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ADVANCED GEOTECHNICAL ENGINEERING

Prof. B. V. S. Viswanadhan
Department of Civil Engineering
IIT Bombay

Lecture No. 31
Module – 4

Lecture – 2 on Stress = strain relationship and Shear strength of soils

Welcome to lecture series on Advanced Geotechnical Engineering. We introduced ourselves to Module 4 which is on Stress-strain relationship and shear strength of soils.

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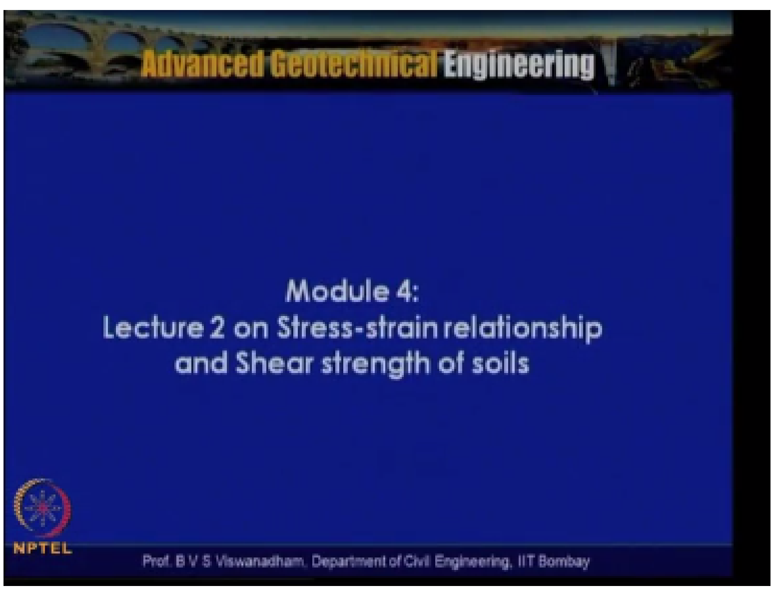

And this is lecture 2 in Module 4. So we have discussed about the contents and then Introduction to the Mohr's Circle Analysis. So in this lecture, we will try to do a detailed discussion on the

Mohr's Circle Analysis and its applications and determination of the pole in the Mohr's Circle and how this can be used to find out the stress on any given plane within the element.

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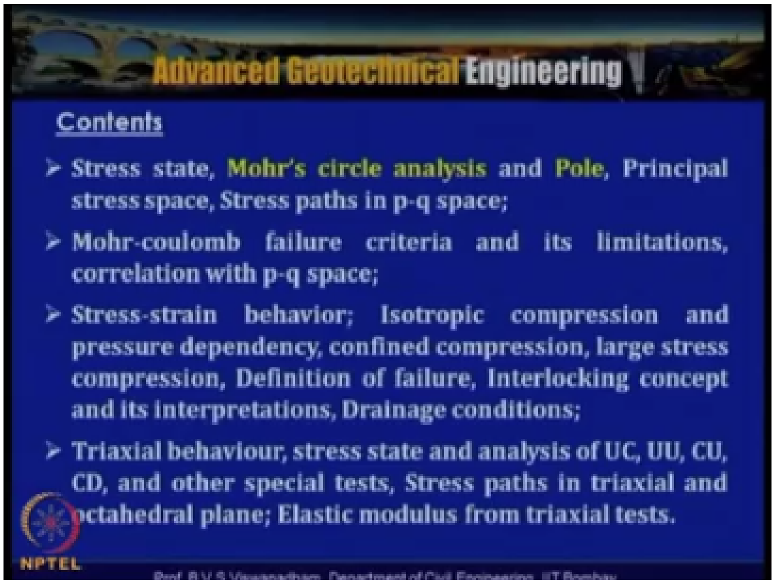

And thereafter we will introduce ourselves to the principle stress space and stress paths in p q space.

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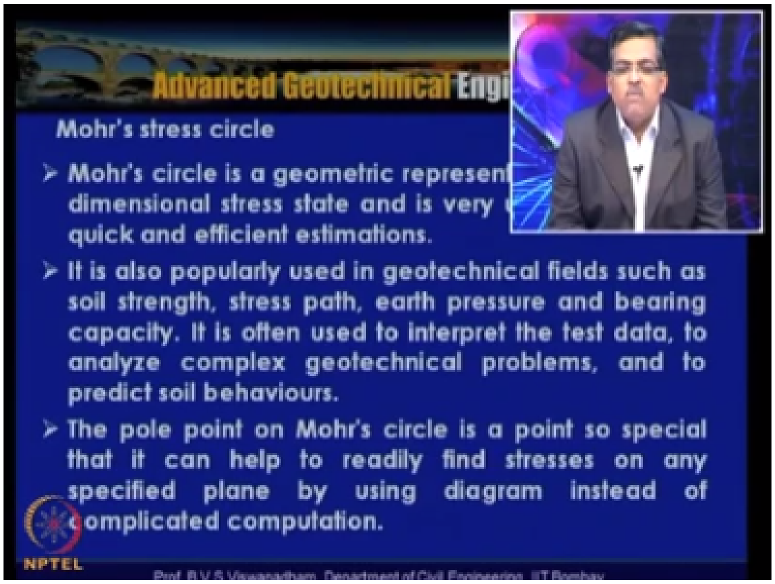

So as we have discussed, the Mohr's Circle is a geometric representation of the two dimensional stress state and is very useful to perform quick and efficient estimations, and it is also popularly used in geotechnical fields such as soil shear strength that anyhow we are going to discuss and earth pressures, varying activation of active conditions and passive conditions are discussed and also in the bearing capacity theories.

So it is often used to interpret the test data and analyse the complex geotechnical problems and predict the soil behaviour and in these the pole point on the Mohr's circle is a point so special that it can help to readily find the stresses on a specified plane by using the diagram instead of complicated computations.

So the topic which we are going to discuss the pole point on Mohr's circle is a point so special that it can help to readily find stresses on any specified plane by using diagram instead of complicated computation. Now coming to the pole points, a pole is unique point located on the periphery of the Mohr's circle.

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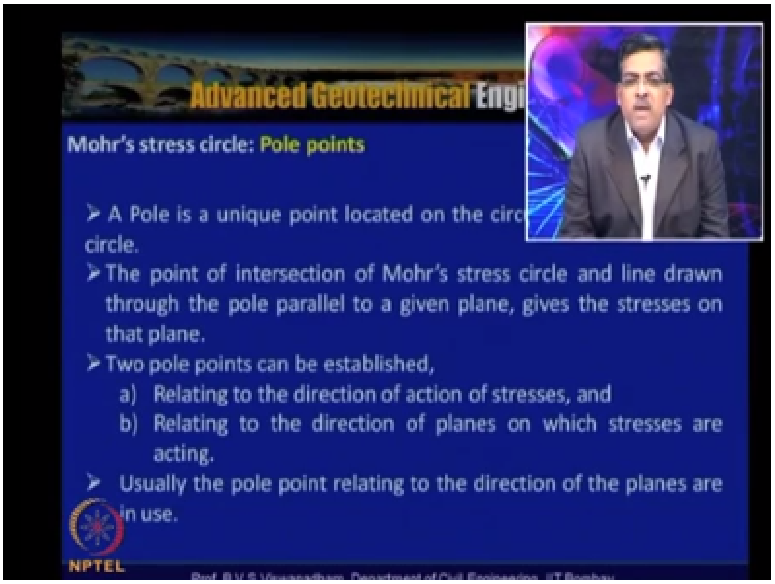

The point of intersection of Mohr's stress circle and line drawn to the pole parallel to a given plane gives the stresses on the plane. So this pole point is also called as the origin of planes. So there are two ways the pole points can be established. One is by relating it to the direction of action of stresses. The other one is the relating to the direction of the planes on which the stresses are acting. Suppose if there is a horizontal plane, if there is a normal stress acting on that.

Then we are referring a horizontal plane and if there is a vertical plane on the element and the normal stress is acting on that then that is also what we are actually referring as the direction of the plane on which the stresses are acting. Usually the pole point relating to the direction of the planes are in use. The direction of the planes whether it is horizontal or vertical is in use.

Now here consider the procedure to establish a pole point for stresses. So what we need to do is that we have to draw the Mohr's circle on the tow sigma space.

