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ADVANCED GEOTECHNICAL ENGINEERING

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Lecture No. 38

Module - 4

Lecture – 9 on Stress-strain relationship and Shear Strength of soils

Welcome to lecture series on advanced geotechnical engineering and we are discussing module 4 and which is on the shear strength of soils.

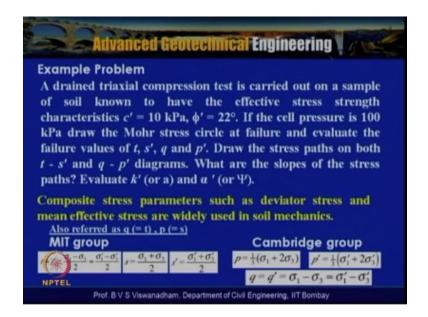
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In the previous lecture we introduced also stood different types of triaxial test they are namely consolidated undrained triaxial test consolidated drained triaxial test unconsolidated undrained triaxial test and un confined compression test and we have also discussed that you know there is a sort other possibilities also with the triaxial test where in we can also performed this extension test in order to measure the tenses strength extension characteristics of a soil or tense strength of a soil so we are you know discussing about the stress paths in triaxial and thereafter we will introduced also to Octo header plane and more columb condition.

And thereafter we will discuss about how we can actually determine elastic more less from the triaxial test.

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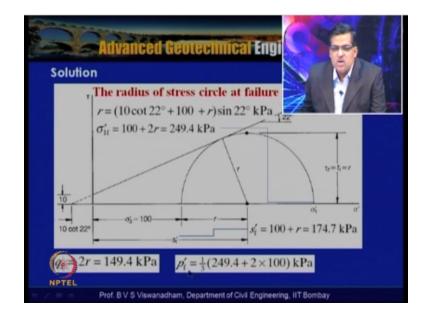
So before looking into the you know the discussion let us take an example problem where in the problem rates like this a drain triaxial compression test is carried out on a sample of soil known to have the effective strength effective stress strength you know characteristics as you know effective stress parameters and they are basically $c_{-} = 10$ KP and $\varphi = 22^{\circ}$ if the cell pressure is 100KP and draw the more stress circle at failure and evaluate the you know failure values of t s-q and p-.

And draw the you know the stress pass on both ts- and qp- diagrams so and what are the slopes of the stress paths evaluate k- and a and α - or ϕ so we should actually you know refer here that that two types of you know parameters composite stress parameters one is actually put forwarded by MIT group and Cambridge group and both the groups actually have used unfortunately same you know symbols they are q and p so for convince here q is also defined as t and p is also defined as s.

So as far as the composite stress parameters deficient such as deviator stress and new effective stress are widely they are basically why used in soil mechanics and in the course of the lectures also here and there we are used both MIT group and Cambridge group you know q and p symbols but here are distinguishing with MIT group and Cambridge group the symbols put forwarded by MIT group and Cambridge group where t = t- means that is nothing but you know $\sigma 1 - \sigma 3/3$ which is = q to the original you know definition put forward by MIT group sc = $\sigma 1 + \sigma 3/2$ so in case of sc = s- = $\sigma 1 + \sigma 3/2 = \sigma 1 + \sigma 3/2$.

But in case of q it is $q = \sigma 1 - \sigma 3/2 = t$ and whereas the Cambridge group if you look into it is $p = 1/3^{rd}$ of $\sigma 1 + 2 \sigma 3 / 1 / 3^{rd}$ of $\sigma 1 + 2 \sigma 3$ where p' = 1 third of $\sigma 1 - t 2 \sigma 3$ so $q = q' = \sigma 1 - \sigma 3$ according to Cambridge group definition of composite stress parameters.

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Now from the given data the cell pressure was actually given as 100KP and the soil; actually was found to have in has been given as the Mohr failure analog was given in the problem, so what w have done is that we have drawn the Mohr column failure analog and on the y axis that is the τ and the x axis it is σ - so here this you know ordinate is 10KP which is the you know the quotient ordinate and the 22^o is the friction angle, now for the we know that the σ 3 is 100Kp that is actually at this point it is given.

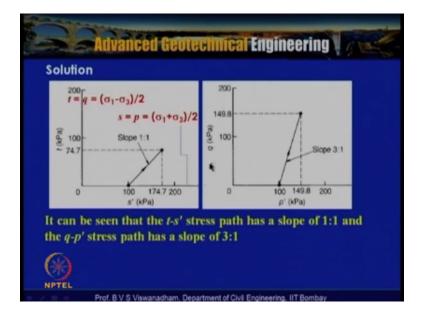
Now what we do is that the radius of the stress circular failure can be obtained by this, so radius is nothing but the radius of the circle is nothing but which is you know if you take this triangle you know which is nothing but 10 core 22 + 100 + r you know Sin 22 so if you take this one the radius is actually obtained here and once the radius is obtained with this as a center and with this is one of the points, the more you know circle actually is drawn and σ 1f can be obtained as 100 is the σ 3.

It is 100+2r that is you know the radius 2 times radius we get the σ 1-f that is the major principles stress at failure and once we get the major principles stress at failure you know this s-f can be obtained as that id nothing but σ 3 + σ 1 – σ 3/2 which is nothing but 100+r0 is about 174.7KP and

qf is nothing but 2r = 149.4KP qf is nothing but you know that is $\sigma 1 - \sigma 3/2$ into 2 so because $q = \sigma 1 - \sigma 3$ so $r = \sigma 1 - \sigma 3/2$ so what we have done is that qf = 2r = 149.4KP and p- f = 1 third of we know that this $\sigma 1$ - which is $174.7 + 2 \times 100$ you know with this what we get is that to that is $\sigma 1$ - is nothing but 249.4 so $\sigma 1$ - is nothing 249.4 + 2 x 100 we get the p- so according to the problem.

We have been asked to determine q- and p- that is according the Cambridge group we have determined like this $qf = \sigma 1 - \sigma 3$ p-f = 1 3rd of $\sigma 1$ - f $\sigma 1$ -f is nothing but 100+2r so this is this point plus 2 x 100 we got this one okay.

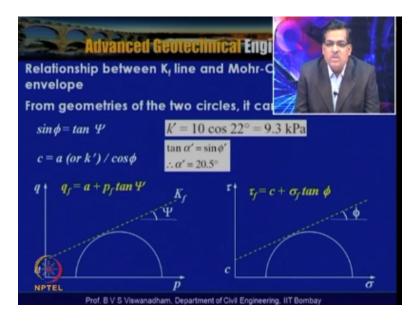
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Now what we actually next to we have said is that we can actually get the you know the stress paths you know have been asked to draw the stress part for you know ts and you know qp- so for both the things we have been given that these are the this is p- and this is q and this we know that this slope is actually will be 3:1 and 31 so the q is 149.8KP that is you know 149.8KP that is this one 149.4 that is here it is plotted here and this is 100 you know up to the you know that is the cell pressure which is applied.

