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**NATIONAL PROGRAMME ON
TECHNOLOGY ADVANCED LEARNING**

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ADVANCED GEOTECHNICAL ENGINEERING

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**Lecture No. 30
Module – 4**

Lecture - 1 on Stress - strain relationship and Shear strength of soils

Welcome to lecture series on advanced geotechnical engineering and we are in module 4 which is on stress- strain relationship and shear strength of soils. Primarily we have actually discussed about the previous lecture about the Mohr circles, but in this module 4 lecture 3 will be concentrating and introducing ourselves to you know what is stress path and how these stress paths can be plotted and introduction to p-q space. The introduction to principal stress space and stress spots n p-q space.

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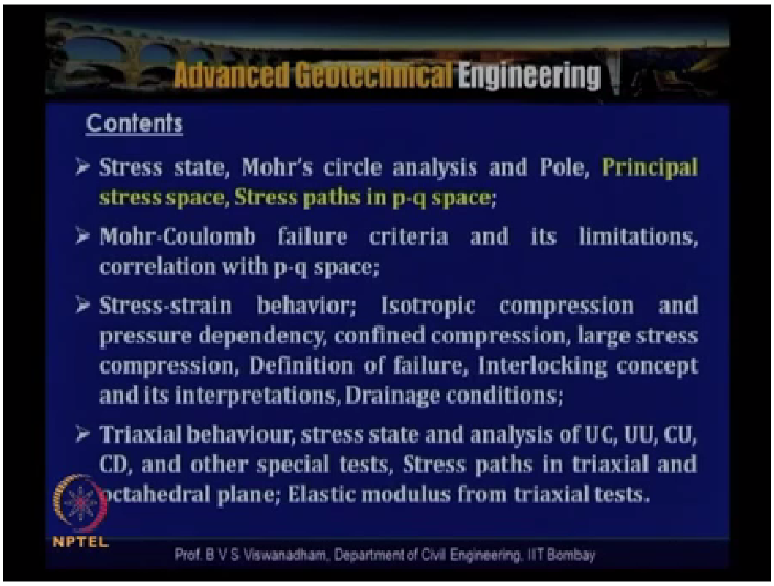

So in this particular lecture of the module 4 we will be concentrating on principal stress space and stress paths in p-q space.

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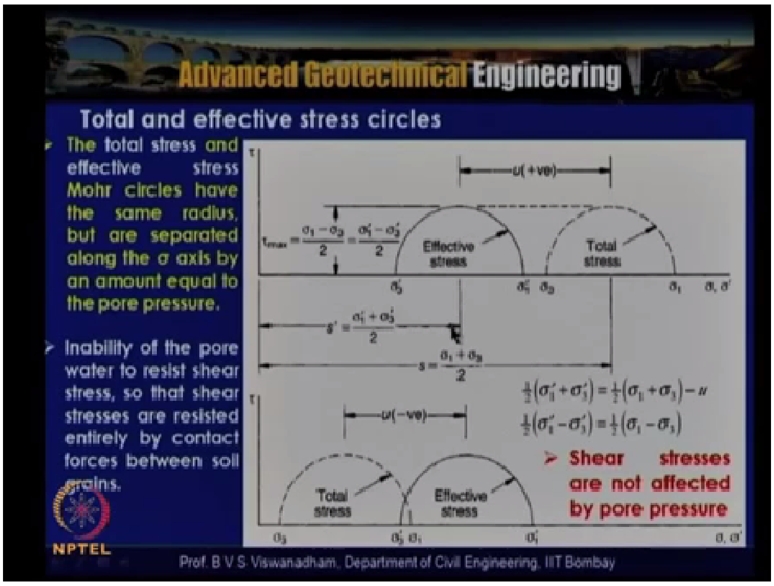

So in the previous lecture we have discussed about the total stress circles and effective stress circles and we said that both in case of total stress circles total stress and effective stresses the Mohr circle diameter remains same and when the pore water pressure is positive when u is positive the total stress circle, effective stress circle is on the left hand side of the total stress circle, but when the pore water pressure is negative then the effective stress circle is on the right hand side of total circle. So it can be seen here tow vs. σ and σ dash and wherein the effective stress circle is actually obtained by using this half σ 1 dash plus σ 3 dash is equal to half σ 1 σ 3-u and half σ 1 σ 3-u and half σ 1-σ 3.

So if you look into this here the total stress and effective stress Mohr circles have the same radius or same diameter, but are separated along the σ axis by amount equal to the pore water pressure, either it is positive or negative. In case of negative the effective stress circle on tow σ space will be on the right hand side of the total stress circle and in case if the pore water pressure is positive then the effective stress circle is on the left hand side of the total stress circle. So the total stress and effective stress Mohr circles have the same radius and, but are separated along the σ axis by an amount equal to pore water pressure, and inability of the pore water to resist to shear stress you know so that the shear stresses are resisted entirely by the contact forces between the soil.

So this explanation is given, you know the shear stresses are not affected by the pore pressure, you know you can see that the shear stresses are not affected by the pore water pressure they are actually same both in case of total stress and effective stresses and this can be physically explained by the fact that inability of the pore water pressure to resist shear stress. So that the shear stresses are resisted entirely by the contact forces between the soil grains only. So you know we can see from this you know graph and then for the discussion we had in the previous lecture. We can understand that the shear stresses are not affected by the pore water pressure. So this can be explained physically by the fact that inability of the pore water pressure to resist to shear stress and so that the shear stresses are resisted entirely by the contact forces between soil grades.

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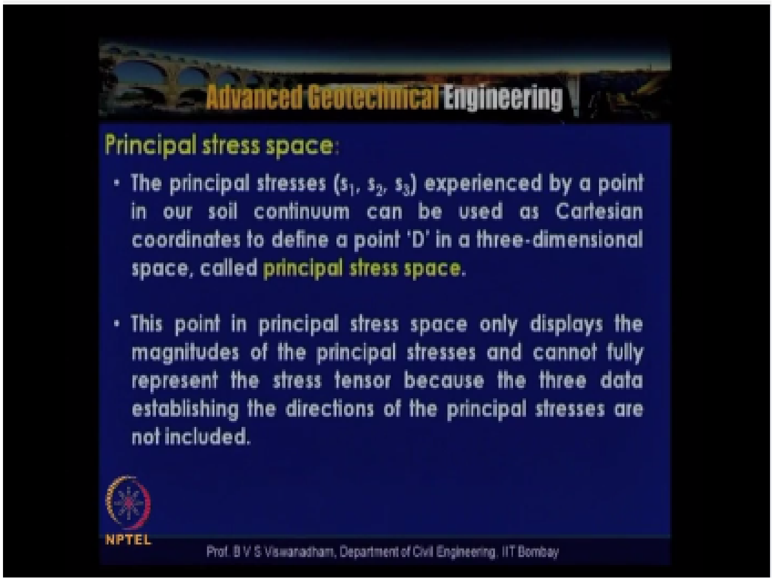

So now let us introduce ourselves to principal stress space. So the principal stresses σ 1, σ 2, σ 3, experienced by a point in our soil continuum can be used as Cartesian coordinate to define a point D in a three-dimensional space, and which is actually called as principal stress space. So we said that σ 1 which is the major principal stress and σ 3 is the minor principal stress and σ 2 is intermediate principal stress. So this point in the principal stress space only displays the magnitudes of the principal stresses and cannot fully represent the stress tensor because the three data establishing the, you know directions of the principal stresses are not included.

So the point in the principal stress space you know display the magnitudes of the principal stresses and cannot fully represent the stress tensor because the three data establishing the directions of the principal stresses are not included.

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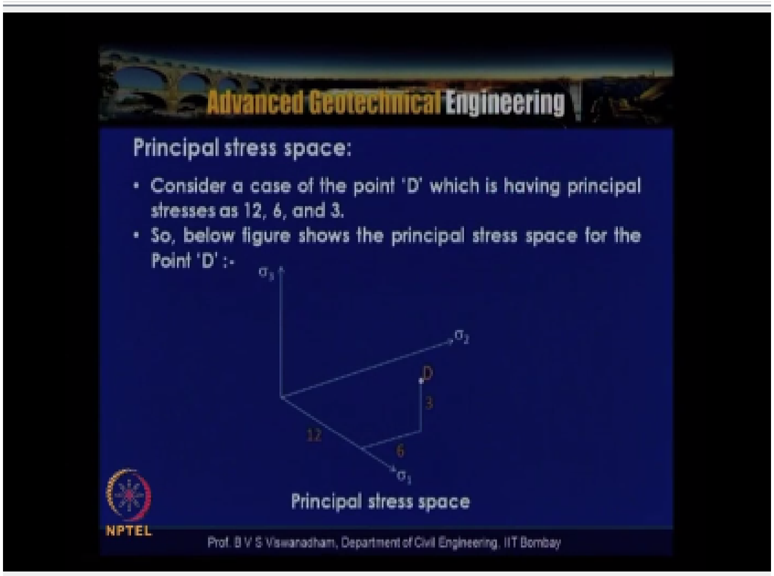

Now let us consider for example a case of the point D where you know it has actually has coordinates like along σ 1 12 and along σ 2 axis 6 units and along σ 3 axis let us say it has got three units. Then this forms the, as can be seen from this figure here and d is the point in the principal stress space were σ 1 is in this direction and σ 2 and σ 3. So you know this d is the you know represented in the principal stress space with 12 units of along σ 1 axis and 6 units along σ 2 axis 3 units along the σ 3 axis.