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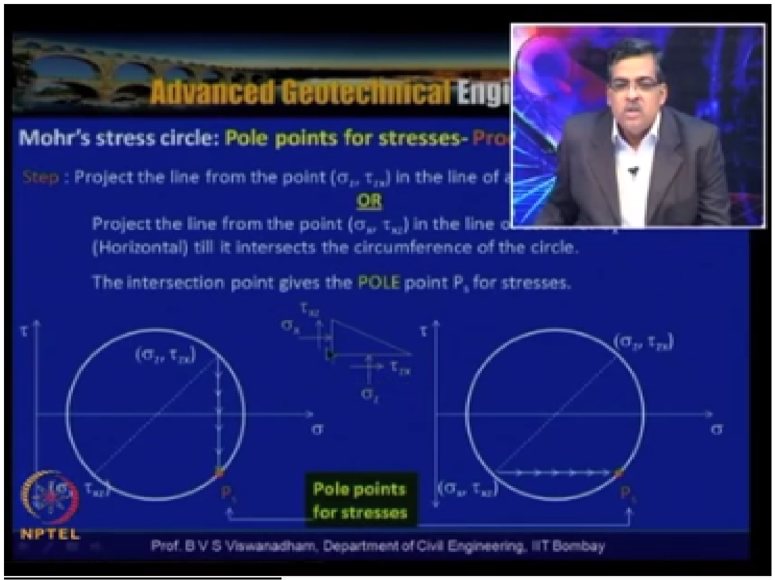

By knowing the sigma z and tow zx and sigma x and tow xz. So these are actually sigma x is normal to the vertical plane, you can see here and the sigma z is normal to the horizontal plane that is sigma z here and this is tow zx.

So if you look into this, the centre of the element if you take moments, we get that tow xz = tow zx because we have to maintain the equilibrium of the element from the moment equilibrium point of view. So passing the line by knowing these two points, draw the Mohr's circle on the tow sigma space then from this point sigma z, tow zx is acting in the direction of the vertical direction, drop a line wherever it actually cuts the Mohr's circle and that is actually pole point from the stresses.

So in this direction if you are able to cut then this is actually pole point from the stresses. Similarly next if you look into this, from here when you have the sigma x which is acting in the horizontal direction, but it is acting on the vertical plane then you actually get a point here which is again same PS. So this is actually pole point from the stresses.

The projection of the line from the point sigma x tow xz in the line of action of sigma x that is horizontal till it intersects the circumference of the circle, or projection of the line from the point sigma z tow zx in the line of action of sigma z vertical but both actually gives the pole point with reference to the for the one method of establishment that is actually called pole point for stresses.

The pole points for the planes that is that, the projection of the line from the point sigma z tow zx, so here you can see that by following the usual procedure we actually have drawn the Mohr's circle and here the sigma z and tow zx and sigma x tow xz. So from this point, so this sigma z is actually acting which is perpendicular but it is acting on the horizontal plane.

So in the direction of this plane draw a line which is parallel to this horizontal plane and till it intersects the Mohr's circle and that point is regarded as the pole point. The intersection point gives the pole point PP for planes. So what we can notice here is that, this is the point pole PP and from the definition of pole point from the planes, similarly when you look into this sigma x tow xz here, so here this is sigma x tow xz is, this plane is vertical here.

So in the direction of this vertical plane, draw the line till it actually intersects the point here. Then so if you again when you draw in this horizontal direction or in this direction, you actually have a point here and from this point, when you put this, this is again the pole point from the planes.

Now when we join this point with the major principle stress here, sigma 1 by intercept with sigma x is at tow=0 is the major principle stress and here minor principle stress that is sigma 3 which is here. So when you draw a line like this, when you draw a line from here to here and on this plane there is minor principle plane is acting. On this plane now the major principle plane is acting.

Now if you are actually trying to find out, wanted to find a stress on a particular plane within the element, let us say an angle alpha within the element, that can be found out by just drawing or dropping a line such that you can actually get these stresses that is sigma and tow, at this particular any element state here. So with that we will be able to find out the stresses in the elements, the stress states in the elements at any given plane within the element.

So in the Mohr's stress circle, when we said that pole points for the planes and which is actually usually adopted wherein what we have to draw, how to establish a pole point is that, project the line from the point sigma z tow zx in the direction of the plane on which these stresses are acting till it intersects the circumference of the circle and intersection point gives the pole point PP for planes.

Now what we have further discussed is that when we join this with the major principle stress magnitude here and this plane is nothing but the major principle plane on which the major principle stress is actually acting and here this is minor principle plane. So these principle planes are the planes on which the shear stresses are zero. Shear less planes are called principle planes and we have major principle plane and minor principle plane in which actually the two dimensional case where by ignoring the intermediate principle stress, what we say is that major principle stress and minor principle stress on principle planes can be identified.

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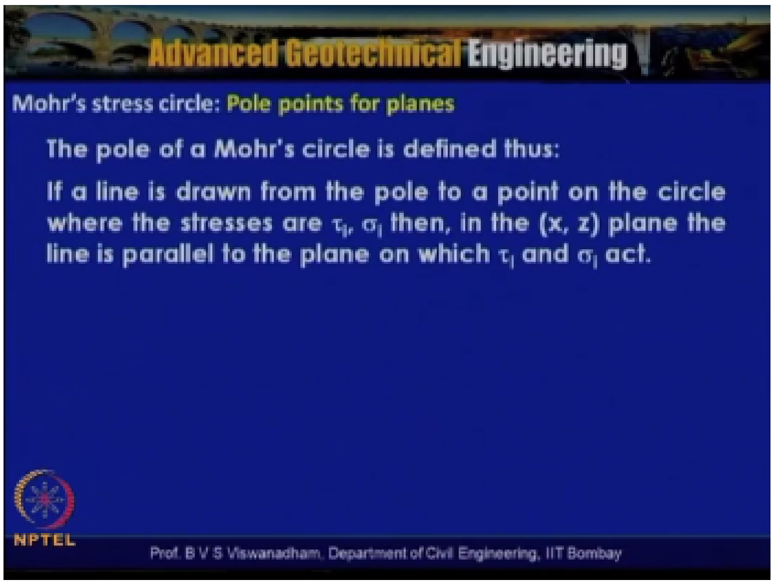

Now the pole of the Mohr's circle is defined as thus now if a line is drawn from the pole to a point on the circle where the stresses are tow i and sigma i, then in that XZ plane the line is parallel to the plane on which the tow i and sigma i act.

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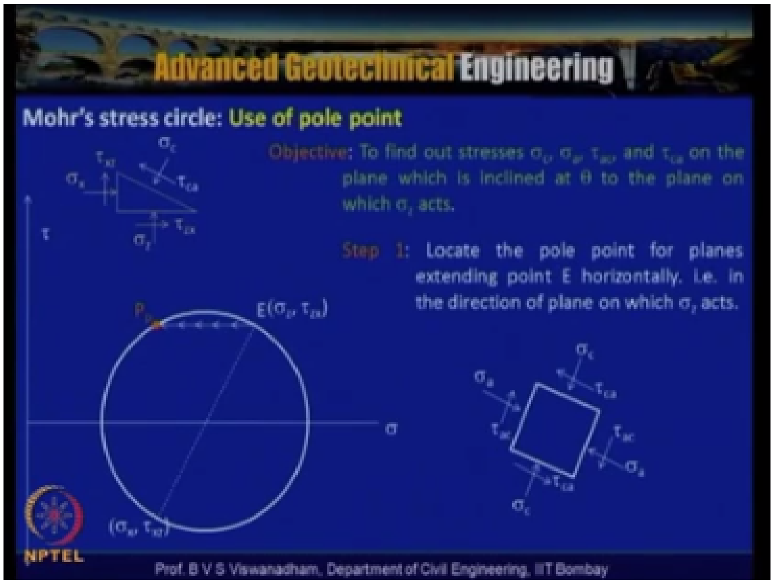

Now use of the pole point if you look into is, basically to find out the stresses sigma c, sigma a, tow ac and tow ca on the plane on which is inclined at theta, to the plane on which sigma z acts. So now we can see that, this is sigma z is actually acting on this plane and sigma x is actually acting on this vertical plane and there is tow zx is acting here and tow xz is acting here.

So in the step 1, locate the pole point for the planes extending point a horizontally. So what we do is that, we locate the pole point that is we know that the stress state here is that sigma z tow zx and sigma x tow xz. Construct the Mohr's circle with this as the diameter and from this the sigma z is actually acting on horizontal plane. So extend this line, project this line till it intersects the periphery of this circle, that is the circumference of the circle, then that point is regarded as the pole point PP.
Now what we do is that, locate the pole point for the planes extending point E horizontally in the direction of the plane on which the sigma z is actually acting, that is horizontal, so we have drawn the horizontal line.

Then, what we have to do in step 2 is that draw a line parallel to the plane with sigma cx. So now draw a line parallel to the plane on which sigma cx, this sigma c is actually acting at an angle, let us say that certain inclination. So draw a line parallel to this line and here we draw a line. So extend a line from point D through the centre of this circle till it intersects the circle.

