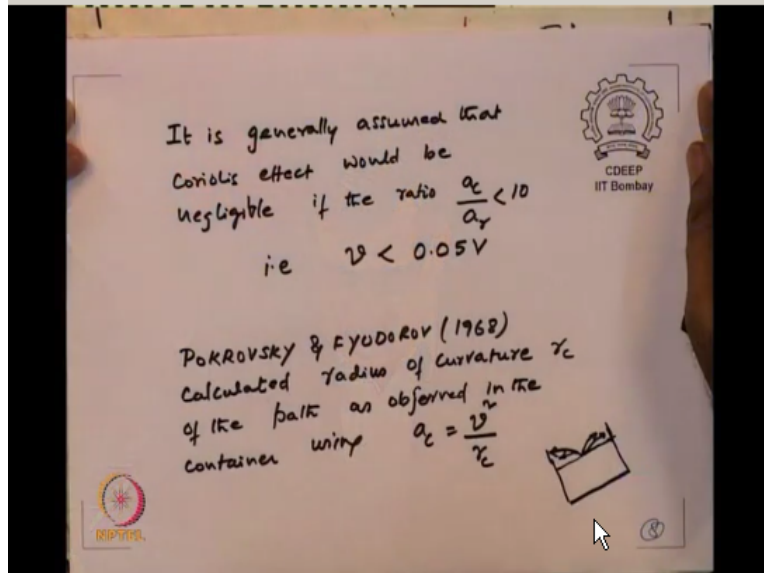
Because we actually assume that this model is subjected to some parted buns what you do some shaking let us say in that case this is actually due to inertial effect in horizontal direction when you look in to the element or in the vertical component  $\frac{d^2 r}{dt^2}$  is the inertial effect in vertical direction then  $r \frac{d\theta}{dt}^2$  which is nothing but the ng terms that is nothing but  $r \omega^2$  that acceleration acting towards the element acceleration acting towards the center.

So here in this particular derivation by taking from accelerations from global coordinates to the local coordinates and the local accelerations have actually exposed has the we have got two components in x direction two components in y direction and in the x direction what we have absorbed that the term which is called as  $2v \omega$  that is the local acceleration term which is call the coriolis acceleration term.

Then we have the you know the Coriolis Effect which is actually defined as ratio of coriolis acceleration to the radial acceleration that is  $a_c$  by  $AR$  so which is nothing but we can write now  $a_c$  is nothing but  $2v \omega$  by  $AR$  let us assume that up to this  $AR$  which is nothing like we have discussed the  $R$  can be effective radius so it is  $RE \omega^2$  so by simplifying using  $v \omega = v/r$   $v/re$  we can write that coriolis effect is nothing but  $2v/v$  so what we are doing is that we are actually comparing two velocities.

One is small  $v$  is nothing but the model velocity and capital  $v$  is nothing the velocity of the particles within the model is can be due to  $c+$  velocity are it can be due to some wall movement the soil particles are moving or it can be due the particle velocity which are generated because of the some seismic pertebuance and then capital  $v$  is nothing but the module velocity, the velocity between the module is locating okay, so if u further you know analysis this one it is actually generally said that if the you know percentage error due cohesion is effect  $AC/AR$  if it is say less than 10% if that actually happens at least that if it is less than 10%.

(Refer Slide Time: 55:31)

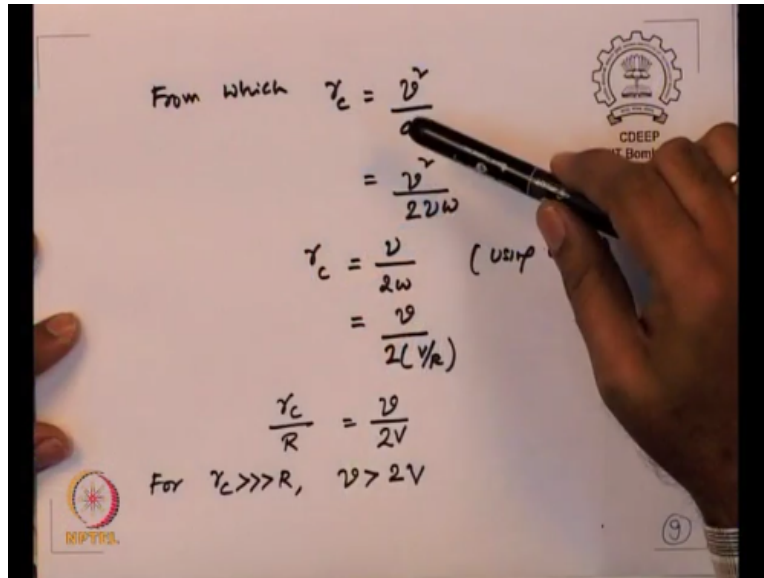


Then what it actually means is that the coriolis effect can be negligible so if the ratio of  $V$  at least that for  $AC/R$  less than  $\approx 10\%$  what we need to that  $v$  has to be  $\approx 0.05 v$  if this is less than  $\approx 0.05 v$  we can say that the coriolis effect can be you know legible that is the module is not subjected to coriolis effect now there are also you know many events, the test which can actually takes place.

Let s say that we actually investigating the effect of you know neglecting blast mode on the you know performance of some structures berried structures so when the explosive is say subject is neglected models that what you say that agenda which is actually thrown with very high velocities, but this particles soil particles are thrown very high velocities, so you need to calculate this radius of the container obtained by this particles.

Pokrovsky and Eyudorov (1968) they have given an entire expression they saying that corilios isolation is given by  $v^2/R$  square  $V$  is the velocity which is the particles actually has been going on when the  $RC$  is nothing but the various been venture they tried to complete the various courage the particles has level has with the radius of courage which the module is been rotating .From which what they have reducing is that  $RC = B^2 / A_c$  .

