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ADVANCED GEOTECHNICAL
ENGINEERING

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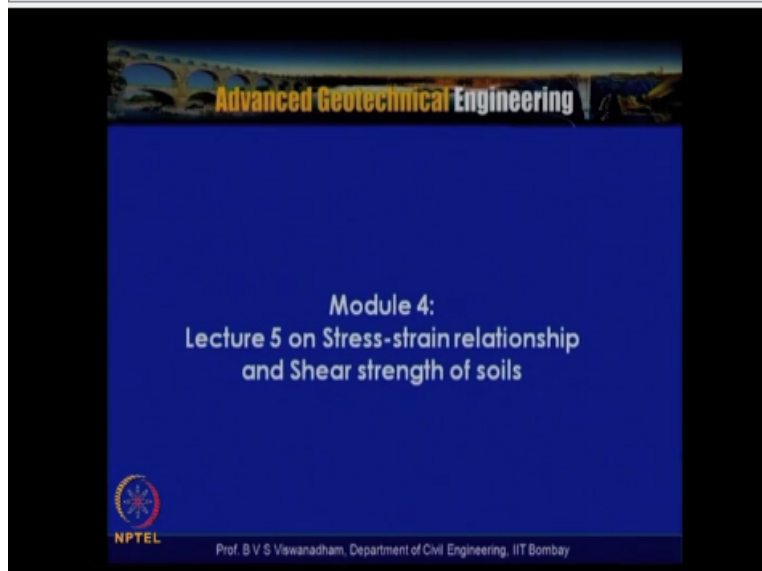
Lecture No. 34

Module – 4

Lecture – 5 on Stress – strain
relationship and Shear
strength of soils

Welcome to lecture series on advanced geotechnical engineering being produced by IIT Bombay.

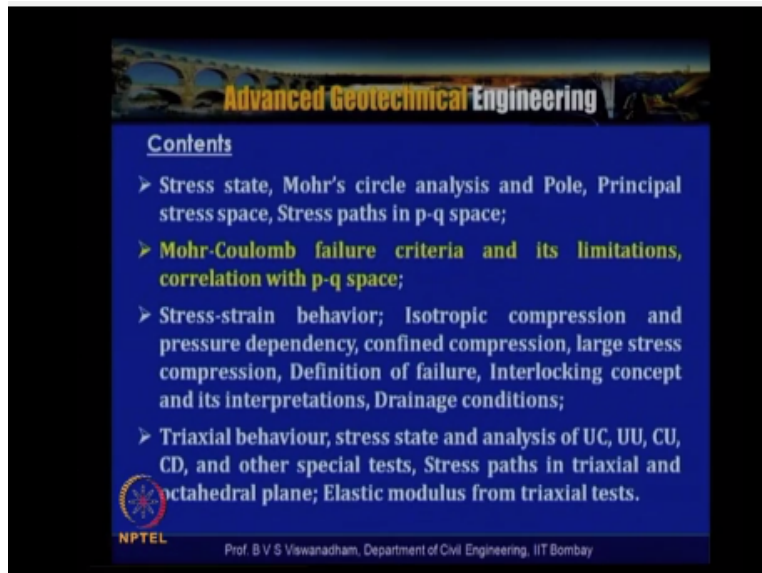
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So this is module 4 lecture 5 on stress strain relationship on Shear strength of soils. So in the previous lecture we try to understand about the different stress strain behavior of mild steel and also we try to see and try to understand strain hardening and strain softening and elasto plastic behaviors. So after having discussed about the stress strain relationships now we will introduce into the MORE Colombo criterion and we will discuss about the relevance to the how we can actually determine by knowing the state of stress on the element.

How actually we can determine the stresses at failure particularly the shear stress and normal stress along the failure plane. So this is the contents which are given.

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The slide features a dark blue background with a title banner at the top showing a bridge over water. The title "Advanced Geotechnical Engineering" is written in a bold, yellow font. Below the title, the word "Contents" is underlined in white. A list of topics is presented in white text with yellow arrowheads. At the bottom left is the NPTEL logo, and at the bottom center is the text "Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay".

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Contents

- Stress state, Mohr's circle analysis and Pole, Principal stress space, Stress paths in p-q space;
- **Mohr-Coulomb failure criteria and its limitations, correlation with p-q space;**
- Stress-strain behavior; Isotropic compression and pressure dependency, confined compression, large stress compression, Definition of failure, Interlocking concept and its interpretations, Drainage conditions;
- Triaxial behaviour, stress state and analysis of UC, UU, CU, CD, and other special tests, Stress paths in triaxial and octahedral plane; Elastic modulus from triaxial tests.

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Then which we are going to concentrate in this lecture which is more Colombo failure criteria and its limitations.

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Stress-strain relationships and Failure criteria

- The little hump in the stress-strain curve for mild steel after yield is an example of work hardening.
- Many soils are also work-hardening, for example, compacted clays and loose sands. Sensitive clay soils and dense sands are examples of work-softening materials.

➤ At what point on the stress-strain curve do we have failure?

➤ In some situations, if a material is stressed to its **yield point**, the strains or deflections are so large that for all practical purposes the material has failed.

➤ This means that the material cannot satisfactorily continue to carry the applied loads. The stress at "failure" is often very arbitrary, especially for nonlinear materials.

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So as we have seen in you know in the previous lecture, you know the sustain relationship of a mild steel you know as a little hump you know after it is proportional limit. So the little hump in the curve after a yield is an example of work hardening. And many soils also exhibit work hardening, for example compacted clays and loose sands when they are subjected to shear and they know exhibit the so work hardening phenomenon.

So sensitive clay soils and dense sands for example that means that after retaining certain peak values there is you know a set of decrease in the stress. So at what point on the stress-strain curve do we have failure, we have to understand. In some situation what will happen is that if the material stress is = yield point the strains or deflections are so large all practical purpose that the material has failed.

So this means that the material cannot satisfy continue to carry the applied loads often every arbitrary especially for non linear materials and particularly you know those materials undergoing like compacted clays and loose sands. We take arbitrary strain 10% or 15%.

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Stress-strain relationships and Failure criteria

- > With these materials, we usually define failure at some arbitrary percent strain, i.e. 15% or 20%, or at a strain or deformation at which the function of the structure might be impaired.
- > Now we can also define the strength of a material.
- > It is the maximum or yield stress or the stress at some strain which we have defined as "failure."
- > There are many ways of defining failure in materials; or put another way, there are many failure criteria.
- > Most of the criteria don't work for soils.

The most common failure criterion applied to soils is the **Mohr-Coulomb failure criterion.**


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So when these materials like you know which is going work hardily we usually define failure at some arbitrary % strain 15% or 20% or it is strain or deformation at which the function of the structure might be implied. So that we define the strain it can be all say 5%, 10% 15 or 20% as a strain or deformation at which the function of this structure you know affected. Now we can also define the strength of the material, it is the maximum and the yield stress or the stress at some strain which we called as failure.

So that is the strength of the material, so there are many ways of defining failure in materials or put in other way there may be many failures of criteria are there but most of the criteria's do not work for soils. So the most common you know failure criterion which can be applied to soils is the lower Colombo more hypnosis is been given and then the Colombo as come out with you know on the base on the experience with construction of the walls you know during his time.

He has come out with made attempts to determine the shear strength of the soils so then it turns out be the more Colombo failure criterion which is actually is wisely given today.

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Mohr-Coulomb Failure Criterion

Charles Augustin de COULOMB (1736-1806) is well known from his studies on friction, electrostatic attraction and repulsion.

Christian Otto MOHR (1835-1918) hypothesized (1900) a criterion of failure for real materials in which he stated that materials fail when the shear stress on the failure plane at failure reaches some unique function of the normal stress on that plane:

$$\tau_{ff} = f(\sigma_{ff})$$

where τ is the shear stress and σ is the normal stress.

The first subscript f refers to the plane on which the stress acts (in this case the failure plane) and the second f means "at failure." τ_{ff} is the shear stress along the failure plane at failure of the material.

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So the Charles Augustine is well known for his study on friction, electrostatic attraction and repulsion, so he has put forward you know contributed to the more Colombo criterion similarly OOTO of failure for the real materials in which he stated that the material fails when the shear strength on the failure reaches some unique function of the normal stress on the plane so hypnosis's states that you know the failure for the real materials in which he stated the real fail when the shear stress on the failure plane reaches some unique function of the normal stress on the plane.