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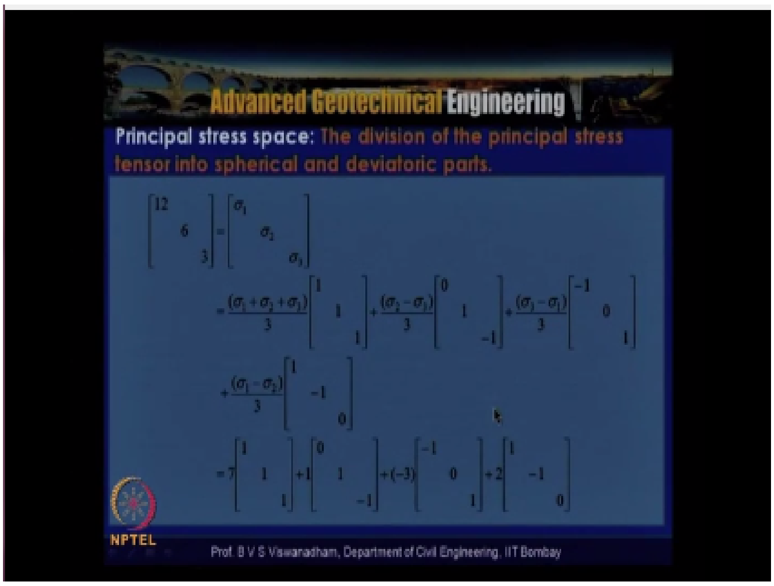

Now this principal stress space the division of principal stress tensor into spherical and deviatory part can be done like this within the matrix 12, 6,3 which is equal to matrix of 6 σ_1, σ_2, σ_3 and which is equal to σ_1+ σ_2+ σ_3 by 3 into matrix of 1, 1, 1 + σ_2- σ_3 by 3 were matrix with 01-1+ σ_3- σ_1 by 3 with matrix of -1, 0, 1 + σ_1 - σ_2 by 3, 1, -1, 0. So by putting the respective units like for σ_1, 12 the units which we have defined here 12, 6, 3 in the example here then we actually get σ_1 plus σ_2 plus σ_3 and once we get simplified by that this 7 will come, then σ_2 - σ_3, and σ_3- σ_1, and σ_1- σ_2.

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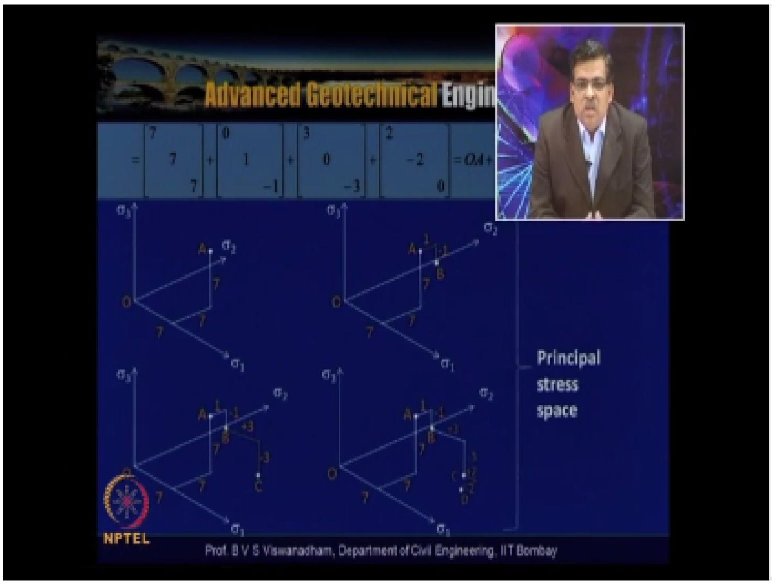

After simplification we this represents you know the matrix like 7, 7, 7 + matrix of 0, 1, -1 +3, 0 -3 +2- 2, 0. So this represents in the principal stress space OA, AB, BC, CD. We so which is nothing but OA. OA is from here to here which is you know has 7 units here, 7 units here that point A and the AB which is having you can say that towards the σ 2 axis 1 unit here and towards the σ 3 axis minus 1 unit here. So this is B here and then when we have BC which is similarly 3, 0 minus 3. So that is represented here similarly CD which is nothing but which is represented here then which is equivalent to OD. So OD is nothing but what we have actually represented in the principal stress space.

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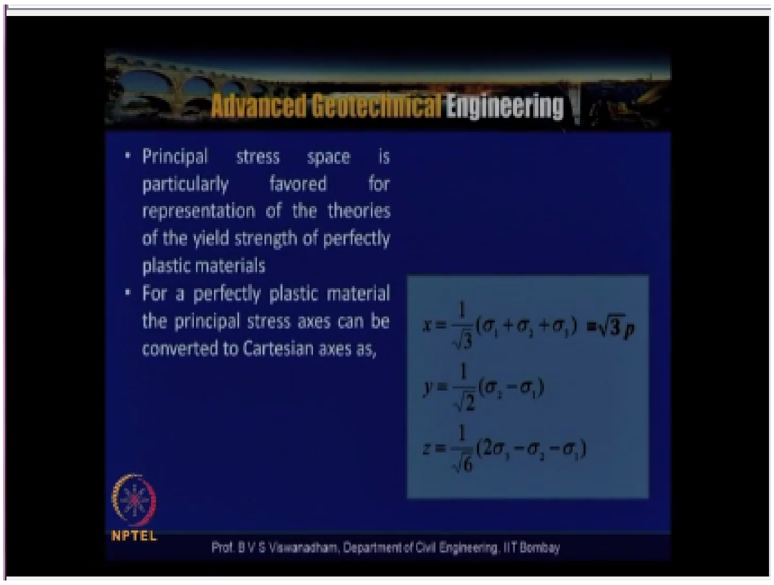

So the principal stress space is particularly favoured for the representation of the theories of the yield strength of perfectly plastic materials and for a perfectly plastic material the principal stress axis can be converted into Cartesian axes as, X=1 by root 3 σ 1+ σ 2+ σ 3 is equal to root 3 P and similarly Y is equal to 1 by root 2 σ 2 minus σ 1 and Z =1 by root 6 2 σ 3- σ 2- σ 1. So the principal stress space is you know particularly favoured for representation of the theories of the yield strength of half perfectly plastic materials.

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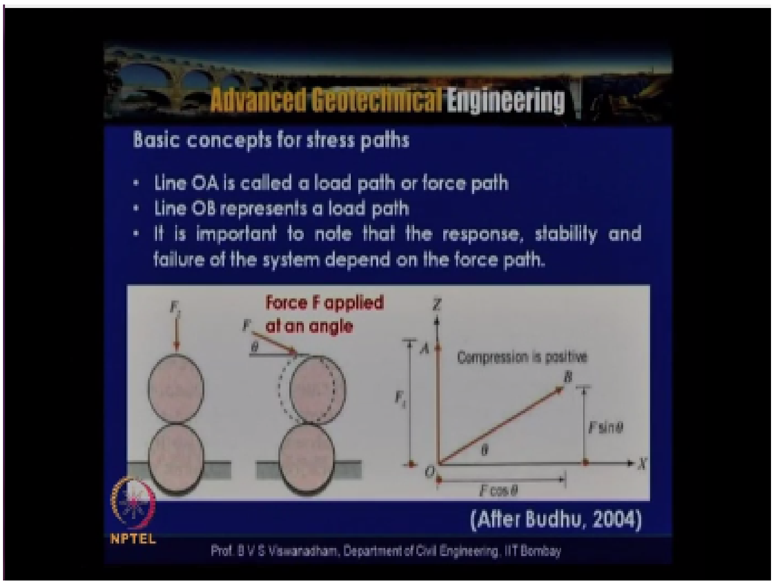

Now let us look the introduction to the stress parts in PQ space so before that let us consider, note two examples. We have on the left hand side in the figure 2 marbles or which can be represented like a 2 spherical grains which are actually you knows in figure on the left hand side here which is actually one grain is sitting on the other and assume that there is a vertical force is acting in the direction of the vertical direction.

