

NPTEL
NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

CDEEP
IIT BOMBAY

ADVANCED GEOTECHNICAL
ENGINEERING

Prof. B.V.S. Viswanandham

Department of Civil Engineering
IIT Bombay

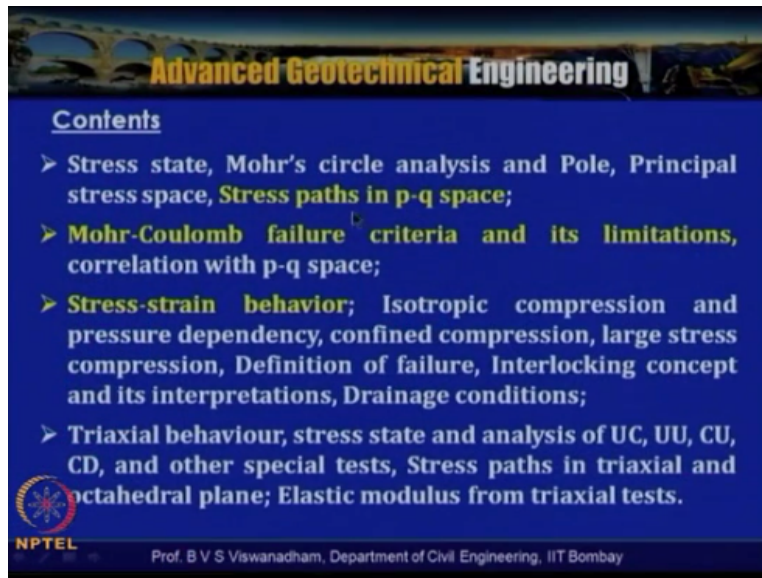
Lecture No. 33

Module-4

Lecture-4 on Stress-strain
relationship and Shear
strength of soils

Welcome to lecture series on advanced geotechnical engineering and we are in module 4 and discussing about the stress strain relationships and stress strain of soils and in previous lecture we introduced ourselves to how to you know draw the conditions now in this lecture we further you know discuss on the stress parts.

(Refer Slide Time: 00:44)



Advanced Geotechnical Engineering

Contents

- Stress state, Mohr's circle analysis and Pole, Principal stress space, **Stress paths in p-q space**;
- **Mohr-Coulomb failure criteria and its limitations**, correlation with p-q space;
- **Stress-strain behavior**; Isotropic compression and pressure dependency, confined compression, large stress compression, Definition of failure, Interlocking concept and its interpretations, Drainage conditions;
- Triaxial behaviour, stress state and analysis of UC, UU, CU, CD, and other special tests, Stress paths in triaxial and octahedral plane; Elastic modulus from triaxial tests.

NPTEL
Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

And then afterwards we introduce ourselves to the difference stress strain behavior of materials the stress strain behavior of different materials and there after we will try to introduce ourselves to Mohr- coulomb criteria and there after we will discuss it about its limitations and correlations with the pq space.

So in this particular lecture will be discussing about the further discussion on the stress parts in pq space then we will introduce ourselves to stress strain behavior of different materials and there after we introduce ourselves to Mohr- coulomb failure criteria.

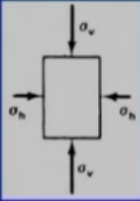
(Refer Slide Time: 01:27)

Advanced Geotechnical Engineering

Different stress paths for initially non-hydrostatic stress conditions ($\sigma_v \neq \sigma_h = 0$)

Initial conditions: $\sigma_v \neq \sigma_h = 0$
(Non-hydrostatic compression)

$$q_o = \frac{\sigma_v - \sigma_h}{2}$$

$$p_o = \frac{\sigma_v + \sigma_h}{2}$$


During loading or unloading →

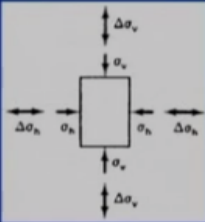
For stress path A:
The final co-ordinates of path A are

- So $\Delta q = \Delta \sigma_v / 2$ and
- $\Delta p = \Delta \sigma_h / 2$

$$q_f = \frac{\sigma_v + \Delta \sigma_v - \sigma_h}{2}$$

$$p_f = \frac{\sigma_v + \Delta \sigma_v + \sigma_h}{2}$$

Hence stress path A is inclined at 45°



NPTEL Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

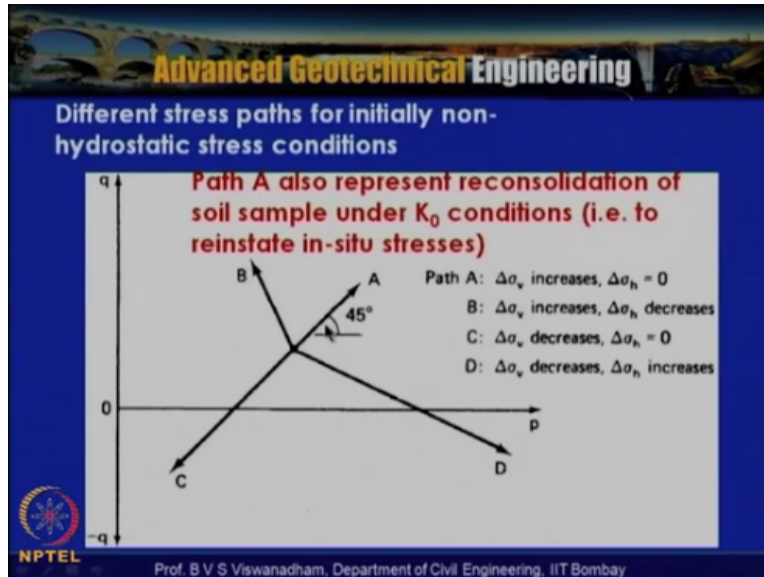
So as we have discussed in the previous lecture you know we have considered well example where the hydrostatic stress conditions prevail the hydrostatic stress conditions $\sigma_v = \sigma_h = 0$ but now let us consider we have a sample which is not having identical stresses in vertical stresses in vertical and horizontal direction that means that σ_v is not equal to σ_h is gone to 0 or we can say that you know the ample which has been taken as been reconsolidated in such a way that in order to represent the initial to stress conditions.

So in this particular slide a sample which is actually shown with the σ_v vertical on the active vertically and σ_h is acting horizontally so the initial condition here is that σ_v is not equal to σ_h and not equal to 0 and the this is called as the non hydrostatic compression so we are actually taking that initial we have not hydrostatic compression then q_o is equal to $\sigma_v - \sigma_h / 2$ that we have defined earlier.

And $p_o = \sigma_v + \sigma_h / 2$ so during loading and unloading we have a tendency of increasing $\Delta \sigma_v$ and by keeping $\Delta \sigma_h$ constant or by you know you can actually very vertical increases in the vertical stresses or horizontal stresses now we are actually required to you know develop a stress path for a actual compression were initially a sample is in non hydrostatic compression that means that σ_v is not equal to σ_h and which is not equal to 0 so the final coordinates of the stress path A you know r we will say that $q_f = q_o + \Delta \sigma_v - \sigma_h / 2$.

So here we actually increase σ_v and here also we increased in the p nothing but $\sigma_v + \sigma_h/2 + \Delta\sigma_v/2$ so if you see that Δq from the initial conditions they both are actually increased by about $\Delta + \Delta\sigma_v/2$ and $\Delta + \Delta\sigma_v h/2$ so the stress path for this condition is something like this.

(Refer Slide Time: 03:59)



When you have the condition where we do not have you know hydrostatic compression previously when we have a hydrostatic compression we actually on the p axis but now because of the prevalence of the non hydrostatic compression we have you know certain ordinate here and from there for path A is that $\Delta\sigma_h = 0$ and you know if you increase the $\Delta\sigma_v$ then you know you actually have this condition here.

So with here what we have done is that $\Delta\sigma_h = 0$ and $\Delta\sigma_h = 0$ so because of that you know we have increases in $\Delta\sigma_v$ so we have q and p they are the final coordinates of you know the stress path A and with that we actually got the you know inclination of the stress path is also because this both are same the $\Delta q/\Delta p$ of this line slope of this line is actually comes to 1 that means that the stress path actually you know are is inclined that 45° .