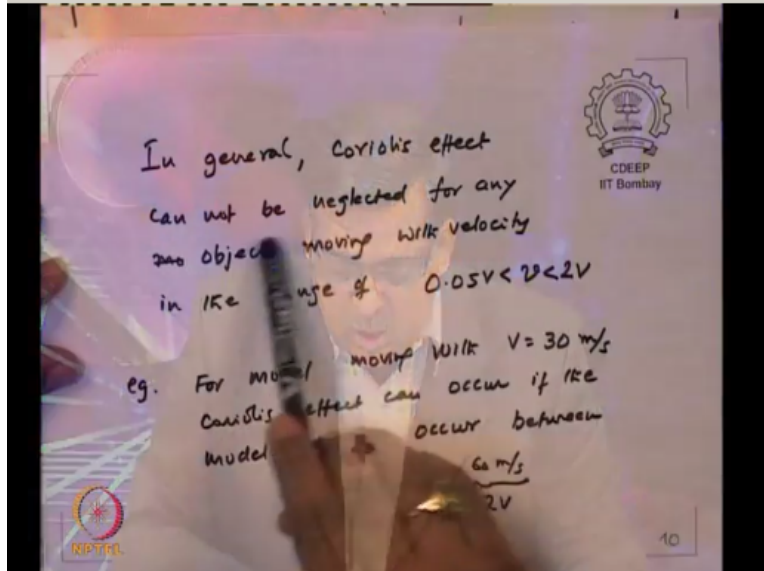
(Refer Slide Time: 57:07)



And AC by substituting you get  $2 V \omega$  so you can that  $RC = V/2\omega$  so using  $\omega = V/R$  RE what we can get this  $V/2 = V/RE$  by taking RE this side  $r_c/r = v/2v$  so it actually replaces is that if RC the radius of the courage obtained by the particles is say much higher than radius of the courage of the substitutes then the effective radius then V is suppose  $2V$  at least that the particle will be thrown with a very high velocity and it is subjected to you know then heat the conditioner boundary and falls.

(Refer Slide Time: 57:57)



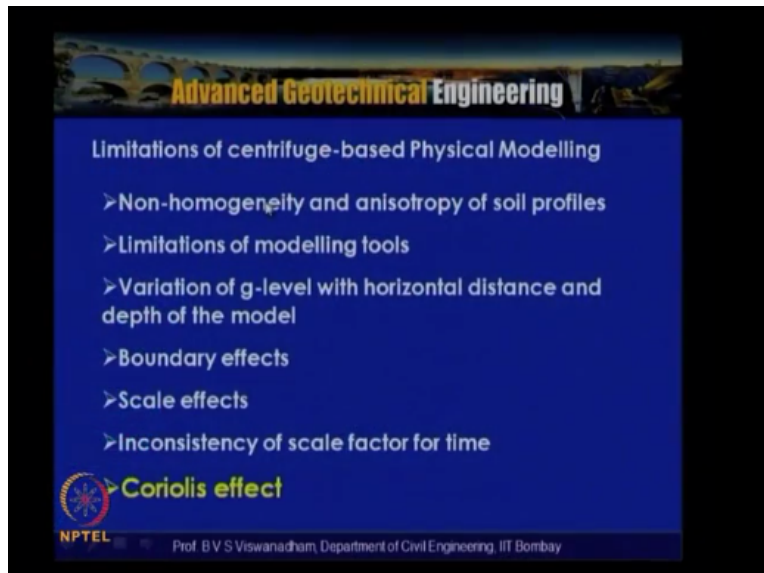


In general Coriolis effect cannot be neglected for any object moving with the velocity range that is that if the velocity is within the  $0.05v < v < 2v$  then we can say that Coriolis effect cannot be neglected if the velocity is actually more than  $2v$  then we can say that the Coriolis effect is non-adjusted that means that the model will not be affected by the Coriolis effect.

So for example if you have a model with  $v = 30 \text{ m/s}$  the Coriolis effect will occur. Keep the model velocity in the range so as to be  $0.05v$ ,  $0.05$  times that  $v$  is  $1.5 \text{ m/s}$  and  $2v$  that is  $60 \text{ m/s}$ . So if the velocities of the moving object within the model are within  $1.5 \text{ m/s}$  to  $60 \text{ m/s}$  then the model is subjected to the Coriolis effect, otherwise the model is actually free from the Coriolis effect.

So in this particular lecture what we tried to understand is that how this model can be established and we also tried to look into some aspects of you know how to calculate the effective radius and then we discussed about the limitations of centrifuge-based physical modeling as we have said in this particular slide.

(Refer Slide Time: 59:10)



Where we have got non-homogeneity and anisotropy of soil profiles but the advance the techniques is also being a you know boundary modeling physical model and engineering and some limitations of modeling tools and boundary effects and scale effects inconsistency of scale factors which we are going to discussed in detail and then we actually had a discussed on the coriolis effect.

## **NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING**

**NPTEL  
Principal Investigator  
IIT Bombay**

Prof. R. K. Shevgaonkar  
Prof. A. N. Chandorkar

**Head CDEEP**  
Prof. V.M. Gadre

**Producer**  
Arun Kalwankar

**Project Manager**  
Sangeeta Shrivastava

**Online Editor/Digital Video Editor**  
Tushar Deshpande

**Digital Video Cameraman**  
Amin Shaikh

**Jr. Technical Assistant**  
Vijay Kedare

**Project Attendant**  
Ravi Paswan  
Vinayak Raut

**Copyright**  
**CDEEP IIT BOMBAY**