So it travels from here to here to failure similarly when we take $t = q = \sigma - \sigma 3 / 2$ sc $= p = \sigma 1 + \sigma 3/2$ so this travels like you know 100 + you know this is nothing but $\sigma 1 + \sigma 3 / 2$ so this can be obtained like this, this is nothing but 174.7KP so that is indicted here so the slope of this you know is 1 so the t failure is round 74.7KP and this is nothing but 174.7Kp so it can been seen that t s- stress has a slope of 1:1 and qp- stress path as a slope of 3:1 so ts you know with the MIT group definition the stress path actually as a slop of 1:1 with Cambridge group definition of q and p' it is 3:1.

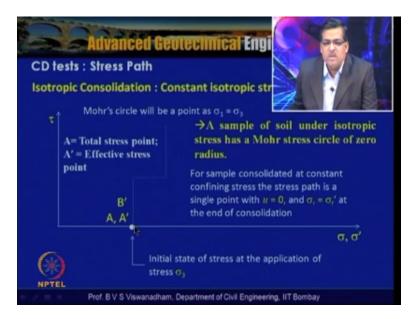
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Now we have been asked to determine k- and you know k- and α - so we know that we have a relationship between k fine and Mohr coulomb failure envelope when we have the q and p and when we have the τ and σ when we compare the from the geometry of the two circles we can actually we have derived that sin - = Tan ϕ = Tan α - and where c = you know this ordinate c = a / a or K' Cos / - so by comparison from here from the two circles we can get k' = from this equation k- = 10 that is you know c is given as 10KP.

10 cross 22 that is around 9.3KP so that means rate to be are actually getting that a or -k- as 9.3KP in q and p you know q and p space and Tan α - = Tan ϕ = Sin ϕ so Sin ϕ - given as Sin22 ϕ - is 22⁰ Tan α ' = ϕ 22⁰ which is nothing but α ' = 20.5⁰ so this α ' = ϕ = the inclination of the you know kf line is 20.5⁰ so like this you know we are need to you know look into the same for getting the you know different parameters at failure from the basic data which is actually reduced from the you know triaxial test.

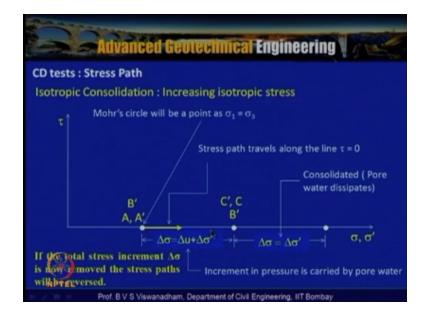
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Now let us consider the consolidated drained test and the stress paths and when we have the isotropic consolidation the constant isotropic stress that means that we have got equal stress in vertical and horizontal direction, then in that case the more circle is nothing but a point where the sample of soil and the isotropic stress basically has more circle of 0 radius, so it is nothing but when we have sample of soil under the isotropic stress conditions the most stress circle has 0 radius and which is nothing but a point.

So we a is nothing but the total stress point and a- is nothing but effective stress point and here of the sample consolidated at constant confining stress the stress path is a single point with u = 0 and $\sigma f = \sigma$ or dash $\sigma r = \sigma$ - the end of consolidation so this is the initial state of stress and the application of the you know stress $\sigma 3$ then you know.

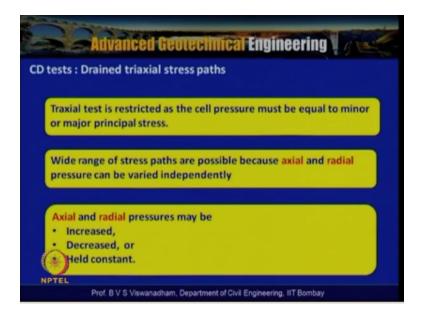
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The Mohr circle will be point that we have discussed now you know when the stress travels to along the line with $\tau = 0$ so this increment in pressure is carried by the pore water itself increment in pressure us carried by the pore water itself, so it actually travels along this σ line or σ - line and further you know once at the end of the consolidation the pore water pressure you know dissipates, so if the in increment of pressure is actually carried by the pore water pressure is here and once at the end of the consolidation that du Δu actually dissipates.

So if the total stress increment $\Delta \sigma$ is now removed that stress paths will be reversed in the opposite direction, so it as actually travels like this but if the total stress increment $\Delta \sigma$ is now is removed that stress paths will be reverse.

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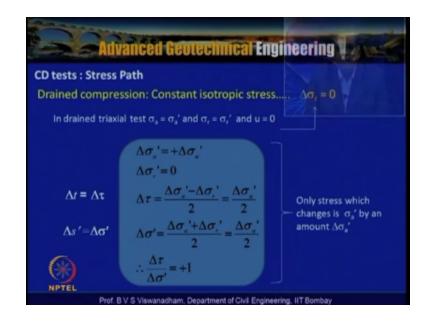
So triaxial test is restricted as the cell pressure must be equal to the minor or major principles stress and wide range of stress paths are possible because of axial and radial pressure can be varied independently and you know axial and radial pressures may be increased or decreased or held constant, so with that you know what we are actually deliberating is that you know number of types of tests which are possible and can lead to different stress paths, so the Cd test with drained triaxial stress paths.

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CD tests : Drained triaxial stress paths
Compression: Constant isotropic total stress. Ao _r = o
Compression: Constant axial total stress $\Delta \sigma_a$ = 0
Extension: Constant isotropic total stress $\Delta \sigma_r = 0$
Extension: Constant axial total stress $\Delta \sigma_a = 0$
Possible combinations of Drained loading on triaxial sample
Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

Compression where constant isotropic total stress, so here $\Delta \sigma r = 0$ compression where constant axial stress is 0 that is $\Delta \sigma a = 0$ and you know then extension the constant isotropic total stress where is $\Delta \sigma r = 0$ and extension where constant you know axial stress with $\Delta \sigma a = 0$ so possible combination of drain loading in triaxial sample are discussed here.