So $\tau_{ff} = f(\sigma_{ff})$ now if you see this the 2 f which are there, two suffixes the t is the shear stress the σ is the normal stress. So the 1st f refers to the plane on which the stress acts in this case failure plane the stress acting towards the failure plane the 2nd f is nothing but the at failure. Suppose if you are actually trying to see t along the tf we write then it indicates that the shear strength along the failure plane may not be at failure.

The normal stress on the failure plane are the possibilities of the failure plane, so when we write the MORE states that the t is the function of σ_{ff} where the shear stress at along the failure plane τ_{ff} , σ_{ff} at failure. So you know these are which are actually put forwarded.
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Mohr-Coulomb Failure Criterion

- A failure theory is required to relate the available strength of a soil as a function of measurable properties and the imposed stress conditions.
- The Mohr-Coulomb failure criterion is commonly used to describe the strength of soils.
- Its main hypothesis is based on the premise that a combination of normal and shear stresses creates a more critical limiting state than would be found if only the major principal stress or maximum shear stress were to be considered individually.

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And it is basically required to relate the function of soil measurable properties and their imposed stress conditions, so the MORE Colombo is commonly used to describe the strength of the soils and it is main hypnosis is based on the premise that a combination of normal creates a more critical limiting stay, maximum shear stress. So it is something like a combination of you knows the normal stress and shear stress along the failure plane are considered.

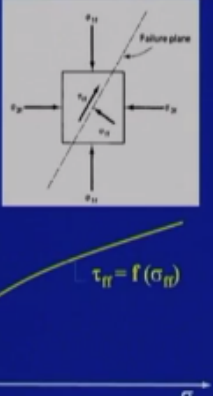
Rather than you know the major principle stress or maximum shear stress what need to consider individually, it is required to relate the strength of a soil as a function of soil measurable properties and their imposed stress conditions, so the MORE Colombo is commonly used to describe the strength of the soils and it is main hypnosis is based on the premise that a combination of normal creates a more critical limiting stay then found if only the major principle stress or you know the maximum shear stress what need to consider individually. So here in this particular slide an element which is actually subjected to.

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Mohr-Coulomb Failure Criterion

- An element at failure with the principal stresses that caused failure and the resulting normal and shear stresses on the failure plane.
- We will assume that a failure plane exists, which is not a bad assumption for soils, rocks, and many other materials.
- If we know the principal stresses at failure, we can draw a Mohr circle to represent this state of stress for this particular element.



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You know major principle stress that is σ_1 that is σ_3 is shown and the failure claim which is you know it incline let us say here α and this is the σ_{ff} is the failure claim, normal stress along failure, τ_{ff} is nothing but you know so according to MORE it is a non linear where the τ σ is ringing for the ranges of the normal stresses in a non linear way. So you can see that this envelope actually extends like this, this is the equation for $\tau_{ff} = f(\sigma_{ff})$, so these stresses are acting here this is τ_{ff} and this is the σ_{ff} normal stress.

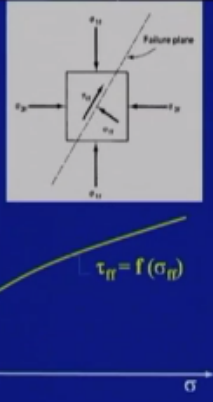
So if you see this element at failure with the principles stresses which causes failure in the resulting normal stresses and shear stress on the failure plane. So we will assume that the failure plane exist which is not bad assumption for soils and rocks and many other. So this plane basically if there are no you know stresses along the major principle then there is the possibility that you can actually have.

We will have failure planes in both the direction, so if you know that the principles stresses failure we can actually draw a more circle to represent this state of stress in this particular element. So with that we will be able to get the 10000 material at failure. So in this particular slide what we have shown is that a more hypnosis.
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Mohr-Coulomb Failure Criterion

- An element at failure with the principal stresses that caused failure and the resulting normal and shear stresses on the failure plane.
- We will assume that a failure plane exists, which is not a bad assumption for soils, rocks, and many other materials.
- If we know the principal stresses at failure, we can draw a Mohr circle to represent this state of stress for this particular element.



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The envelope is non linear and which actually indicates $\tau_{ff} = f(\sigma_{ff})$ and this is actually shown here and this is the failure plane and which is the assumed for it exist you know for the soils and rock and many other materials.

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Mohr-Coulomb Failure Criterion

→ Mohr hypothesis: the MOHR-COULOMB FAILURE CRITERION failure point of tangency defines the angle of the failure plane in the element or test specimen.

→ The Mohr failure hypothesis is illustrated for the element at failure shown.

→ Stated another way: the Mohr failure hypothesis states that the point of tangency of the Mohr failure envelope with the Mohr circle at failure determines the inclination of the failure plane.

The diagram illustrates the Mohr-Coulomb failure criterion. It shows a Mohr circle on a σ - τ plot. The failure envelope is a straight line passing through the origin. The point of tangency between the Mohr circle and the failure envelope is labeled 'Point of tangency'. The failure plane is shown as a line passing through the pole of the Mohr circle and the point of tangency. The angle of the failure plane is indicated as α . The principal stresses σ_1 and σ_3 are also shown. Below the Mohr circle, a stress element is shown with normal stresses σ_1 and σ_3 and shear stresses τ . The failure plane is shown as a line passing through the element at an angle α to the horizontal.

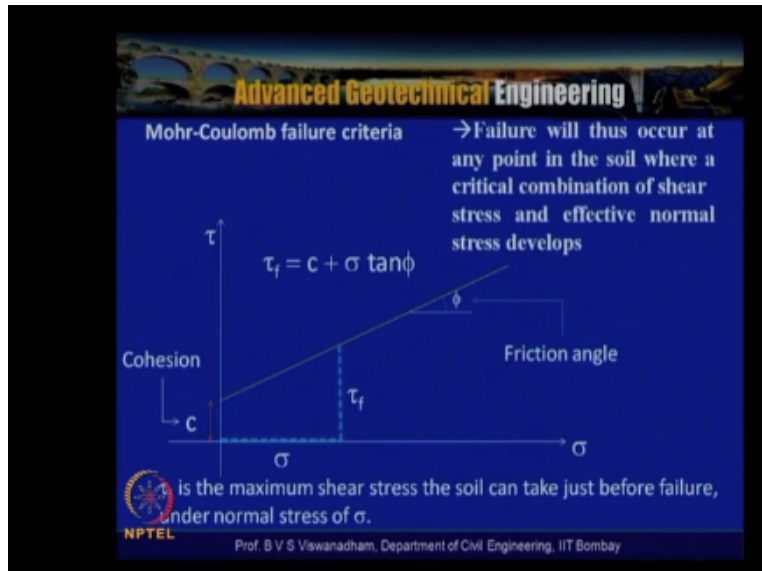
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Now if you look into this slide as we told that if you more circle though we are actually indicating upper half of the more circle for the convince but if the material is actually is having you know the homogeneity then this kind of planes are positive, the failure plane one can claim like this other bottom of the circle it can have like this, so this is the which is appear like this, so the more Colombo failure point of tangency defines the angel of failure plane.

So the way this is the major principle stress and then when you draw this is the pole from here when you draw a line where the bore is in contact with the you know the more circle here and here and this inclination indicates the angel of failure plane. So this is you can see that along this these are the stresses which are τ_{ff} and σ_{ff} and these are actually because of the major principle and minor σ_3 .

And when we have you know the homogeneity then there is possibility that you know you also have in this direction, so this is actually parallel to this one. So the more is instructed for element for failure is actually shown in this particular figure and the more failure states that point of tenancy of the more failure determines the inclination of the plane. So in another way more hypnosis can be stated the points of the tenancy of more failure envelope with the more circle at failure determines the angel of inclination of the failure plane.

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So in this you know the combination of now, you know the coulomb what he has said that for that he said that there are two types of parameters are there one is stress depend parameter and other one stress independent parameter and these stress dependent parameter is nothing but some friction between the grains and stress independent parameter you know the equation essentially interaction develops between the soil grains.

