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**ADVANCED GEOTECHNICAL  
ENGINEERING**

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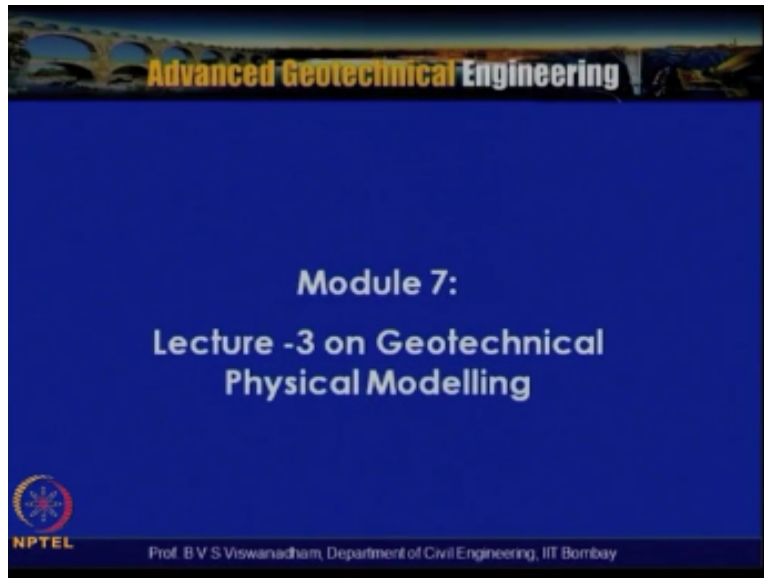
**Lecture No. 52**

**Module – 7**

**Lecture – 3 on Geotechnical  
Physical Modeling**

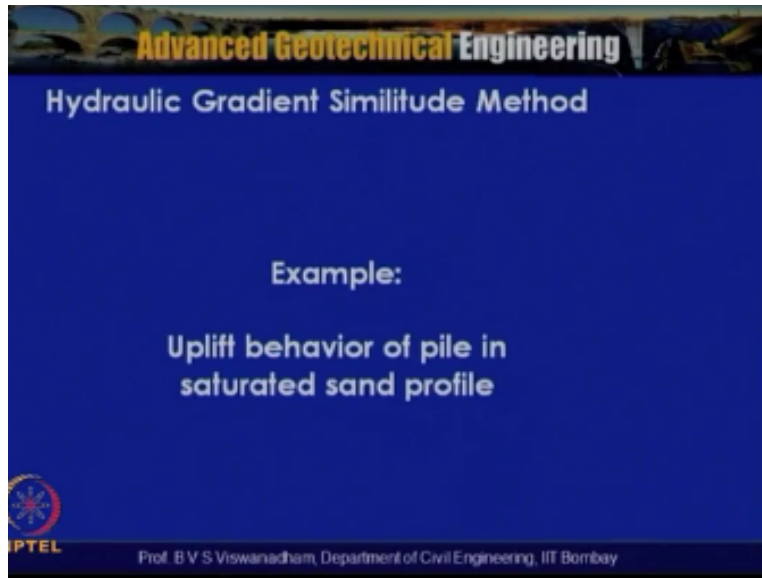
So welcome to course on advanced geotechnical engineering module 7 and titled geotechnical physical modeling and in this module we are actually going to discuss about the centrifuge based physical modeling, mechanics of centrifuge based physical modeling even listen to that we will actually try to solve a problem which we have actually based on the discussion we had in the previous lecture on hydraulic gradient simulation method.

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So this is module 7 lecture 3 on geotechnical physical modeling, so after having discussed I hydraulic gradient similitude method we said that with the help of by maintaining the differential pressures between top and bottom of the soil sample.

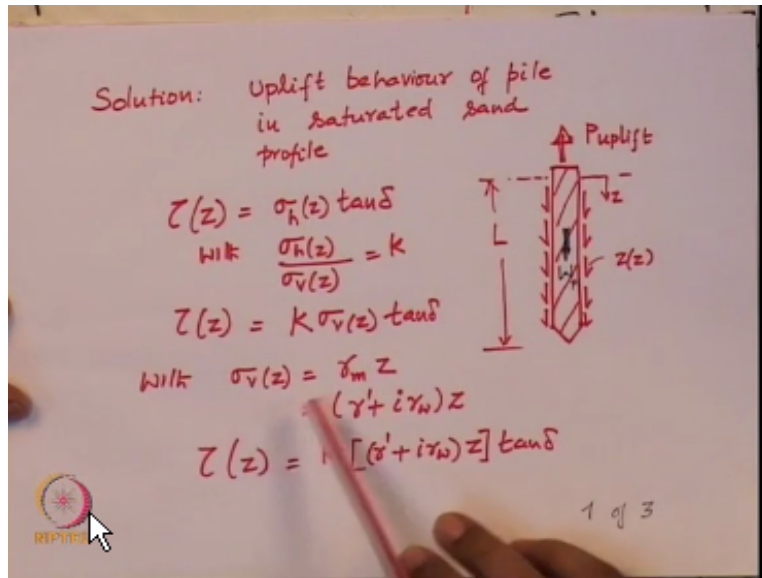
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Predominantly saturated granular soils there is a possibility that we can actually replicate this stress as that in the prototype. And in connection with that let us look I to the example wherein uplift behavior of a pile in saturated sand profile that means that in this particular example we are considering a pile of certain length ember in a saturated sand deposit when it is subjected to uplift.

So this piles can be subjected to uplift like for example if they happen to be a foundations of transmission line towers for example the river bed in a saturated sand profile can do exist then the piles can be subjected to uplift or what we say is also pull out. So this we would like to understand to investigate true hydraulic gradient similitude method, so for that how we need to model the uplift capacity and how we have to get the bond stress or the shear stress around the pile soil interface. So let us look in to the solution like this.

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The problem here before us is uplift behavior of pile in saturated sand profile so here this is the pile of diameter  $d$  this is the ground surface and this is the length of the embedment when the pile is actually subjected to uplift the first resisting force will be the self weight of the pile and on the periphery of the pile there will be shear stress which are actually acting in the downward direction because the uplift will be in the upward direction.

So along with the self it this shear stress will be acting in the downward direction and they will be along the length of the pile and there will be along the periphery of the pile. Now we can write by using relationship like  $\tau z = \sigma h z \tan \delta$  this  $\sigma h z$  is nothing but the horizontal stress which is acting at a particular depth  $z$  below the ground surface,  $\sigma h z$  is nothing but the horizontal stress acting at a particular depth that is  $z$  below the ground surface.

And  $\delta$  is the pile soil interface friction angle this is for in case of uplift generally with me on the lower side, so the  $\delta$  is the interface friction angle. Now we have relationship with  $\sigma h z = k \times \sigma v z$   $\sigma v z$  is nothing but vertical stress at a particular depth  $z$  by with the substitution of by substituting  $\sigma h = k \times \sigma v z$  we can actually substitute here then we get  $\tau z = k \sigma v z \tan \delta$ ,  $k$  is coefficient of earth pressure this depends upon the type of the pile which is you know installed if it is like precast driven pile it will have a different value it is like it will have a higher value if it is a bored cast-in-place pile it can have low value because the soils surrounding the pile is not stressed because of that you know it can have low  $k$  values.

But in case of a precast driven pile which is driven into this sand deposit there is a possibility that soils surrounding the pile will get stress because of that there is a possibility of you know higher  $k$  values. So now we can write that  $\sigma_v z = \gamma_m x z$  where  $\gamma$  is nothing  $\gamma_m$  is nothing but effective unit weight of the soil, so we have defined in hydraulic gradients similitude method  $\gamma_m = \gamma' + I \gamma_w$  this  $I \gamma_w$  term is actually obtained from the so called you know the hydraulic gradient if you are actually having a hydraulic gradient of let us say 50 or 100 depending upon the definitions whatever  $e$  had that means that it is possible that the  $\gamma_m$  can be increased by 50 x the  $\gamma$  in the prototype.

So by writing  $\sigma_v z = \gamma_m x z$  we can write  $\gamma' + I \gamma_w z$  so by substituting this here in this we can write  $\tau_z = k \gamma' / I \gamma_w z \times \tan \delta$ , so we what we have done is that we substituted for  $\sigma_v z = \gamma' + I \gamma_w z$ . Now let us see further now what we do is that in order to get you know simulation between model that is in hydraulic gradient simulation method and corresponding  $\tau_z$  in prototype what we do is that.