(Refer Slide Time: 05:12)

Advanced Geotechnical Engineering

Different stress paths for initially non-hydrostatic stress conditions ($\sigma_v \neq \sigma_h = 0$)

For stress path D: (For $\Delta\sigma_v$ decreases while $\Delta\sigma_h$ increases)

Initial conditions: $\sigma_v \neq \sigma_h = 0$
(Non-hydrostatic compression)

The final co-ordinates of path D are

- So $\Delta q = -\Delta\sigma_v/2 - \Delta\sigma_h/2$
- $\Delta p = -\Delta\sigma_v/2 + \Delta\sigma_h/2$


$$q_o = \frac{\sigma_v - \sigma_h}{2}$$

$$p_o = \frac{\sigma_v + \sigma_h}{2}$$

$$q_f = \frac{(\sigma_v - \Delta\sigma_v) - (\sigma_h + \Delta\sigma_h)}{2}$$

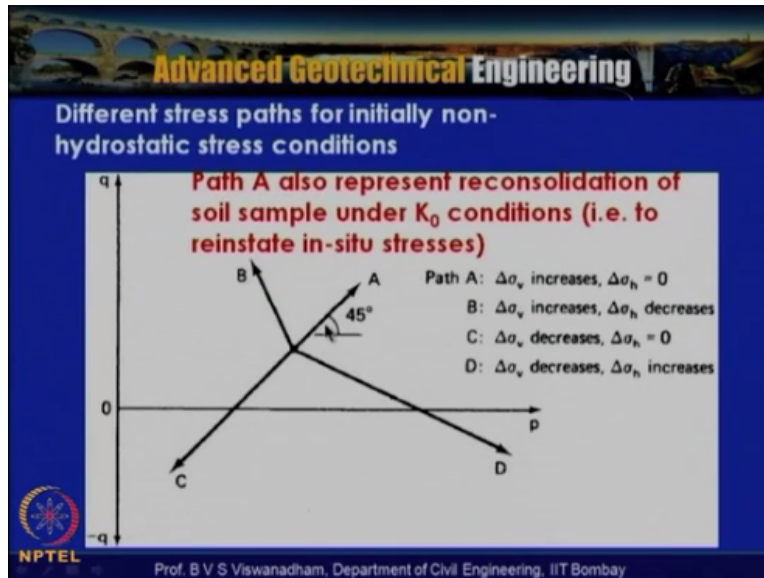
$$p_f = \frac{(\sigma_v - \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2}$$

➤ The actual slope of the stress path D depends on the relative magnitudes of $\Delta\sigma_v$ and $\Delta\sigma_h$, but in general it trends down and out

 Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

So the stress path A is inclined at 45° now for stress path D varying the $\Delta\sigma_v$ decreases while the $\Delta\sigma_h$ increases that means that we can see here stress path D.

(Refer Slide Time: 05:21)



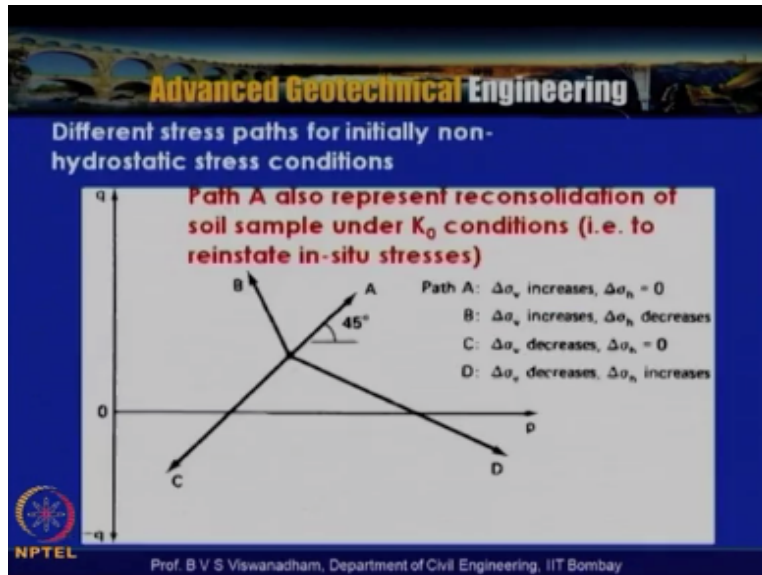
So stress path D were $\Delta \sigma_v$ decreases and $\Delta \sigma_h$ increases suppose if you say that for $\Delta \sigma_v$ decreases and $\Delta \sigma_h$ equal to 0 then this stress path C is actually in this direction and this is also inclined 45° and you know this you know this similarly by using the same concepts what we have discussed you know when we have $\Delta \sigma_v$ increases and $\Delta \sigma_h$ decreases.

Then we have a stress path in this direction so let us try to derive you know what is the stress path for D that is in this condition we have $\Delta \sigma_v$ decreases and $\Delta \sigma_h$ increases so the stress path D the condition what we have is that $\Delta \sigma_v$ increases and $\Delta \sigma_h$ increases so the $\Delta \sigma_h$ increases that is the condition which we are having here.

So initially $\Delta \sigma_h$ is not equal to σ_h is equal to 0 so initial conditions are that $p_0 = \sigma_v - \sigma_h/2$ and $q_0 = \sigma_v + \sigma_h/2$ and the final coordinates of the path D we can look into here $\Delta \sigma_v$ is decreasing so we can write the final coordinates of the point D as $q_f =$ one of the coordinates is equal to, $q_f = \sigma_v - \Delta \sigma_v - \sigma_h + \Delta \sigma_h$ because this is positive because $\Delta \sigma_h$ is increasing here divided by 2.

And similarly $p_f = \sigma_v - \Delta \sigma_v$ because $\Delta \sigma_v$ is decreasing and $\Delta \sigma_h$ is increasing so $\sigma_h + \Delta \sigma_h/2$ by simplifying this we get you know $q_f - q_0$ if you take that is Δq which is nothing but $-\Delta \sigma_v/2$ and $-\Delta \sigma_h/2$ and $\Delta p = -\Delta \sigma_v/2 + \Delta \sigma_h/2$. So the actual slope we are actually not defining the $\Delta q/\Delta p$ is the slope of the stress path line D but the actual slope of this correspond D depends on the relative magnitude of $\Delta \sigma_v$ and $\Delta \sigma_h$ but in general it trends downward and out. So the actual slope of the stress path D depends on the relative magnitudes of $\Delta \sigma_v$ and $\Delta \sigma_h$, but in general it is actually you know trading downward and out.

(Refer Slide Time: 08:05)

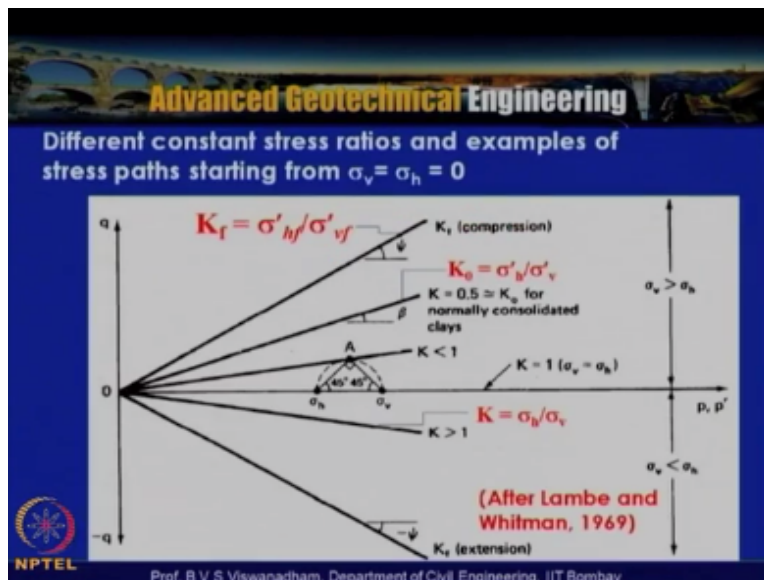


So what we have done is that in this particular example in continuation of our previous example where hydrostatic stress conditions are there, there we have drawn this stress path ABCD and similar to this what we have done is that when initially we have say initially non hydrostatic compression conditions then we have deduced and derived here stress path A and stress path B and we have said that this stress path B the inclination depends up on the relative magnitudes of $\Delta\sigma_v$ and $\Delta\sigma_h$.