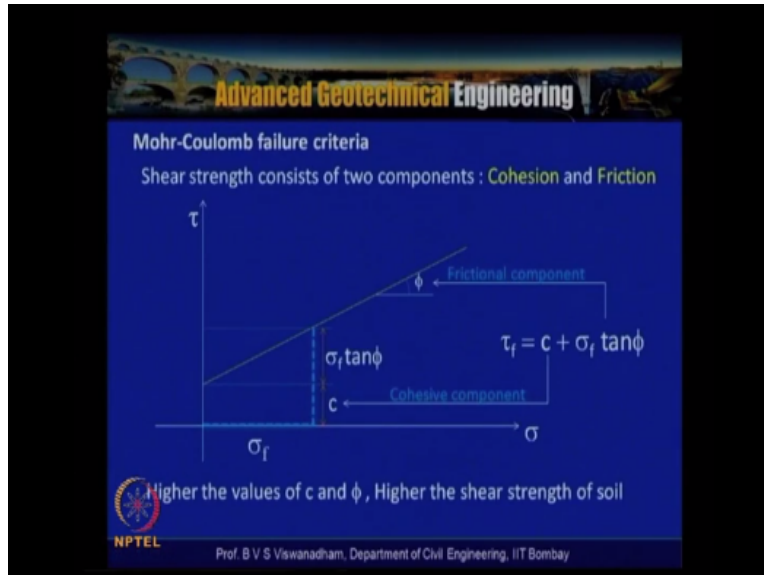
So this actually when it is combined with more and Column if you know interacted has a linear relationship within the stress range which beyond that higher stresses the validity of this more column this thing cannot be guaranteed but further the you know definition belevority whatever he assumed is actually valid for the you know the assume stress range and this tau σ plot here this indicates that you have to a combination σ and such a way that these stress failure. $\tau_f = c + \sigma \tan \phi$.

So if this happens to failure stress then it will be $\tau_f = c + \sigma \tan \phi$ if the failure will does will occur at point in the soil were a critical combination of stress and effective normal stress develops so basically the failure would actually develop at any point in the soil where a critical combination of shear stress and effective normal stresses you know develops so τ_f is the agnostics the soil can take just before failure.

And the loner stress of σ , σ so the equation which is popular is actually is $\tau = c + \sigma \tan \phi$ which is like $y = mx + c$ where are the intercept here c is cohesion intercept and the angle of this Mohr-coulomb failure and allow is the angle of internal friction that is the friction angle you can can be

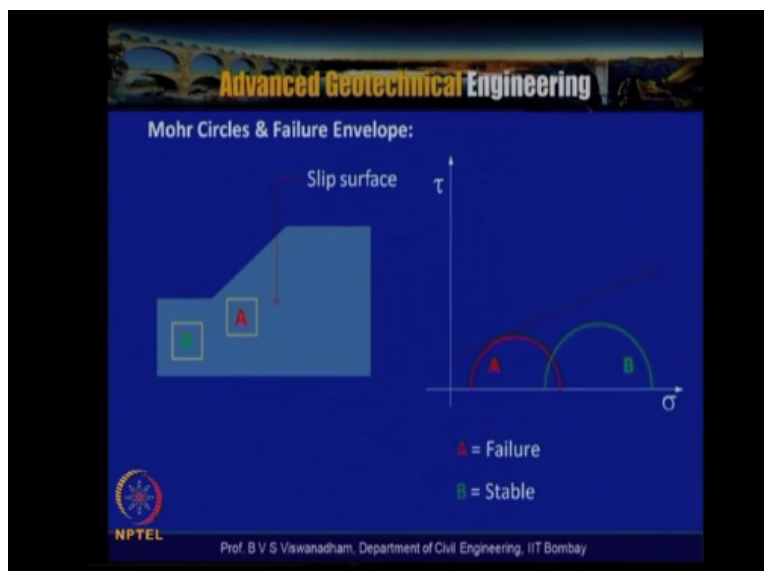
seen here so when you have the combination of you know τ and σ the failure will occur at any point in the soil where a critical combination. Shear stress effective normal stress develops

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So further it has been shown here the same with cohesion components which is the shear strength contributed with two components one is the cohesion component and other one is friction component so we have $\tau = c + \sigma \tan \phi$ another higher value of c and ϕ the higher the soil so if you have which actually can develop good cohesion good friction when the vertical actually is have higher resistance.

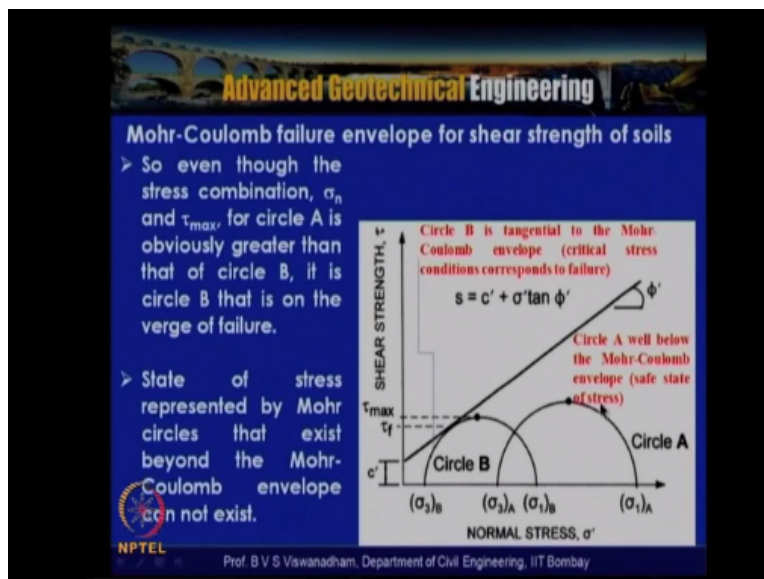
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So let us consider some typical example where more circles and failure envelopes how you can actually construct so let us see if you look into here we have two Mohr circles return one Mohr circle a and circle b and there is a line which is you know the Mohr coulomb failure envelope which is indicated has straight line and this indicates that the circle b is stable though it actually has got the stresses more than you know the combination is such that.

The circle a is actually failure and you can see that possibility that circle a can be located here that element here and circle b is somewhere here so it is indicates that envelope slope if you consider this can be a depth so where you have the possibility of the failure here and the circle b is location ever from the you can see that this is can be under a stable condition.

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Further discussed here where we have we show here the strength tau and normal stress on the x axis and failure plane, failure envelope which the c-and ϕ - and these are the effective strength parameter which are actually called and we have a circle a and circle b and circle a is well below the you know Mohr coulomb failure envelope and also it is said that the combination of

stresses which are major principal σ_1 and σ_3 is such that you know it is in the safe state of stress but you look into the circle b you know it is tangential to the Mohr coulomb envelope and it is actually having a point of you know this point of coherence here and when you draw the line this is the angle of inclination of the failure plane and you can see that these is the Mohr's.

And this is the Mohr's that stress is at failure so if you look into this here even though the stress combination you know σ_n and Mohr's of circle A are obviously greater than the circle b it is actually circle B that is on the verge of failure so if you look into that circle a and circle b even though the stress combinations of σ_n and Mohr's of this circle A is actually you know greater than the that of circle B it is the circle B that is on the edge of failure.

So circle A is on the safe state of stress so the state of stress represented by the more circles that exist beyond the you know Mohr's envelope you cannot exist so that means that if you are having a Mohr's you know the Mohr's above the failure envelope that means that it indicates that the failure would have already take place so if you are having a you know Mohr's circle which is well below the failure envelope then it is said that it is in the safe state of stress the Mohr's is stable.

And when it is in when it comes in contact with the Mohr's envelope there is the possibility that the failure at times so that is at failure that is the Mohr's circle at failure and then there is no possibility of existence of Mohr's circle above the failure envelope because by the time already failure would have taken place so here we actually define the factor of safety which is nothing but τ_{ff} that is available and divided by these shear stress which is actually applied.

So the shear stress can be due to the driving stresses either because of the sulphate loading or because of seepage or due to you know certain or due to some external loading so the factor of safety is τ_{ff} available to τ_f applied so if the stress increase so that the failure occurs then the Mohr's circle becomes tangent to the Mohr's failure envelope that is what we have seen with the circle B the combination is such that actually regarded as the failure circle so according Mohr failure, failure occurs on the plane inclined that α and when the shear stress on the plane is τ_f so this is not the largest or the stress in the element.