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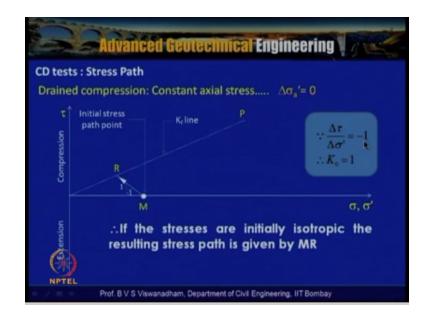


Now let us consider the drained compression the constant isotropic stress that $\Delta \sigma \tau$ that means that the change in radial stress is 0 $\Delta \sigma \tau$ is actually 0 so in a drain triaxial test the $\sigma a = \sigma'$ -a and $\sigma r = \sigma \tau$ -so and then u = 0 this is the pore water this question is 0, so $\Delta \sigma r a' = \Delta \sigma a$ sorry $\Delta \sigma a = \Delta \sigma a$ a- which is positive and $\Delta \sigma r = 0$ so if you look into this $\Delta t = \Delta \tau = \Delta \tau = \Delta \sigma a - \Delta \sigma \tau$ that is nothing but $\Delta \sigma 1 - \Delta \sigma 3 / 2$ so if you look into this here $\Delta \sigma r = 0$ with that no what we have get is that $\Delta \sigma a - / 2$ and this is $\Delta \tau = \Delta \sigma a - / 2$.

And when you have $\Delta \sigma s = \Delta \sigma$ - then $\Delta \sigma = \Delta \sigma a$ - that is nothing but a $\Delta \sigma 1 - + \Delta \sigma r$ - which is nothing but $\Delta \sigma 3' / 2$ so with $\Delta \sigma r = 0$ what we have got is that this $\Delta \sigma a - / 2$ so the slope of this one $\Delta \tau / \Delta \sigma$ - is nothing but the positive that is nothing but if they get canceled then $\Delta \tau / \Delta \sigma' = + 1$ so only the stress which changes is σa - by an amount $\Delta \sigma$ - that is nothing but the axial stress is actually increases and the rest everything with $\Delta \sigma r$ - which is $\sigma 1' = 0$ and in the drain stress path compression constant isotropic.

Stress in where $\Delta \sigma r = 0$ and with that you know the pore water pressure dissipation both during the shearing is 0 so this is can b indicated.

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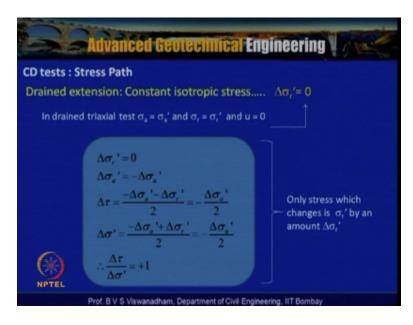
Whatever we have deliberated with drained compression with constant isotropic stress with $\Delta \sigma r = 0$ with compression on upper axis and extension blow so σ and σ axis here, so m is that point where initial stress point and then what we have said is that you know the stress path actually travels is to m to p and this slope of this line is $\Delta \sigma / \Delta \sigma \Delta \tau / \Delta \sigma$ - = positive 1, so there is nothing but because of this deliberation what we have discussed here in the slide the slope the slope of this line is you know 1.

The slope of this line is 1 vertical I horizontal 1 vertical 1 horizontal and this is the k of line so if you look into the stress path is a straight line the slop 1:1 that is actually 45° failure rate point p, so the slope is actually meeting the kf line at point p, so p is the stress so mp is the stress path is the straight with slope 1:1 failing that point p, now drain compression constant axial stress that means that here $\Delta\sigma a' = 0$ that means that there is no change in the axial stress, so here what we have what is that.

You know the only stress which changes is $\Delta\sigma$ - by an amount $\Delta\sigma$ - that means that w are actually increasing let us say $\Delta\sigma$ r' so $\Delta\sigma'=0$ and $\Delta\sigma$ r' = - $\Delta\sigma'$ so $\Delta\tau$ is nothing but $\Delta\sigma$ a' $\Delta\sigma$ r' / 2 with that you know what we get is that $\Delta\sigma$ r' / 2 and $\Delta\sigma' = \Delta\sigma$ on dash a + - $\Delta\sigma$ r'/2 so with that what we get is that $\Delta\sigma$ r' = 0 so with that what we get that $-\Delta\sigma$ r' / 2 so when we have let us say the sample $\Delta\sigma$ a' = 0 and when keep on increasing $\Delta\sigma$ - the sample experiences you know the $\Delta\sigma$ $\Delta\tau / \Delta\sigma'$ = the slope will be.

You know there is -1 so that means that you know 1 vertical -1 horizontal you can see here so in this case when we have the constant axial stress $\Delta\sigma a_{-} = 0$ that means there is a constant axial stress and if the stresses are initial isotropic the result in stress path is given as mr that is if the mr is the resulting stress path which is actually you know which actually got a $\Delta\tau / \Delta\sigma' = -1$ with $k_0 = 1$.

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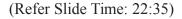


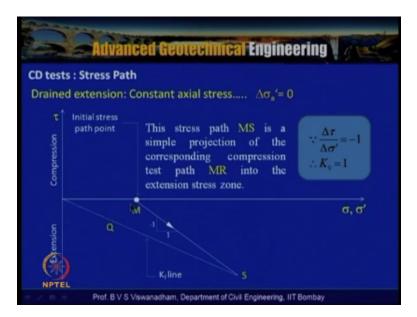
Now drained extension the constant isotropic stress where $\Delta\sigma r' = 0$ that means that here in the drained axial test $\sigma a = \sigma a'$ and $\sigma r = \sigma r'$ and u = 0 so here the only stress which changes is $\sigma r'$ and by an amount $\Delta\sigma r'$ so you know the only stress which actually changes is $\Delta\sigma r$ - by an amount $\Delta\sigma r$ - so in this case $\Delta\sigma r$ - = 0 that is and then $\Delta\sigma a' = \Delta\sigma a'$ and $\Delta\tau = -\Delta\sigma a - \Delta\sigma r' / 2$ and with $\Delta\sigma r$ - = 0 what we get is that $\Delta\tau = -\Delta\sigma a / 2$ and $\Delta\sigma'$ = with $\Delta\sigma r' = 0$ what we get is that $-\Delta\sigma a' / 2$ so $\Delta\tau / \Delta\sigma' = +1$ so if you look into this here.

What we have got is that $\Delta \tau = -$ and $\Delta \tau / \Delta \sigma'$ is equal to also – so the stress path is here Mq is a simple extension of the corresponding compression test Mp into the extension test zone, so here the $\Delta \tau / \Delta \sigma' =$ positive but they have 1 – with – 1 vertical – 1 horizontal and towards the action, so Mq is the simple extension of the corresponding and compression test Mp into the extension test zone, so for the drained extension the you know the constant isotropic stress is $\Delta \sigma r'=0$ is actually given as the stress path Mq is a simple extension.