And so in the similar lines you know we can reduce the stress paths for B and C the procedure is same first is that we getting the initial coordinates that is q_0 and p_0 and then depending up on the condition we have, we have to get the final coordinates of the stress path then we have to get the Δq and Δp which is nothing but Δq is nothing but $q_f - q_0$ and Δp is nothing but you know $p_f - p_0$ and with that we will be able to get the our $\Delta q / \Delta p$ we will be able to get the slope of the stress path and we also can draw the stress paths different you know it on the qp space. Now one more you know information here is that the path A which is actually here also represents because it is initially you know non hydrostatic stress conditions so the initial in situ stress conditions in for a sample when it when we have you know σ_v not equal to σ_h and this represents the initial in situ stress conditions.

So path A also represent the re-consolidation of soil sample under k_0 conditions, so this path A this is also indicates the re-consolidation of a soil sample under k_0 condition then we do because under k_0 condition means we do not have the identical stresses and σ_v for example for a normally consolidated soil we have got you know $\sigma_h = k_0 \sigma_v$ and where $k_0 =$ it can be for 0.5 so in that case you know it will actually represent the path A also represent the re-consolidation of soil sample under k_0 conditions.

(Refer Slide Time: 10:27)



So we will try to you know look into for the different you know stress ratios it is connected to express this in the form of stress ratios and σ_h / σ_v where σ_h is nothing but you know vertical stress σ_h is nothing but the horizontal stress σ_v is nothing but the vertical stress and you know it depends up on like you know with the example of stress paths starting from 0 where $\sigma_h = \sigma_v = 0$ then you know we have the you know we have written drawn different stress paths here and the one which is actually here which is indicates that $k=1$ that is what actually what we said is that the hydrostatic compression where $\sigma_h = \sigma_v$ where $k=1$ condition. Suppose if you are having a sample under water then you know that $k=1$, which is actually along this line.

Then you know we have you know the line which is you know $k < 1$ that is for you know where we have got $k > 1$ which is actually indicates for the you know over consolidated soil samples and 0.5 to 1 is actually normally consolidated place and up to maximum 0.8 for normally

consolidated soils, but for normally consolidated clays this is k_0 condition where $k_0 = \sigma'_h / \sigma'_v$ and where in this zone above this p axis or pp' axis $\sigma_v < \sigma_h$.

And below this pp' axis it is $\sigma_v < \sigma_h$ so in case of let us say that your over consolidated soil that can be you know very high you know horizontal stress because of the stress locking which actually can take place because of the passed stresses history conditions or when we have what let us say the hard crust and where the soil has been subjected to very high amount of over consolidation stresses because of the process of drying.

And there also it can be possible to that the some stresses can be locked and then and where the σ_h is actually can be more than σ_v . And we also have you know this particular line which is the upper most line which is actually called as you know k_f line and which is the failure line which is actually inclined at ψ and this line is actually inclined at an angle β so $k_f = \sigma'_h / \sigma'_v$ and by σ'_v so this is nothing but horizontal stress at failure and vertical stress at failure.

So the effective horizontal stress at failure effective vertical stress at failure, so and the samples below here this also indicates that you know when the sample actually has been subjected to extension that failure then we have the stress path actually extending below the pp' axis and where with the angle of inclination is negative ψ and this is also k_f line under extension condition what we define and this is the k_f condition k_f line under the compression conditions.

(Refer Slide Time: 13:49)

Advanced Geotechnical Engineering

Stress paths in p-q space

- Constant stress ratios appear as straight lines in the p - q diagram. These lines could also be stress paths for initial conditions of $\sigma_v = \sigma_h = 0$ with loadings of K equal to a constant σ_h / σ_v
- Note that $q/p = \tan \beta = \frac{1-K}{1+K} \rightarrow$ In terms of $K = \frac{1-\tan \beta}{1+\tan \beta}$
- Where β is the slope of the line of constant K when $K < K_f$
- At failure, the slope of K_f line is indicated by the symbol ψ
- At any point when we know p and q , σ_h and σ_v can readily found graphically.

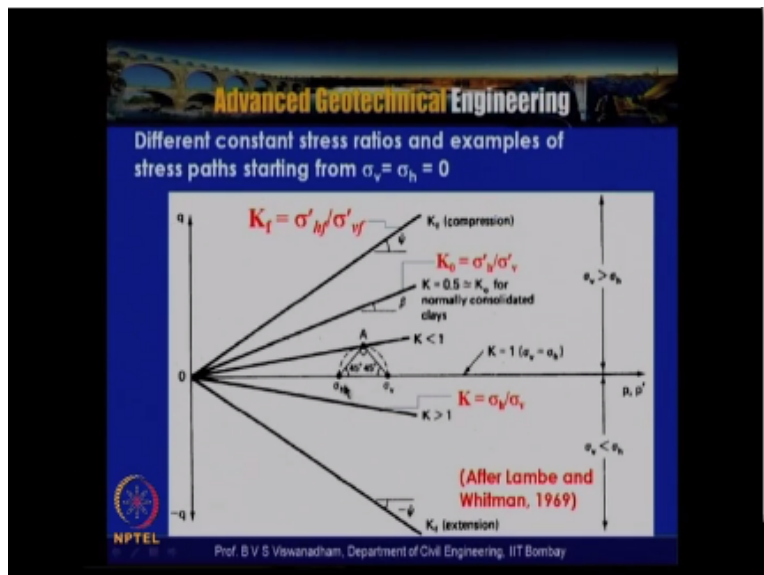
For $\sigma_h > \sigma_v$, q is negative and $K > 1$

NPTEL Prof. B V S Viswanadham, Department of Civil Engineering, IIT Bombay

So the constant stress ratios appear as these straight lines on pq diagram, so there could be you know these lines could also be stress paths for initial conditions of $\sigma_v = \sigma_h = 0$ with loading of $k = \sigma_h / \sigma_v$, with loadings of $k =$ a constant equal σ_h / σ_v . So note that $q/p = \tan\beta$ so we can see that when you have what k you know here the $q/p = \tan\beta$ which is nothing but $1 - k / 1 + k$ in terms of k we can write that $1 - \tan\beta / 1 + \tan\beta$ in terms of k this expression can be written as $1 - \tan\beta / 1 + \tan\beta$.

Where β is the slope of the line of constant k where k is less than k_f at failure the k of line is indicated by the symbol ψ at failure the k_f line is indicated by symbol ψ in compression or in you know in extension. But any point when we know p and q and σ_h and σ_v can readily be you know found out graphically, that means at any point once we know the p and q .

(Refer Slide Time: 15:06)



Let us say at this point and by drawing 45° from here and here and you know we can actually find out what is σ_h and σ_v , so with these particular conditions. At any point when we know p and q the σ_h and σ_v can readily be found graphically, but $\sigma_h > \sigma_v$ q is – and k is actually > 1 .

So $\sigma_h > \sigma_v$, that is $\sigma_v < \sigma_h$. $k_0 < 1$ - what it indicates that $k > 1$, so this actually indicates the instant and this actually indicates the normal consolidated state.

So this was the discussion about the test paths in the pq space, the test paths during sedimentation sampling of consolidated clay where k_0 is >1 if you consider.

(Refer Slide Time: 16:05)

Advanced Geotechnical Engineering

Stress paths during sedimentation and sampling of NC clay ($K_0 < 1$)

- When soils are deposited in a sedimentary environment like a lake or the sea, there is a gradual build up of overburden stress as additional material is deposited above.
- If the area of deposition is relatively large compared with the thickness of the deposit, then it seems reasonable that the compression is 1D.