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Mohr-Coulomb failure envelope for shear strength of soils

Factor of safety = $\frac{\tau_r(\text{available})}{\tau_r(\text{applied})}$

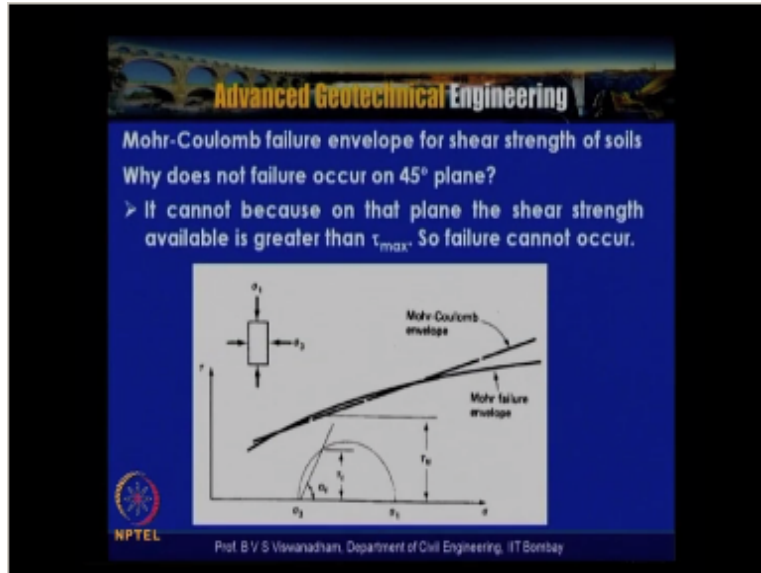
- > Now, if the stresses increase so that failure occurs, then the Mohr circle becomes tangent to the Mohr failure envelope.
- > According to the Mohr failure hypothesis, failure occurs on the plane inclined at α_f and with shear stress that plane of τ_{ff} .
- > This is not the largest or maximum shear stress in the element!!!
- > The maximum shear stress acts on the plane inclined at 45° and is equal to: $\tau_{\max} = \frac{\sigma_{1f} - \sigma_{3f}}{2} > \tau_{ff}$

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So we can see that in the circle B here the stress if you look into it that is actually is more than the failure stress but the combination is such that you know where you have got normal stress and τ_f is such that it actually you know becomes tangential to the Mohr circle and then it is regarded as the.

So this is according to the hypothesis where if the stress increase so that the failure occurs and then the Mohr circles becomes tangent to the Mohr failure envelope so this is not the largest or stress in the element so the τ_f actually less than the τ_{\max} but the stress if you look into the τ_{\max} on the plane incline that $45^\circ = \tau_{\max} = \frac{\sigma_{1f} - \sigma_{3f}}{2}$.

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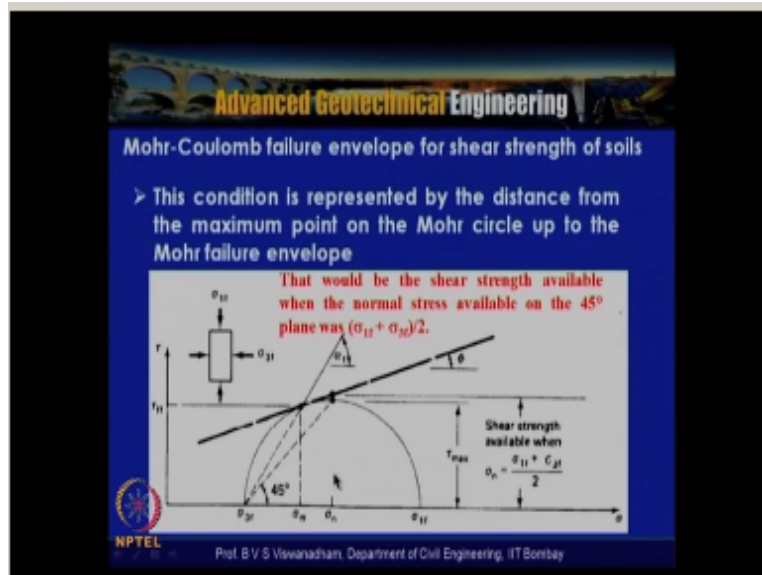
So if you look into this when we take you know at this point this is $\sigma_1 - \sigma_3 / 2$ and then it is acting on the plane which is inclined at 45 degrees here see it is not on the plane so the stress acts on the plane inclined at 45 degrees and it is equal to $\sigma_n f - \sigma_3 f / 2$ which is actually greater than τ_{ff} which we are actually have just now discussed now let us you question might come is that why you know does not failure occur on 45 degrees plane.

So when it is actually chased is maximum along that failure plane why not actually the failure is actually unique the failure begin why it is not common it cannot because on that plane the shear stress the availability is greater than the τ_{max} because the shear strength the available is if you look into it here when you see here if you see the combination of this $\sigma + \tau_{max}$ the shear virtually available is actually more than you know τ is on this 45 degrees plane.

The shear stress is actually maximum the shear strength available to avoid the failure is actually you know greater than you know τ_{max} so because of that you know what it implies is that the failure cannot actually occur on the 45 degrees plane so here we have shown more cooled envelope which is a linear version which is actually assumed in the more failure envelope is you know you can see the non linear variation here.

Now if you look into you have got a circle which is as a major principle stress σ_1 and σ_3 and this is you know the τ_f and this is circle is recorded as stable because the τ_{ff} you know is here this vertical orbit is τ_{ff} the shear stress which is actually there is only τ_f .

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Now let us consider you know if you are having a you know like more circle where this is the envelope which is like this and then we are having the maximum shear stress which actually acts on the 45 degrees plane so this is maximum shear stress and failure is actually occurring on that plane which is actually σ_f which is actually different from the 45 degrees maximum shear stress then we discussed that the combination is such that.

The shear stress is actually available you know on you know this plane is actually more than water actually you know this shear stress is so this condition is actually represented by the distance from the maximum point of the more circle up to the more failure envelope or more column failure so this would be the shear stress available when the normal stress available on the 45 degrees plane was $\sigma_1 + \sigma_3 / 2$

So this is the shear strength available when $\sigma_n = \sigma_1 + \sigma_3 / 2$ that is of the strength that is at this point so available is actually here what you can be seen is that point is actually the shear stress available and the maximum shear stress on the 45 degrees plan is τ_{max} here so because this ordinate vertical ordinate is actually if new look into this on the Mohr circle τ σ plot it is actually more than the τ_{max} so it also explains you know why you know the failure plan will not be on the 45 degrees plan.

But there are some exceptional cases when you are actually having saturated and untrained conditions for a given soil where this is then possibility that the failure plan occurs actually on

the plane of maximum shear stress where that is possible for when you are actually doing you know attempts to make determine the stress in the laboratory where you actually are you have a condition where you have saturated conditions then the particularly for either for case or front this conditions provide.

And there is the possibility that you actually we have the you know occurrence of failure plane at 45 degrees but with these two you know discussion what we have so this is you a stable circle but when the circle actually you know comes in contact with the more coolant failure analog then that is the point this pointy from σ_3 to this it is actually the angle of the failure of the plane what we are calling so this is the angle of failure plane.

So the question is that when we have this shear stress maximum shear stress on 45 degrees plane why not actually the failure plane you know occurs along the you know the plane of maximum shear stress so for that what we actually trying to explain if you are having a given state of stress which are actually shown here from the graphically it is actually shown here you can see that this is you know.

The shear stress available when the normal stress that is at the center here so this is 0 to this is $\frac{\sigma_1 + \sigma_3}{2}$ so that becomes $\sigma_n = \frac{\sigma_1 + \sigma_3}{2}$ so when this actually combination is actually there so you know what we have through attain failure is you know this much should be there but what we have is only τ_{max} so because of that you know the failure cannot actually takes place on the so called plane of maximum shear stress.

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Mohr-Coulomb failure envelope for shear strength of soils

- The only exception would be when shear strength is independent of normal stress, i.e., when Mohr failure envelope is horizontal and $\phi = 0$.
- Such materials are called purely cohesive for obvious reasons or this may result in completely saturated and un-drained conditions.