And in this case you know the grain which is the bottom grain is has the same position, but upper grain actually being pushed with a force applied at an angle theta. So in order to represent this one let us say that we keep on increasing this from 0 to certain F. Then the path of this load what we said that the load path or force path travels 0 to A in this direction. That means that in the vertical direction. In this case the force is actually applied at an angle which is theta so which actually travels in this direction.

So this is for the path of the you know the experienced by this marble where it can undergo tilting it can undergo riding and depending upon the differential interaction between the two grains. So line YA is called load path or force path and the line OB represents a load path for you know the example which is actually shown here with a force supplied at an angle.

So it is important to note that the response stability and failure of the system depends on the force path. So for the point B were which actually has got Y intercept of 5 in 5 F sign theta X intercept of F cos theta. That is actually represented here. So in this particular example for the two grains which are either grains spherical grains or marbles when it is actually applied force in the vertical direction the load path is actually shown here and similarly the load path OB is actually shown with a force applied at an angle theta and this is for the force applied at an angle.

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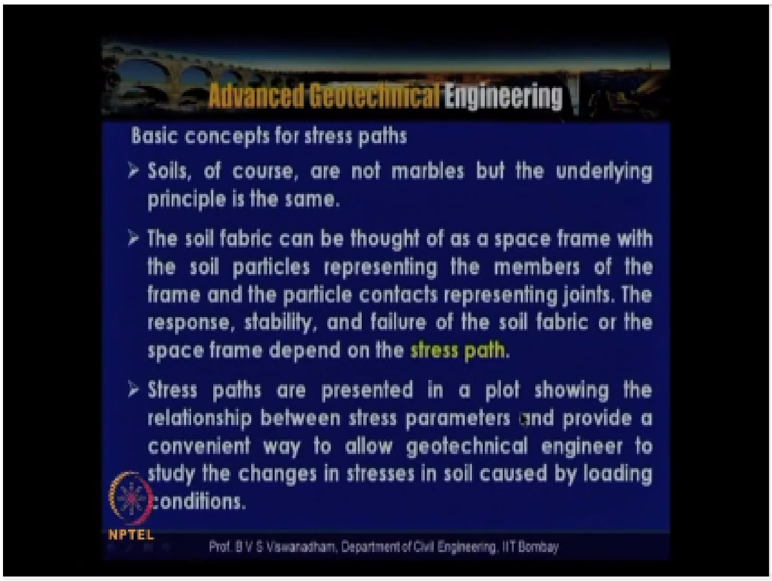

Now we know that soils of course are not marbles, but the underlying principal is same. So when you extend this principal the soil fabric can be thought of as a space frame with soil particles representing the members of the frame. So soil particles represent the members of the frame and the particle contacts representing the joints. So here what we are having is that the soil fabric can be thought of has a space frame with the soil particles representing the members of the frame and the particle contacts represents in representing joints. So the response stability and failure of the soil fabric or the space frame depend upon the stress path to which the soil specimen or soil mass is subjected.

So what we have done is that from the example what were we have taken spherical grains or marble grains we are connecting to you know from the our soil where the soil fabric and is you know assumed as space frame, you know the soil fabric means the particle arrangement assumed as a space frame with the soil particles representing the members of the frame and particle contact points are represented as the joints. So the frame is actually connected with the joint and this is these joints are actually represented in the soil as the contact points at the grain to grain contacts and members as actually represented by soil particles.

So the response and stability of a failure of the soil fabric of the space frame depend upon the stress path. So the stress paths are presented in a plot showing the relationship between the stress parameters and provide a convenient way to allow geotechnical engineer to study the changes in stresses in soil caused by loading conditions. So the stress paths are represented in a plot showing the relationship between the stress parameters and provide a convenient way to allow geotechnical engineer to study the changes in stresses in soil caused by the different kinds of loading conditions and different combinations can be considered.

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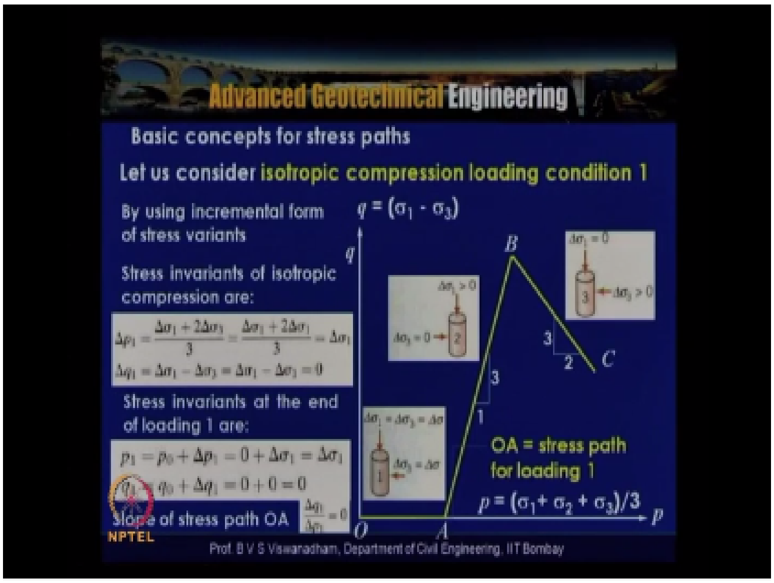

So let us consider you know a isotopic compression loading condition in one. This loading condition is defined as one where in the isotropic loading condition where Δ σ 1 is equal to that means that vertical stress and horizontal stress σ 1, Δ σ 1 and Δ σ 3. That is in the horizontal direction they both are same. so here in this particular chart were Q which is represented as P is equal to σ 1+ σ 2 + σ 3 by 3 and Q is equal to σ 1 - σ 3. So here consider the isotropic loading condition 1 so by using incremental form of the stress variants.

So we can say that the stress invariants of isotropic compression are so Δ P1. Δ P1 is nothing but you know incremental increase in P1 the P space is equal to Δ σ 1 + 2 Δ σ 3 because Δ σ 2 is equal to Δ σ 3, we have taken it as Δ σ 1 + 2 Δ σ 3 by 3 is equal to were because the Δ σ 3 is equal to Δ σ 1 then we are putting in terms of Δ σ 1 so because of the Δ σ 1 + 2 Δ σ 1 by 3 which simplifies to Δ σ 1.

And Δ Q1 is equal to Δ σ 1 - Δ σ 3 which is equal to Δ σ 1 and Δ σ 1 so it is zero and the stress variants at the end of the loading one are so it increments with p(0). So it increments with p (0) so p (0) + Δ p (1), so where p(0) is equal to zero initial conditions are zero so Δ σ 1 that Δ p(1) is equal to Δ σ 1 and q(1) is equal to q(0) + Δ q(1) both in initial and at the end of the loading they both are same so in that case what will happen is that q(1) is zero.

So the stress path for this loading condition where isotropic compression is involved it traverses from O to A. So it traverses from O to A where the slope of this line is equal to zero because Δ σ 1 that is Δ σ 1, Δ q (1) by Δ p (1). So Δ q (1) is equal to zero and Δ p(1) is Δ σ 1. So zero by Δ σ 1 it is zero.

So the slope of this line is zero and this is YA is equal to stress path for the loading 1 that is loading 1 is isotropic compression which is indicated on the, where we have got the stresses which are identical in you know all directions. Then that is you know indicated as path OA. so if you are having at pressure a trust condition and with cannot is equal to 1 let us say and then in that case vertical condition and with cannot is equal to 1 let us say and then in that case vertical stress is equal to horizontal stress. In that case with isotropic compression conditions are simulated. Then you know the stress path actually is along this p line along the p axis with Δ q is equal to 0. So this is for you know for this OA which is for isotropic compression.