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$$\frac{[\tau(z)]_m}{[\tau(z)]_p} = \frac{k_m}{k_p} \cdot \frac{[\gamma' + i\gamma_w]_m}{[\gamma' + i\gamma_w]_p} \cdot \frac{z_m}{z_p} \cdot \frac{\tan \delta_m}{\tan \delta_p}$$

With  $k_m = k_p$ ;  $\frac{z_m}{z_p} = \frac{1}{N}$ ;  $\frac{\tan \delta_m}{\tan \delta_p} = 1$

$$\text{or } \frac{[\gamma' + i\gamma_w]_m}{[\gamma' + i\gamma_w]_p} = N$$

We get  $\frac{[\tau(z)]_m}{[\tau(z)]_p} = 1$

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We take a ratio of  $\tau z$  in model  $\tau z$  in prototype = from the previous expression we have  $k \gamma' + I \gamma_w z \tan \delta$ , so what we have done is that we have given written in the model dimension stress and prototype dimensions  $m$  indicates the model and  $p$  means prototype or equivalent full scale structure. So  $k_m / k_p \gamma' + I \gamma_w$  model  $\gamma' + I \gamma_w$  suffix prototype  $z_m / z_p \tan \delta_m / \tan \delta_p$ .

Now with  $k_m = k_p$  assuming that all the stress is history and other aspects have been taken care and because of the reducer scale model the  $z_m$  in model is  $z_p \times n$  that is  $z_m / z_p = 1 / n$  and when we resemble this when we have identical stress histories and assuming that the friction angles interface friction angles in model in prototype are identical that is  $\tan \delta_m / \tan \delta_p = 1$  and  $\gamma' + I \gamma_w$  model divided by  $\gamma' + I \gamma_w$  suffix  $p = n$ .

So this is by virtue of the you know the hydraulic gradient similitude method by substituting this in this particular expression what we get is that this is  $n$  times this is  $1 / n$  time this is one and this is one with that  $\tau z$  in model to  $\tau z$  in prototype = 1, so this indicates that the interface stress in model and you know equivalent full scale structure if are they pile in saturated sand profile with identical stress histories is found to be identical. Now let us see after having deduced of the scale factor for interface stress.


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$$P_{\text{uplift}} = W_p + \pi D \int_0^L \tau(z) dz$$

$$P_{\text{uplift}} = W_p + \pi D L \cdot \tau(z)$$

$$\Rightarrow \frac{[P_{\text{uplift}}]_m}{[P_{\text{uplift}}]_p} = \frac{1}{N^2} \quad \left[ \begin{array}{l} \text{with } [Z(z)]_m = [Z(z)]_p \\ \frac{D_m}{D_p} = \frac{L_m}{L_p} = \frac{1}{N} \\ \& \text{Neglect } W_p \end{array} \right]$$

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In model in prototype let us see how we can actually get the uplift  $p$  uplift that is nothing but the uplift force is nothing but  $w_p + \pi d w_p$  is nothing but the self weight of the pile  $\pi d \int_0^l \tau z dz$ . Now by integrating this is what we get is that  $\pi d l$  because  $\tau z$  and then when you take the limits here  $\pi d l$  is nothing but  $\pi d$  is the prototype and  $l$  is the length of the pile  $\times \tau z$ .

So neglecting here as we said that one of the limitations of hydraulic gradient simulated method is that the self head to the pile is not enhanced in the model it will be smaller but the self head of the pile is not enhanced. So with that neglecting this term then  $p$  uplift in model in prototype we can obtain as  $1/n^2$  with  $\tau z$  in model =  $\tau z$  in prototype that is we how reduce just now wherein we said that by simulation of  $\gamma_m = n \gamma_p$   $\gamma_m$  is the effective unit weight of the soil to  $n$  is nothing but which is induced because of the simulation of you know higher hydraulic gradient.

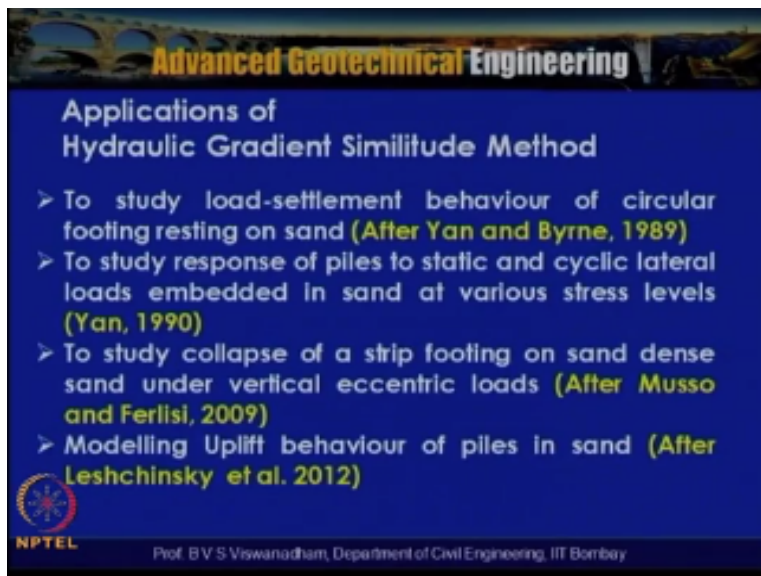
And  $\gamma_p$  is nothing but the bulk unit weight or submerged unit weight of the soil then with  $\tau z$  in model =  $\tau z$  in prototype and  $d_m / d_p = l_m / l_p = 1/n$  and neglecting  $w_p$  says that  $p$  uplift in model in prototype =  $1/n^2$ , that means that if you have let us say a pile which is actually taking for example 1200kn which is an hypothetical uplift load but if you are trying to look in to let us say  $n = 50$  that means that you require about  $1200n$  that is about 120 kg is sufficient for you to impose the type of higher load.

Here what we have deduced is that by using this solution we try to reduce the scale factor for the  $\tau z$  which we found that it is identical as that of in the prototype then the uplift capacity of a pile

in  $k_n$  is scaled as  $p$  uplift in model =  $p$  uplift in prototype divided by  $n^2$  so this is how you know one need to you know develop the scaling lose for the different applications of this you know in the modeling particularly the modeling which is different from a full scale that is when you do in a small scale physical model 1:  $n$ .

Now after having discussed about the method and also discussing about you know a problem a example problem in fact the problem which we have just now discussed is after Leshchinsky et. at all 2012 and the application of the hydraulic gradient simulated method some of the selected application areas are actually given here basically to study the load settlement behavior of circular footing or square fittings resisting on sand.

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The slide is titled "Advanced Geotechnical Engineering" and lists the following applications of the Hydraulic Gradient Similitude Method:

- To study load-settlement behaviour of circular footing resting on sand (After Yan and Byrne, 1989)
- To study response of piles to static and cyclic lateral loads embedded in sand at various stress levels (Yan, 1990)
- To study collapse of a strip footing on sand dense sand under vertical eccentric loads (After Musso and Ferlisi, 2009)
- Modelling Uplift behaviour of piles in sand (After Leshchinsky et al. 2012)

The slide also features the NPTEL logo and the text "Prof. B.V.S. Viswanadham, Department of Civil Engineering, IIT Bombay" at the bottom.

And to study the response of piles in static and the cyclic at lateral loads embedded in sands at various stress levels. So this is in this particular study by Yan at all 1990 what they have done is



that they have simulated this you know these different stress levels by increasing the you know hydraulic gradients and then simulating the values from  $n = 1$   $n = 5$   $10$   $100$  up to  $100$  they have done. So and then also they also studied this static response as well as the cyclic response of this you know piles and another application is that which is after Musso and Ferlisi 2009 is the to study the collapse of a strip footing on sand or it is a dense sand under vertical eccentric loads.

And As I told you the recent example by this Leshchinsky et al is on the studying the uplift behavior of a piles embedded in sand. So like any technique we have limitations of the hydraulic gradients simulated method and these limitations some of the limitations are listed here but what need to really understand about are ensure that identical or uniformed distribution of hydraulic gradient and the sustain you know folding of these pressures is very important and it can be this is base can be use to model only the behavior of saturated sand and uniform of fine sand uniform tend to yield more uniform gradients across the soil layers.