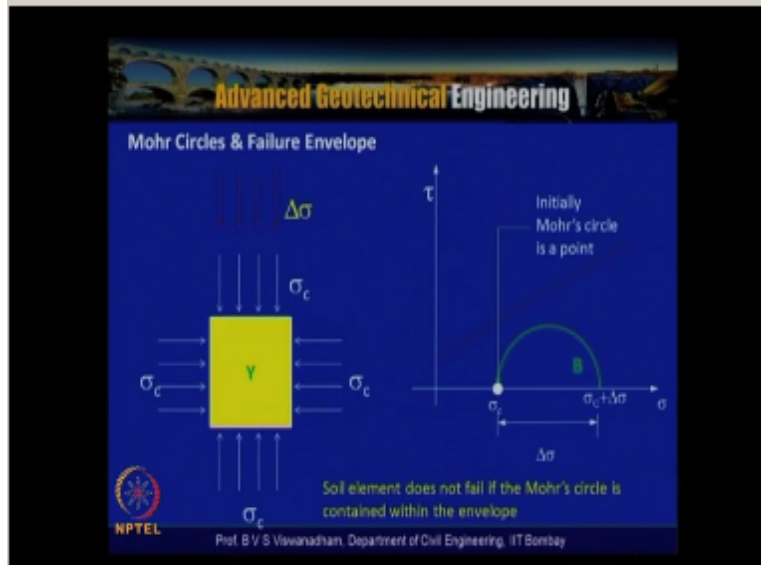
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But it has been told that the only exception would be when shear strength is independent of normal stress that means that when the Mohr circle Mohr failure envelope horizontal and $\phi = 0$ that means that when you have $\tau = c$ that means that when you have you know sometimes you have got an element such a way that the lateral that is the minor principle stress is 0 that is $\sigma_3 = 0$ and.

Then you know this is the you know the stress at failure in that case also you can see that when this is the point then this will be the Mohr coulomb failure envelope which indicates that angle of inclination of the Mohr column failure envelope is 0 and at this intercept is nothing but c so which is nothing but $\tau = c$ and this point which is you know is a failure plan which is actually you know $\sigma_f = 45$ degrees here.

So such materials are called qt quotient or obvious reason or this may result in completely saturated untrained conditions so this materials are called as soils are cure clay soils when they exhibit these types these things then it is actually you know results in you know the failure plan occurs along the plan of maximum shear stress.

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So now consider a element which is actually subjected to the state of state which is actually shown here σ_c is the confining pressure initial confining pressure let us assume that we have vertical and horizontal direction these can be you know achieved by pressurizing water and so that we will be able to get the identical pressures on the principle stresses and that is principle planes both horizontal and vertical and.

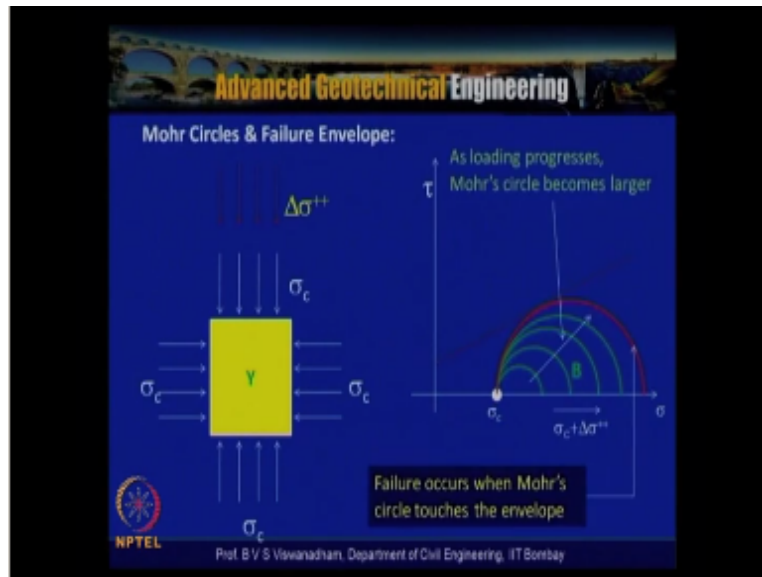
So initially when we have applied the σ_c so initially the Mohr circle is actually is a point then with that pressure when we try to increase $\Delta\sigma$ let us say that so this is actually possible when we have you know a certain element at a certain ground when what will happen is that when is that element is actually subjected to certain increase in pressure due to foundation loading or in a construction.

There is increase in stress if that increase in stress is such a way that these state of the stress in the element is such a way that the more circle actually becomes tangential to the Mohr column failure envelope and there is a determination of failure in failure and then that combination actually give the shear strength at failure so if that situation actually happens in the field either from the stability of this slopes in dams or in the pressure problems the failure state is actually attained.

So here a typical stress element is shown here and with increase in the lope which is $\sigma_c + \Delta\sigma$ the Mohr stile where here and this state is such that it is still stable but when the $\sigma_c + \Delta\sigma$ is increased to such an extent that when it becomes tangential to the Mohr column failure envelope then the

soil element tends to the failure so the soil element does not failure if the Mohr circle is actually contained within the envelope that is that this is well below the envelope than the you know the failure is not at attained in the sample.

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Now let us assume that the Mohr circle actually Mohr circle travels because of the increase in $\sigma_0 + \Delta\sigma$ so $\sigma_c + \Delta\sigma$ σ_c is initial applied then $\Delta\sigma$ is actually increased continuously then the circle migrates towards right with you know what we are doing is that we are actually maintaining same σ_c and then started increasing so with that what will happen is that you reach a certain straight you know in the element in the failure plain where it actually attains τ_{ff} that is this point.

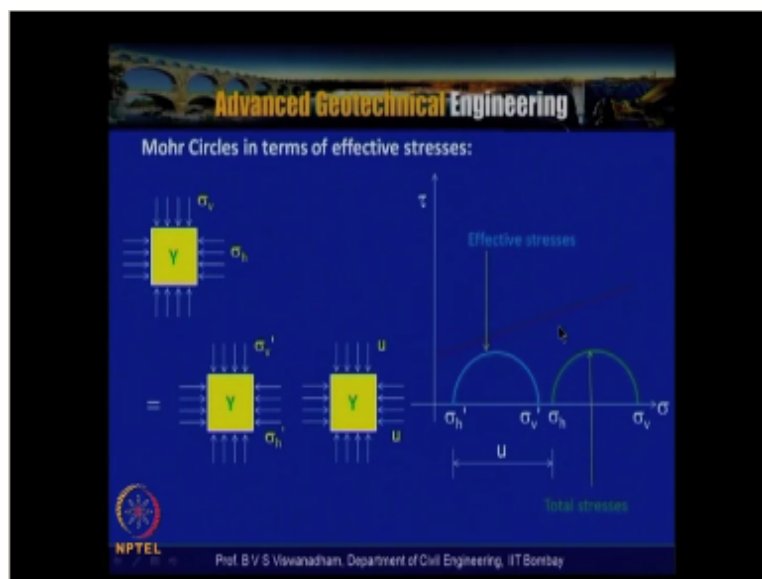
And the where ever actually they become tangential so that is actually is α_f that is you know inclination of failure plane where the you know the shear stress failure is actually acting this is the stress at failure plane and it is actually on this plane so failure actually occurs when more circle touches the envelope so the circle travels so initially when we have when you prepare the sample and apply σ_c although out if the Mohr circle condition is nothing.

But a point on the $\tau\sigma$ envelope and then once you continue to increase the you know the vertical stress that is $\Delta\sigma^+$ then there is a possibility that as a loading progresses the Mohr circle becomes larger and larger now so if you further look into and take the circle at failure let us say that $\sigma_c + \Delta\sigma$ combination that is at the end failure and the σ_c automatically it becomes at failure for the given state of stress.

So this is the loading plan orientation this is actually acting on the vertical plan that this parallel to this one parallel to this one this is perpendicular so it is actually acting on this so this is parallel to this plane and parallel to this plane is this one on which actually this stresses are acting.

So if there are no shear stress definitely they are eligible to be called as you know the principle planes major principle plane and minor principle plane and the failure plane is actually oriented at $45 + \frac{\phi}{2}$ with the horizontal so that is the you know inclination from the pole here when we draw that is actually $45 + \frac{\phi}{2}$ and the plane of maximum shear stress is accurate 45 degrees.