Of the corresponding compression test Mp into the extension system that means that we actually have discussed you know the this one which is in the compression test and this is actually resulting in the you know in the extension zone, now we have another case that drained extension the constant axial $\Delta\sigma a' = 0$ and you know $\Delta\sigma a' = 0$ where $\Delta\sigma r = + \Delta\sigma r'$ so $\Delta\tau = \Delta\sigma a' - \Delta\sigma r / 2$ where with $\Delta\sigma a' = 0 \Delta\sigma a' = 0$ so what we have is that you know $-\Delta\sigma r / 2$ and $\Delta\tau' = \Delta\sigma a + \Delta\sigma r / 2$ with $\Delta\sigma a = 0$.

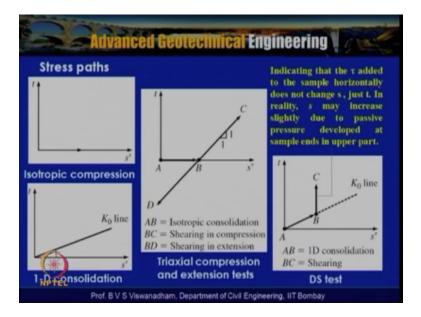
What we get is that $\Delta \sigma r'/2$ with the slopes $\Delta \tau / \Delta \sigma'$ which is nothing but – 1 and nothing but – 1 so the only stress which actually changes is you know $\Delta \sigma r'$ by an amount $\Delta \sigma'$ with what here $\Delta \sigma a$ - that axial stress is actually is actually not change.





So with that what actually we get is that the stress path MS is a simple extension of the corresponding compression with stress path MR that is what we have indicted like this into the extension stress zone, so this is nothing but the where $\Delta \tau / \Delta \sigma$ - = - 1 for k₀ - 1 condition that these for isotropic case.

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So we actually have seen you know various stress path for in this particular slide a simple isotropic compression stress path is actually shown we said that it actually travels along ts- that is S- line and when you have got only consolidation one dimensional consolidation where you know there is only vertical strain and $\sigma \epsilon r = 0$ as symmetric case where you have that k0 line is like this, this is the k0 line so this you know on this the you know only the consolidation actually happens.

But you know when you have you know let us say for a direct shear test you know when the direct shear test when you draw initially you know this ab is actually very similar as the k0 line, so it travels from to b you know because you know v in triaxial test what we do is that w initially there is you know some vertical test is actually apply and before commencement or before shear there is elastic equilibrium or k0 condition is established, so ab is that you know along you know the k0 line.

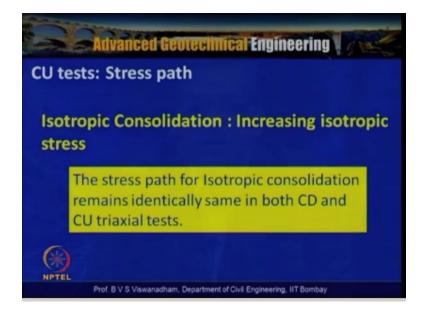
So there is point b where you know when the shear started you know what you can see is that bc is the path which actually takes the stress path is actually takes in direction shear test during shearing, so indicating that the you know the τ added to the sample horizontally and it does not change you know s does not basically s but just only p will change then what you see is that the these changing what is actually happening is that the it only the t changes s will not change that means that the line travels.

Vertically upwards this is in case of a you know direct shear test, so in reality basically s may increase slightly due to the pass through passive pressures which are actually developed at the samples in the upper portion of the sample, so that means indicating that the τ added to the sample you know horizontally does not change so this figure which is actually the stress apth which is actually shown indicated that you know that τ which is actually subjected to the sample you know it does not change.

Horizontally with is but only that t is actually changing so in reality s may be subjected to some increase due to you know passing pressure developed by the sample and in the upper part, so this is actually one of the limitations of the direct shear test this we have discussed earlier now when you have got that triaxial compression and extension test what we have discussed is that either with one vertical one horizontal 45° case and in this same this is you know in the towards the compression side this is towards the extension.

So this is shearing in compression and this is actually shearing in the extension, so shearing and extension means how it comes so at ab isotropic consolidation isotropic consolidation is here and then you know when it actually bc it actually changes from here that is shearing in the stress paths bc is the shearing in compression and stress path bd is shearing in extension.

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So consolidated under in triaxial test so till now we have actually discussed about the stress paths for the consolidated drain test, so the in the in case of consolidated under in triaxial test what we have disused is that during consolidation the draining is allowed but during shear the drainage is not allowed so that there is a possibility of the buildup of the pore water pressure and then we also discussing that deepening upon the stress history and the type of the soil deposit like a louse sand order den sin.

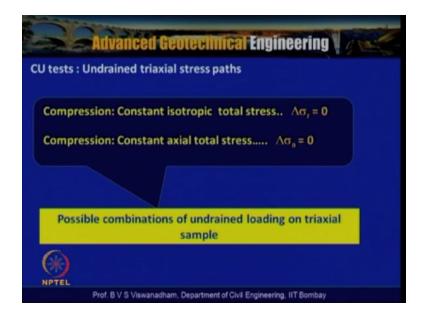
Or normally consolidated clay or more consolidate clay in case of loose sand and normally consolidate clay there is a possibility that the pore water pressure axis pore water pressure developed during shear is positive and when you have den sand and water consolidate clay you know the pore water pressure developed you know is in negative in nature that is berceuse of the phenomena of the dilation, so the isotropic consolidation in increasing the isotropic stress path for isotropic consolidation domains.

And identically same in both cd and cu triaxial test for a isotropic compression case initially the both the cd and cu they are actually one at the same, so this we have discussed some in Mohr circle actually initial a point. (Refer Slide Time: 27:33)

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CU tests :	Stress Path	
Isotropic	Consolidation : Increasing isotropic stress	
τ	Mohr's circle will be a point as $\sigma_1=\sigma_3$. Stress path travels along the line $\tau=0$	
	B' C', C A, A' B'	
-	$ \begin{array}{c} \leftarrow \Delta \sigma = \Delta u + \Delta \sigma' \longrightarrow \sigma, \sigma' \\ \uparrow & \uparrow & \downarrow \\ \end{array} $	
	Increment in pressure is carried by pore	water
	Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay	

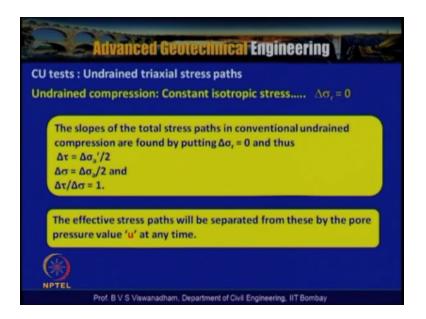
You know with 0 radius and stress path travels is along the $\tau = 0$ this we have discussed during you know our discussion on the consolidated drain triaxial test, so this is at the end of dispersion of pore water pressure it actually you know they stress of the in the sample increases by $\Delta\sigma$ which is nothing but the amount of pore water pressure which is actually dissipated during consolidation and here we have two cases when compression the constant isotropic.