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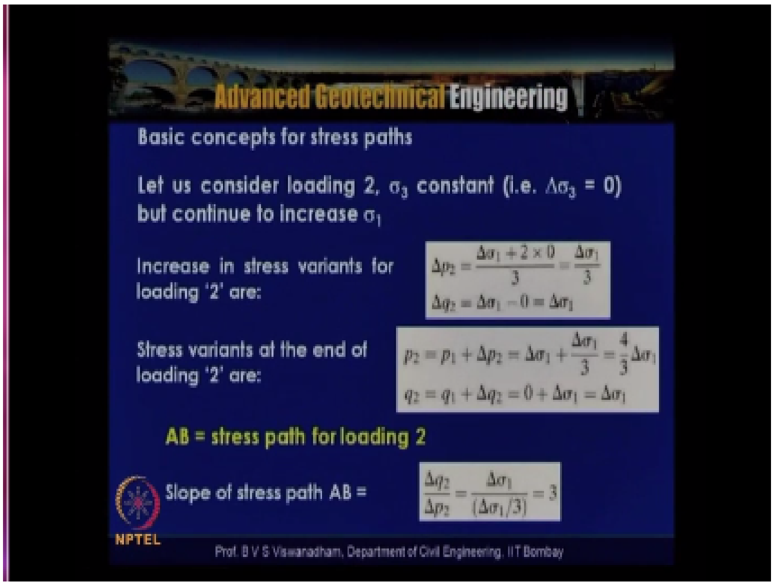

Let us consider the loading condition 2 where σ 3 is constant that means that there is a σ 3 which is applied, but there is no change in Δ σ 3. The σ 3 is constant means rate of change of Δ σ 3 is constant. so Δ σ 3is equal to zero which is the loading condition is indicated within two here and Δ σ 1 is greater than zero that means that once we held with certain σ 3 without constant σ 3.

Then you know that we increase the pressure. so in that case this loading condition is 2 were σ 3 is equal to constant and with a Δ σ 3 is equal to zero, but continue to increase σ 1 in that case the increase in the stress variants for loading 2 are Δ p (2) is equal to Δ σ 1 + 2 into Δ σ 3 that is set Δ σ 3 is equal to zero.

So what we have Δ σ 2 is equal to Δ σ 1 and the stress variants for loading 2 are p(2) is equal to p(1) + Δ p(2) where p(1) is obtained as Δ σ 1that is here in this case in the previous condition Δ σ 1and Δ σ 1 by 3. So this simplifies to 1+ 4/3 into Δ σ 1, so with simplifies to 4/3 into Δ σ 1 and q(2) is equal to q(1) + Δ q(2) which is nothing but q(1) initially we have seen in the previous case q(1) is equal to zero. So what we can say that q (1) is equal to zero + Δ q (2) is equal to you know what we have done here increasing the stress variant for the type of loading condition which we have considered.

It is Δ σ 1. So q (2) is equal to Δ σ 1. So AB is the stress path for the loading to and the slope of the stress path AB is Δ q (2) by Δ p (2). Δ q (2) is Δ σ 1 and Δ p (2) is Δ σ 1 /3. So you know by simplifying this what we get is the 3. so that means that the slope which is you know which actually has got 3 vertical, 1 horizontal is the slope for this line stress path from A to B for the condition were the loading were Δ σ 3 is equal to zero and σ 1 is continue to increase.

So this is for a type of loading condition, as can be seen this is initial condition isotropic compression and there is an increase like this, this is the stress path and for the next this thing what we to loading condition 3 where Δ σ 1 is equal to 0, but we continue to increase let us say Δ σ 3.
That means that we are rate of change of Δ σ 1 is not there. Δ σ 1 is maintained constant. That is σ 1 is maintained constant. Δ σ 1 is equal to zero.

In that case here what we are doing is that we are actually trying to see some sort of you know expansion where Δ σ 3 is actually it also represents something like a squeezy, so in this case for the loading 3 what will happen is that σ 1 is constant that means that Δ σ 1 is equal to zero, but continue to increase Δ σ 3 and that means that Δ σ 3 is greater than zero.

So increase in the stress variants for loading 3 are Δ p (3) is equal to 0 + that is Δ σ 1 is equal to zero + 2 Δ 3/2 Δ σ 3/3 which simplifies to 2/3 of Δ σ 3 and Δ q (3) is equal to 0 - Δ q σ 3 which is nothing but 0 - Δ σ 3. So there is an increase Δ σ 3 is being increased so you know Δ q (3) is equal to minus Δ σ 3. And stress variants at the end of loading 3 are P3 = P2 + Δ P3 = 4/3 Δ σ 1. P2 was obtained has here in this case P2 is obtained as 4/3 Δ σ1 + 2/3 Δ σ3 which is actually ΔP3 here. So Q3 =Q2+ Δ Q3. Q2 was obtained as Δ σ1. So by substituting here Δ σ1- Δ σ3 with Δ σ1=0 what will have is - Δ σ3. So the BC, the stress path for the loading 3 which is shown here.

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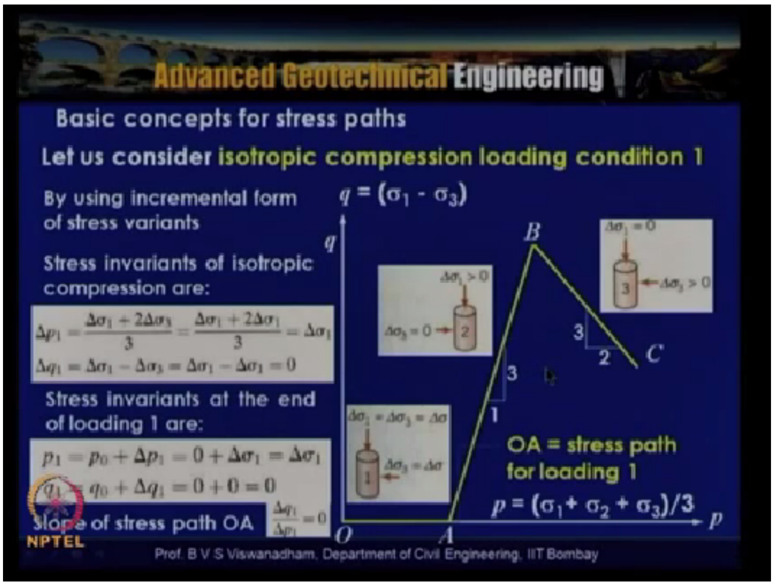

The stress path for the loading 3 is in this direction so you can see that this path now takes a downward turn here, because the loading condition here is that there is no change in σ1 that is Δ σ1, change of σ 1 is zero. Δ σ is zero. But Δ σ3 is being increased that is ≤ 0. So with that you can see that how the slope of the stress path is BC is nothing but Δ Q3 Δ P3 which is nothing but – Δ σ3/ 2/3 of Δ σ3 with that what you get is that -3/2. So -3/2 the slope is actually is indicated here 3 vertical 2 horizontal that is the slope which actually runs down here for stress path BC for the loading condition with Δ σ1 =0. σ 1 is not… σ 1 is constant and a Δ σ1 is zero and Δ σ3 is continue to that is ≤ zero. So that is for loading condition 3.

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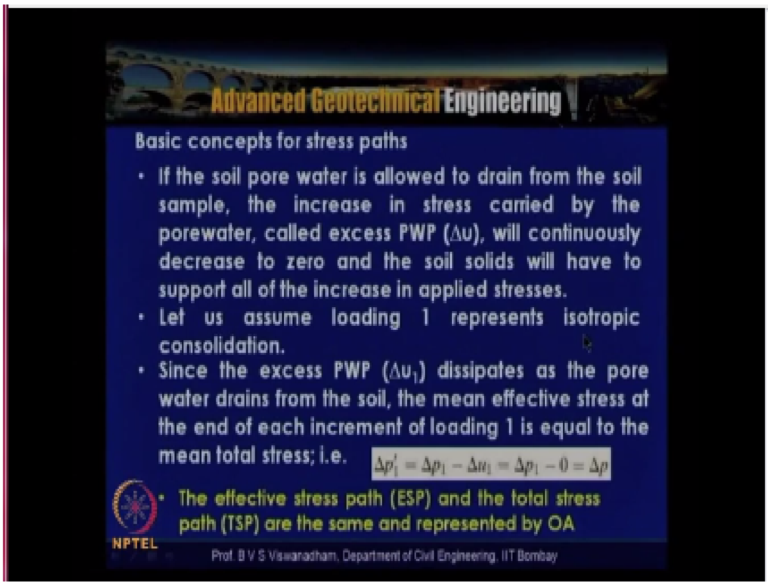

But till now this is an example which we have taken and then we try to plot the stress paths, but when we connect it to soil when it is under drained condition or undrained condition it can be under, during consolidation stage if the drainage is allowed then the consolidation takes place and during shearing if the drainage is not allowed then the excess pore water pressure develops and this is also depends upon the type of the soil. If you are having normally consolidated soil then you have positive pore water pressure and when we have water consolidated soil there can be possibility that the native pore water pressure they do develop.

So if the soil pore water is allowed to drain from the soil sample the increase in the stress carried by the pore water is called excess pore water pressure Δ U will continuously decrease to zero and the soil solids will have to support all of the increase in the appropriate places. So if the soil pore water pressure is allowed to drain from the soil sample and the increase in the stress carried out by the pore water is called the excess pore water pressure and we will continue this excess pore water pressure will continue to decrease G2 0 and the soil solids will have to support to the, all the increase all the pressure which is increased in the applied stresses.