So from here draw this circle so that you get these sigma a and tow ac that is on these stress states and then we get sigma c and tow ca. So what we have drawn is that, we have drawn a line parallel to this plane on which the sigma c is acting. Now what we have established is that, we established element at the D and D' here and these are the stresses at this state on the sigma c and tow ca which are nothing but here and here it indicates that sigma a, and τ ac.

So when we join, let us say again here to the major principle stress here minor principle stress, then this will become the major principle plane and this will become the minor principle plane.

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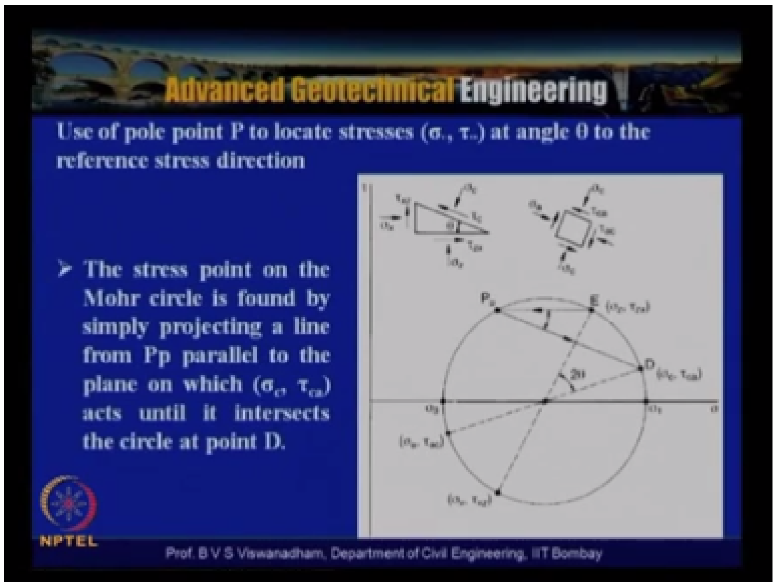

Now this is actually shown diagrammatically here once again, use of the pole point to locate the stresses sigma and tow at an angle theta to the reference stress direction. So here what we have done is that we have got Mohr circle on the tow sigma space and we with sigma z tow zx which are actually acting here and sigma x and tow xz by following the sign conventions which are defined yesterday we actually defined in the previous lecture, we can actually draw, indicate this in tow sigma space and draw a Mohr's circle with this as the diameter.

Then this sigma, wherever it intersects the sigma axis, this will become the major principle stress that is measured from here origin to this point and this is the magnitude that is measured from here to here that is sigma 3. Now the stress point on the Mohr's circle is actually now from here we draw a line parallel to this, so you actually can locate this pole that is PP, or you can draw a line parallel to this plane, so you this vertical plane you will actually again get from either from this or this, establish the origin of the planes.

Then from here what we need to do is that, we wanted to determine the stresses sigma c and tow ca at an angle theta, so draw a line parallel to the plane on which sigma c and tow ca are acting. So with that what we get is that, we get this stress states like sigma c from after locating D by drawing an angle theta, below with PPE.

Then drawing D and D' passing through the centre of the Mohr's circle, we actually get the stress state sigma a tow ac, that is sigma a tow ac and then we get sigma c and tow ca. So by using this procedure, like use of pole point p to locate the stresses sigma c and tow ca at an angle theta to the reference section is demonstrated in this particular slide.

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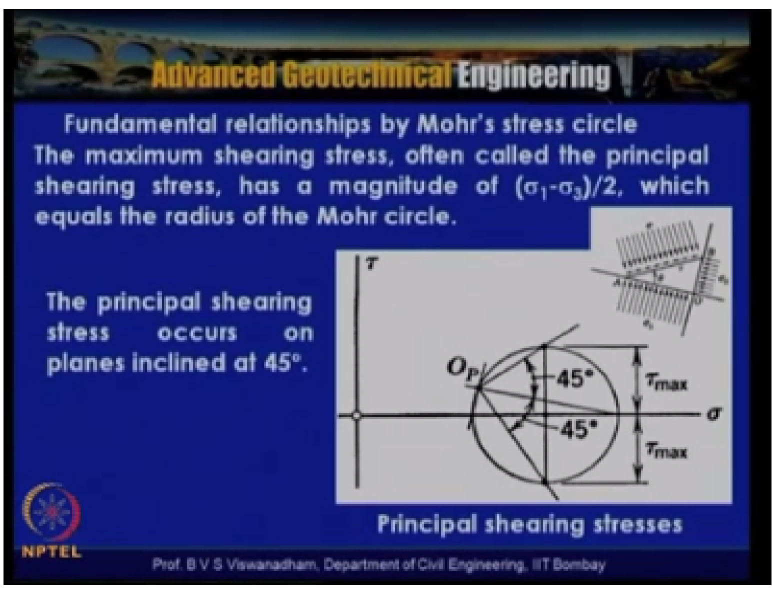

Now the fundamental relationships by the Mohr's circle can be discussed. So here tow sigma in space in Mohr's circle which is actually shown here where you can see that the stresses are actually acting sigma 1 and sigma 3 are acting on this, sigma 1 is actually acting on plane OA and sigma 3 is actually acting on plane OB and which is inclined at an angle theta. So what we need to do is that again by doing sigma 1 and sigma 3, we can draw the Mohr's circle and from here to locate PP or OP that is the pole, from here draw the line parallel to this plane on which this is actually acting.

So this is inclined at theta. So draw a line. So from here what we can actually draw is that, if we look into this and the shear stress will be maximum here that is tow x = sigma 1 and sigma 3/2 which is nothing but the radius of this Mohr's circle and this point is sigma 1, measured from here to here and this point is sigma 3 the minor principle stress. So what we get is that, this is the major principle plane and this is the minor principle plane.

The principle shearing stress is actually occurring at that line joining this point to the point where the major shear stress occurs, that is 45 degrees. So we have tow max which is on the upper portion of the Mohr's circle and the bottom portion of Mohr's circle. So the maximum shear stress often called the principle shearing stresses has a magnitude of sigma 1 – sigma 3/2 and which equals the radius of the Mohr's circle. So the maximum shearing stress often called the principle shearing stress has a magnitude of sigma – sigma 3/2 which equals the radius of the Mohr's circle.

So the principle shearing stresses occur on planes inclined at 45 degrees. Now another aspect is that, the conjugate shearing stresses. That is that the shearing stresses and planes at right angles to each other are numerically equal but are opposite side. So numerically equal because the element has to be maintained equilibrium by t.

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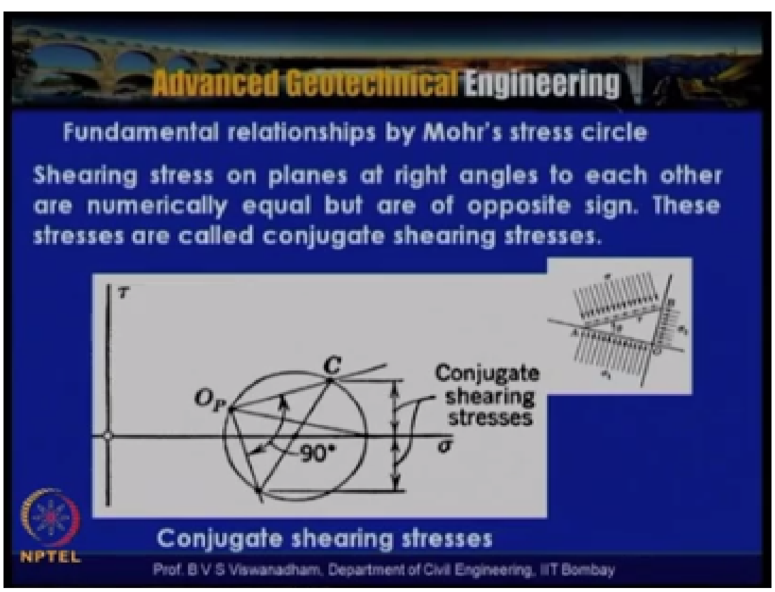

If you take a moment about the centre of the element we get tow zx=tow xz or tow xy=tow yx or tow yz=tow zy. So these stresses are called conjugated shearing stresses. So here the conjugate shearing stresses are represented here. So we can see that, if you are having these shearing stresses at this and at this point, so we call these are, which are on the upper, they are equal in magnitude but only thing is that they are opposite in the sign. So one is positive, other one is negative here. So this is indicated here. These two are called as conjugating shearing stresses.