So if there is you know some non uniform deal the ensuring the uniformity in the hydraulic gradients applied is questionable and or may not be consistent.

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**Advanced Geotechnical Engineering**

### Limitations of Hydraulic Gradient Similitude Method

- It can be used to model only the behaviour saturated sand. Uniform fine sands or uniform silty sands tend to yield more uniform gradients across the soil layers, and more consistent test results. The hydraulic gradient technique was found to be particularly sensitive to the distribution of the hydraulic gradient in the soil profile.
- Limited to model with horizontal ground surface only.
- It can not account for self-weight of the foundation materials (like footing/pile/etc..)

In conclusion, the hydraulic gradient technique that is presented herein offers a viable, inexpensive scale modelling approach that may be useful for model simulation of a variety of geotechnical engineering problems.

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So uniform fine sands or uniformed graded fine sands or uniformly grades silty sands ten to yield more uniform gradients across the soil layers. And it try to give the more consistent results and the hydraulic gradient technique was found to be particularly sensitive to the distribution of hydraulic gradient in the soil profile.

And more details are actually given by Yan 1990, wherein they have actually discussed about the you know different issues as for a as the you know with the distribution of hydraulic gradient which is in the soil profiles are concern. And this hydraulic gradient method is limited to model with horizontal ground surface only mostly the geo static stress condition that is with horizontal ground surface only can be done.

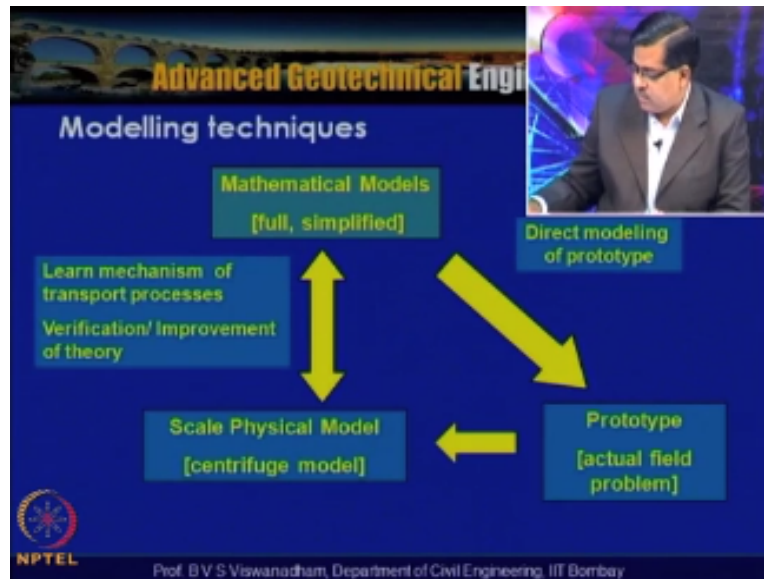
And it cannot account for the self weight to the foundation materials like footing or pile okay, so in case of uplift particularly you know it actually matters as for as the compressive load behavior is concerned we generally not will not consider but in case of uplift one of the you know remedial measures for increasing the uplift capacity to the pile is to increase the size by increasing the sulfate of the pile.

So accept that you know but the some selected you know limitations whatever has been highlighted here the hydraulic gradient technique this is presented here is a viable and inexpensive scale modeling approach that maybe useful for simulation of verity of geo technical engineering problems and without you know altering you know the gravity level one can actually ensure the identical stresses in model in prototype.

So ion conclusion what we can say about the hydraulic gradient technique is that is a you know viable and inexpensive scale modeling technique where identical has that in the model as that in the prototype can be you know simulated in the model and it is useful for a simulation verity of geo technical engineering problems. So the more research is warranted in this particular area where say large size models can be developed for understanding different types of problems under identical stresses as that in the prototype.

So after having discussed about the different modeling techniques like we said that we have theoretical models, mathematical models, and a numerical model, and in this particular the flow chart which is actually given here we have situation like sometimes from the mathematical model full and simplify.

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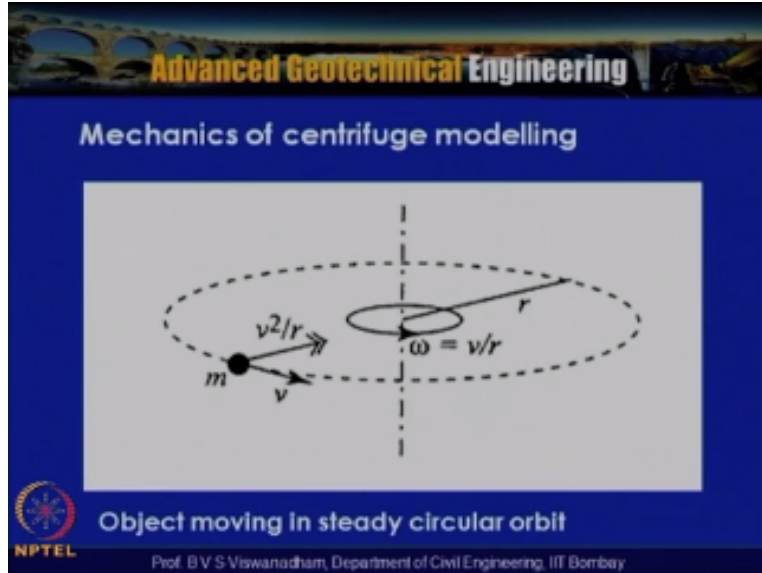


And what we tend to do is that we tend to do the prototype actual field problem will be studied and so it is actually like a direct modeling of prototype and if the prototype or full scale structure is existing and which needs to be validated then one of the alternative what we said is that the scaled physical model that is which is different from a full scale physical model one of the viable options in apart from you know the hydraulic gradient simulate method what we said is that the centrifuge based physical model.

So here the centrifuge based physical model which is actually shown here, so it can be helpful for a modeling some prototypes or modeling some prototypes which are nonexistent, suppose if you are actually talking about a new phenomenon and that you know involves you know in assumptions of a fractious prototype and it actually helps us to you know understand and simplify this mathematical models and basically learning the mechanism of transport process verification or improvement of the theory.

So with that what will happen knowledge which is actually gain from this scale physical models or of the actual prototypes or the fructuous prototypes it actually helps to understand the land mechanisms basically the mechanisms of the you know behavior can be you know ascertained.

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So you know before going in to the in-depth understanding about the centrifuge based physical modeling let us understand that you know the centrifuge based physical modeling is a physical modeling technique in which a body is rotated about a vertical axes in a horizontal plane. So when this is in the process of rotating about a vertical axes in horizontal plane there is a centripetal acceleration acts towards the center which is actually shown here for a body which is actually having mass  $m$  a centripetal acceleration acts towards the center.

And  $v$  is the tangential velocity and  $\omega$  is the you know the angular velocity about a vertical axes, so earlier you know when investigating this putting forward this technique many alternatives like vertical rotating about a vertical axes or rotating about a horizontal axes you know this aspects have been studied but now you know it is well established that in the this is object moving in a study circular orbit in a horizontal plane about a vertical access is did point of discussion.

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**Mechanics of centrifuge modelling**

If a body of mass  $m$  is rotating at constant radius  $r$  about an axis with steady speed  $v$  then in order to keep it in that circular orbit it must be subjected to a constant radial acceleration  $v^2/r$  or  $r\omega^2$ .

In order to produce this acceleration the body must experience a radial force  $mr\omega^2$  directed towards the axis.

By normalizing the centripetal acceleration with earth's gravity  $g$  and state that the body is being subjected to an acceleration of  $Ng$  where  $N = r\omega^2/g$

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So here if body of mass  $m$  like which we have shown in the previous slide is rotating at a constant radius  $r$  that is the radius from the center of the shaft to the within the body about an axis with the steady speed  $v$  then in order to keep it in that circular orbit it must be subjected to a constant radial acceleration that is  $v^2 / r$ ,  $r = r \omega^2$  which is acting towards the center. So in order to produce this acceleration which is actually acting towards the center of the shaft or a body about an axis which is rotating, the body must experience radial force that is  $mr \omega^2$  directed towards the axis.