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So in terms of effective stresses this we have discussed already but to instead further we have got let us say vertical stress and horizontal stress are acting on a sample σ_h and σ_v and when we have the pore water pressure if is when we have is subtracted from the total stresses and we have

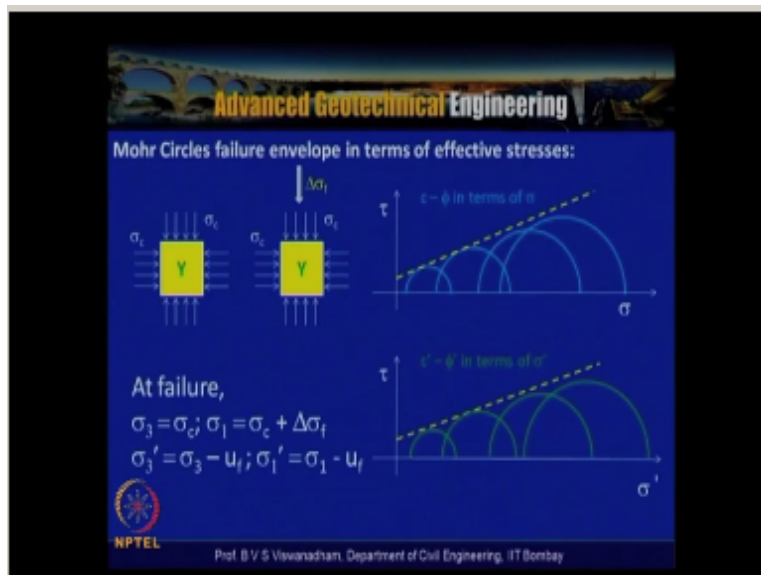
got effective stresses in vertical direction and horizontal direction and the pore water pressure acting in all directions.

So for this you know when you have got you know the total stresses the Mohr circle actually shifts by difference actually that is u so $\sigma_h = \sigma_v - u$ so by that you know the circle actually becomes shift and so if you look into this here we actually have failure envelopes for effective stress conditions and you know in total stress conditions they are different so they can actually yield different combinations of.

So if you are actually having under total stress conditions if you are actually getting a strength called parameter which is actually in a stress in depend parameter c and you know stress dependent parameter friction and then it is called c and ϕ and if you are actually doing with untrained conditions then it is called as c_u and ϕ_u c suffix u .

And u indicates undrawn ϕ indicates the angle of internal section or a friction angle and untrained conditions so but when we have you know the effective stresses conditions then it is actually said as c_1 and the ϕ_1 where these are called effective cohesion and you know effective friction angle or effective angle of internal friction.

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So these are actually shown here in order to get these parameters either in terms of total stress parameters or in terms of effective stress parameters we need to have different combinations of you know the stress conditions need to be applied to the soil because with one circle we will not

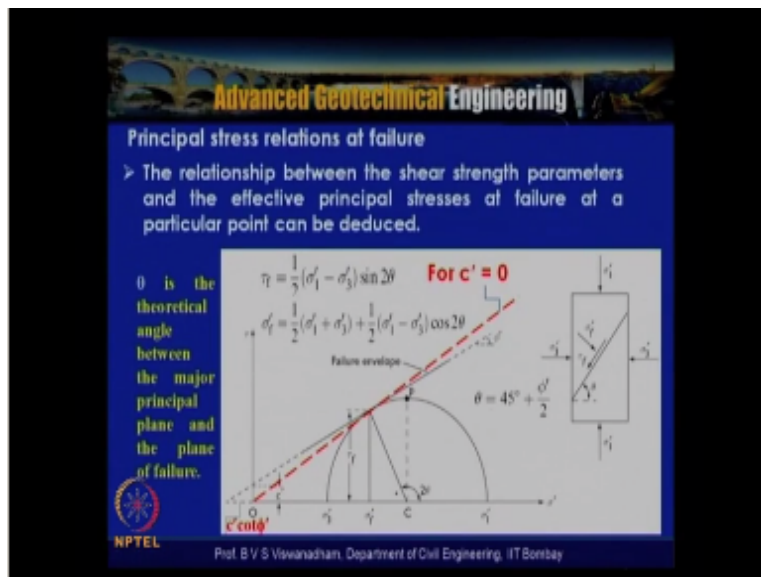
able to you know certain you know for a given element so in that case what we need to do is that we have to do it at a different combinations here in order to build up.

This Mohr column failure envelope either in total stress or in effective stress conditions what is exactly we have to do is that σ_{31}, σ_{11} and then we have to do in on σ_{32} and σ_{12} and then we can actually do it on σ_{33} and σ_{13} and the other one for combination is that σ_{34} and σ_{14} so when we put together you know when the circles actually attain these are the when there are actually obtained as at failure.

Then you know we draw the envelope you know tangential to the these all this Mohr circles and that envelope is actually constructed where we can actually yield to the inclination of this envelope is actually lead to give friction angle and then this intercept actually gives you know the cohesive intercept but in for some family consolidated soils and you know sands when you are actually having particularly loose sands under drainages conditions.

There is a possibilities that the $c=0$ then the envelope is actually you know is passing through the horizon so in terms of σ_1 so the same when you measure the water pressure for pore water pressure and when you subtract this from the total stress then we will actually get effective stress circles so when you have the combination of this Mohr circles and then can lead to construction of a failure analog here which is shown here.

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So then you know we can actually further use this knowledge and try to get the principle stress relations at failure and these are actually worked out by using this you know Mohr circle which is actually shown at failure where we have got τ versus σ and this is the element which has been subjected to σ_1 on the major principle plane and σ_3 on the minor principle plane and the failure surface is actually assumed to be inclined at θ or the another notation which you are following these α_f where f is the failure plane and then this is the normal stress acting on the failure plane that is σ_{1f} and τ_f is the principle stress acting the shear stress acting on the failure plane.

So when we have this Mohr circle let us say that when we have got certain position intercepts v_1 and the angle of internal section ϕ_1 that is the effective stress Mohr circle and so the combination is that σ_1 which is nothing but $\sigma_3 - u$ when we that we have got σ_3 and here $\sigma_1 - u$ that is σ_{11} so here you know this point is the point of tangent c and.

So we can write from the by following the geometry $\tau_f = \text{major}$ you know shear stress which is actually is $\frac{\sigma_1 - \sigma_3}{2}$ so we can write $\tau_f = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$ and $\sigma_{1f} = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$ so we have combinations of σ_f can be you know at failure so this τ_f is actually given by his audience vertical and σ_f is you know.

This is nothing but $\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}$ so by using the geometry of the from here we can actually get the τ_f and σ_{1f} and if you are consulting here c_1 as this vertical intercept and with the inclination of ϕ_1 this is small horizontal incepted is actually called as c_1 caught ϕ_1 so θ is the theoretical angle between the major principle plane and the plane of failure.

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Principal stress relations at failure

Now
$$\sin \phi' = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c' \cot \phi' + \frac{1}{2}(\sigma_1' + \sigma_3')}$$

Therefore
$$(\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3') \sin \phi' + 2c' \cos \phi'$$

➤ The following equation is referred to as the Mohr-Coulomb failure criterion:

$$\sigma_1' = \sigma_3' \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) + 2c' \tan \left(45^\circ + \frac{\phi'}{2} \right)$$

With $c' = 0 \rightarrow$

$$\sigma_1' = \sigma_3' \tan^2 \left(45^\circ + \frac{\phi'}{2} \right)$$

In the special case, when $\phi = 0$:
$$(\sigma_1 - \sigma_3) = 2c$$

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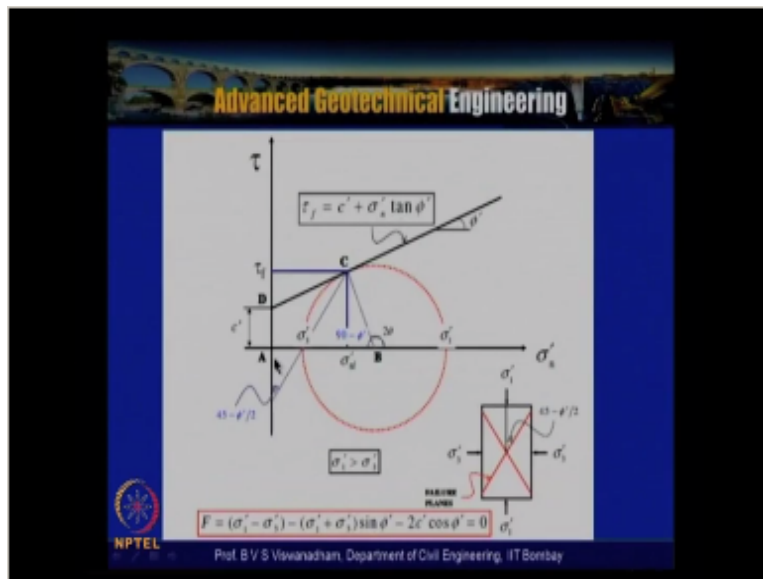
So from the you know the geometry we can actually extend from here that $\sin \phi$ you know from this here $\sin \phi$ can be written as $\frac{1/2(\sigma_1 - \sigma_3)}{c + \cos \phi (\sigma_1 + \sigma_3)/2}$ so that is $c + \phi + 1/2(\sigma_1 + \sigma_3)$ because this is $\sigma_3 + \sigma_3/2$ with that we will be able to get this horizontal ordinate so now $\sin \phi = \frac{1/2(\sigma_1 - \sigma_3)}{c + \cos \phi (\sigma_1 + \sigma_3)/2}$ so $\sin \phi = \frac{1/2(\sigma_1 - \sigma_3)}{c + \cos \phi (\sigma_1 + \sigma_3)/2}$.