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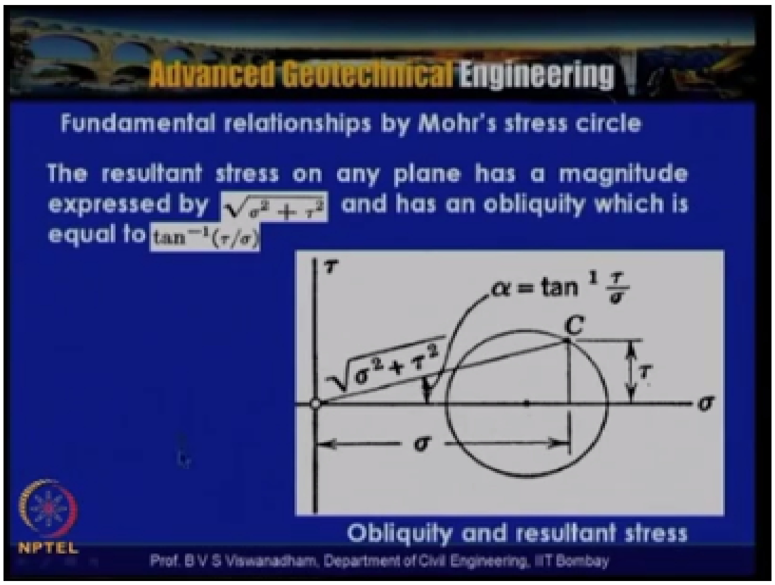

Now the other aspect is that, what is the obliquity and the resultant stress? So the resultant stress on any plane has a magnitude expressed by, we have an element, let us say sigma n tow, so the resultant is actually nothing but root over sigma^2 + tow^2 at any particular point on the Mohr's circle and has an obliquity which is equal to tan inverse of tow/sigma.

So if we look into, let us say that this is the point C on the Mohr's circle which we are referring, now what we do is that from the origin that is where, from here to this point, so this is actually called as the maximum obliquity angle which is nothing but alpha = tan inverse tow/sigma, this is tow and this ordinate is sigma and this magnitude is nothing but root over sigma^2 + tow^2 which is the resultant. So the angle of obliquity is nothing but alpha = tan over tow/sigma.

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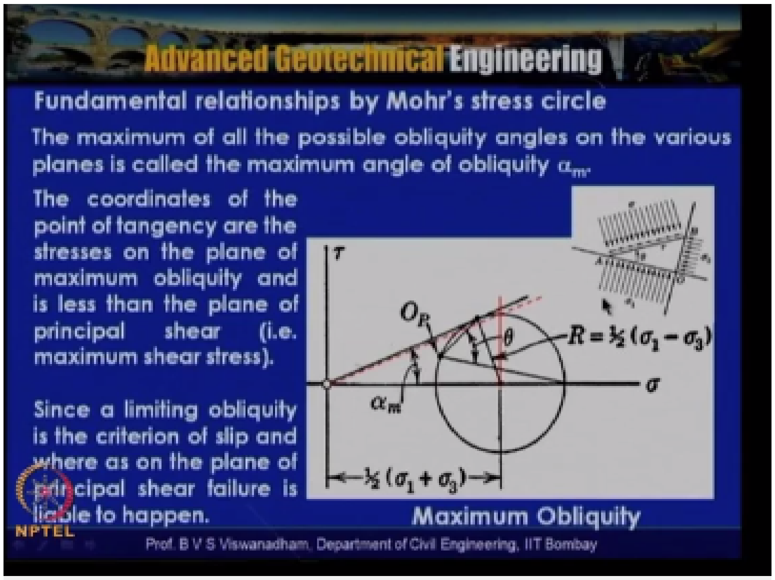

Now further if you look into it, in this particular slide where it has been shown that we have got in the same situation that Mohr's circle, but here if you look into this here, when indicate this sigma 1 and sigma 3 acting at an inclination theta. So what we have drawn is that we have located the pole and from here what we have drawn is that we try to determine the stresses on any plane at inclination theta, so that is this point, point where it actually touches the, the point where it is tangential to this line.

When you draw from the origin, and if the point where it actually gets tangential and that point and this point when you join and when you look into that this is also, so this angle is actually called as, this angle joining from this point to this point is actually called as the maximum angle of obliquity. So the maximum of all the possible obliquity angles on various planes is called maximum angle of obliquity, which is alpha M. The coordinates of the point of tangency or the stresses on the plane of maximum obliquity and is less than the plane of principle shear.

So if you look into this, the principle shear is nothing but the maximum shear stress. So here the maximum shear that means that the failure is most likely to happen, but when you say that, limiting obliquity is that it is the best criterion which is used to initiation of the slip, indicate the initiation of the slip. So the maximum obliquity is less than the plane of the principle shear because since the limiting obliquity is the criterion of the slip and whereas in the plane of principle shear is actually liable to happen.

So the coordinates of the point of tangency are the stress on the plane of the maximum obliquity which is actually the angle which is joining to the maximum shear stress is actually less than this maximum angle of obliquity. So if you look into this, this is not the maximum shear stress, the maximum shear stress is here. So here at the point of maximum shear stress with radius r = sigma 1 and sigma 3/2 the failure is actually most likely to happen.

But here, this point which actually indicates the maximum limiting obliquity angle is basically is the criterion which is actually specified for initiation of the slip and whereas in the plane of principle shear is actually is liable to happen.

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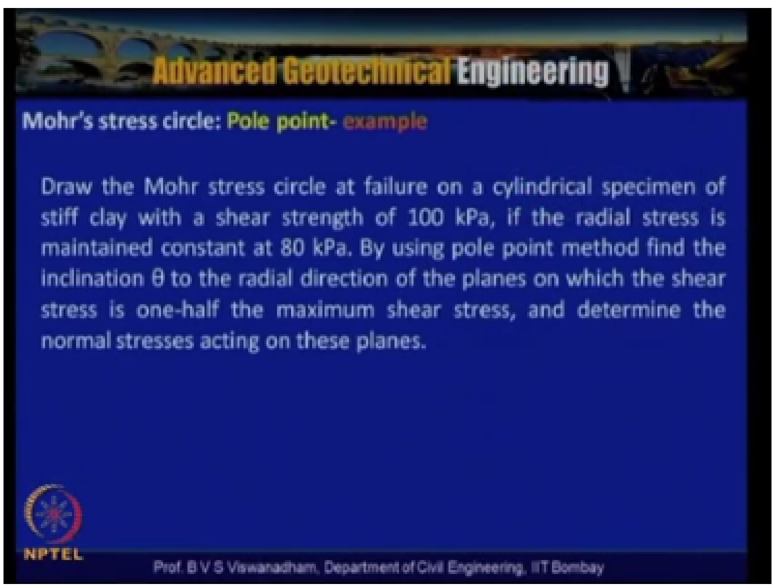

So now consider an example where you may need to draw the Mohr's stress circle at failure on a cylindrical specimen of stiff clay with a shear strength of 100 kPa, and if the radial stress is obtained constant at 80 kPa, by using the pole point method find the inclination theta to the radial direction of the planes on which the shear stress is one-half of the maximum shear stress and determine the normal stresses acting on these planes.

So we need to determine the magnitude of the normal stresses acting on these planes and the condition which is given here is that find the inclination theta to the radial direction of the planes on which the shear stress is one-half of the maximum shear stress.

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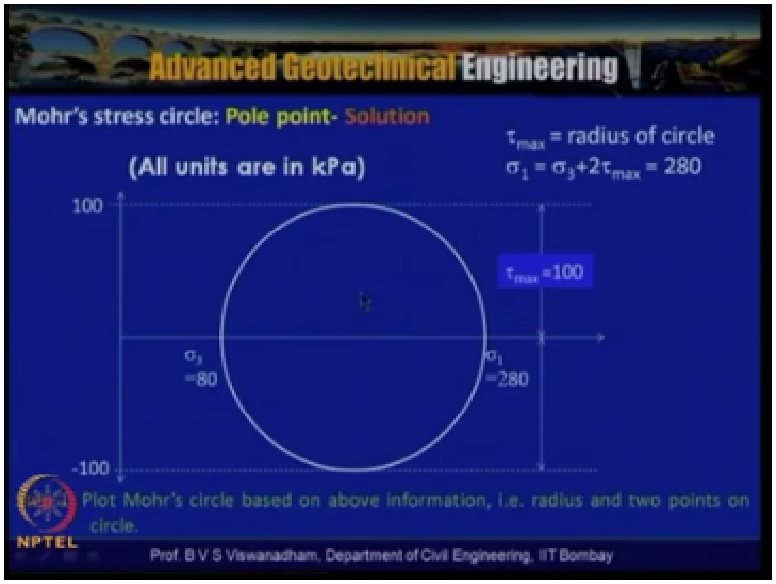

Now what has been given is that, the maximum shear strength which is nothing but 100 kPa, so that means that the maximum ordinate is actually sigma – sigma 3/2 that is tow max=100, so with this as radius sigma – sigma 3/2 as the radius, draw the Mohr's circle on tow sigma space, so then we have got the maximum shear stress with 100 kPa here minus 100 kPa here and the circle in 1, it actually intersects with sigma axis at sigma 1 it is 280 kPa and sigma 3 is 80 kPa.