Now let us assume that loading condition one represents the isotropic consolidation and since the excess pore water pressure Δ U1 dissipates as the pore water pressure drains from the soil. The main effective stress at the end of each increment of loading 1=mean total stress. Because Δ P1- =Δ P1-Δ U1 and at the end of consolidation because when the pore water pressure excess pore water pressure which is generated is dissipates to Δ U1 when it is tending to 0 then we can say that Δ P1=Δ P1 which is nothing but which is both the effective stress path ESP and total stress path both are identical for the isotropic consolidation and which is represented by OA.

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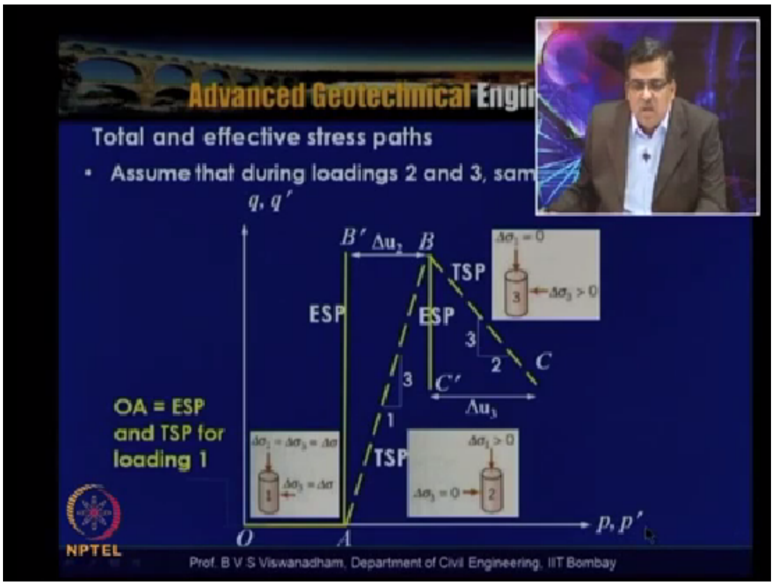

That is for during the isotropic consolidation both OA represents for the effective stress path as well as the total stress path. So in case of isotropic consolidation the slope of the stress path which is zero and which is along the P or P – axis and which actually the effective stress path and total stress path are identical and they follow in one line and this is for the total stress path and effective stress path for the isotropic consolidation. Now assume that during loading 3 the sample is un-drain. Now during loadings 2 and 3 the sample is un-drain then we can see how things will change.

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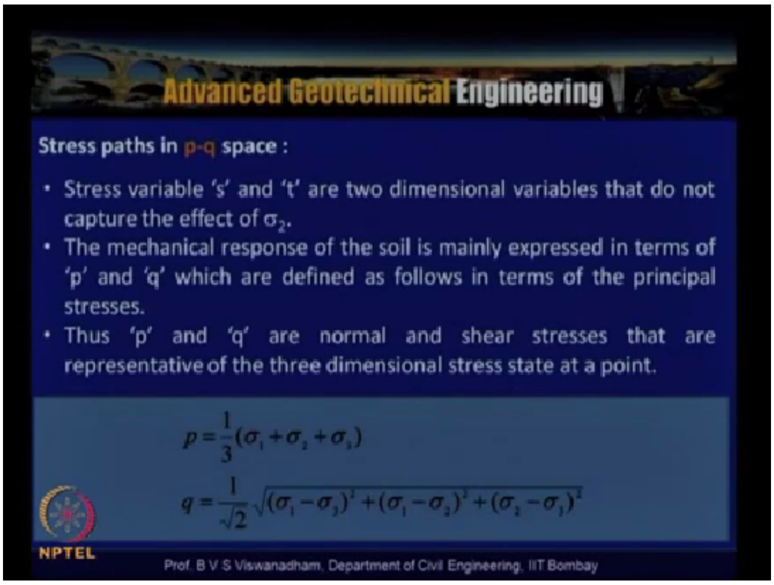

Now let us assume that if the during loading, so this is AB represents the stress path for total stress path and BC what we have reduce that is for the total stress path and then for effective stress path with sample remains un-drain the effective stress path travels from A to B dash and the effective stress path for this condition where $\Delta \sigma 1=0$ and $\Delta \sigma 3 \leq 0$. The effective stress path travels as vertically down with slope almost 90 degrees. You can see that BC dash and AB dash. AB dash for the loading condition where $\Delta \sigma 3 =0$ and where $\Delta \sigma 3 \leq 0$ we can see that how this path travels as downward like this and this travels as the upward like this.

But however this we will discuss while after having introduced the traxial test, For this stress path in PQ space stress variable S and T are the two dimensional variables that do not capture the effect of σ 2 and the ,whereas mechanical response of soil is mainly expressed in terms of P and Q which are defined as follows in terms of principal stresses, so this P and Q mechanical response of soil is mainly expressed by taking into σ 1 σ 2 σ 3 and these stress variable S and T are basically they are two dimensional variables and basically they do not capture the effect of σ 2 and thus the P and Q are normal and shear stresses that are representative of the three dimensional state stress stated a point.

So the P and Q are the normal and the shear stresses and that are represented of the three dimensional stress state at a point where P is given by 1/3 σ 1+σ 2+σ 3, q= 1 by root 2, root 1 over σ 1 σ 2 whole square + 1 – σ 2 whole square + σ 2 – σ 3 whole square, So in the two dimensional when you wanted to convert let us say that this is in terms of Q which is 1 by root 2 into root over σ – σ 3 whole square + σ 1 σ 2 whole square = σ 2 σ 3 whole square.

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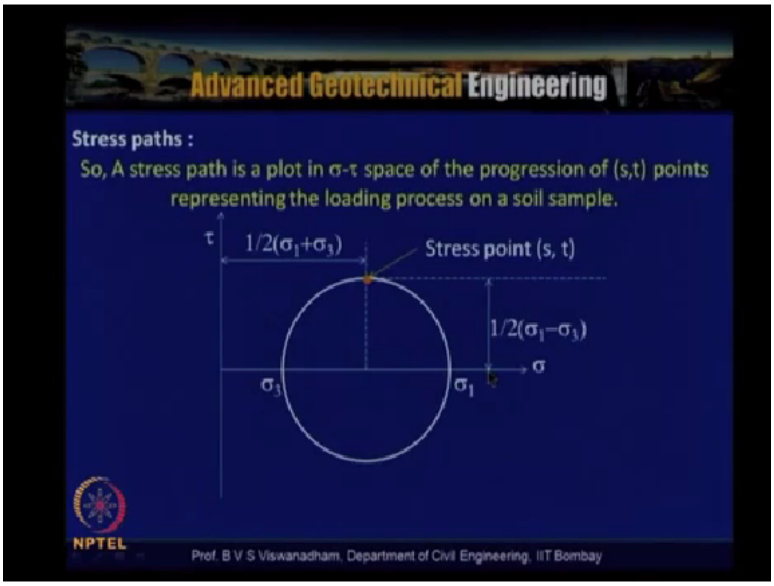

Now let us say that we have Mohr circle with the σ 1 as the major principal stress and σ 3 as the minor principal stress and σ 1 – σ 3 is the diameter of the Mohr circle and the centre of the Mohr circle on the σ axis at a distance σ 1+σ 3 /2 and let us assume that this is the stress point ST and here we can see that this point A which is nothing but the, when you join the pole here this is the major shear stress plane, this is called the major shear stress plane and this coordinate of this point is along the PX σ X =σ 1+σ 3/2 along the τ axis it is σ 1-σ 3/2.

So when you have let us say with constant σ 3, when σ 1 is increasing. In that case what will happen is that with constant σ 3 the Mohr circle actually travels like this. So you will actually get σ 1 11, σ 1 12, σ 1 13 like this. So whenever we actually join these points which are actually of maximum shear stress, than that indicates something like when you join this line and this is actually claimed at 45 degrees.