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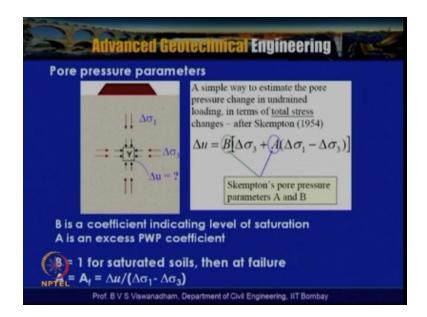
Total stress $\Delta\sigma r' = 0$ and constant axial total stress where $\Delta\sigma a_{-} = 0$ so possible combinations of under in loading on the triaxial test are shown here where both are compression one is that constant isotropic total stress $\Delta\sigma r' = 0$ there is no change in the radial stress and in other cases that there is no change in the axial stress, so first case is that you know the constant isotopic stress are $\Delta\sigma r' = 0$ when you consider.

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Then we have you know the slopes of these stress paths in conventional undrained compression or found to be by putting $\Delta \sigma r = 0$ and then with that we get $\Delta \tau = \Delta \sigma a' / 2$ and $\Delta \tau = \Delta \sigma = \Delta \sigma a / 2$ and then the ratio of $\Delta \sigma \Delta \tau / \Delta \sigma = 1$ so the effective stress paths will be separated from the from by these pore water pressure value u at any time.

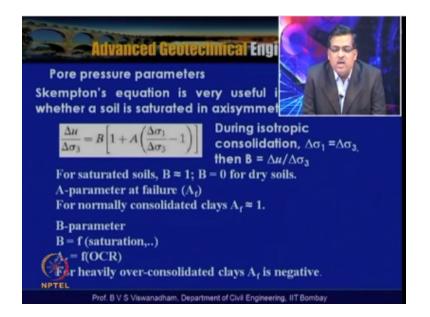
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So before you know discussing this stress paths of you know the you know in case of consolidate and drained type the triaxial test we should actually also introduce ourselves to two parameters which are actually put for skempton they are called as you know this skempton's pore water pressure parameters and like in simple way to estimate the pore water pressure change in undrained loading in terms of total stress is actually given by skempton 1954, and which is given by an equation $\Delta u = \text{this}$ is $\Delta u = b \times \Delta \sigma 3 + a \times \Delta \sigma 1 - \Delta \sigma 3$.

A and b both are called as skempton's a parameter A and B parameter, so B is a quotient indicating the level of saturation, so if B = 0 is basically a dry soil and B = 1 indicating that the soil is completely saturated and A is nothing but the axis pore water pressure position A is nothing but the excess pore water pressure position, so B = 1 for saturated soils when you put b = 1 when b = 1 and then you know Δu / then we can actually write you know A = and $Af = \Delta u / \Delta \sigma 1 - \Delta \sigma 3$ so at failure at failure with B = 1 we can write $A = Af = \Delta u / \Delta \sigma 1 - \Delta \sigma 3$.

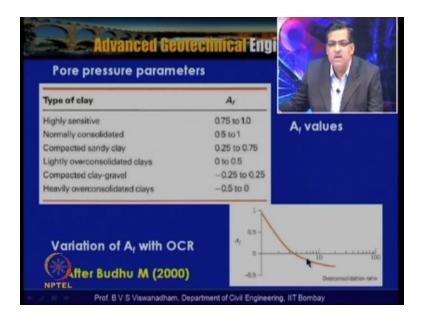
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So what we have done is that further this particular equation we have taken $\Delta u = B \Delta \sigma$ or 3 B into brackets $\Delta \sigma 3 + A$ into $\Delta \sigma 3$ let us divided this entire skempton equation by $\Delta \sigma 3$ so what we get is like this $\Delta u / \Delta \sigma 3 = B$ into 1 + A / 1 + A into $\Delta \sigma 1 / \Delta \sigma 3 - 1$ so skempton's equation is very useful in determining whether the soil is saturated in access symmetric test, so during isotropic condition $\Delta \sigma 1$ is $\Delta \sigma 3$ so during a isotropic condition $\Delta \sigma 1 = \Delta \sigma 3$ so when you substitute in this what we get is that $B = \Delta u / \Delta \sigma 3$ when $B = \Delta u / \Delta \sigma 3$.

So when during the isotropic consolidation we maintain the $\Delta\sigma 1 = \Delta\sigma 3$ so with that you know when you substitute here $\Delta u / \Delta\sigma 3 = B$ what we obtain, so for saturated soils B = 1 and B = 0 for dry soils, so A parameters at failure Af is nothing but for normally consolidate clays Ar = 1 so we normally consolidate soils they Ar = 1 and fro heavily in over consolidated soils Af will be negative the excess pore water pressure position at failure will be negative, so the b parameter basically is a function of saturation.

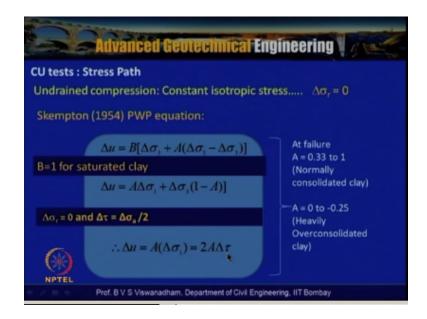
And A parameter is basically function of over consolidation pressure so over consolidation ratio if it is high then it actually changes into a negative for over consolidated case that Af will be negative. (Refer Slide Time: 32:34)



So this particular table which is actually gives you know typical Af values for a heavily highly sensitive soils you know 0.721 and normally consolidated soils clays 0.521 and if you look into this compacted sandy clay that Af values is 0.252.75 and likely over consolidate is actually have got 0 to 0.5 and compacted clay gravel is actually has got -0.25 to +0.25 heavily over consolidate clays can actually go up to -0.520 so if you look into this shear when you plot which is actually gives.