So by normalizing this centripetal acceleration with earth's gravity that is  $g$  and the state that the body is being subjected to acceleration of  $ng$  where so what we have done is that the so-called  $mr \omega^2 / mg$  when we normalize we get  $n g = r \omega^2$  so this is the fundamental you know equation which we use in centrifuge based physical modeling this helps if you are actually having you know a certain angular velocity  $\omega$  when it is when we wanted say this implies that if we want the higher values of  $n$  then increase the angular velocity.

So by increasing the angular velocity there is a possibility that we will be able to get higher values of the  $n$ , so it is a relationship which will actually allow us to you know select you know a required amount desired angular velocity for modeling you know let us say  $n = 10$   $n = 50$  or  $n = 100$  or  $n =$  some 500.

And basically this is further explained it is like you know if you consider string if you consider a string attached to the sphere having certain weight.

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### Principle of Centrifuge modelling

- For example, the string attached to the sphere experiences a tension that can be felt by the hand holding the other end. The tension in the string applies the centripetal force on the sphere pulling it toward the centre of rotation.
- This force must be balanced by another force, otherwise the sphere would move toward the centre. This can be explained by means of a "fictional" or "inertial" force that acts on the sphere radially outward (Madabhushi, 2014)

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This string you know experiences tension and that can be felt by the hand holding the other hand so the tension in the string applies the centripetal force on the sphere pulling it toward the center of rotation so the tension the string applies this centripetal force on the sphere pulling it toward the center of rotation. So this force must be balanced by another force otherwise the sphere would move toward the center.

So otherwise what will happen is that suppose if you are rotating about a certain point and at a certain radius  $r$  and it is subjected to you know force of a  $mr \omega^2$  central you know the centripetal force.

So the force must be balanced by another force otherwise what will happen is that the sphere would move towards the center so this is can be explain by means of a frictional or inertial force that acts on the sphere radial outward. So the force must be balanced by another force otherwise the sphere would move towards the center.

So this is according to Madabhushi 2014 so this is actually explain that the force must be balanced by another force so otherwise the sphere would move toward the center this can be explained by means of a you know that inertial force that acts on the sphere radially towards. So you know that is who you know the so-called the centrifugal acceleration what we actually get

from this Newton's third law and we actually utilize that in the centrifuge based physical modeling. By Newton's third law the centrifugal force and the centripetal force must be equal.

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### Principle of Centrifuge modelling

- By Newton's third law the centrifugal force and the centripetal force must be equal and opposite. The centrifugal force causes a centrifugal acceleration that acts radially outward.
- In the example of the car going around a corner, we can say that the tires of the car will exert centrifugal forces on the road acting radially outward.
- Centrifugal forces are exploited in a wide variety of engineering devices such as centrifugal pumps and centrifugal clutches (Madabhushi, 2014)

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And opposite the centrifugal force causes a centrifugal acceleration that acts radially outward, in the simple example of the car going around a corner we can say that the tires of the car will exert centrifugal forces on the road acting radially outward. So the centrifugal forces are exploited in a wide variety of engineering devices such as centrifugal pumps and centrifugal clutches. So this is an extension which we actually used in you know geo technical based centrifuge modeling. So before going in to that let us consider you know from the mechanics point of view rotation of a rigid body about vertical axis.

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### Rotation about a fixed axis

Consider a rigid body which rotates about a fixed axis  $AA'$  and let  $B$  be the projection of  $P$  on  $AA'$

And  $PB = r \sin \phi$

$\Delta s = PB (\Delta \theta)$

$v = ds/dt = r \Delta \theta / (\Delta t) \sin \phi$

The velocity  $v$  of  $P$  is a vector perpendicular to the plane containing  $AA'$  and  $OP$ .

$v = \omega \times r$

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Here what we see is a vertical axis which is  $aa'$  and this is the horizontal plane about which the  $p$  is rotating about a vertical axis in a horizontal plane. And at point  $p$  consider a rigid body which rotates about a vertical axis  $aa'$  and this is  $x$  direction this is in  $y$  direction this is in  $z$  direction, and this is the angle of the last  $e$  so  $\omega = \theta \cdot k$  and this is in  $j$  vector and this is in  $i$  vector. And there is point  $p$  and this vertical axis in this plane that angle subtended is  $\pi$ , and when it moves from this place to this place this angle which is subtended by the moving particle  $p$  or body is  $\theta$ .

Now when you consider the rigid body which rotates about a fixed axis  $aa'$  and let  $b$  will be the projection of  $p$  on  $aa'$  that piece that  $b$ ,  $b$  the projection of  $p$  on  $aa'$  what we can write is that  $pb = r \sin \phi$   $r$  is the position vector that is  $op$  is the position vector so we can write that  $pb = r \sin \phi$ , so for a small displacement along this thing the small length  $\delta s$  arc length  $\delta s$  can be written as  $pb \times \delta \theta$ ,  $\delta \theta$  is the small angle which is subtended from by moving from point 2 from this point to this point let us say.

So  $\delta s$  is the small distance along this curve, so with that what we can actually get is that  $\delta s = pb \times \delta \theta$  where  $v = ds / dt = r \delta \theta / \delta t \times \sin \phi$  what we have done is that we have divided  $\delta s = pb \times \delta \theta$  / both sides by  $\delta t$  and when  $\delta t$  tends to 0, we can write  $v = ds / dt = r$  that is  $pb$  and  $\delta \theta / \delta t \times \sin \phi$ .

So this is the scalar product of a vector which is you know acting perpendicular to the plane of you know axis  $aa'$  and  $op$  that means that this triangular if we take a you know plane and in to that plane this vector tangential velocity is acting in this direction. So that means that we can



write that  $v = \omega \times r$  is the vector along this is equal to  $\theta$ .  $k$  is the vector along this axis and  $r$  is the position vector of this, so  $v = \omega \times r$ .

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**Rotation about a fixed axis**

For determining the acceleration of the particle  $p$

$$a = \frac{dv}{dt} = \frac{d}{dt} (\omega \times r) = \frac{d\omega}{dt} \times r + \omega \times \frac{dr}{dt}$$

$$= \alpha \times r + \omega \times (\omega \times r) \quad (\text{where } \alpha = d\omega/dt)$$

$$= a_t + a_n$$

For constant angular velocity,  $a_t = 0$

For uniformly accelerated motion,  $a_t \neq 0$

$\Rightarrow$  Relevance to centrifuge based physical modelling:  $a_t = 0$  for tests conducted at constant angular velocity and for uniformly increasing g-level,  $a_t$  is present.

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Then in order to for determining the acceleration of the particle  $p$  what we do is that we differentiate this  $v = \omega \times r$  the cross matriculation of  $\omega \times r$  then we get the  $dv / dt = d / dt \times \omega \times r$  that means that  $d \omega / dt \times r + \omega \times dr / dt$ , so here we see that the acceleration is divided in to two components so  $d \omega / dt$  we can write it as you know this is nothing but angular acceleration rate of change of angular velocity change in angular velocity.

So which is nothing but  $\alpha \times r = \omega \times dr / dt$  so her what we can do nis that this  $dr / dt$  we can write it as  $v$  and  $\omega \times v$  that  $v$  can be substituted by  $\omega \times r$  with that we can get is that  $\alpha \times r + \omega \times r$ , so here the first component is tangential component and  $\omega \times \omega \times r$  is the normal component of the acceleration.

So here  $\alpha \times r$  which is angular acceleration and you know the position vector again it is acting in the direction of tangential velocity because when you take the projection of  $aa'$  to the point  $p$  and with the position vector which is there and it will be perpendicular to that plane then you know it will be acting towards the you know in the direction of tangential velocity.

Whereas the normal component where we have the central access  $aa'$  and the vector which is perpendicular to that plane  $\omega \times r$  is the  $v$  which is the tangential velocity vector and the

perpendicular to this plane that means that is acting towards the center that means that we have  $\omega \times \omega \times r$  which is actually nothing but a centripetal acceleration done.

Now here the as for as the centrifuge based physical modeling is concerned and if you are actually increasing the  $\omega$  let us say uniformly accelerated motion then that means that the both tangential component as well as the normal component of the you know acceleration they do exist. And but if you are having a constant angle velocity that means that immediately after retaining certain speed and if you are able to maintain the constant angle of velocity, in that situations that  $a_t = 0$  it means that the constant angle of velocity is attain.