So tow max is the radius of the circle, so sigma 1 = sigma 3 + 2 tow max, so tow max is actually given as 100, so with that what we can actually find out is that sigma 1 so which is nothing but 280. So the diameter of the Mohr's circle is nothing but 280-80 that is 200 kPa, the radius is nothing but tow max = the sigma 3/2 that is 100 kPa.
So the plot, Mohr's circle based on the above information and radius in two points on the circle. Once we do that, now the condition has been given is that 50% of maximum shear stress, draw a line.

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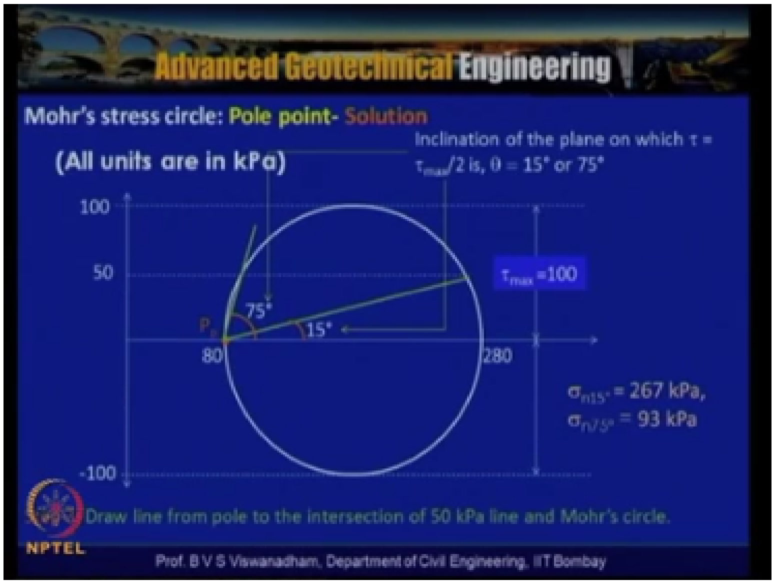

That means that when you are actually doing draw a line which is intersecting the Mohr's circle, so draw a line at tow=50 kPa which is half of tow max. Now as the principle stresses are acting on edges on the sigma 3, so here this plane is actually horizontal because the cylindrical sample, so sigma 1 is acting on the horizontal plane, sigma 3 is acting on the minor principle plane.

So this itself will become like a, PP is nothing but the pole here. So from here draw the line which intersects at point where sigma 3 is identified and that is pole point PP. Now from here what has been asked is that the inclination of the plane on which tow = tow max/2 which by drawing a line from here, this point is indicated by the condition which we have given and with that we can actually get on this 15 degrees and at this particular point.

We can actually get sigma n 15 as 267 kPa and another ordinate is nothing but 50 kPa, then you can see that when we draw a line at 75 degrees, now not possible because this point actually intersects at these two points, so when you draw the line here, from these 75 degrees with horizontally, it is at 75 degrees with horizontal and this magnitude at this stress sigma 1 75 degrees indicates that 93 kPa. So draw the line from the pole to intersect the intersection of 50 kPa, a line on the Mohr's circle.

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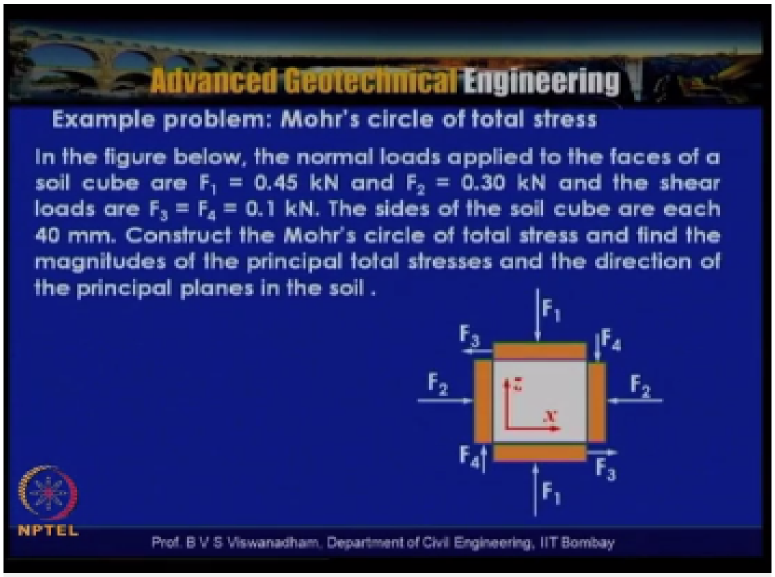

So what we have actually based on the given data, we try to establish and determine the normal stresses acting on these planes.

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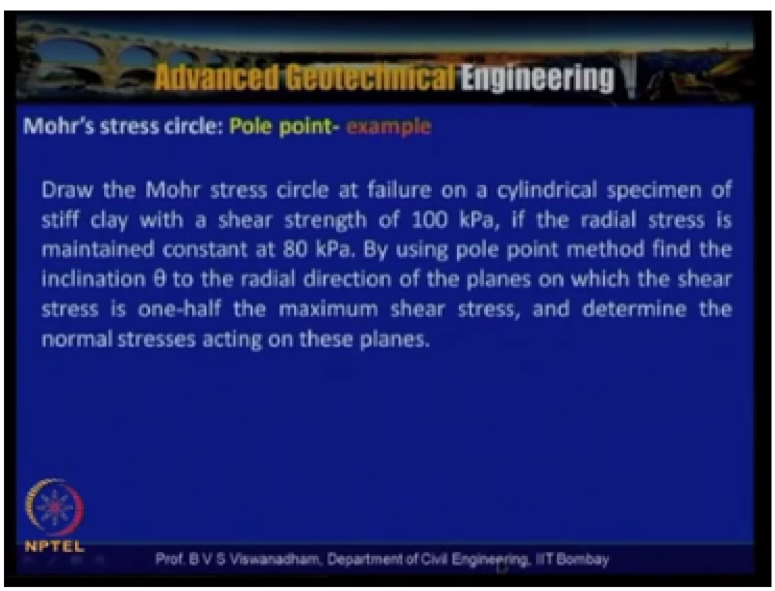

Now here with the given information two planes are possible so we could actually get for this same shear stress two different normal stresses can be seen. One is that plane inclined at 75 degrees, the other one is actually at the 15 degrees.

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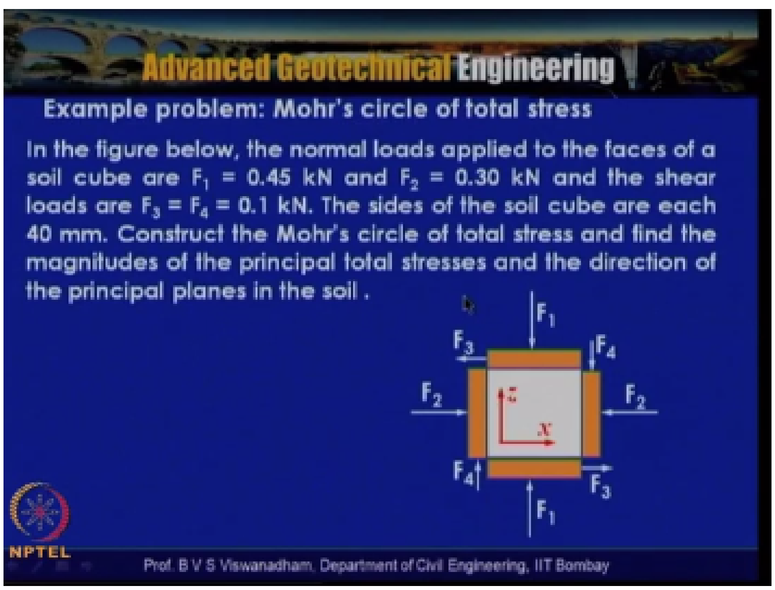

Now consider an example problem where the Mohr's circle of the total stress and element having a size of 40mm/40mm is a cube. So a two dimensional state has been actually represented here and the x axis and z axis are actually shown within the element and for estimating the shear forces and all, with muscle color filled rectangles are actually shown basically to show, you can see that this is conjugate shearing stress shear force F4, and this is the conjugate shear force F3.

So here the element is actually subjected to shear like this and these are the normal stresses and these are the normal forces F2, and this is actually normal force F1.So in the given figure, the normal loads are applied to the faces of the soil cube having the 40mm/40mm/40mm dimensions and F1 is 0.45 k Newton, F2 is 0.3 k Newton and the shear loads are F3=F4=0.1 k Newton's. The sides of the soil cube are each one is actually having 40 mm. To construct a Mohr's circle of the total stress and find the magnitude of the principle total stresses and the direction of the principle planes in the soil.