➤ $\sigma_h/\sigma_v = K_0 = 0.4$ to 0.6 for granular soils, $K_0 < 0.5$ up to 0.8 or 0.9 (Average is 0.5)

NPTEL
Prof. B V S Viswanatham, Department of Civil Engineering, IIT Bombay

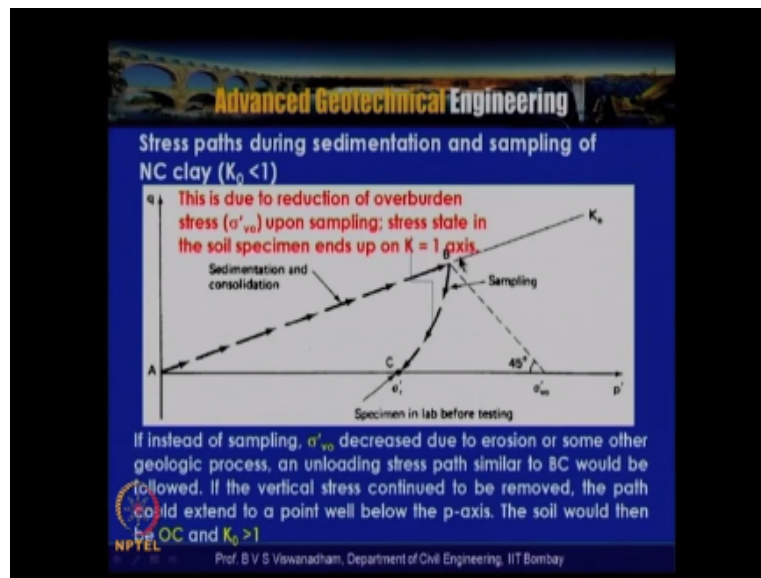
So this normally consolidated clay the conditions is that when the soils are deposited in the sedimentary environment like a lake or the sea and the sea in the marine environment, and the lake will be you in the soils are called as leg stream soils and there is the gradual build up of overburden stress as a additional deposited above. So as a deposition actually happens continuously there is the gradual build up of overburden stress as the issue is deposited above.

So if the area deposition is relatively large compared to the thickness of the soil deposit it seems that we can reasonably estimate that one dimensional compression takes place. So $\sigma_h / \sigma_v = k_0 = 0.426$, $k_0 < 0.5$ at maximum it can extend upto 8 or 0.9, so good average value what you can

say is 0.5. So the stress path during sedimentation and samplings of normal construct clay we are actually entering to draw.

When the soils are deposited in the sedimentary environment like a lake or the sea there is the gradual build up of overburden stress as a additional deposited continuously. So if the deposition is actually large comparing to the to the thickness of the soil deposit it seems that we can reasonably estimate that one dimensional compression and with that $\sigma_h / \sigma_v = k_0 = 0.426$, $k_0 < 0.5$ at maximum it can extend up to 8 or 0.9 depending upon you know the type the soil, the average is actually about 0.5.

(Refer Slide Time: 17:54)



Then we know these stress path is actually drawn like this with q versus p- where you know we have because of the sedimentation consolidation this stress path follows the k_0 condition because of the one dimensional it is along the k_0 line. So it is along the k_0 line then you know if you are sampling is there and there is the possibility that the stress will get reduce. So you can see that you know the stress path actually takes this particular direction just because of the sampling.

So here this is actually due to the reduction of overburden stress upon sampling stress path actually basically this path actually takes place because you know this is the reduction of overburden stress because we have taken the ample out. The stress stale in the soil upon the specimen ends up on $k = 1$ axis. So initially in the along the k_0 condition then you know we have actually collected the sample.

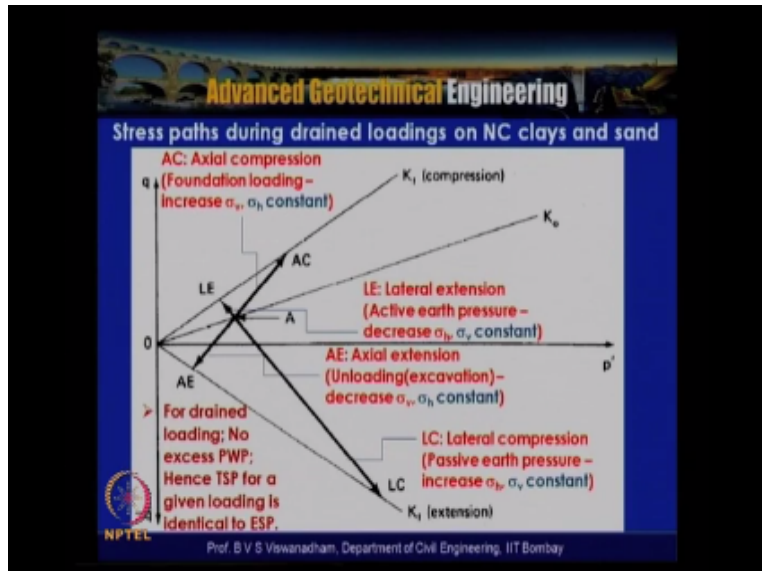
And then overburden stress so because of that what happens is that the path tends to travel like this and hits the p- axis that is the $k = 1$ axis. This is $k = 1$ axis what we have discussed, so instead of sampling σ_v decrease, let us assume that the overburden stress is getting relieved because of the continuous erosion process or some of the geologic process. Then unloading stress path is similar to BC would be follow.

So BC is also like you know unloading stress path and if the vertical stress continued to be removed then the path could extend to the point well below the p axis and then the soil would be, so when the vertical stress continued to be removed, then the stress path migrates towards the below the p axis and soil $k_0 > 1$. So in this particular slide what we have actually drawn is the stress path during the sedimentation and sampling of clay.

So when you have the continuous sedimentation is happening and say when no sampling is actually taking place and no erosion is taking place or no other geologic process is actually taking place, then the stress path due to the sedimentation process and consolidation process is in this direction. But in case at any point when the sample is actually is collected or in erosion or any other geological process will be subjected.

Then there is the reduction in the overburden stress upon sampling or due to the process what we discussed then the stress taking soil in the specimen and $k = 1$ axis.

(Refer Slide Time: 21:05)



Now here in this a condition where we have actually given you know the stress path during drain loading normally consolidate clays and sand. So drain loading in sense in that in during shear we also actually assuming that it increase in the excess pore water pressure is close to g . that means that there is no change in the excess pore water pressure and the sample is allowed to drain. So in this case we have different stress path which is actually listed here and this is our k axis, $k = 1$ axis.

And you know this is k of line compression and this is our k_0 line where we just now discussed about the sedimentation and the consolidation which actually happens on k_0 conditions and this is the k_f line for extension case, this we actually already introduced. Now let us say that we have got a sample which is again subjected to the axial compression and the geotechnical engineering example is that we have a foundation and the loading is actually increasing and the σ_h is constant.

Like we have 2 stories building the column is subjected to say increase in loading, so in that case the element AC is subjected to axial compression and the example for this is actually the foundation during increasing a that indicates for σ_v and σ_h constant. So then the case the stress path actually follows you know for this condition where you have k_0 condition that is just now we derived further non hydro static compression condition.

And AC follows that 45 you know this particular path this is the stress path, so action compression for foundation engineering increased where foundation loading increase and σ_h

constant. And then the next example is that LE there is lateral extension is happening that means that when active pressure condition actually prevail when the wall actually moves, the soil element is actually subjected to extension under a vertical stress.

So this is actually indicated as active pressure is decreasing σ_h with the σ_v constant, so because you know initially the wall is under elastic equilibrium σ_h with σ_v connected with k_0 conditions $\sigma_h = k_0 \sigma_v$. but what happen is that when the wall when it is actually moving that is decreasing in the sense that condition is called active pressure condition with the decrease in the σ_h at a constant σ_v .

So this is you know the path is actually followed as LE and this condition is actually indicated here, so this is actually an example in the geotechnical engineering active pressure condition. Then we have another condition called this is the stress path which is actually indicates per AE you can see that this is the stress path for the AE where axial extension is taking place that means that here we do not have a wall but what is actually happening is that?

We are actually extending the soil, that means that we have got certain ground surface and the soil the excision is actually happening and because of that there is the decrease in the horizontal unloading you know, because of that there is the decrease in the σ_v and σ_h constant. So this condition actually will lead to you know the stress path actually like this AE.

Then we know when we have this particular example of you know LC which is the stress path in this case the wall is actually moving towards backfill in this horizontal elements undergoes compression where σ_h with the σ_v constant. So in this particular case actually what is happening is that you know the lateral compression actually takes place? So stress path for that is actually indicated as the stress path line LC.