So the as far as the relevance of centrifuge based physical modeling is concerned  $a_t = 0$  for a test which is actually conducted at a constant angle velocity and for a uniformly increasing g level  $a_t$  is present, that means that if you are actually increasing this can be very short duration as far as the equipments are concerned but relevance to the centrifuge based physical modeling is a that if you are actually having a constant angle velocity the tangential component will not exist and for uniformly increasing g level  $a_t$  is present both  $a_t$  as well as  $a_l$  will be present.

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### Principle of centrifuge modelling

- It is to generate similitude of stresses and strains between models and their real or hypothetical prototype by simply increasing acceleration levels proportion to the reduction in linear dimensions.
- - Can be explained through a slope stability problem

At failure (FS = 1),  $N_s = c_u / (\gamma_s H) = c_u / (\rho_s g H)$

The stability of the slope depends on  $c_u$ , H, and  $\gamma_s$

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So let us look into the simple after having discussed about the mechanics of you know the centrifuge based physical modeling let us look at the principle of centrifuge based physical

modeling with an example of a slope which is having saturated undrained clay. As it is to basically the principle is to generate the similitude of stresses and strain between models and their real or hypothetical prototype by simply increasing acceleration levels proportion to the reduction in linear dimensions.

That in order to increase the acceleration we have a body rotating about the vertical axis in a horizontal plane. So this can be explained through a slope stability problem let us assume that we are having a stability of slope which is actually found to have the factor of safety of the slope found to have dependence on let us say, a slope inclination as well as you know as  $\theta$  as well as the undrained quotient, as well as  $\gamma$  and if it is having certain depth below the toe then  $d$  the height of the slope.

So when we do the dimensional analysis and when we you know apply the Taylor's theory like factor of safety is found to function of  $\theta$ ,  $C_u/\gamma h$  and  $d/h$  and slopes for which are having inclinations more than  $53^\circ$  what we say is that the factor of slope failure will not tend to occur. In that case the factor of safety is found to depend only on  $\theta$  and  $C_u/\gamma h$ . So we said that for the simulation of similarity in model prototype that means that you have the identical factor safety in model prototype.

And we said is that  $\theta$  in model =  $\theta$  in prototype, so that can be achieved by somehow by means of ensuring the identical configuration of the slope but  $C_u/\gamma h$  will be model in prototype that means that  $\gamma$  which is nothing but the  $\rho_s g$   $\rho_s$  is nothing but the mass density of the soil. So at failure the factor of safety = 1 what we can write from the Taylor's estimating number is that  $N_s = C_u/\gamma_s h$  where  $C_u/\rho_s g h$ . So you know this you know when you expand further.

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**Principle of centrifuge modelling**

- In order to model the slope failure for 1/50 physical model at 1g, it becomes necessary to produce a material having  $[c_u/\gamma] = 1/50$  of the prototype material. [i.e. for  $(N_s)_m = (N_s)_p$ ]

→ It is almost impossible to produce a material with  $c_u/\gamma = 1/50$

For a 1/50 physical model at 50 g,  $N_s = c_u / [\rho_s(50g)H/50]$

$$= c_u / \rho_s g H = c_u / \gamma_s H$$

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In order to model slope failure for 1:51 1/50 in physical model at 1g it becomes necessary to produce a material having  $C_u / \gamma h = 1/50$  for ensuring identical stability number in model prototype what it says is that when h is reduced by 1/50 times and in order to have  $C_u / \rho_s g h$  in model prototype to be identical that is the stability number in model prototype will be = means the identical factor safety that is = at failure = 1.

In order to ensure that it says that if you are actually taking about 1:50m scale model at 1g it says that  $C_u / \gamma = 1/50$  that means that it implies that if you are not making the soil weight less then you know one the alternative is to very low equations. So if you are actually having you know let us say  $C_u$  50k Pascal in prototype and this means that at 1:50 a  $C_u$  of 1k Pascal that means that almost above limit.

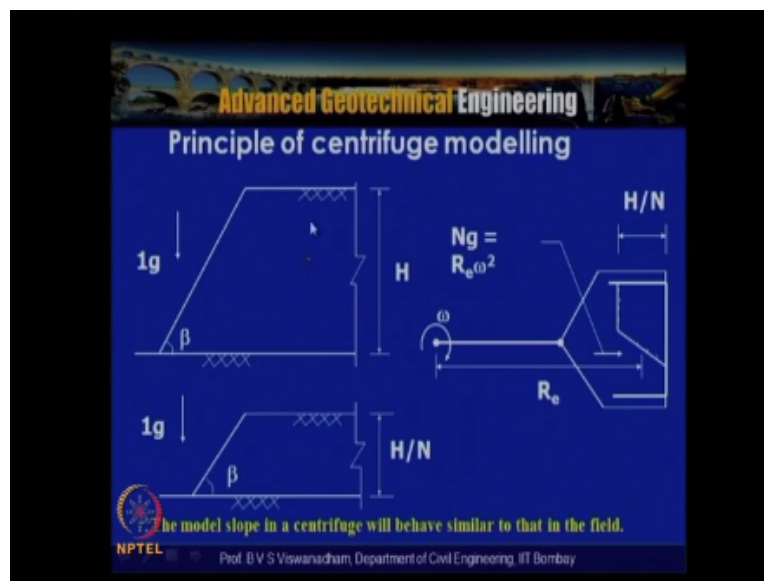
You know the soil strength is almost 0, so it is almost impossible to produce material with  $C_u / \gamma = 1/50$  and you know the moment we remove the scaffolding for the what we maintained for achieving the slope inclination there can be a failure. So for a 1/50 Pascal model at 50 let us say and the  $N_s$  will in model the stability number model will be equal because here what is actually happening is that the  $C_u / \gamma$  is reduced by 1/50.

But by ensuring identical  $C_u$  in modern prototype what as actually happens is that  $\gamma$  which is nothing but  $\rho_s \times g$  and by maintaining as the same soil as that in the prototype and increasing the  $g / 50$  times because we are able what we do is that we rotate about vertical axis in horizontal plane to you know induce a constant angular velocity. So with that what will happen is that for a

$1/50$  model at  $50g$   $N_s = C_u / \rho_s 50g \times h / 50$  which actually ensures that  $C_u / \rho_s g h = C_u / \gamma s h$  which is nothing but  $N_s$  in model prototype are identical.

So this is one of the classical examples which was used to bring out the you know the merit of centrifuge modeling by using this particular examples, the stability of a slope of undrained composed of undrain clay so in this particular slide.

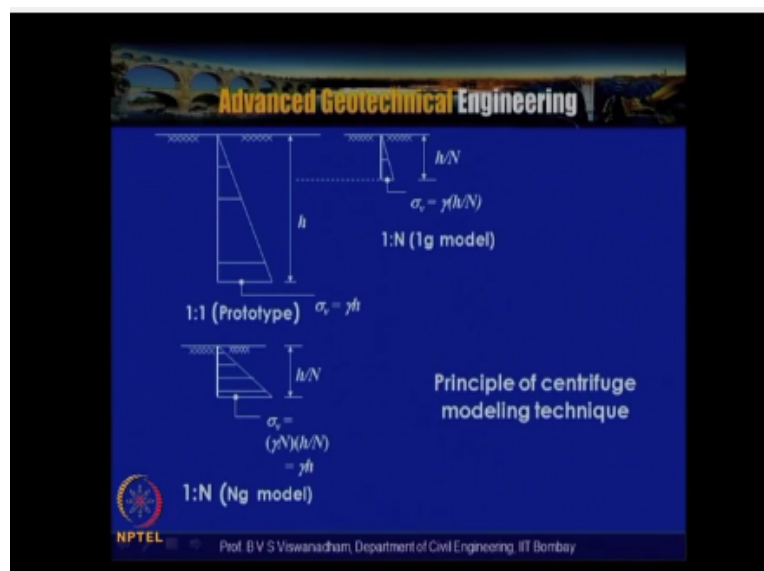
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A slope with inclination of  $\beta$  which is actually shown here having I height h and the same when it is reduced by  $1/n$  times the slope height is actually is  $h/n$  times, so this and this actually the correspondence will not be there because you know there is the vast difference of you know because we are not able to reduce by  $C_u/\gamma$   $1/50$  the behavior is next to impossible. But the behavior is difficult to model but here if you see the  $1g$  model what you can see is that by rotating a model about vertical axis in a horizontal plane.