Variation of Af with OCR with increase in OCR where you can see that here it is 1 2 and 3 so OCR actually more than 3 that is from OCR = greater than 2 is set as likely more than lightly over consolidated clays, so when you actually have got OCR in the value in the range of more than 3 the Af value will be negative, so this is actually know that this discussion we actually had earlier if you connect and this is because of the pore water pressure development which actually takes place in case of you know during shear in case of you know the over consolidated r than see dens deposit to soils.

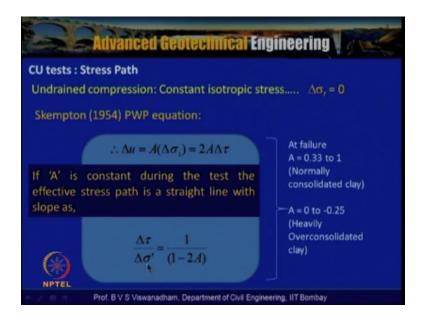
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So the undrained compression constant isotropic stress now what we with the background of the skempton's pore water pressure parameters we actually write the pore water pressure equation again with $\Delta u = B$ into $\Delta \sigma 3 + Aa$ into $\Delta \sigma 1 - \Delta \sigma 3$ and it failure A = 0.33 to 1 for normally consolidate clay and so for B = 1 for saturated clay when you substitute here what we get is that $\Delta u = A \Delta \sigma 1 + \Delta \sigma 3$ we have taken common into -1 - A, so here what we have what is that $\Delta \sigma 3$ into 1 - A so here $\Delta u = A \Delta \sigma 1 + \Delta \sigma 3$ into 1 - A so $\Delta \sigma r = 0$ and $\Delta \tau = \Delta \sigma a/2$ so this is you know $\Delta \sigma r = 0$.

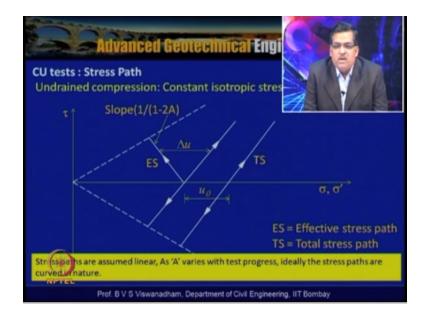
There is no change in the radius stress but $\Delta \tau = \Delta \sigma a/2$ so when you substitute this $\Delta \sigma a$ is nothing but $\Delta \sigma a = 2 \text{ x}$ you know $\Delta \sigma = 2 \text{ x} \Delta \tau$, so when you put here $\Delta u = A$ into $\Delta \sigma 1$ A into $\Delta \sigma 1$ with this when there is no change this become 0, so what we have is that $\Delta u = A$ into $\Delta \sigma 1$ so $\Delta \sigma 1$ is subtitled by 2 into $\Delta \tau$ so what you get is that $\Delta u = 2A \Delta \tau$ so A = 0 to 0.25 for heavily more consolidated clays, now further you know this $\Delta u = A$ into $\Delta \sigma 1 = 2A$ into $\Delta \tau$ and where if A is constant during the test with effective stress path.

There is a straight line with slope as you know $\Delta \tau / \Delta \sigma = 1 / 1$ ' 2A so here if you looking to it, here you go $\Delta u / \Delta \sigma 1$ which is nothing but 2A/ $\Delta \sigma$ so what we get is that. (Refer Slide Time: 36:29)



If A is constant during the test the effective stress path is a straight line with slope is nothing but $\Delta \tau / \Delta \sigma' = 1 / 1 - 2A$, so this is nothing but the slope of the you know the stress path.

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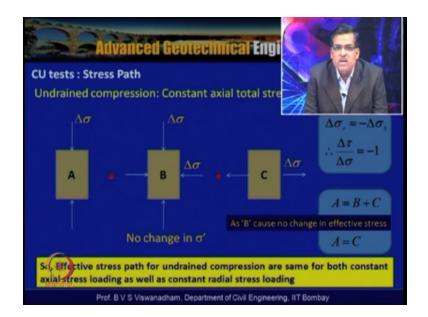


Which is indicated in this diagram and her in case of the this is the total stress path and this is the effective stress path you can see that and this difference is actually because of the you know Δu during the shear so the slop of this line you know if A is constant the slope is nothing but 1/1 - 2A the slope of this line so the stress path assumed to be linear if A- actually varies with the test with the once the test in progress once the test is in progress then ideally the stress paths are curved in nature.

So the effective stress paths will be curved in nature that is what we have shown in the previous discussion by the effective stress paths are curved in nature because the value of A will be not constant that is the stress path the effective stress the skempton's pore water pressure parameter will not be constant that is the slope will be changing, so because of that what will happen if it is constant then it is linear and if a is constant the slope will be constant and then this is in effective stress path but.

If there is a you know if there is A varies with the when the test is in progress then the ideally stress paths are curved in nature.

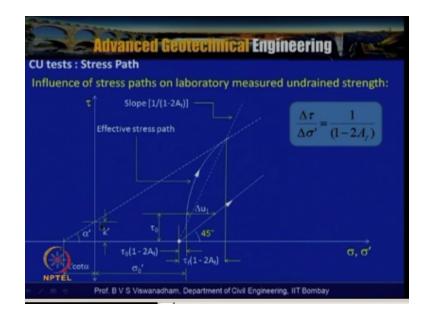
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Now you know with the constant axial stress once you look into it so when we have a $\Delta\sigma$ then which is actually nothing but is indicated by A and which is nothing but when you have $\Delta\sigma$ and then no change in $\Delta\sigma$ ' effective isotropic condition and plus when actually see that is $\Delta\sigma$ is there that you know then A = B + C where $\Delta\sigma r' = -\Delta\sigma r' = \Delta\sigma 3$ so $\Delta\tau / \Delta\sigma = -1$ and with this actually we can say that as B cause no change in the effective stress and we can say that A = C so the effective stress path for undrained compression.

Same as both constant stress excess test loading as well as the constant radial stress the effective stress path for the undrained compression are the same for both constant axial stress loading as well as constant radial stress loading, so here with this slide what you are saying is that the effective stress paths for undrained compression or same for both constant axial stress as well as the constant radial stress loading.

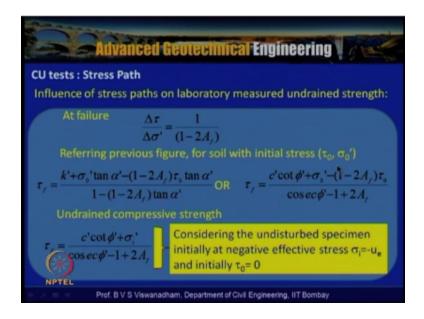
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So when this is the constant axial stress once you look into it so total stress path you know σr then constant σa increase then it is like this and when you have the unique effective stress path is actually possible like this with variation of the Af and total stress path you know total stress path σa constant and σr decrease then it is actually you know like this, so influence of the stress paths on the laboratory measured undrained strengths which we actually can be given by you know this particular $\tau \sigma$ plot here.