So when we simplify further this is reduced into $\sigma_1 - \sigma_3 = \sigma_1 + \sigma_3 \sin \phi + 2c \cos \phi$ that is simple by cross multiplication we have got the flowing equation you know is referred to as the Mohr column you know failure criteria and which actually gets simplified to $\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2) + 2c \tan (45 + \phi/2)$ which is also called as $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$ and where α is the angle of intimation of the failure to plane and this is also called as balance equation $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$ or in terms of you know we also indicate by n_s
 So in that case you know if you write $n \phi = \tan (45 + \phi/2)$ you can write $\sigma_1 = \sigma_3 n \phi + 2c / n \phi$ so when we have let us say that $c = 0$ then in that case $\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2)$ so that indicates that the envelope runs like this when you have $c = 0$ this component is actually equal to zero so with that what we get is that you know $\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$ so this is actually also when $c = 0$ and these identity is also varied for also soils.

But in the special case when $\phi = 0$ when ϕ actually becomes 0 then what we say is that $\sigma_1 - \sigma_3 = 2c$ so when $\phi = 0$ you know then we indicate that this is actually $\sigma_1 - \sigma_3 = 2c$ so what we have said is that for in case of soil $\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2)$ but when we are having you know the general equation which is actually defined as the Mohr column failure ingredient referred as the Mohr column

failure as $\sigma_1 = \sigma_3 \tan^2 45 + \frac{5}{2}$ and $2c \tan 45 + \frac{5}{2}$ and this is also indicated as $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$ where α is angle of inclination in the failure plane.

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So here further this is actually indicated here where in we actually have got you know the failure plan which is actually inclined with you know the conjugate plain also shown here this is the conjugate failure plain other when the samples actually homogenous as it hole this is actually is possible and so you actually have the failure planes which are you know which can actually have inclination $45 + \frac{\phi}{2}$ and $45 - \frac{\phi}{2}$.

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Principal stress relations at failure

The essential points are:

1. Coupling Mohr's circle with Coulomb's frictional law allows us to define shear failure based on the stress state of the soil.
2. The Mohr-Coulomb criterion is: $\sin \phi' = \frac{\frac{1}{2}(\sigma'_1 - \sigma'_3)}{\frac{1}{2}(\sigma'_1 + \sigma'_3)}$

$$\frac{(\sigma'_1)_f}{(\sigma'_3)_f} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2(45 + \phi'/2)$$

$$\frac{(\sigma'_3)_f}{(\sigma'_1)_f} = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2(45 - \phi'/2)$$
3. Failure occurs, according to the Mohr-Coulomb criterion, when the soil attains the maximum effective stress obliquely, $(\frac{\sigma'_1}{\sigma'_3})_{max}$.

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So the important points which actually have cropped up from the discussion are that the coupling Mohr circle with Coulomb's frictional law allows us to define the shear failure based on the stress state of the soils so that is what we call as the Mohr-Coulomb failure will be a criteria so by coupling Mohr circle with Coulomb's frictional law we are able to define this shear failure based on the stress state of the soil so this stress state of the soil in level can be raised due to you know.

The how we actually apply the loading so the Mohr-Coulomb criterion which is you know for a let us say for a soil when it comes out with that this is $\sin \phi' = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}$ so this can be simplified so we get the relationship that is oblique relationships which are actually $\sigma'_1 = (1 + \sin \phi') \sigma'_3$ so these from the diametric identities when we use this is nothing but $\tan^2(45 + \phi'/2)$ and similarly when we have $\sigma'_3 = \sigma'_1$ then it is $1 - \sin \phi' / 1 + \sin \phi'$ where in we have $\tan^2(45 - \phi'/2)$.

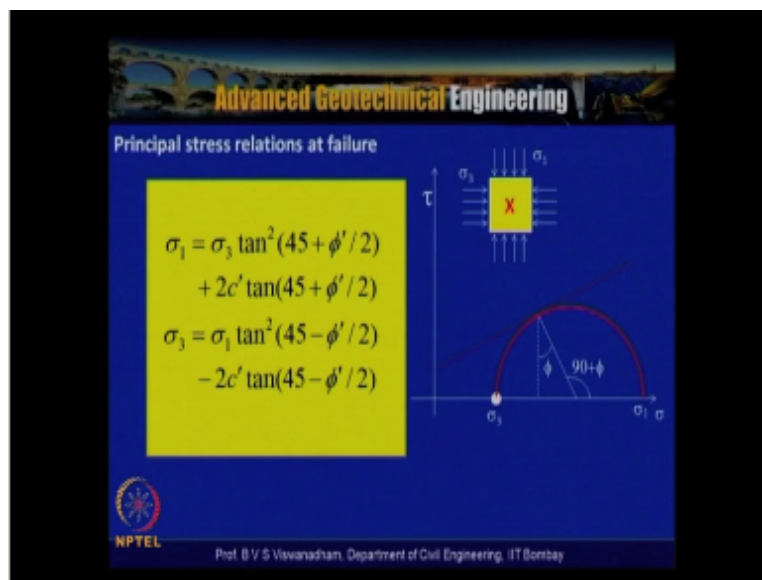
So failure occurs according to the Mohr-Coulomb criteria when the soil attains the maximum effective stress obligated that is $\sigma'_1 + \sigma'_3$ when it becomes maximum then the failure occurs so failure occurs according to Mohr-Coulomb criteria when the soil attains the maximum effective stress only to the ratio has to be maximum that is σ'_1 / σ'_3 as to be maximum so what we understood from the discussion is that coupling Mohr circle with Coulomb's frictional law allows us to define the shear failure based on the stress.

Based on the soil then we have also given come of the relationship which are actually valid for granular soils or a frictional soils is $\sigma'_1 / \sigma'_3 = 1 + \sin \phi' / 1 - \sin \phi' = \tan^2 \phi'$ and σ'_3 sometimes what will have

is that we have let us say that when we have a dry earth pressure condition then the wall is actually moving towards the back fill with a constant vertical stress then there can be horizontal stress that is lateral stress is actually more than you know.

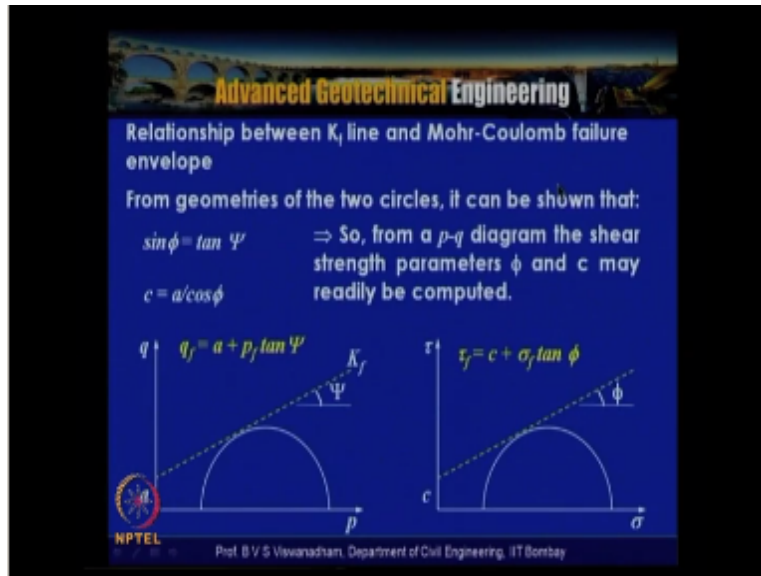
That is you know more than the σ_1 so in that case σ_3/σ_1 so where that identity yield is that $1 - \sin\phi + 1 + \sin\phi$ and which is actually is $\tan^2 45 - \phi/2$ and this is what actually has been indicated here when you have a combination such that you know you actually have failure plane which is actually the such a way that you know we get the so called you know $\sigma_3/\sigma_1 = \tan^2 45 - \phi/2$.