So what you are seeing is the plane view of the you know the model rotating about vertical axis, so here the model is reduced by  $1/n$  times but because of the increased centrifuge forces acting towards this by using the frictional or initial forces acting towards the model that is we take it as  $Ng = r\omega^2$  so these forces they do exist when model is rotating about a vertical axis in a horizontal plane in order to keep that in certain orbit.

And that is nothing but what we say  $mg = r\omega^2$  so if you are actually having this constant angel velocity conditions what we can that the model slope in centrifuge will behave into the category prototype, that means that a correspondence between the model this particular method which is at  $1g$  at full scale level can be achieved.  
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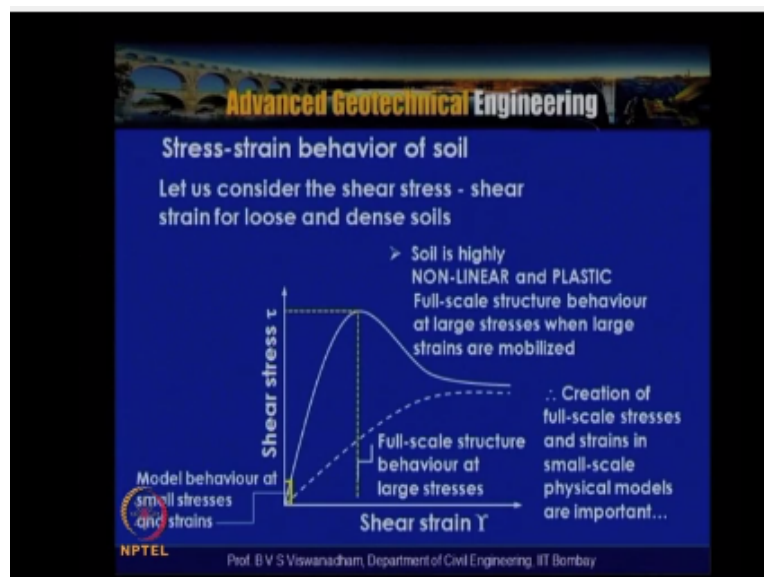
So consider this with as far as the geo static stress distribution as concerned let us say that we are having a soil profile having unit weight  $\gamma$  and at the depth  $h$   $1:1$  prototype of full scale structure the  $\sigma_v$  the ordinate is here is  $\gamma h$  where  $h$  is the you know the vertical height and for similar situations for same soil having  $\gamma$  with  $1:n$   $1g$  model with the stress is nothing but  $\sigma_v \gamma \times h / n$ . so here what we have is that very low stress compared to the stress which is actually existing in the prototype.

But contribute into this when we have a model subjected to  $n$  gravities though the you know here for the example the vertical ordinate is  $h$  and horizontal is  $\gamma h$ , so here both vertical ordinate and horizontal ordinate are effected here in the vertical ordinate is reduced but the horizontal ordinate

that is  $\sigma_v$  is identical as that in the prototype that means you can see that  $\sigma_v = \gamma \times h/n$  because  $n$  is now  $\gamma$  is  $n\gamma$  with that what will happen is that  $\sigma_v = \gamma h$ .

So this was also possible in hydraulic symmetry method by with the help of  $C$  plus forces what we said that this can be achieved but we also discussed that merits and the demerits of the technique.

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So further we will try to explain the importance of you know the simulation of you know the stresses so consider a shear stress behavior for loose and dense sandy soils let us say that we have got this relationship by testing a loose and dense sandy soils. So this is the shear stress and this is the shear strain and this for dense sand and this is for the loose sand you can see that there is the hardening.

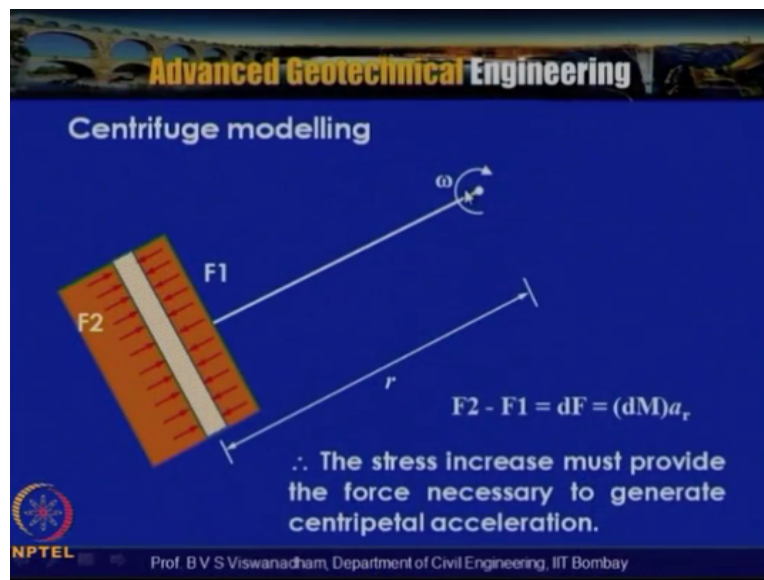
And then soft behavior and here the continued hardening actually and then there will be peak will not be you know exhibited here now we have the mode at small stresses and strains we can see that the stresses are very low because in the previous line we said that the  $\sigma_v$  model is equal to  $\sigma_{vp}/L$  so what you can see is that the soil initial layer the stiffness is very high.

And in the stresses are very low and but similarly when you look into this full scale structure behavior at large stresses you can see that the stiffness is low as a stresses are high so here this

the soil is we can look into is highly non-linear and plastic and full scale structure behavior large stresses when large strains are mobilized.

So creation of full scale stresses and strains in small scale physical models are very important and particularly when you wanted to model to capture the response of a you know particular full scale structure having embedded in certain type of soils which is every important to capture the you know the identical stress strain behavior of a soil in the model has that in the prototype.

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So consider let us saying the centrifuge's modeling let us that you consider the model rotating at a vertical axis what we are seeing is you know the centre of the shaft the model rotating about the vertical axis with the constant now consider the small element to  $dR$  having the thickness along the radial direction and this you know force acting on the area  $A$  the area perpendicular to plane on this figure is  $A$ .

And the force acting on this is  $F_1$  and  $F_2$  and  $F_2$  is more than  $F_1$  because the stress increases as you go down so the stress increases must provided the force necessary to generate the centripetal acceleration so  $F_2 - F_1$  that is a small  $dF$  is equal to we can write it as  $dM \cdot \text{acceleration}$  that is  $A r$  with reference to  $r$  from the centre of the shaft measure from the centre of the shaft  $B$  the centre of the element.

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**Centrifuge modelling**

$$dM = \rho (dV) = \rho (dZ \cdot A)$$

$$d\sigma_v = dF/A = \rho ((dZ \cdot A)/A) \cdot a_r \quad \text{Using } a_r = Ng$$

$$\sigma_v = \int_0^{z/N} \rho(Ng) dz = \rho gz$$

⇒ Thus stresses are identical at geometrically equivalent points in the prototype and centrifuge model, provided the linear scale in the model is the inverse of acceleration scale  $N_g = 1/N_l = N$

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Further what we can write is that we can write  $dM$  as  $\rho \cdot dV$   $\rho$  is the mass instead of soil which we are used in the particular example so where  $Dv$  we can write it as  $dZ \cdot A$  that means that volume a small volume of the element having thickness the  $Z$  and area  $A$  then we can write stress as  $d\sigma_v = DF/A$  the substitution for  $DF = \rho dZA/A \cdot a_r$  so by using  $a_r = ng$  that means that when the model is rotating above the vertical axis the gravity is when we actually accepted.

So what we can say is the  $\sigma_v =$  when you integrate from 0 to  $z/n$   $\rho \cdot Ng dz$  when we integrate from 0 to  $z/n$  what we get is that  $\rho gz$  so that what we have discussed in the previous slide the stresses are identical at geometrically equivalent points in the prototype and centrifugal model provided the linear scale.

The model is inverse of acceleration scale that is  $N_g = 1/N_l = N$  there is also called as one condition  $l_m/l_p = 1/n$  and  $n_l$  is nothing but length scale factor which is  $l_m/l_p = 1/N$  and  $g_m/g_p$  that is  $N_g$  is nothing but acceleration due to gravity scale factor with this  $g_m = ng_p$  so when we said that you know  $g_l = 1$  that means that length is reduced to times of acceleration due to gravity  $g$  increased by  $N$  times.