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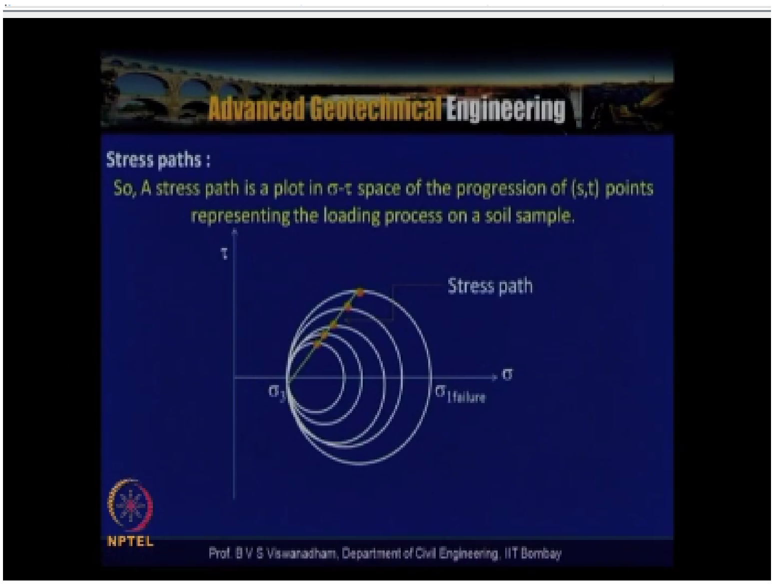

So a stress path basically is a plot which is drawn in tow σ space which is connecting a progression of ST where S is equal to, S and T which we have to find from the 2 dimensional point of view representing the loading process on a sample. So where you have the S = half σ 3 by σ 2 and T = half σ 1 – σ 3/2.

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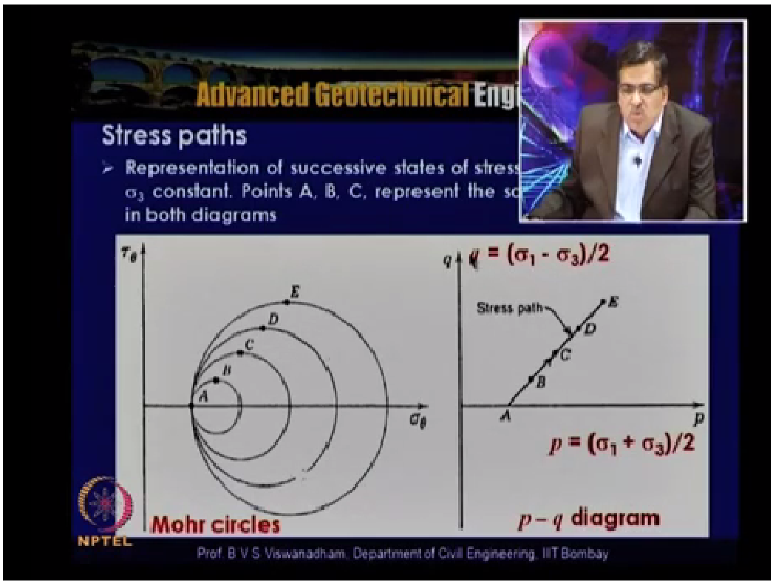

So this is again shown here with different notation, which is P is nothing but S here, S=σ 1 + σ 3/2. T is nothing but Q here, σ 1-σ 3/2. So the representatives of these successive states of the stress as σ 1 increases with σ 3 constant. So when you have the σ 3 constant and this for Mohr circle A, Mohr circle B, Mohr circle C, Mohr circle D, Mohr circle E, so series of Mohr circles. So the representation of the successive states of the stress as σ increases with σ constant, points A, B, C, D, E they represent the same stress conditions in both the diagrams. So this points which are actually because it is not possible to indicate the number of circles.

When we do the conditions with different σ 3 values so when you will be having number of Mohr circles on the tow σ space, so that is difficult and can lead to confusion, so further what it is actually simplified is that the stress path only simplifies a path where a point joining the points where the maximum shear stress on the Mohr circle is generated, so that points are actually picked up here and the A, B, C, D, E is actually known as the stress path for the condition where the σ 3 constant and σ 1 increases. So this is ST diagram or QP diagram where Q=σ 1, σ 3/2 and Q=σ 1-σ 3/2 where Q=to also and T=σ 1-σ 3/2.

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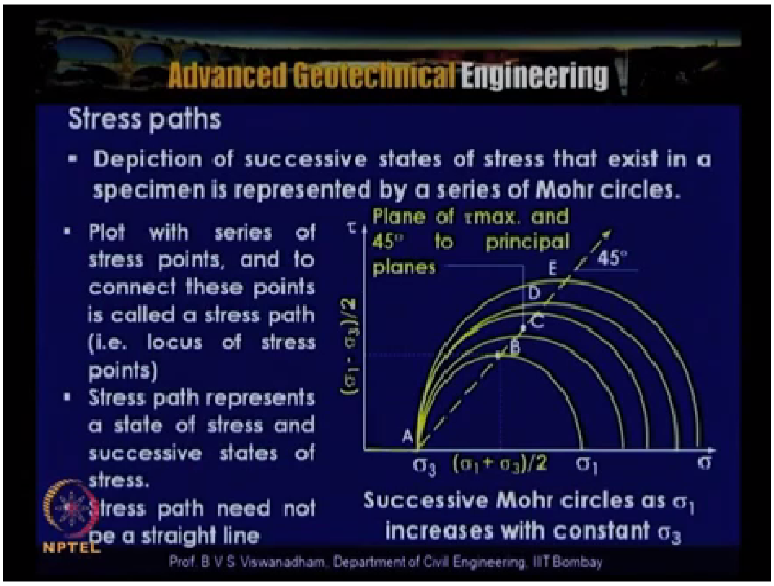

So the duplication of successive states of stress that exists in a specimen is represented by a series of Mohr circle, that is what we have discussed, and plot with the series of stress points and when you connect these points is called the stress path, the locus of the stress points, this is also called as the locus of the stress points, so the stress path is nothing but a plot with a series of stress points and when you connect these points and that is called as a stress path, which is here indicated again as A, B, C, D, E, which is actually inclined at 45 degrees because this is nothing but the plane of tow max and 45 degrees to the principal plane, so the major principal plane is here and minor principal plane is here, so it is actually 45 degrees to the principal plane.

So in this case if you look into it this is the major principal plane and this is the minor principal plane, so this stress path for this condition is inclined at 45 degrees to the principal plane, so this is nothing but the plane of tow max and 45 degrees to the principal plane, so the stress path represents a state of stress and successive states of stress and stress path need not be a straight line depending upon how we actually vary the incremental stresses sometimes we can actually vary nonlinearly also, in that case the stress path actually is also formed as a curve. So stress path need not be a linear version it can be also need not be a straight line it can be nonlinear also.

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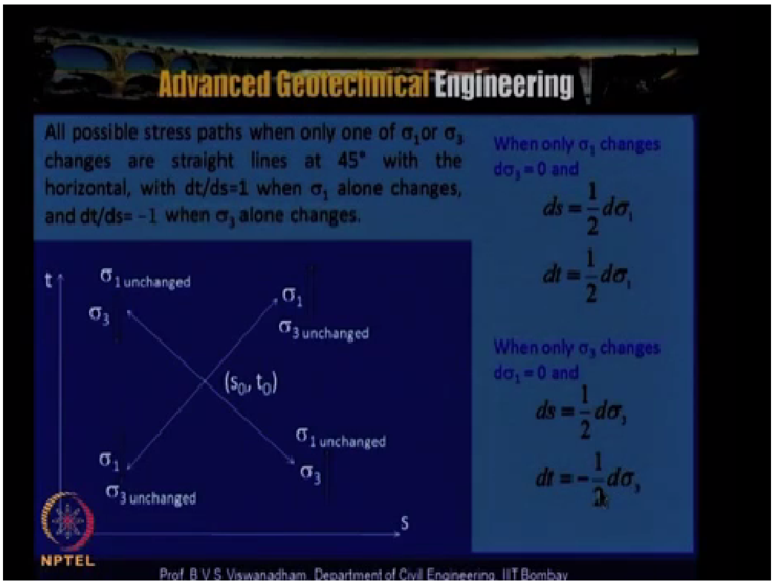

So here when we have that DS, which is nothing but small change in σ 1+σ 3/2 that is off the σ 1/DS is indicated as D σ 1+D σ 3/2 and DT is indicated off D σ 1-D σ 3. When σ 1 changes, D σ=0. When σ 1 changes that means that you are having σ 3, but D σ 3 is 0 then in that case DS=off D σ 1 and DT=off D σ 1, so you can see that DS and DT both are same. The change in DS and change in DT both are same. When σ 3 changes and D σ=0 then DS=D σ 3/2 and DT= -1/2 x D σ 3, so DT is equal to if you look into here in this direction σ 3 is unchanged here, so σ 1 increased then the path goes like this and σ 3 is unchanged, σ 1 decreased then the.

Path comes down like this and when you have σ unchanged that is D σ 1=0 and σ 3 decreases that means that you are actually releasing the confining pressure then this comes down like this. When you have σ 1 unchanged there is certain σ 1 and σ 3 continuous then the path actually goes down to 0 here, so the stress path actually changes from this direction either this side or this side depending upon how the σ 1, σ 3 the stress condition, which are actually changing on the sample.