And this line is K_f line this line is Kf line this line is the K_f line the failure line and this intercept is Kf' r A according to our notation α - or ϕ and K- cot α that is this ordinate, so we can actually get the $\Delta \tau / \Delta \sigma'$ as the slope of this line slope of this rectors path is 1/1 - 2Af, A_f is the you know the this slop of this line at A_f is the pore water pressure skempton's pore pressure parameter at failure Af, so $\Delta \tau / \Delta \sigma' = 1/1$ ' 2Af that is the slop of this line, now if you look into this you know.

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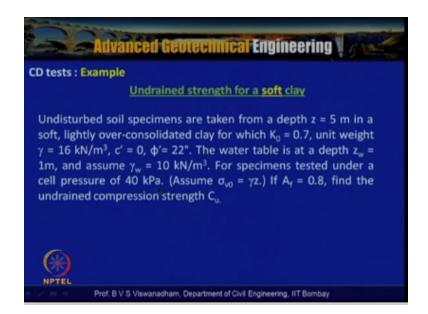


The at failure $\Delta \tau / \Delta \sigma' = 1/1 - 2Af$ so at failure which is like this different to the previous figure basically for the soil with initial stress $\tau 0$ and σ_0 ' so we can write $\tau f = k - so k$ ' is nothing but this ordinate $+ \sigma_0$ Tan α that you know the vertical axial ordinate okay and -1 - 2Af you know τ_0 Tan α - so what we have written is that this particular vertical ordinate we have written and that ordinate is actually working to be k' + c' σ_0 Tan $\alpha - 1 - 2Af$ τ_0 Tan α that is from the this slope is actually known to you.

And with that we can actually calculate so we are subtracting we are taking the net height that is you know τ_f and so with this we can actually simplify saying that $\tau_f = c' \operatorname{Cot} \varphi' + in$ terms of c' and φ' parameters, so we converting from the k_f line to the $\tau \sigma$ space completely where with that $\tau_f = c' \operatorname{Cot} \varphi' + \sigma_0' - 1 - 2\operatorname{Af} \tau_0/\operatorname{Cos} \sec \varphi' - 1 + 2\operatorname{Af}$ so undrained compressive strength can be obtained you know $\varphi' = 0$ undrained compressive strength is equal to can be obtained as you know $\tau_f = c' \operatorname{Cot} \varphi' + \sigma_1' / \operatorname{Cos} \sec \varphi - 1 + 2\operatorname{Af}$, so here the considering the under stable sample initially.

At negative effective stress there is $\sigma i = -U_e$ and initially $\tau_0 = 0$ initially $\tau_0 = 0$ so with that you know term will get you know when this will be eliminated then what we have is that this $\sigma' = \sigma i$, $\sigma 1$ ' so what you have a c' Cot $\phi' + \sigma'$ you know I / Cos sec $\phi' - 1 + 2Af$ that actually remains here.

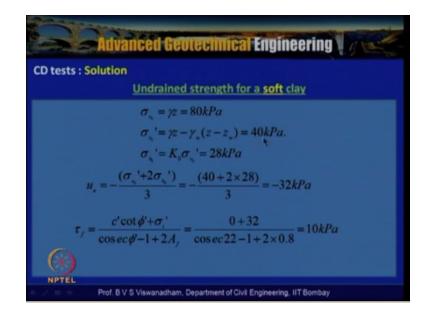
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So based on the deliberation let us look into the you know 1 problem here undistributed soils specimens are basically taken from a depth of say at z = 5m in a soft lightly over consolidate clay for which $K_0 = 0.7$ undistributed soils specimens are taken from a depth z = 5m in a soft lightly over consolidate clay for which K0 = 0.7 and the unit weight of the soil is 16KN/m³ and c'= 0 ϕ ' = 22⁰ and the water table is at depth zw = 1m and assume that $\gamma w = 10KN/m3$ for specimens tested under some pressure of 40KP.

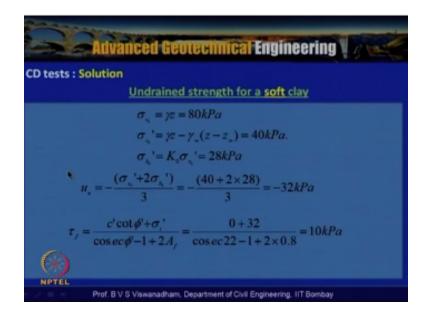
Assume that you know $\sigma v_0 = \gamma z$ and if f = 0.8 find the undrained compressions strength at Cu undrained compressions turns u so here you Af that is the r2 this stress parameters skempton's parameter at failure is given as 0.8, so the solution actually works out like this.

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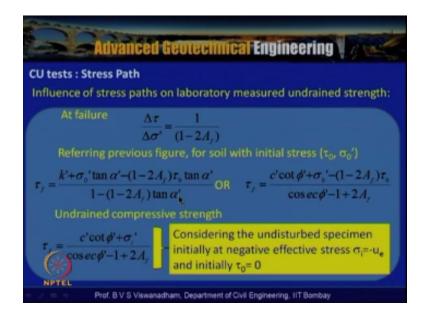
We can first get the vertical stress which is nothing but 16KN/m³ and 5m depth so it is 80KP the effective stress because water ground table is given at 5m the ground water table you know water table is depth of 1m so by taking this consideration what we get that a right stress, so once you get once you know.

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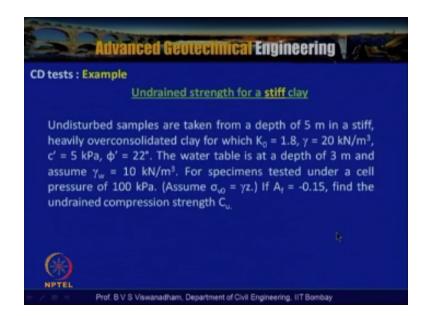
The effective stress and k0 is given as 0.7 and $\sigma'v0$ is actually 40kPa so with that what we get is that 28kpa, so ue that can be obtained by $\sigma'v0+2\sigma h$ you know $\sigma'h0/3$ that is you know the σv and $+2\sigma h$.