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Element of soil (a) at surface of the earth and (b) on centrifuge

a.

area  $A = 1$   
density  $\rho$

b.

area  $A = 1$   
density  $\rho$

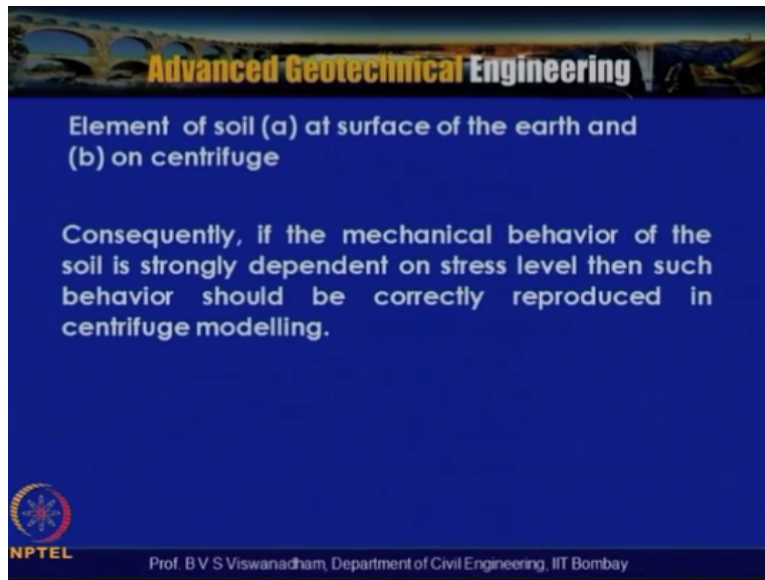
$$\sigma_v = \int_0^z \rho g dz = \rho g z$$

$$\sigma_v = \int_0^{\frac{z}{N}} \rho N g dz = \rho g z$$

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Now in this particular slide further that discussion is given where in when you are actually having the element of the soil at the surface of the earth there is in the prototype we have  $\sigma_v = \rho g z$  you know  $\rho g z$  so similar condition is actually  $q$  with  $\sigma_v = 0$  to  $z/n$   $\rho N g dz = \rho g z$ .


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Element of soil (a) at surface of the earth and (b) on centrifuge

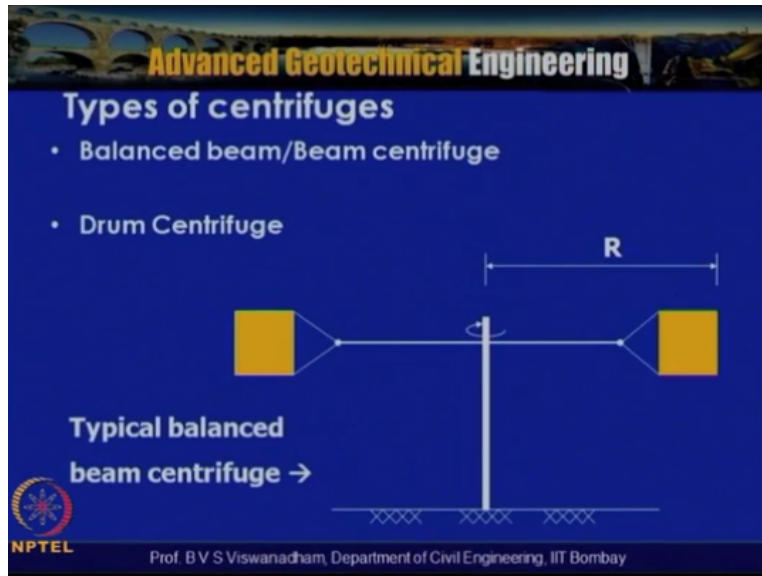
Consequently, if the mechanical behavior of the soil is strongly dependent on stress level then such behavior should be correctly reproduced in centrifuge modelling.

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So consequently the mechanical behavior of the soil is strongly dependent on the stress level that such behavior could be correctly reproduced the centrifuge modeling that is what actually we have said is that if the mechanical behavior of the soil is strongly dependent upon the stress levels then such behavior could be you know correctly reproduced in centrifuge modeling.

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So in order to do this there is number you know the equipments actually have been developed in the world they are actually the balanced beam and beam centrifuge and another type of centre is diffused in fact the initial attempts of this technique were reported in 1936 by fedro of this centrifuged based physical modeling a first to the part of this centrifuge the first international conference of soil mechanics in 1936.

And subsequently about after long gap then 1970 onwards with the help of the UK, USSR and other countries the further you know the work on this actually as convinced and since then there is number of equipments of the developed and every four years you keno the conferences on this centrifuged based of physical modeling nowadays they are actually calked as physical modeling in geo-techniques.

So we have actually two equipments one is called balanced beam centrifuged a or beam centrifuged so in this particular slide what we see is the you know a vertical shaft nd this is called the arm or beam of the centrifuged nd this is called the basket and initially at normal gravity there it is beam is 0 the basket will be in this posit6ion but s the rp increases as the rotation of a this particular system takes place about vertical axis in horizontal plane. When the basket swings up what we call this is called swinging basket because the basket swings up and this angle from this point to this axis along the arm is actually called as hamming angle and that the time which takes to the reach this thing horizontal level is actually called s ramping time as this attached to the planet.

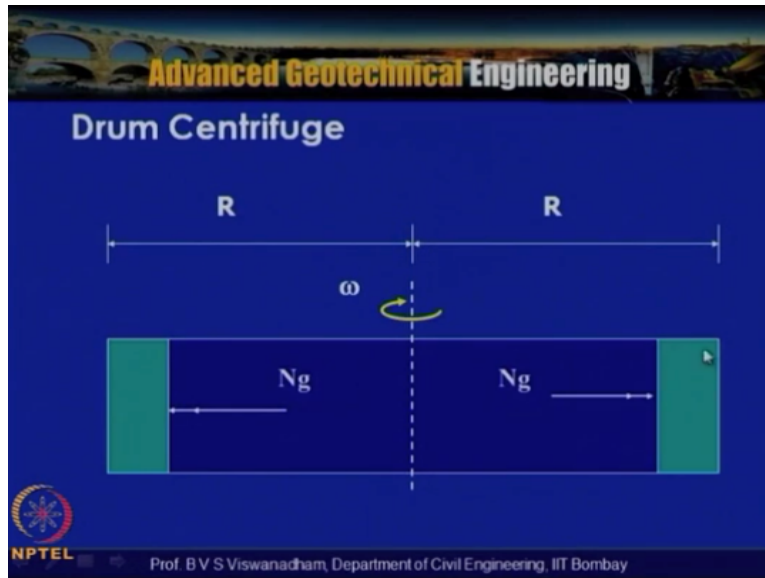
So what we have is this virtually what we have is really the  $Ng$  which is actually rotation of the rigid body of the vertical axis active on the both the sides and  $1g$  acting downwards but when you look into this resulting effect of the  $Ng$  as  $Mg$  this is nothing but the radial acceleration feel is nothing but  $\sqrt{n^2+1}g$  but if you look into higher values of  $N$  that  $N=N'g$  which is almost consider equivalents.

So this is what we see is the balanced beam centrifuge where you have got two swing basket the idea was to carry you know two different models in simultaneously but subsequently considering the power saving requirements and the aerodynamics point of view this basket is actually placed by the counter weight and which is actually most depending upon the weight of the model say the purpose of this balanced beam and beam centrifuge is that the  $\sum mr=0$  that means if you are having a mass the mass is here  $m_1$  and  $r_1$  and  $m_2$   $r_2$  the  $\sum mr$  has to be 0.

So this is you know typical you know the balanced beam centrifuge then we also have another category of type of equipment called in the drum centrifuge but in the balanced beam centrifuge in this basket the container is placed and there are two types of continuous one is called static containers another open called you know static test and which are rigid nature.

And if you are actually doing the equipments like dynamic expressions like earthquakes another studies then there is the possibility that in order to prevent the reflection of you know the beam that requirement that we have to use you know some sort of containers which actually identical motions as that in the prototype.