So in this particular slide what we try to explain is that DS=off D σ 1+ D σ 3/2 and DT=off D σ 1-D σ 3/2, so here when σ changes and only D σ 3=0 then DS=D σ 1/2 and DT=D σ 1/2. When only σ 3 changes D σ=0 then DS=D σ 3/2 only σ 3 changes and D σ 1 there is no change, σ 1 remains unchanged then DT=shear stress actually decreases, so you see that the path is actually coming down.

See here, it is coming down. So all possible stress paths when only one of the σ 1 and σ 3 changes are straight lines at 45 degrees with the horizontal with DT/DS-1 when σ 1 alone changes. So here important point to be noted is here is that all possible stress parts when only one of the σ 1 and σ 3 changes are straight lines at 45 degrees and with horizontal and with DT/DS=1 when σ 1 alone changes and DT/DS=1 when σ 3 alone changes. So this observation needs to be noted.

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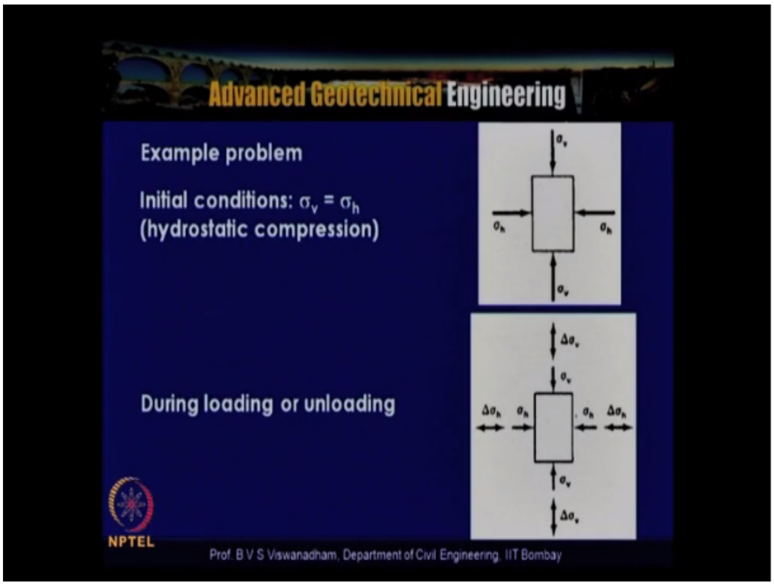

Now let us take an example problem where we have got initial condition which is hydrostatic condition σ V=σ H, you have got some cylindrical sample where you have applied vertical stress and horizontal stress, that means that the sample is confined with a hydrostatic pressure which is actually having, that means that the sample is actually with a pressurized system and then it will be subjected to σ V in vertical direction, σ H in horizontal direction.

But when the σ when it is with the hydrostatic condition, the σ V=σ H and during loading or unloading it can be like when you have the initial conditions and Δ σ V can increase or you can actually have Δ σ H can increase or decrease. That means that here the conditions are that during loading and unloading the sample will be experienced something like increasing in Δ σ 1 and decreasing in Δ σ 1 and/or when this is maintained constant there is increase in the Δ σ H and decrease in the Δ σ H.

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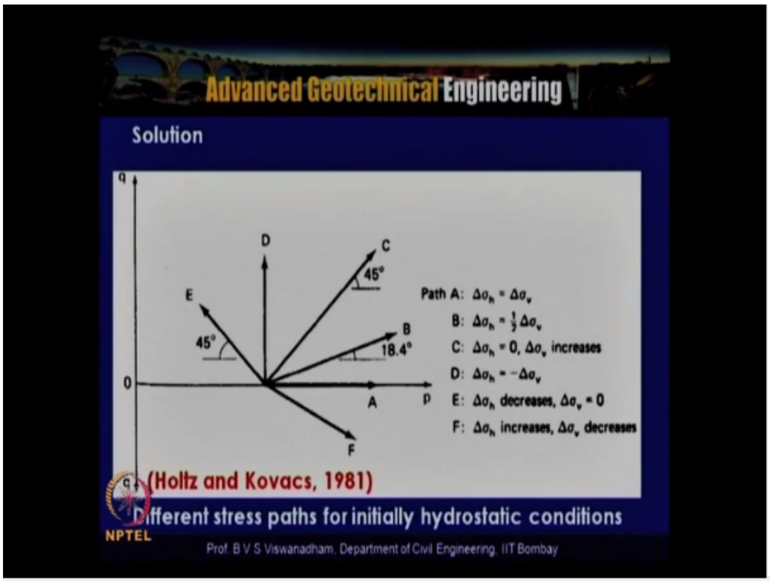

So the stress paths for this case with different stress points paths for initial hydrostatic condition. As we said that initially hydrostatic condition means you know that path actually starts along the p axis or this is s equal to σ on σ 3 by 2, q is equal to t is equal to σ on σ 3 by 2 so you can see that you have got different path now a that is the if this is a let us say a point t ta, tb, tc, td, te, and tf are the different stress paths and out of this you can see that path a which is with the condition Δ σ h is equal to Δ σ v and path b what we have done is that we maintain Δ h is equal to half Δ σ v and in this stress path c what we have done is the Δ σ h is equal to 0 and Δ σ v equal to 0, that is what we have discussed in the previous example.

And path b where Δ σ h is equal to - Δ σ v, here we are actually maintaining Δ σ has - Δ σ v and e where Δ σ h decreases as Δ σ v is equal to 0, that is there is certain σ v and the sample and the Δ σ h is being decreased, and in path f where Δ σ h increases and Δ σ v decreases so this example, you know this is the you know the solution for this example problem when you actually have got you know variable loading condition and this can be worked out like this.

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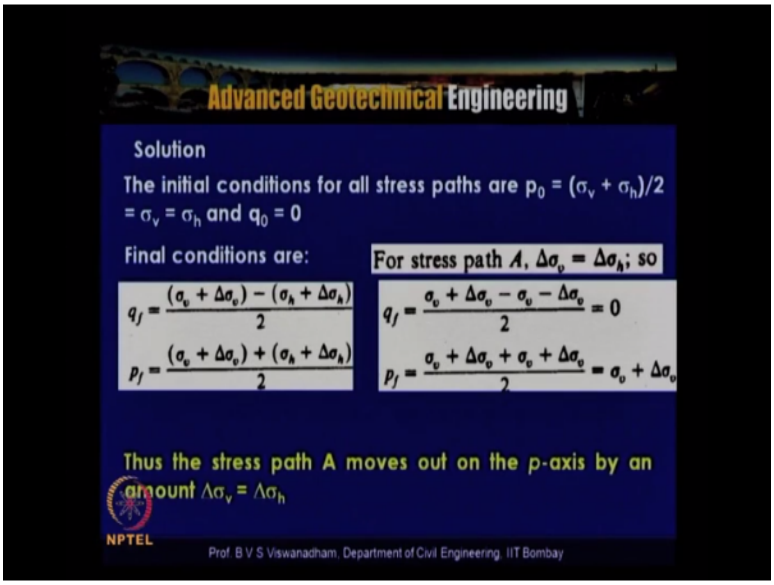

The initial conditions of the for all stress path are p 0 is equal to σ v plus σ h by 2 and with that σ v is equal to σ h so this equivalent to when you have σ v is equal to σ h which is nothing but p 0 is equal to σ v or σ h and q 0 is equal to 0 and the final conditions are which can be given as final condition which is indicated for qf is nothing but σ v plus Δ σ v minus σ h plus Δ σ h by 2 and pf is equal to σ v plus Δ σ v plus σ h plus Δ σ h by 2.

So when we have a you know this, when we substitute for this conditions what we have for this stress path a you know obtain the stress path a what we do what we have been given the loading conditions is that initially σ v is equal to σ h and Δ σ v is equal to Δ σ h, that means that qf is equal to σ v plus Δ σ v minus σ v.

Because σ h is equal to Δ σ v minus Δ σ v by 2 so we that what we have got this that qf is equal to 0 what we of got qf is equal to 0 and pf which is σ v plus Δ σ v plus σ hi substituted as σ v and Δ σ h as Δ σ v so when I substitute this which is nothing but 2 * σ v plus Δ σ v by 2 that get simplify 2 σ v plus Δ σ v so you can see that the stress path a moves out of the p axis by an amount equivalent to Δ σ v is equal to Δ σ h.