So what we have been asked is that, we have been given forces, so each area of the face is nothing but 40mm/40mm in meters it is when you convert into millimeters into meters it is area of each phase is 0.016 meter^2, so by putting let us say, F1 divided by that area, we will get the stress sigma Z and sigma X is nothing but F2 by this area then we can get stress F4 on the plane divided F4 divided by that area and similarly F3 divided by this area you will get the shear stress on the sample.

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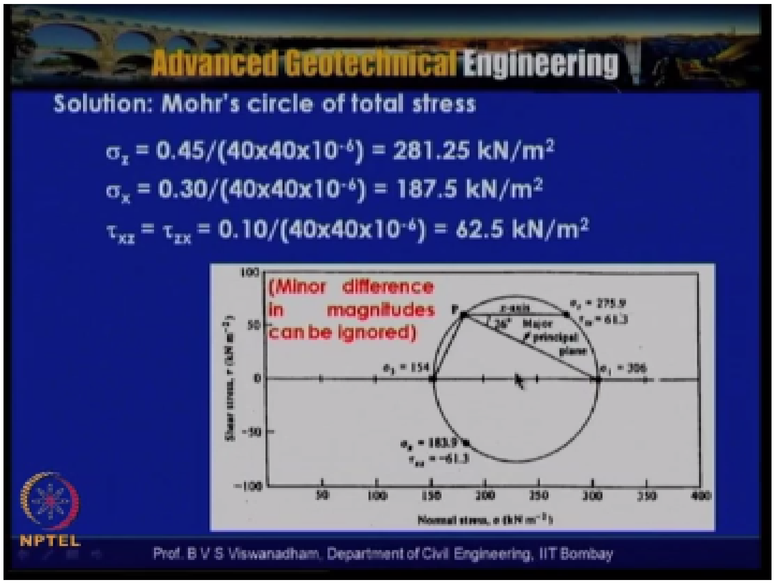

So now we do is that as we discussed we determined sigma Z which is 281 km/m2 and sigma X is 187. 5 km/m2. Now tow zx = 0.1 divided that is due to F3 and F4. So with that we have got the data, which is given based on the given data we establish sigma Z and sigma X and the tow xz and tow zx which are actually acting on the element. Now what we have done is that by knowing the stress rates here that is the magnitudes are little bit different, but however they can be good and so here with this stress stated like 281.25 km/m2 and 62.5 km/m2 and this point, as well as this point by the appropriate using the sign convention.

We can actually draw the Mohr circle and on the tow sigma space which is drawn to the selected scale, then what we can do is that from this because it is actually acting on horizontal plane, so draw a line to locate the pole, so this is the point P, which is the pole. Now this is the major principle stress, the ordinate is with the given dimensions which are given in the figure.

The sigma 1 magnitude is 306 kPa and sigma3 is 154 kPa, which is measured from here to here and which is measured from here to here, so the line joining this when you draw a line to this one, this is the major principle plane, which is inclined at 26 degrees with the horizontal line, which is drawn within the Mohr circle to locate the plane and this is inclined at 90 degrees plus 26 that is about 116 degrees, 90 plus 26 and this is nothing but the minor principle plane.

So in the given problem we have been asked to calculate the magnitude of the major principle plane and minor principle planes and also the direction of the planes which are indicated in this particular problem, which is the solution, which is actually given for the problem.

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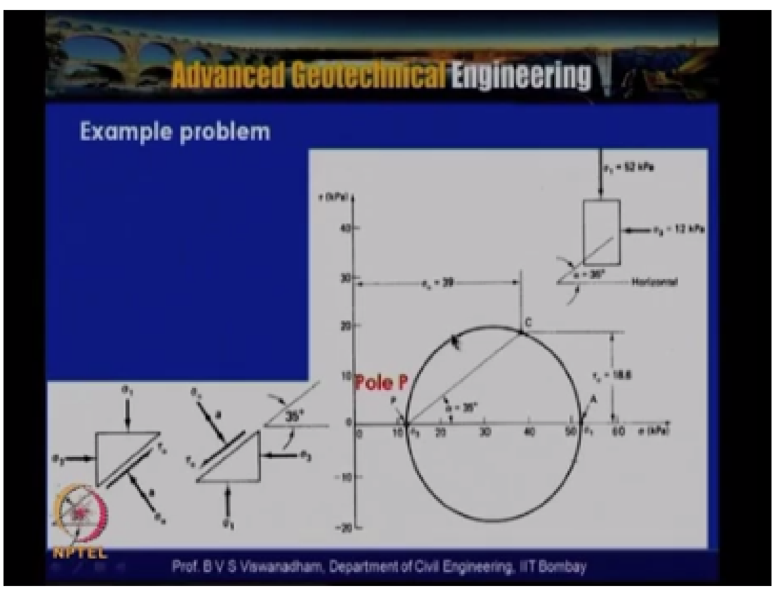

Now, another example here if you look into it, there is an element which is actually subjected to a cylindrical element, subjected to the minor principle stress is 12 kPa and major principle stress is 52 kPa. You can see that these elements are not having shear stresses so hence they can be called as shear less planes, so this is a major principle stress and minor principle stress. The major principle stress is actually acting on the horizontal plane and minor principle stress is actually acting on the vertical plane.

So as can be noted here, the sigma 1 with the sigma 1, 52 kPa and sigma 3, so with 52-12 that is about 40 kPa as the diameter draw the Mohr circle. Once you draw the Mohr circle, so this is the point A which is on the sigma axis which is 52 kPa is actually indicated here, this is the minor principle stress, so this is actually acting on horizontal plane, so from here draw a line to locate the pole P, so this is the pole P.

Now what we need to do is that we need to let us say that the angle alpha is 35 degrees, so on this angle 35 degrees, when you draw from this point to this point, so point intersects at C, so at point C wherever it actually intersect the Mohr circle, we can actually interpret the sigma A and tow A, so that is nothing but 39 kPa and this is 18.6 kPa.

So what we have actually got from this problem is that we actually have for a given plane, which is actually inclined at alpha is equal to 35 degrees within the element, so first we have drawn the Mohr circle based on the information, which is given that is sigma 1 is given 52 kPa, sigma 3 is given 12 kPa.

So with 40 kPa as a diameter, we have drawn a Mohr circle and then we have identified the pole P and from the pole P, we can actually draw a line at 35 degrees, suppose if it is let us say 60 degrees then that line will be located at this point. This is the point where you can actually get the stresses in the element at the particular plane of inclination, and another important point is that in this the major principle plane is this and minor principle plane is actually indicated by a line perpendicular to this major principle plane, so major principle plane is horizontal here and minor principle plane is here, which is perpendicular to the major principle plane.

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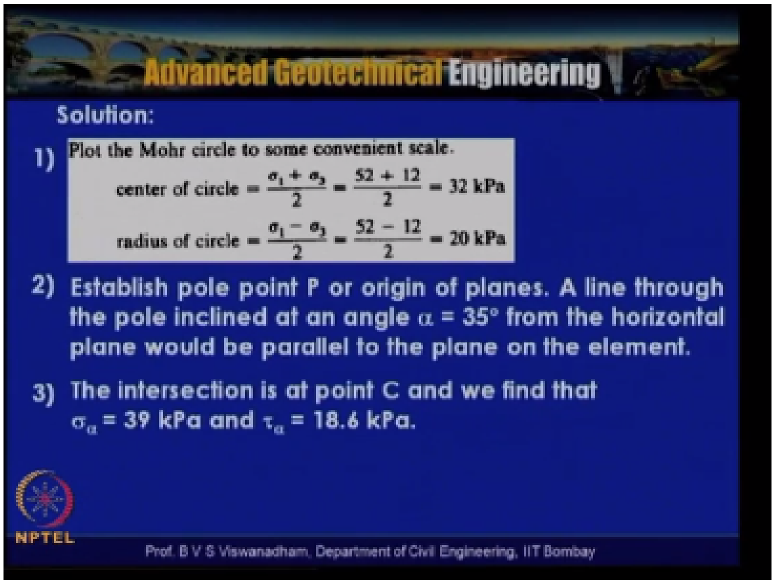

Now the same procedure is actually explained here, plot the Mohr circle to some kinetic scale and establish the pole point or the origin of the planes. A line through the pole inclined at an angle alpha is equal to 35 degrees from the horizontal plane would be parallel to the plane of the element and that is of interest and the intersection is at point C and from there we can calculate the sigma alpha, which is 39 kPa and tow alpha is 18.6 Pascal's which are actually acting along the plane, which is inclined at 35 degrees with the major principle plane.