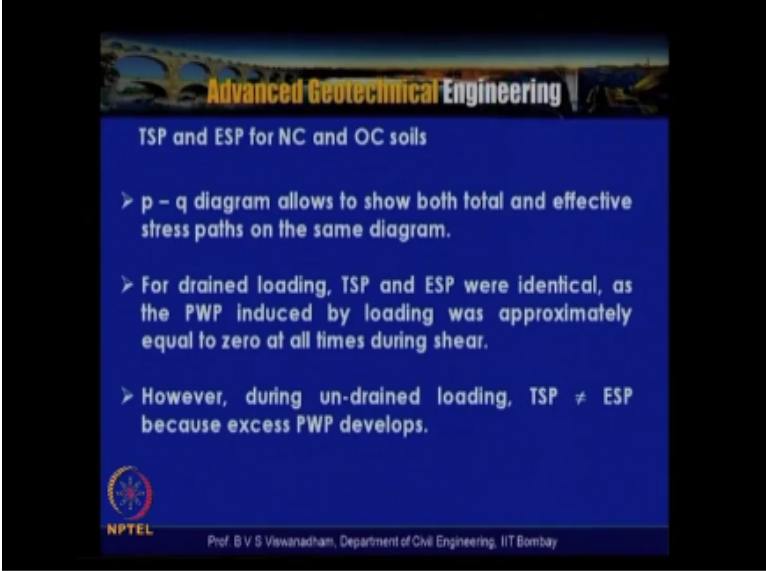
And where as active pressure line actually meets ultimately the k_f compression line and similarly the actual extension line meets also the k_f extension line ultimately when it actually attains the like example of unloading or excursion takes place. So we are LC basically with lateral compression and LE with lateral extension you know the stress path LE and LC which indicates the lateral extension and lateral compression.

And here one thing to be noted is that these stress paths for drain loadings and hence actually you know for drain loading as there is no change in the excess pore water pressures. So hence total

stress path for a given loading is, so in case of drain loading these whatever stress path we have drawn we are actually valid for both you know total test conditions as well as effective test conditions. So drain loading as there is no excess pore water pressure during the shear loading.

So the total stress path for the given loading is identical through the effective stress paths, so the total stress path for a given loading is identical through the effective stress paths.

(Refer Slide Time: 27:42)



The slide is titled "Advanced Geotechnical Engineering" and has a blue background. It contains the following text:

TSP and ESP for NC and OC soils

- $p - q$ diagram allows to show both total and effective stress paths on the same diagram.
- For drained loading, TSP and ESP were identical, as the PWP induced by loading was approximately equal to zero at all times during shear.
- However, during un-drained loading, TSP \neq ESP because excess PWP develops.

NPTEL
Prof. B V S Viswanathan, Department of Civil Engineering, IIT Bombay

So $p-q$ diagram allows you know one advantage with the total stress paths and effective stress paths for NC and OC soils the $p-q$ diagram will allow you to show both the total and effective stress path, the same diagram because for normally consolidated soils particularly under drain conditioned as we do not have the variation you know the excess pore water pressure, so both are identical.

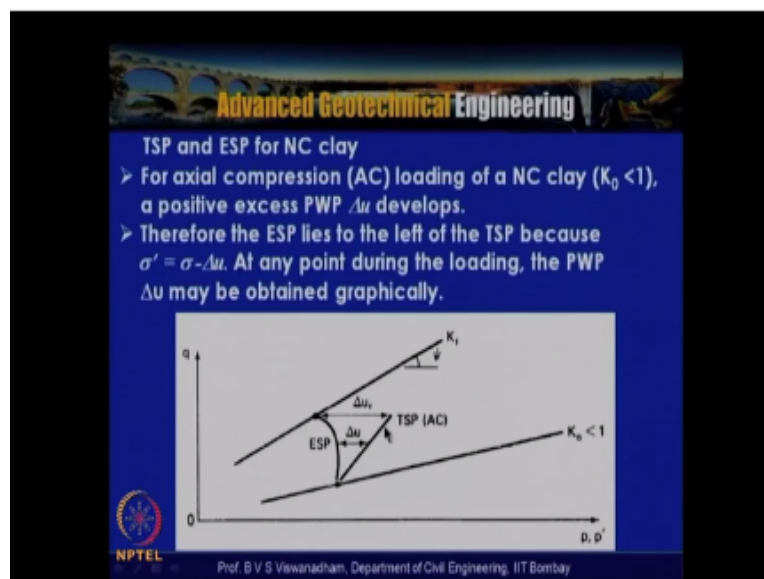
The $p-q$ diagram will allow us to show both total and effective stress path in the same diagram and for drain loading the total stress path and effective stress path are identical as the pore water pressure will increase by the approximately $= 0$ at all times during shear. So why you the total stress path effective stress path are identical, the point to be noted is that the pore water induced by the loading was approximately $= 0$ at all times during shear.

So because it actually allowed to drain pore water induced was approximately $= 0$ at all times during shear. However during un drained loading that means that the sampling not allowed to

drain when there is no volume change is permitted then the total stress path is not equal to effective stress path because excess pore water pressure develops because of the undrained conditions.

So because of the undrained loading the total stress path is not equal to effective stress path because excess pore water pressure develops. Now if you look into the total stress path.

(Refer Slide Time: 29:16)

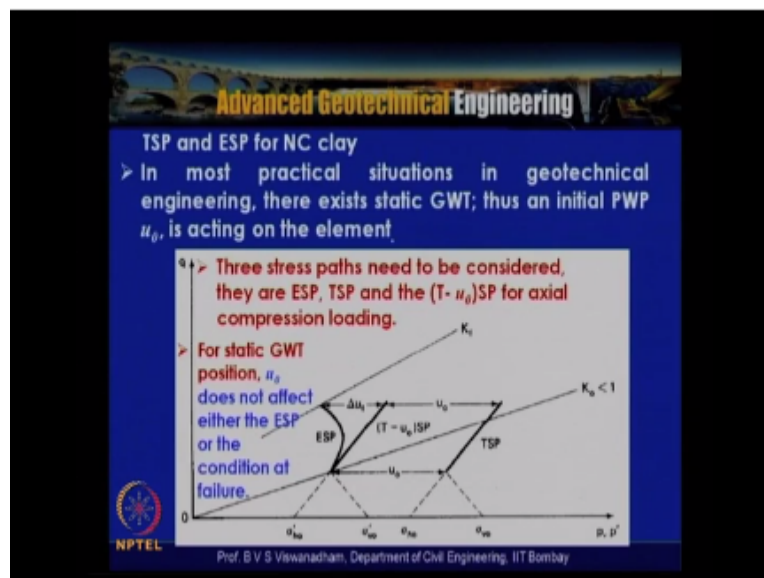


Effective stress for normally constraint clay basically for you know actual compression loading for normally constraint clay where k_0 is actually < 1 , so this is the $k_0 < 1$ line and this is the you know the k_f line and you know in the normally constraint clay when we do not allow the takes place under conditions the excess pore water pressure develops is the positive in nature because you know there is the sample tends to compress.

So for actual under compression, $k_0 < 1$ so positive excess pore water pressure Δu and therefore the effective stress path actually lies to the left of the total stress path because $\sigma' = \sigma - \Delta u$, where Δu excess pore water pressure which is actually developed. So at any point during loading the pore water pressure Δu may be obtained graphically. So when we draw this is the total test path what we have drawn but when we actually have the effective stress path by considering Δu .

So we can see that this stress path is actually seem to be left here in the way actually this is shown here, so at any point during loading we can actually determine the pore water pressure graphically we can say that by sustaining this we can actually determine the pore water pressure as the loading is a happening. So this is the case for the total stress path and the effective stress path for normally constrained clay where this is the total stress path AC and this is the effective stress path because for this actually true for normally clay.

(Refer Slide Time: 31:22)



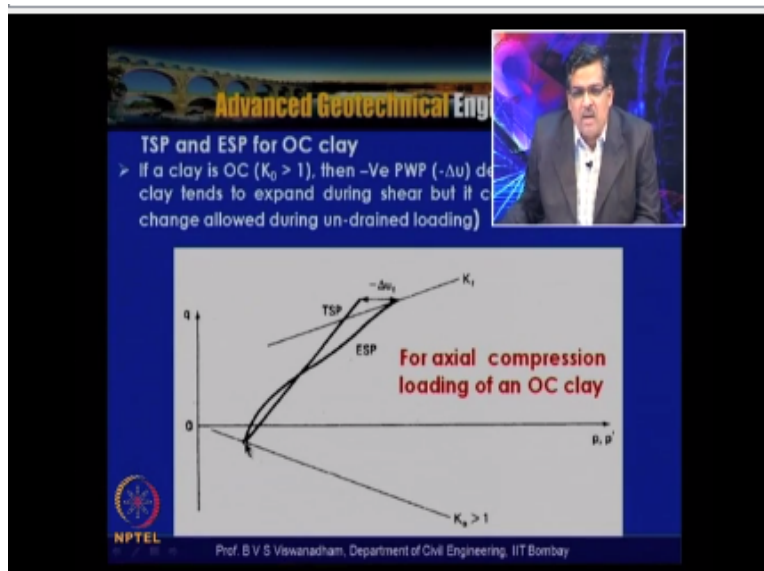
So let us say that in most practical situations you geotechnical engineering there exist static down water table always, so that means there is some initial pore water pressure u_0 will always be there in acting on the element. so in the case what will actually happen is that when you have K_0 conditions which is $k_0 < 1$ and the total stress path you know this we have discussed by taking the initial pore water pressure into consideration then you know we have this $T - U$ SP is actually this one.