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So in drum centrifuge in this particular slide what actually has been shown is that there will be periphery drum and the channel will be fixed here and there will be central shaft and the tool table so here in the drum centrifuge what will happen is that the tool table and as well as the drum rotate above the vertical axis and synchronized manner.

So here what is happening is that is possible for us to do a number of know one dimensional problems and is also possible to you know stimulate the elastic off space conditions in soil mechanics particularly when you take you know  $\pi d$  the perimeter which is the along the peripheral of the drum then it is possible that you reveal to model you know large infinite soil extends on the surface of the soil.

And so here what we actually have is that a soil model which is actually created during flight hither with clay or with the sand by special techniques varying will be able to you know get the you know this desire ground surfaces and also the stress strain although things can be matched and in addition it also helps that you know in a given with the identical stress when we have it is possible that this technique this particular equipment can be used to do multiple experiments with identical stress. So the vertical stress in the radial acceleration filed.  
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**Vertical stress in the radial acceleration field**

$dr = r_b - r_t$   
 $dM = \rho(dV) = \rho(dZ \cdot A)$   
 $d\sigma_r = dF/A = \rho((dZ \cdot A)/A) \cdot a_r$   
 Using  $a_r = r\omega^2$   
 $\sigma_r = \rho\omega^2 \int_{r_t}^{r_b} r dr$   
 $\sigma_r = \frac{\rho\omega^2}{2} (r_b + r_t)(r_b - r_t)$

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Can be obtained now is basically nothing but consider once again the rotation of the rigid body about a vertical axis where  $z$  is the depth from the top surface, so you can see that and consider an element having  $dz$  in terms of dimensions  $z$  discussed. Previously we actually said it has  $DR$  and where  $R$  is the radius up till here and when you consider the  $z$  it is nothing but  $Rt + z$  into the element.

And  $Rb$  is nothing but the radius upto the base of the model and  $Rt$  is nothing but the radius upto the top of the model which is nothing but  $dr = Rb - Rt$ , so again by using this but this is in terms of radial coordinates when we substitute here what we do is that  $d\sigma_v = df/A$ ,  $\rho dZ A/A \times Ar$ , so here by using  $Ar = r\omega^2$  what we can write is that  $\sigma_i = \rho\omega^2 r dr$ , so with that what we get is that  $\rho\omega^2/2 Rb + Rt \times Rv - Rt$  where in  $v$  actually say that you know these verticals stress in radial acceleration field is not linear it is actually non linear.

That means that what is actually happening is that because of the radial acceleration field there is a certain degree of non linearity is actually you know encountered. So here you can also observe that when you look into the variation of the  $G$  level with the depth at the top of the model it is  $Rt\omega^2$  at the bottom of model it is  $Rb\omega^2$ . So if you look into this you know there is the variation of the  $g$  level with the depth.

Similarly when you have the you know at this particular point the centre of the model it is  $Rt\omega^2$  at the edge of the model the gravity which is actually imposed is  $Rt + \text{this } \frac{1}{2} \text{ set which is say } \Delta h\omega^2$  and so there is a possibility that the gravity level is actually varied along the horizontal

distance and this with the vertical depth and if you consider the model in what we get is that you know there is a model which actually calls with the identical surface as the top.

So in that actually variation of the g level with depth can be related but however both in drum the variation g level with you know the depth is existent. So we further when you look into this vertical stress.

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**Vertical stress in the radial acceleration field**

$$dM = \rho (dV) = \rho (dZ \cdot A)$$

$$d\sigma_v = dF/A = \rho ((dZ \cdot A)/A) \cdot a_r$$

Using  $a_r = Ng = (R_t + z)\omega^2$

$$\sigma_v = \int_0^z \rho \omega^2 (R_t + Z) dz$$

$$= \rho \omega^2 (R_t + z/2)z$$

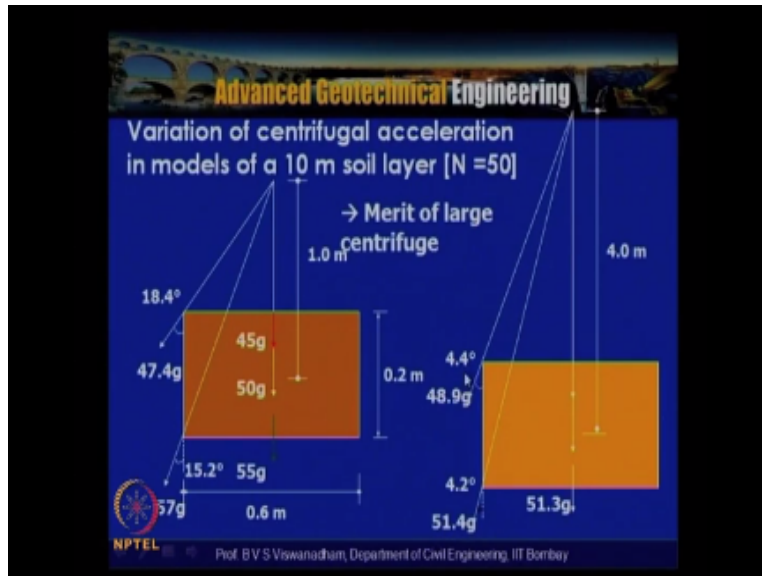
⇒ This indicated that non-linear variation of vertical stress resulting due to model rotating in a radial acceleration field.

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In the radial acceleration field if you look into this even from the reference to in terms of Z coordinates also  $\sigma_v = 0$  to  $z \times \rho \omega^2 R_t z \times dz$ , so this expression also indicate that there non linear variation of the vertical stress results due to the model rotating about a radial acceleration field so further you know.

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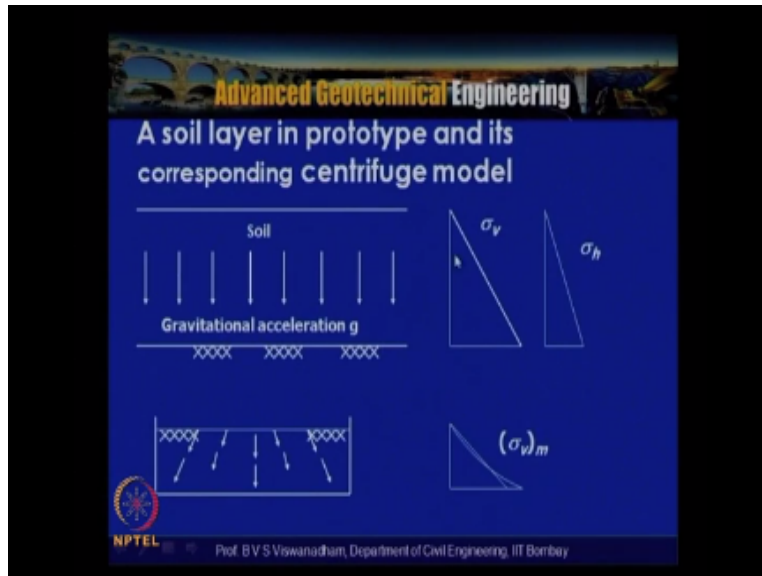




This particular slide shows about the merit of the large centrifuge which is actually having a small model, small radius and this is the large radius what you can see is that when you have a large radius the angle which is subtended to the centre is very low and in the small centrifugal the angle is very high. So what we have is that you know here you have 45g here you have 55 g in this case what you can see that the variation 0 level is marginally negligible here.

There is the variation but it is highly traceable, so you can see that here 48.9 and here 59.4, 15.3 here 48.9 so variation g level is very less. So this indicates that you know the g level can vary with the depth and here also when you compare a soil layer in prototype.

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And the corresponding centrifuge model what we said is that if you have not a prototype with this the  $\sigma_v$  and this is  $\sigma_h$  let us say for normally consolidated soil condition when you have the similar centrifuge model when you subjected to rotation of vertical axis is horizontal plane because of the radial acceleration field there is a small minor variation which is actually resulted because of this radial acceleration field effects.

So in this particular lecture what we have discussed about is the example on the hydraulic stimulated method and further we actually looked upon how the mechanics centrifugal base model can look into you know the further you know the understanding about the centrifugal base model and we said that if mechanical way of the soil is important to capture then the centrifuge base physical modeling is one of the viable option or it is the tool available for geotechnical engineering to understand about the you know geotechnical structures.

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