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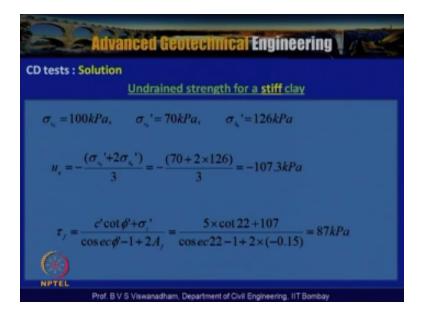
So that is you know this one we are actually estimating and that is actually given as estimated as -(40+2x28)/3 is -32kPa and τf now is the s $\cot \varphi + \sigma'$ that is nothing but you know which is obtained as which is obtained as you know this one as 32kPa that is ue= that σ'_A that is 32kPa so $\csc \varphi$ that is φ' is given as $22^\circ \csc 22$ -1+2x0.8 so with this the shear strength that failure works out to be 10kPa.

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So similarly another problem where undistributed samples are taken from a depth of 5m in a stiff heavily over consolidated clay for which K0=1.8 and because this is a wall consolidated clay the k0 will be high and γ 20kN/m³ and c'=5kPa and $\varphi'=22^{\circ}$ and water table is at a depth of 3m and assume that γ w=10kN/m³ and for specimen tested under a cell pressure of 100kPa and if Af=-0.15 so you can see that the Af is negative air and find the undrained compression strength Cu.

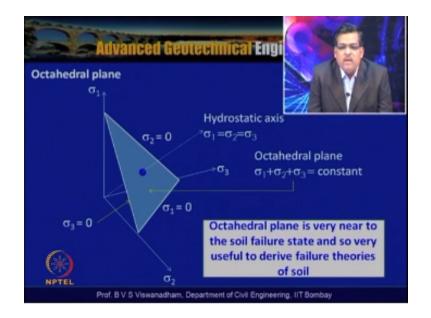
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So the solution again $\sigma v0$ is actually 5m that is a unit weight is 20 so 100kPa and the effective stress can be obtained as 70kPa and with 1.8 we will able to get $\sigma h'$ as 126kPa similarly here now by –(ue) is actually calculated where the clay is over consolidated so $\sigma v'+2\sigma'h0h'/3$ so with that what we get is that -107.3kPa so $\tau f=c' \cot \varphi + \sigma'/\csc \varphi' - 1+2Af$, so by substituting this $\varphi \cot 22+107$ because here $\sigma'=$ - this minus of minus will become plus here.

And the cosec22-1+2x(-0.15) so with that the shear strength of the soil works out to be around 87kPa. So what we have done is that we actually have you know try to deduce the discussed about in the consolidated undrained trail test and then we actually have you know obtained this you know deliberation about.

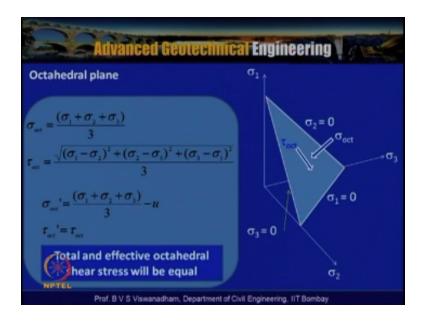
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So then you know we also have this octahedral plane and where we have for, if you are having the $\sigma 1$ axis and $\sigma 2$ axis and $\sigma 3$ axis and then you know when you actually have this lines indicate that $\sigma 3=0$ and $\sigma 2=0$ along this line and this $\sigma 1'=0$ and hydrostatic axis actually passes from the median of this that is hydrostatic axis= $\sigma 1=\sigma 2=\sigma 3$ when ot actually happens, and $\sigma 3'$ actually passes to this one and $\sigma 1'$ actually passes through this vertex and $\sigma 2'$ is actually passes through this vertex.

But along this $\sigma' 3=0$ along this you know $\sigma 1'=0$ along this $\sigma 2'=0$, so the octahedral plane is equal to $\sigma 1+\sigma 2+\sigma 3=$ constant the octahedral plane is very near to the soil failure state so very useful basically to derive the failure theories of the soil. So this octahedral plane which is indicated as $\sigma 1+\sigma 2+\sigma 3=$ constant and which is a plane which is very near to the soil failure and so very useful to derive the failure theories of the soil.

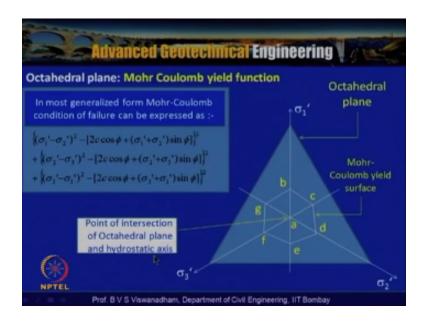
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You know further you know σ octahedral what we can actually get is nothing but $\sigma 1 + \sigma 2 + \sigma 3/3$ and τ octahedral the shear along the octahedral plane that means that on this octahedral plane that normal to the plane that is σ octahedral and along this that is τ octahedral so we can get σ octahedral as nothing but $\sigma 1 + \sigma 2 + \sigma 3/3$ and τ octahedral is nothing but $\sqrt{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2/3}$ so σ' after octahedral is equal to $\sigma 1 + \sigma 2 + \sigma 3/3$ -u so and τ' octahedral= τ octahedral.

So total and effective octahedral shear stresses are equal, total and effective octahedral shear stresses are will be equal.

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So from the Mohr Coulomb yield function in most generalized form for the Mohr Coulomb condition of failure can be expressed as you know which is nothing but $(\sigma 1' - \sigma 2')^2 - 2c\cos\varphi + (\sigma 1' + \sigma 2'\sin\varphi')^2 + (\sigma 2' - \sigma 3')^2 - 2c\cos\varphi + (\sigma 2' + \sigma 3'\sin\varphi)^2 + (\sigma 3' - \sigma 1')^2 - (2c\cos\varphi + \sigma 3'\sigma 1\sin\varphi)^2$ so the Mohr Coulomb yield surface is indicated by you know this surface which is you know indicated as gfedcb and a is the point of the intersection of the octahedral plane and you know the hydrostatic axis, this is the point through which, through point a the hydrostatic axis passes where $\sigma 1 = \sigma 2 = \sigma 3$.

And this is the point of intersection of the octahedral plane, so you can see that these are the point of intersection of the octahedral plane. So this is the octahedral plane and the Mohr Coulomb failure surface is actually defined or limited by this gfedcb surface so this is called as the Mohr Coulomb yield surface. So then you know different failure criterions are actually described.

So what we do is that in the next class we will discuss more about the octahedral plane and in the relevance to the you know how this can be interpreted for you know the so called understanding about the failure criterion and then there after we will try to discuss about you know how we can actually determine e for the e from the shear test data and then we will try to do some example problems on the shear strength of the soils.

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