And from there if it is subjected to the shear loading without allowing the drainage then this stress path, so then we are actually drawing here or showing here 3 stress paths one is the total stress path and other is the $T - U_0$ Sp, where u_0 is actually is the initial pore water pressure because of the static ground water table and then that is the actually called as $T - U_0$ SP and other one is the effective stress path.

So if you see these 3 stress paths need to be consider and they are actually effective stress path and $T - SP$, $T - U_0$ SP but you know one point to be noted is that for static ground water table position U_0 change does not affect the significantly either they are on the effective stress path or on the conditions of, so hence you know this can be ignore and otherwise you can look into that you know for the, static ground water table position U_0 change does not affect the significantly either they are on the effective stress path or on the conditions of.

So hence you know this is actually considering the practically situation we are actually have discussed that how that ground water table can be taken and then we have actually drawn 3 stress path effective stress path and $t - U_0$ SP and TSP but this static ground water table position U_0 change does not affect the either they are on the effective stress path or on the conditions.

(Refer Slide Time: 33:36)



So now after seen for the normally consolidated soil let us look into how it can be for over consolidated clay, the total stress path and effective stress path for over consolidated clay where we have the $k_0 > 1$ condition. So the sample is you know where $\sigma_h > \sigma_v$ where the k_0 is > 1

condition. So if the clay is over consolidated basically what will happen is that there initially there will be some positive for on pressure and then the clay the dense clay tense to expand.

So because of that what will happen the pour water pressure becomes negative the need to pour water pressure develops because the clay tends to expand during the shear but it cannot. So why it cannot because you know the no volume change is actually allowed during undrained loading so because of that what will happen the pour water pressure which is actually you know develops because of the you know un-drained conditions and you know the then the clay which is tending to expand.

So for that condition what will happen is that the pour water pressure excess pour water pressure will be negative that is called negative pour water pressure actually develops. So in that case what will happen in the stress path is this is the total stress path now $\sigma - = \sigma - u$ and then because of you know - sign the effective stress path is actually on the right hand side. You can see that the effective stress path, so this is the effective stress path for over consolidated clay and this is you know the total stress path for the you can see that the origin of this stress path is from $k_0 > 1$ line.

(Refer Slide Time: 35:29)

Advanced Geotechnical Eng

Mohr circle and failure theories

- If the load or stress in a foundation or ear until the deformations become too large, under the foundation or slope, has FAILED.
- In this case we are referring to the STRENGTH of the soil, which is really the maximum or ultimate stress the material can support.
- In geotechnical engineering we are generally concerned with the shear strength of soils because in most of our problems in foundations and earthwork engineering, failure results from excessive applied shear stresses.

NPTEL Prof. B V S Viswanatham, Department of Civil Engineering, IIT Bombay

So now after having discussed about the stress path in pq space so now let us look into you know how this stress path in pq space can be linked with more criteria and the subsequently how we can actually define a failed criteria for soils. So we have already discussed about the bore circles

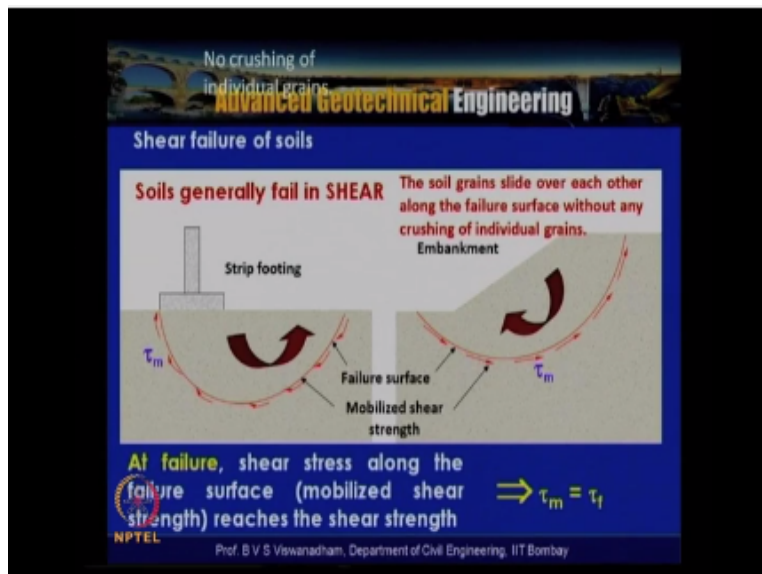
and we said that in given element and we can also determine at the stresses at failure, so these examples we have already looked into the previous lectures.

Now if the load or the stress in foundation at slope is increased until the deformation becomes too large, then we say that the soil under the foundational slope, let us say that we are having a foundation when we continue to load the further there can be possibility that the soil under the foundation or when you increase the instantly to the slope, the slope actually speedily.

So if the load or the stress in foundation at slope is increased until the deformation becomes too large, then we say that the soil under the foundational slope. So in this case we are actually referring to the strength of soil and which is really the maximum or the ultimate stress that can support. You can say that the strength of the soil is nothing but the definition if you look into it is the maximum or the ultimate stresses that can support you know is defined as strength.

Then in geotechnical engineering we are generally concerned with shear strength of the soil because the most of our problems in foundation in network engineering failure results from excessive applied shear stresses. Then the shear stresses you know the driving shear stresses dominate the shear strength of the material then failure occurs. So in geotechnical engineering we are generally concerned with shear strength of the soil because the most of our problems in foundation in network engineering failure results from excessive applied shear stresses.

(Refer Slide Time: 37:40)



So consider here in this particular slide we have a stiff footing when it is actually subjected to excessive loading then there can be possibility that you know this footing undergoes if experience of failure surface, so failure surface there is certain mobilized shear strength to counter this external loading and so this is what we call as failure surface or a slip line and you know this is mobilized shear strength.

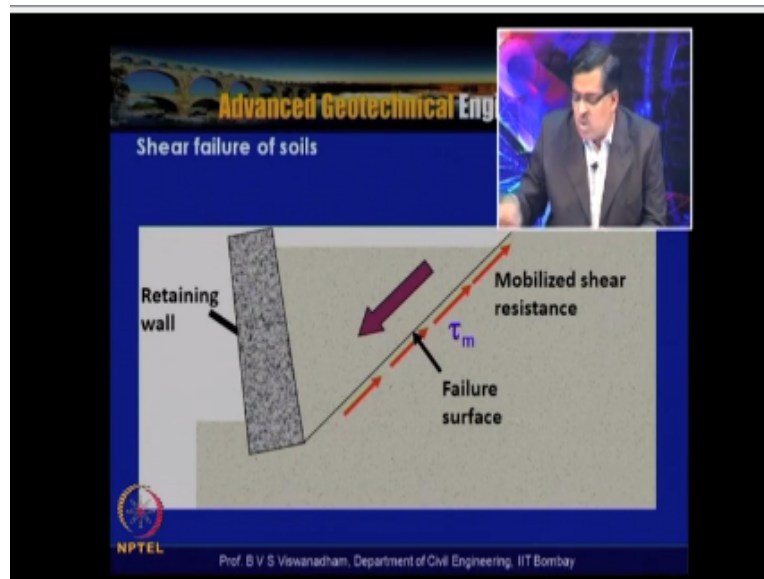
But if you are having a slope which is because of the self weight when it is actually trying to you know counter this driving this is nothing 1000 but the mobilized shear strength to counter the, so this is what we call a failure surface so here the soil grains soil generally fails, the moment actually the driving shear stresses due to this loading or comes the mobilized shear strength when the failure occurs.