Now consider similar example, only thing is that in this particular case, the element in space is inclined at 20 degrees that means that the element is actually considered within the space with 20 degrees inclination with the horizontal, so the element under same stresses have been considered.

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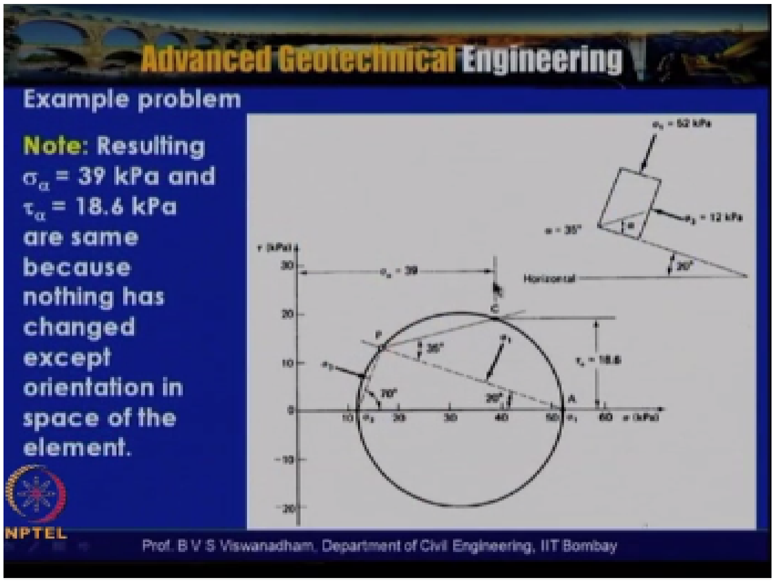

But only thing is that this element in space is actually subjected to an inclination of 20 degrees and again we are interested this along this 35 degrees plane what are the state of the stresses? So with the given information now with 52 kPa and 12 kPa, now draw the Mohr circle so what we get is that we draw the Mohr circle. Now what will happen is that because the inclination of the plane on which the major principle stress is actually changed now, so because of that so parallel to this plane and the location of the pole point changes.

Previously, the pole point was here when it was horizontal now it got lifted up so you can see that draw a line from the P and where it actually intersects, so the pole is actually located here, so from here if you put this 35 degrees then again it will intersect at sigma energy equal to 39 kPa and tow energy is equal to 0.6 kPa so this sigma 1, this is the major principle plane now and this is the minor principle plane with magnitudes which are actually given like 12 kPa and 52 kPa, so this the major principle plane and this is minor principle plane.

So the resulting sigma alpha is equal to 39 kPa and tow alpha is equal to 18.6 kPa are the same because nothing has changed except the orientation of the space of the element, so when you have the orientation of the element is actually changed within the space then the internal stresses will not change but the pole point locations and the inclinations of the planes depend upon on the orientation of the element is under consideration.

So now we will take another example wherein the stresses are actually shown in the figure below.

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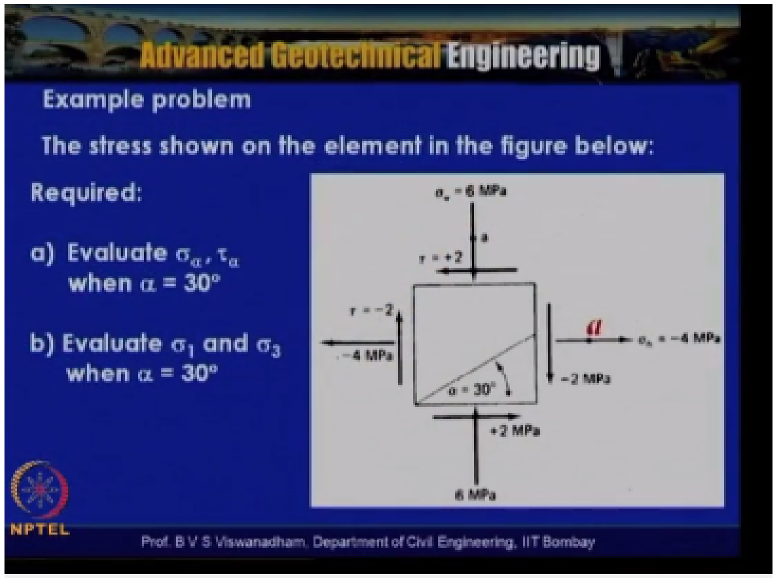

Wherein you can see that this is an element and which is subjected to, this is the horizontal plane where sigma V is equal to 6 mega Pascal's, which is actually subjected here and we have got tow is equal to 2 mega Pascal's and this is minus 2 mega Pascal's and this element is under tension and this is actually the shear stress, which is minus 2 mega Pascal's here and minus 2 mega Pascal's here.

So we have the, in the vertical direction it is compression and the latter direction it is, we have the element under tension. So what we need to determine is that evaluate the stresses sigma alpha and tow alpha when alpha is equal to 30 degrees within the element and as shown in the element and evaluate sigma 1 and sigma 3 when alpha is equal to say 30 degrees, so we need to calculate what is the location of the maximum principles stress and minor principle stress.

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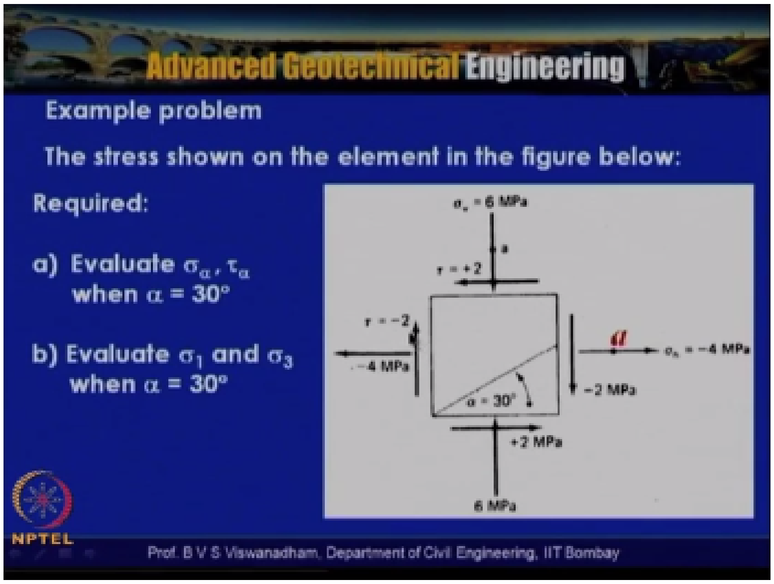

So the procedure is that plot the state of the stress on the horizontal plane that is 62 at point A and note that the shear stress makes a clockwise movement about A, so the shear stress is actually making clockwise movement about A, so from the consideration which we have defined in the previous lecture, shear stress makes a clockwise movement about A, so therefore when the shear stress is actually making clockwise movement then it is actually regarded as positive.

And if you look into this here, the A is a point outside the element, so about this if it actually makes a clockwise movement it is regarded as positive, so in this case if you consider element A it is actually making clockwise movement, but when you take this particular the shear stress is actually counter clockwise movement about this point within outside the element, so this is actually regarded as negative.

So this point is actually is negative and again here this is making clockwise movement so this is actually positive, so these two shear stresses are positive and these two shear stresses are negative.

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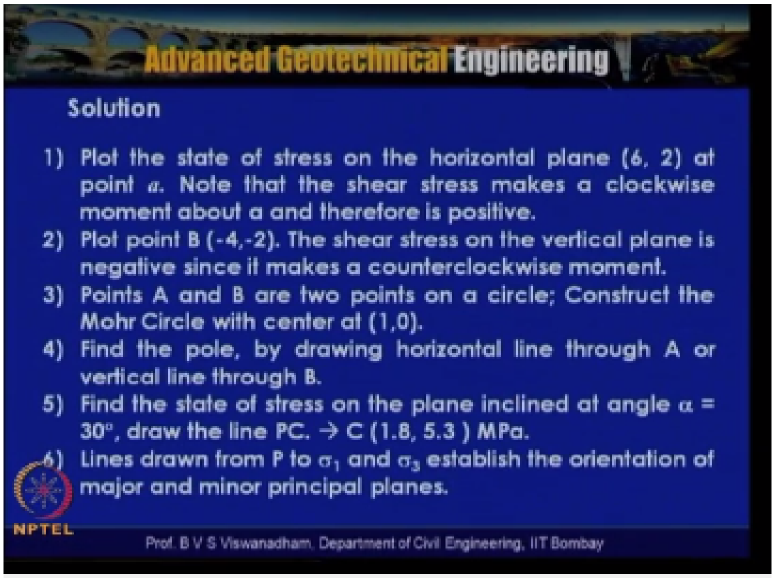

So plot the point B minus 4 minus 2, the shear stress on the vertical plane is negative since it makes counter clockwise movement, so clockwise shear stresses are positive and counter clockwise shear stresses are negative, so point A and B are two points on the circle, so construct the Mohr circle with center at 10 that is 1 mega Pascal's and zero ordinate.