That means that the stress path you know take q by Δ q by Δ pf it is zero 0 by Δ σ v so that means that the stress path actually travels along this and it is moves by distance Δ σ h is equal to Δ σ v and this is nothing but qf and pf are nothing but the core nets where pf is equal to 0, that means that is an point is on the p axis is or x axis which is nothing but s is equal to σ on σ 3 by 2. Similarly we have a you know the stress path v the initial condition are same but what we have done is that here is that Δ σ h is equal to Δ σ v by 2 that means that Δ σ h is equal to Δ σ by 2, the stress path b is actually travel c incarnation of you can see that 18.4 degrees.

So in order to deduce this we follow the same conditions here, qf is equal to σ v plus Δ σ v minus σ h plus Δ σ h by 2 and pf is equal to σ v plus Δ σ v plus σ h plus Δ σ h by 2. Now for substituting σ h is equal to σ v initially so σ v plus Δ σ v minus σ v minus Δ σ v by 2 because Δ σ h is equal to Δ σ v by 2 so with that what we got is that one by four Δ σ v and similarly pf is equal to, when we simplify this we get σ v plus 3 by 4 Δ σ v.

So these points are nothing but coordinates of the end of the stress path, b qf and pf are the points coordinates of the at the end of the stress path b q and p both increase by Δ σ v by 4 and 3 Δ σ v by 4 so both are actually increasing you can see that this is increased by Δ σ v by 4 and this is increased by 3 by 4 of Δ σ v by 4 this implies that stress path b has a slope of you can see that this by this what q by p we get a slope as 1 by 3 or 18.4 degrees. So hence we can actually draw now this stress path b if t tb as the you know with incline at coordinates of what we discuss is 1 by 4 Δ σ v and then 3 by 4 Δ σ v it is increased by that much amount and then that is the you stress path for this loading condition where Δ σ h is equal to Δ σ v by 2.

The third loading condition here for stress path c is Δ σ h is equal to 0 that is no change in the σ h but σ v continue to increase so this we have discussed but again we will actually solve this by using the method which you have discussed just now with Δ σ h is equal to 0 and Δ σ v is greater than zero that is increases by the some amount then we can actually substitute here qf is equal to when substitute σ v plus Δ σ v minus σ v by 2 because Δ σ which is equal to 0 it is get 0 so its becomes Δ σ v by 2 and pf is equal to σ v plus Δ σ v plus Δ σ v by 2 so this also increases by amount you can see that both qf and pf we are coordinates of stress path c and both increase by magnitude switches is equal to Δ σ v by 2 and Δ σ v by 2.

So this implies that the slope is one so q by p which is one which implies that actually the stress path actually has got as slope of 45 degrees so now this tc which is the condition where Δ σ h no change in the set pressure and that is the horizontal pressure and Δ σ v increases and that is nothing but this path tc where the coordinates of this which is increased by that point which what we have shown as Δ σ 1 by 2 and Δ σ v by 2. Now we have stress point d so in this stress path d the condition what is be maintain as the Δ σ h is nothing but minus Δ σ v stress the stress path d where the condition here mention is that Δ σ h is minus Δ σ v.

So by using the same principals substituting in this and writing here with σ h is equal to σ v and you know when we put this what is happened is that you actually have it is increased by Δ σ v so this actually has increased by Δ σ v and this point is nothing but σ v plus Δ σ v plus we have σ v minus Δ σ v because the Δ h Δ σ h is equal to minus Δ σ v so with that what we get is that you know these two will get cancelled then what we have is that two σ v by 2 which is nothing but σ v.

So values of pq are the coordinates of the end of the stress d and qp both increased by Δ σ v and σ v and implies that stress path d has a slope of 90 degrees so that is the reason why we can see that the stress path is actually traversing vertically up with a coordinate here, we can see that Δ σ h is equal to minus Δ σ v which is actually traversing up here with 90 degree slope.

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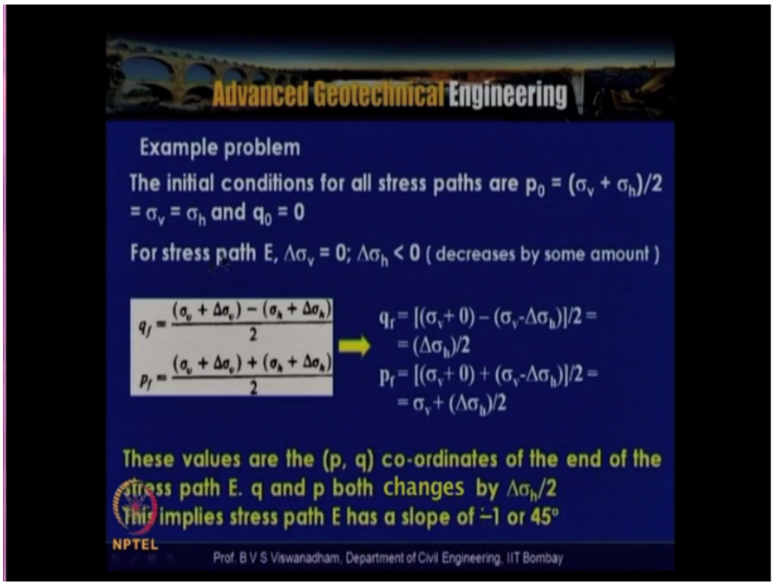

So for e lets as consider first stress path e where Δ σ v is equal to 0 that is the change in the vet has 0 but this self pressure Δ σ actually decreased by some amount so why using the same principals where qf is equal to σ v plus Δ σ v minus σ h plus Δ σ h by 2 by substituting here what we are what will get is that σ v plus 0 that is Δ σ v is equal to 0 minus σ v minus Δ σ h by 2.

So where we in get qf is equal to Δ σ h by 2 and pf is equal to σ v plus 0 plus σ v minus Δ σ h by 2 where in we have got σ v plus Δ σ h by 2 so we can see that the values of q and p are increased both by Δ σ h by 2 so this implies that the slope of e has a slope of minus 1 or 45 degrees so this traverse for this load conditions which is what we have discussed is that like this.

So when Δ σ h decreases and with Δ σ h decreases σ is decreases with Δ σ which would zero you can see that this travels in this direction towards the q axis and which is as inclined 45 degrees and increased by the same amount both in a Δ q and Δ p and when you have Δ σ h is equal to Δ σ v that is said the when you have Δ σ h is equal to zero and Δ σ v into just traverse this direction when you have in this direction that means that here what is actually happening is that with the constant σ 1 when you have and Δ σ h actually when you have sample is actually losing the confinement that means that there is a sort of you know the sample is subjected to attention.

So you can see that that is actually happening here whereas in this case where Δ σ h is actually zero there is no change in the self pressure or horizontal pressure but when Δ σ is going, so this is actually towards the compression and this is towards the tension. So we can look into this the ways what we have done is that we try to you know see you know this plot this stress paths by actually following the fundamental principals wherein this particular this example wherein it is subjected to initially hydrostatic condition are considered but suppose if you not having hydrostatic conditions initially when say the $\Delta \sigma v$ is not equal to $\Delta \sigma h$ as σv is not equal to σh then that means that you know the origin of the stress.

Path location is somewhere are here and from there again it actually follows the you know different path can be drawn. So in this particular example what we have considered is that the initial condition is hydro static compression where we have actually σv is equal to σh and when you have this, when you have got different variation of $\Delta \sigma v$ and $\Delta \sigma h$ then we have actually drawn and then we have actually also seen how this procedure can be adopted for plotting this stress paths.

So you can see that here the stress paths are actually plotted and they are nothing but the locus of the you know points on the Mohr circle and instead of this actually avoids the series of Mohr circles when you have actually different stress conditions, so this stress path are the know the straight line joining the locus of the stress straights on the Mohr circle and they are actually need not be straight line and depending upon how you excise, for example when you have $\Delta \sigma h$ is equal to half let us say $\Delta \sigma v2$ half $\Delta \sigma v2$.

And in this case we have actually undertake as curve also so you have actually get a nonlinear variation also so not necessarily the straight line but when we have this conditions of this you know $\Delta \sigma h$ is equal to $\Delta \sigma y$ by 2 and we have $\Delta \sigma h$ zero and $\Delta \sigma v$ increases and you have $\Delta \sigma h$ decreases and $\Delta \sigma v$ zero then actually follow the actually nothing but the plain of maximum stress.

Wherein it actually has got and it actually inclined at 45 degrees to the principal plane. So in this particular lecture what we have done this that we have discussed very briefly about the principal stress space and then we have tried to introduce ourselves to stress path and seeing some examples where how the stress path can be plotted in pq space or st in the two dimensional space where s is equal to q is equal to σ minus σ 3 by 2 and p is equal to s is equal to σ n minus σ 3 by 2.

So further we will actually discuss on this and then when we introduce ourselves to the traxial compression test then we will try to see when we have got a different drainage condition and un-drained and drained how we can actually draw effective stress paths and total stress paths and with some examples and that makes very clear.

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