Similarly here also the driving shear stresses due to the self weight of the soil mass which is actually subjected to the outer movement like this when this is actually more than then the failure occurs. And then we say that the soils generally fail in shear and the soil grains actually slide over each other along with the failure surface without any crushing of a individual grains.

So what is actually happening is that the along the failure surface these soil grains actually slide over each other along with the failure surface without any crushing of a individual grains. So in failure this shear stresses with along with the mobilized shear strength that means that $\tau_m = \tau_f$. So at failure τ_m mobilized shear strength = τ_f that means that the failure inception takes place.

You always for the adequate stability either due to the condition of the footing or due to condition of a slope at, see that the t_f and t_m as to have certain factors of safety. Factor of safety you know actually is defined as t_f/t_m and if it is actually having adequate then you know it is found to be have stable conditions.

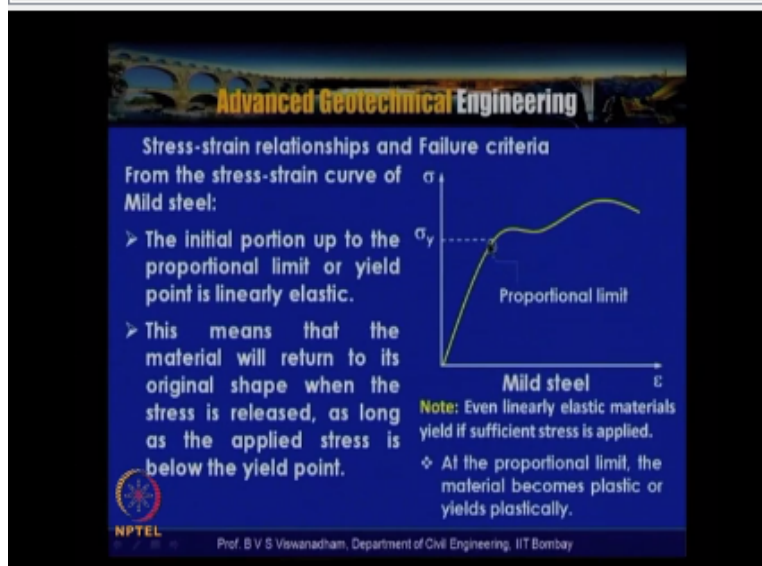
(Refer Slide Time: 40:06)



So consider another example like retaining wall, and when the retaining wall moves way from the backfill, so this is actually back fill and this is the typical example for the retain wall and when it is subjected to say active condition active at pressure conditions walls move away from the backfill. So in that what will happen is that the filled failure actually is countered by the mobilized shear resistance along the failure surface which is actually shown here?

Moment this is dominated by this deriving shear stress is in this case only sulphate is there but it is subjected to external loading and they also contributed to the deriving shear stress and then once this deriving shear stress is dominated this mobilized stress and here also the failure actually occurs. So here also we say that the shear failure of you know soils.

(Refer Slide Time: 41:05)



So now after discussed about the importance of the shear strength, and the shear stresses which are actually caused due to the external loadings, now let us look into certain you know stress strain relationships and failure criteria and let us consider typical stress strain behavior of a mild steel here, where we have σ versus ϵ where we have got a stress strain variation of shown here and this is the point where the yield stress and we called it as the proportional limit.

So you can see that there is the hump which is created and then there is hardening takes place and then some softening take place, so where the softening indicates the increase in the strength and the decrease in the stress and in the hardening in the sense but increase in the gradual increase in the strength. We can see that the initial portion of the proportional limit or yield point is linearly elastic.

The initial portion of the plastic limit or initial portion of the proportional limit or yield point linearly elastic and this means that the material will return to its original shape once the stress is actually removed. So as long as applied stress is below the yield point we can say that the element will be under the elastic conditions. So the initial portion of the stress-strain curve of the mild steel is actually in the linear elastic conditions and this means that the material will return to the original shape.

As long as the applied stress is actually below the yield point, we need to note that even linearly elastic material, even with the sufficient stress is actually applied, so you can see that some yielding of this materials you know at higher order of stresses and at the proportional limit the

material becomes plastic and yields plastically. So from the proportional limit what will actually happen is that materials tends to become plastic and yields plastically.

So the material actually explains a plastic value, so this is actually a yield point what we call and this point is called proportional limit and as the proportional limit the material actually becomes plastic and heals plastically.

(Refer Slide Time: 43:37)

Advanced Geotechnical Engineering

Stress-strain relationships and Failure criteria

- It is possible, however, for a material to have a nonlinear stress-strain curve and still be elastic,
- Note that both these stress-strain (i.e. for mild steel and non-linear elastic) relationships are independent of time.
- If time is a variable, then the material is called visco-elastic.
- Some real materials such as most soils and polymers are visco-elastic.

→ Why can't we use a visco-elastic theory to describe the behavior of soils?

Limitations in the linear theory of visco-elasticity!

Non-linear elastic

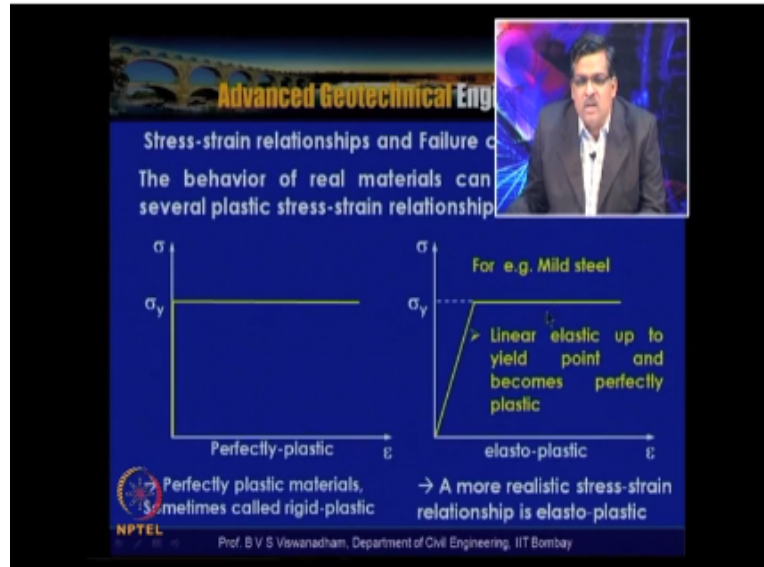
NPTEL Prof. B V S Viswanathan, Department of Civil Engineering, IIT Bombay

Now let us also consider you know we have say non linear elastic variation the sustain variation can be non linear, so it is possible however for a material to have a non linear sustain curve. So note that the both these sustain are independent of time and then the time is variable the material is called visco elastic. So some real materials which as soils and polymers are visco elastic in nature, so some real materials like soils and polymers they exhibit the visco elastic sustain behavior.

So when the time is actually is a parameter but in case if you look into this sustain we a have a mild steel and the sustain behavior which is actually shown here they are independent of time. So people can say that why we cannot actually use visco elastic theory to describe the behavior of soils when we are actually saying that these soils are exhibited as visco elastic behavior but the limitations is that linear elasticity.

So the limitations of the linear theory of visco elasticity because of that we are not able to we have some limitations in describing the behavior using the visco elasticity theory.

(Refer Slide Time: 45:06)



So here a typical 2 examples of effective plastic conditions shown here the sustain behavior and here the perfectly plastic are indicated and sometimes it is actually as rigid plastic, so that means the yield stress is there on the y axis and the σ axis and then the material is actually remains within the increase in strain, the stress is actually maintained as stress. So the behavior of the real materials can be idealized basically by 2 relationships.

One what we call as the perfectly plastic other one is called as the elasto plastic and this elasto plastic actually is more realistic sustained relationships is elasto plastic a linear reason and then now for example when you have at mile steel beyond the proportional limit the particular portion can also be linearize by maintaining horizontal plate here. So example is that the elasto plastic can be you know described, elasto plastic behavior can be idealized.

And linear elastic upto the yield point and then becomes for perfectly plastic, so beyond yield point the yield point and then becomes for perfectly plastic. So this is the more realistic sustain relationship which is called as elasto plastic and example is that for mile steel and which is actually remains upto liner elastic upto yield point then there after it has perfectly state.