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So the principle stress relations at failure is further given here that is for case where $\sigma_1 > \sigma_3$ that is $\sigma_1 = \sigma_3 \tan^2 45 + \pi/2 + 2c \tan 45 + \pi/2$ and when we have situation where $\sigma_3 = \sigma_1 \tan^2 45 - \pi/2 - 2c \tan 45 - \pi/2$ so this is in case when you are having Mohr c and friction angle.

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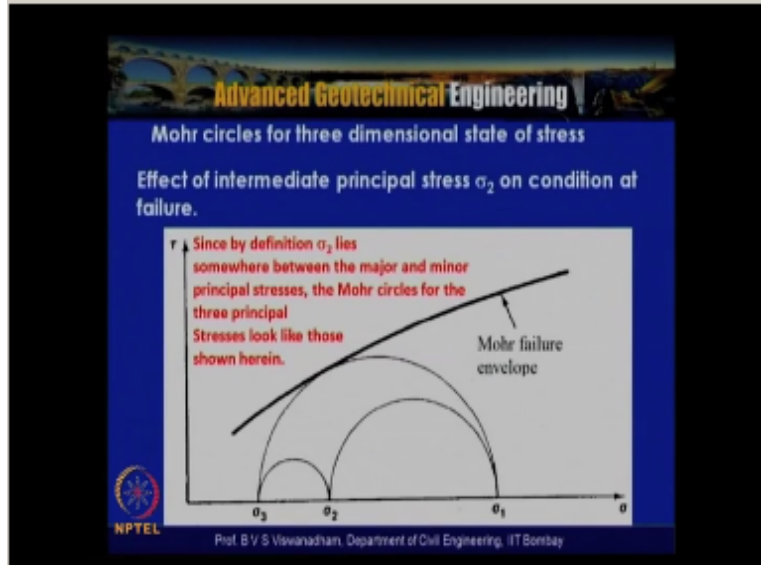


So after having obtained this then we can also link this Mohr column failure envelope with the with you know p - q space and this we actually have discussed in our previous lectures and where p is actually indicated as $\sigma_1 + \sigma_3 / 2$ where $q = \sigma_1 - \sigma_3 / 2$ when we have that you know when we have said that the K_f is the failure line and which is actually indicated at ψ so here also and this indicates this corner indicates that you actually get you the equation the point is actually tangential.

So this intercept a and this vertical intercept is inclination is ψ similarly when you take the Mohr column failure envelope we have τ σ and where we actually have got a Mohr circle with a inclination actually ϕ and c as vertical in intercept and $\tau_f = c + \sigma_f \tan \phi$ so σ_f is that normal stress at failure so from the p - q diagram the stress parameter ϕ and c may be readily be completed.

Suppose if you are having the p - q diagram where $p = \sigma_1 + \sigma_3 / 2$ and $q = \sigma_1 - \sigma_3 / 2$ so from there also we can actually you know determine you know the parameters ϕ and c like by equating c with $a / \cos \psi$ this intercept and then you know the $\sin \phi = \tan \psi$ so $\sin \phi = \tan \psi$ so with that you know we can actually get $\psi = \sin^{-1}(\tan \phi)$ so from this relationship we can actually get so this is you know once you know that the relationship between K_f line and Mohr column failure analog can be obtained.

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So and the another issue which we need to discuss is that what is the effect of you know intermediate principal stress σ_2 on the condition of failure suppose if you are having a cylindrical sample what we say is that $\sigma_2 = \sigma_3$ but if you look into it you know whether this intermediate principal stress actually have brought an effect on the condition of failure because you know the more column failure analog will not actually take this into consideration.

So if you look into this here when you have got a σ_1 and σ_2 you actually have got a circle like this and when you have got a σ_2 and σ_3 , σ_2 is nothing but the intermediate principal stress and the σ_3 is the minor principal stress and σ_1 is the major principal stress so we have the $\sigma_1, \sigma_2, \sigma_3$ they are the principal stresses so by definition if you look into it σ_2 actually lies somewhere between the major principal stress.

And minor principal stress and if you actually talking about you know the failure analog it is the you know the major principal stress the Mohr circle which is actually resulted because of the major principal stress and minor principal stress that is one which is actually you know gets tangential to the Mohr failure analog so the by definition σ_2 lies somewhere.

Between the major principal stress and the Mohr circles for the 3 principal stresses look like those actually shown here and but one which is actually becomes tangential to the you know more failure analog at failure he is resulting due to Mohr circle which is resulting due to σ_1 and σ_3 combination.

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Mohr circles for three dimensional state
Effect of intermediate principal stress σ_2
failure.

- It is obvious that σ_2 can have no influence on the conditions at failure for the Mohr failure criterion, no matter what magnitude it has.
- The intermediate principal stress σ_2 probably does have an influence in real soil, but the Mohr-Coulomb failure theory does not consider it.

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So it is obvious that you know from this discussion that σ_2 can have low influence on the conditions at failure for the Mohr failure criterion and no matter what magnitude because it falls within this two stresses and this intermediate principles stress σ_2 does have influence on real soil but the Mohr column failure theory does not consider it so the Mohr column failure theory does not consider into intermediate principle stress and we also said that that is actually not having influence on the conditions at failure.

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Limitations of Mohr-Coulomb theory:

1. Linearization of the limit stress envelope

Possible overestimation of the safety factor in slope stability calculations,
Difficulties in calibration because of linearization

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So some limitation on the Mohr column theory can be seen where we have got in assume that the linearization of the limit stress envelope so the possible results in possible estimation of the safety factor in slopes calculations and difficulties in calibration because of the linearization and this is also valid for you know usual experimental range in the laboratory where in you can see that you know this linearization actually valid up to you know certain range.

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2. Mohr-coulomb failure criterion is well proven for most of the geomaterials, but data for clays is still limited.
3. Soils on shearing exhibit variable volume change characteristics depending on pre-consolidation pressure which cannot be accounted with Mohr-Coulomb theory.
4. In soft soils volumetric plastic strains on shearing are compressive (negative dilation) whilst the Mohr-Coulomb model will predict continuous dilation.

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And then for we actually have Mohr column failure criterion is well proven for most of the geo materials but data for that clays you know the number of actually been made and the soils and shear exhibit variable volume change in characteristics depending upon frequency pressure which cannot be accounted with Mohr column theory.

And in soft soils volumetric plastic range on shearing are compressive that is day to day while the Mohr column model will be dilation so these are the some of the you know limitations which are actually you know resulting the due to Mohr column failure criterion or Mohr column theory we can actually look an example problem this will be solving in the next lecture.

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Example problem

Figure below shows the soil profile at a site for a proposed building. Determine the increase in vertical effective stress at which a soil element at a depth of 3m, under the centre of the building, will failure if the increase in lateral effective stress is 40% of the increase in vertical effective stress. The coefficient of lateral earth pressure at rest, K_0 , is 0.5.

1m
2m

$\gamma_{sat} = 18 \text{ kNm}^3$
 $\phi' = 30^\circ$

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Where in the figure below shows a soil profile at ψ for a proposed building so determine the increase in vertical effective stress at which a soil element at a depth of 3 meters under the center of the building will failure will fail if the increase in lateral effective stress is 40% of the increase in vertical effective stress the coefficient of lateral earth pressure at rest k_0 is given as 0.5 and the friction angle is given as 30 degrees.

And the saturated unit weight is assumed to be 18 kilometer cube both above groundwater level and below ground water level then we need to calculate what is the you know the $\Delta\sigma$ at which actually the failure occurs for a given conditions so here ϕ is the more failure envelope more envelope is given to us.

So based on that we can actually calculate so in this particular lecture we try to discuss about the in detail discussion we had on Mohr column failure criterion and we have thrown light on how the principles stress relations can be reduced and then we connected with you know created initiator with discussion on the limitations of more to do theory.

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