Now we also have another typical sustain behavior that is called brittle and work hardening and softening different curves are shown here. The curve which is on the left hand side which is actually indicates the brittle variation. So at some point the point of loading collapse and crushes, so this is generally happens or some cement stabilizer soils they tend to exhibit you know the brittle behavior.

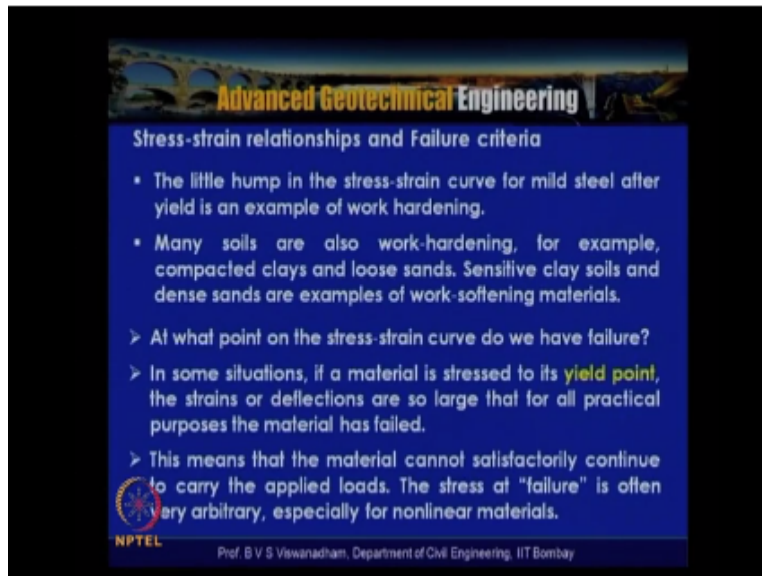
So the brittle behavior at the particular brittle undergoes a sudden collapse or a you know it actually gets crush and example is that the cast straighten or concrete and lot of rocks are brittle in nature. So this is actually a stress strain behavior which is actually shown for a brittle condition but here what we have for σ versus ϵ and conditions like work hardening, so we can see that we have two curves here it actually has work hardening type of behavior.

We can see that we have work softening curve increase in strain there is decrease in the stress and in this case increase in strain there is decrease in the stress you can see. So here there is actually the hardening is happening and there is what we say the softening is happening, work hardening the materials become stiffer because they attain higher modest actually they are strain or worked out.

So work hardening the materials become stiffer as they are strain or worked out, work softening shows a decrease in the stress as they are strained beyond the peak stress. As they are strained beyond the peak stress, so beyond the peak stress what will happen is that they experience the softening condition and with that what will happen is that they what we will call occurs when the beyond the peak stress.

So we have work hardening and work softening conditions in fact you know in the mild steel it is actually we have hardening and then it is actually followed by the softening takes place.

(Refer Slide Time: 49:09)



Advanced Geotechnical Engineering

Stress-strain relationships and Failure criteria

- The little hump in the stress-strain curve for mild steel after yield is an example of work hardening.
- Many soils are also work-hardening, for example, compacted clays and loose sands. Sensitive clay soils and dense sands are examples of work-softening materials.

➤ At what point on the stress-strain curve do we have failure?

➤ In some situations, if a material is stressed to its **yield point**, the strains or deflections are so large that for all practical purposes the material has failed.

➤ This means that the material cannot satisfactorily continue to carry the applied loads. The stress at "failure" is often very arbitrary, especially for nonlinear materials.

NPTTEL
 Prof. B V S Viswanathan, Department of CIVIL Engineering, IIT Bombay

And so when you link this stress strain relationships in failure criteria, so the little hump in the sustain curve for the mild steel after yield an example for work hardening, many soils are also work hardening for example compacted clays and loose sands, and loose sands when they are actually sheared they are actually exhibits also work hardening, when you have got some compactive place they also show you know work hardening.

And the sensitive clay soils and dense sands exhibits the work softening, that means that after the peak stresses what will happen is that there is the work softening situation actually happens here. So at what point at this behavior we do have failure, so that is actually you know what we say that in some situation the material stress then strains are deflection are so large that all the particle will fail.

So in some situation if the material is actually stressed to its yield point then that we say that the strains are deflection are so large that all particle purposes the material is actually failed and this a means that the material cannot satisfactory continue to carry uploads, the stress failure is often arbitrary especially for non linear materials. We take at for some non linear materials we take a certain strain, the material cannot satisfactory continue to carry uploads; the stress failure is often arbitrary especially for non linear materials.

So in this what we are try to see that in some situation material is the strains are deflections are so large that all practical purposes attains the failure condition. So with these we generally usually define some materials which are going some hardening define some failure condition at

some arbitrary % of strains like 10%, 15% or 20% strain or it is the strain or deformation at which the function of the structure might be applied.

So from the point of view we can see and we can actually also define this, so it is the maximum or yield stress or stress at some strain which we have defined as the failure. And there are many ways of defining failure in the materials and most of the criteria basically they will not work for soils, so most common failure criterion what we say that is that they apply to the stress to the soils where you know one of the traditional.

So when the element or the soil mass especially subjected to this shear stress which are actually more than the strength of the material and we are actually saying is that there are many ways of defining failure in materials but that means that set of criteria are there but most of the criteria they basically they fail for soils and one of the common criteria which we are going to discuss is the Mohr Coulomb failure criteria. This was actually you know introduced by the Coulomb 1736 to 1806.

(Refer Slide Time: 53:06)

Advanced Geotechnical Engineering

Mohr-Coulomb Failure Criterion

- An element at failure with the principal stresses that caused failure and the resulting normal and shear stresses on the failure plane.
- We will assume that a failure plane exists, which is not a bad assumption for soils, rocks, and many other materials.
- If we know the principal stresses at failure, we can draw a Mohr circle to represent this state of stress for this particular element.

$\tau = f(\sigma)$

NPTEL Prof. B V S Viswanathan, Department of Civil Engineering, IIT Bombay

And well known from his friction and electrostatic attraction and because of the need for the design of retaining walls, the Coulomb actually as defined the shear strength and then it is used successfully after noticing several failures of the walls you knew the Coulomb has actually formulated the shear strength and also recognized that there are 2 different types of you know inherent properties of the soil.

One is actually stress dependent one other one is stress independent one and with that you know the Coulomb has formulated define the shear strength and Christian Otto MOHR during 1835 to 1918 hypothesized actually a criterion of failure for real materials in which he stated that When the shear stress on failure claim at failure reaches, so more actually as opposes that in which of criterion is that a criterion of failure for real materials in which he stated that When the shear stress on failure claim at failure reaches some unique function of the normal stress.

So τ is nothing but the shear stress and σ is nothing but the normal stress, f indicates that you know refers to the plane on which the stress acts and f is nothing but at failure, so this is nothing but the shear stress at failure along normal stress at failure at failure plane. So MOHR actually as given in relationship that $\tau = f(\sigma)$ and where this is actually given and then it was actually combined with the Coulomb criterion.

Where in the Coulomb actually said that this is what actually that τ is function of σ where τ is function of σ and here it is nothing shear stress at failure and σ is nothing but the normal stress at failure and this can achieved because at element at failure when the shear stress at failure along normal stress at failure at failure plane. So we will assume that failure plane exist which is not exemption for soils or logs any other materials.

Basically we know that the principles at failure we can draw as more circle and to represent the state we also discussed it that by knowing the principle stresses, so when we have different sample stresses we can actually draw the series of more circles. So this particular lecture we introduced discuss about the stress path and OC clays and normally consolidated clays and thereafter we actually have try to understand. Plastic visco plastic and perfectly plastic and elasto plastic and thereafter we introduced ourselves to MOHR coulomb criterion. So we will continue further in this direction.

**NPTEL
NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

**NPTEL
Principal Investigator
IIT Bombay**

Prof. R. K. Shevgaonkar
Prof. A. N. Chandorkar

Head CDEEP
Prof. V. M. Gadre

Producer
Arun Kalwankar

Project Manager
M. Sangeeta Shrivastava

Sr. Cameraman
Tarun Negi

Sr. Online Cameraman
Sandeep Jadhav

Digital Video Editor
Tushar Deshpande

Technical Assistants
Vijay Kedare
Ravi Paswan

Copyright
CDEEP IIT Bombay