Find the pole by drawing the horizontal line through B or vertical line through B and find the state of the stress on the planes inclined at alpha that is 30 degrees and draw the line PC. So this is actually explained here.

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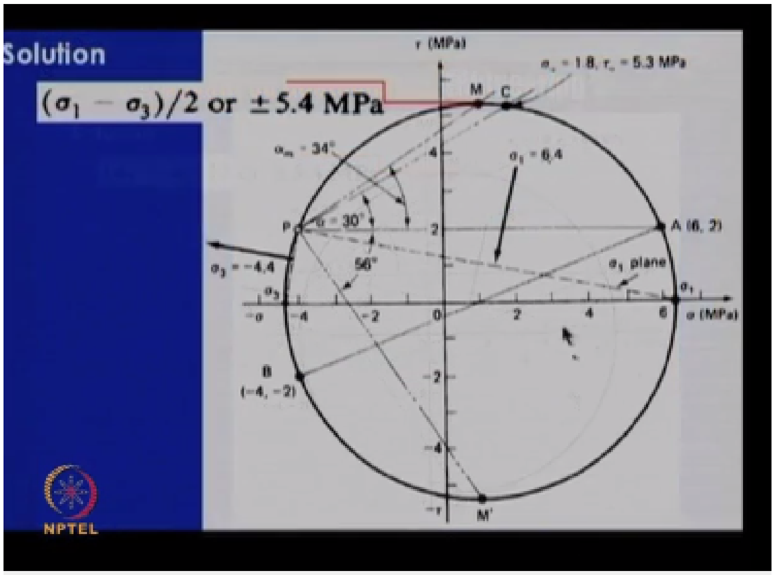

So what we have done is that we know the stress rates upon the defined sign conventions. We have located the A that is 62, which is written here. Plot the state of the stress on the horizontal plane 62 at point A. Note that the shear stress makes a clockwise movement about A and therefore it is positive, so that we have located and now to locate point B what we have said is that plot point B minus 4 minus 2. The shear stresses on the vertical plane are negative, since it makes a counter clockwise movement, so we have located here.

So with this as the diameter and with center as 10, we draw a Mohr circle on the tow sigma space. So now you can see that the origin is here and then the Mohr circle actually I had attended to the native portion also.

Now what next step we need to do is that after having drawn the Mohr circle find the pole by drawing the horizontal line through A, so draw from this known stress state here, draw the line horizontal because the known stress state is here, draw the line parallel to this plane, so it intersects the Mohr circle at point B or you can a draw a vertical line parallel to this plane on which the stresses are acting, so it intersects at P, so this is the pole P, so from here, what we have been asked is that calculate the stresses at an inclination 30 degrees within the element, so from P with PA.

Draw a line which actually intersects with PC, so the point C is nothing but we get the stress state that is sigma N is 1.8 mega Pascal's and the tow A is actually 5.3 mega Pascal's, but if you look into this one, this particular portion M, which is sigma 1 and sigma 3/2, which is nothing but plus or minus 5.4 mega Pascal's and which is M and M- we actually have got these points here, this is the maximum shear and this is also a point of maximum shear.

So this inclination of PM is 34 degrees and magnitude of this actually about 5.4 mega Pascal's. Now the next issue is that to draw a line passing through this point, this is nothing but the sigma 1 and this is sigma 3, so with this we can actually find out what is the magnitude of the sigma 1, so this sigma 1 magnitude is at 6.4 mega Pascal's and this is nothing but in tension that is sigma 2=-4.4 mega Pascal's.

So this is the minor principal plane and this is the maximum plane. The inclinations with the planes are actually indicated here. So in the figure what are being asked is that we actually need to find out the straight of this stress when the element is actually having plane inclined at 30 degrees and lines drawn from P to sigma 1, sigma 3 established the orientation of the minor principle planes.

That we have actually done through here in this particular exercise in the problem where we actually identify and calculated what is the maximum principal stress and minor principal stress and also when the given elementary actually subject to the different state of stresses and we have actually try to understand what is the, how we can actually determine and interpret the several parameters very efficiently.

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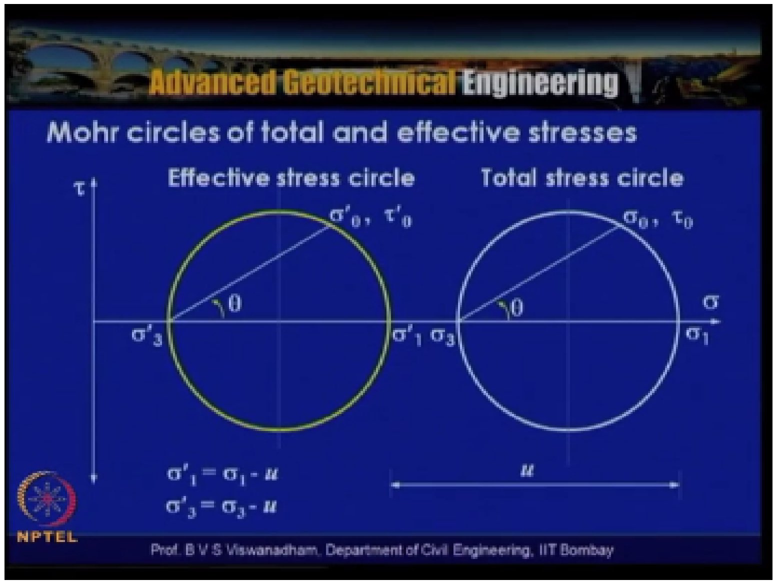

Now as we know that the total stress is nothing but effective stress plus pore water pressure. So given this condition we can actually draw a different total stresses and effective stresses. So here in this particular slide wherein we can actually see that more circle for total on effective stresses, so here on the α sigma plot if you look to this on the right hand side the circle in white color indicates the total stress circle and this is nothing but the effective stress circle.

So on the, when you have sigma one as the major principal stress and the sigma three as the minor principal stress with this sigma one, sigma three as the diameter draw the Mohr circle. Now this is the again the pole point here, now from this point at an inclination theta we can actually draw an element where sigma α 0 or α theta are the stresses on that particular plane which are actually acting purpose sigma 3 it has acting perpendicular this plane and α theta actually acting along this plane.

So but when you use sigma one - = sigma one –U where U is the pore water pressure and sigma -3 = sigma 3-U, when we do that the circle actually shifts to the left hand side, so in case of effective stress circle which is on the left hand side of the total stress circle and with the magnitude of the effective major principle stress is nothing but sigma one -= sigma 3 sigma1-U and this effective minus principle stress is nothing but sigma 3- = sigma 3-U. So the difference is nothing but U, again the U actually located the pore point. The pore will not change and from this inclination theta and this stress state sigma – theta and tow – theta we can see that the difference between these two horizontally, these two are separated by distance U.

You can see that both in effective stress circles and total circle states the Mohr circles are actually having identical diameters.

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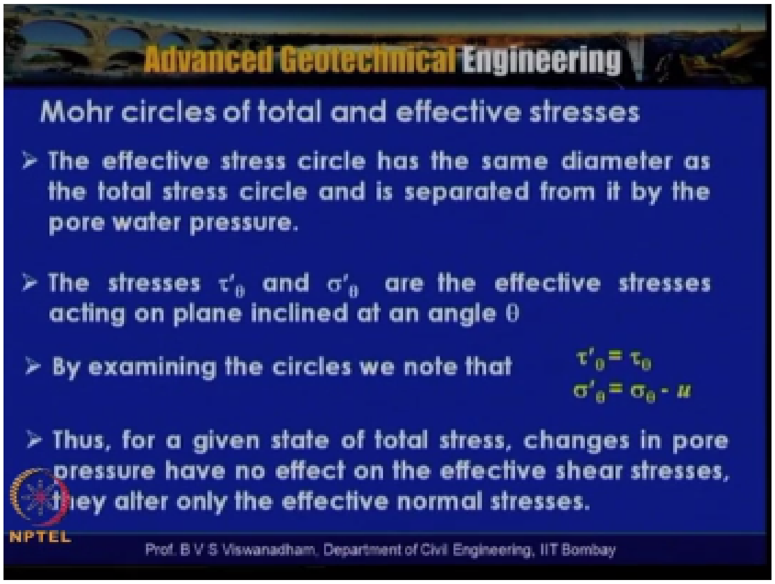

So we can actually summarize the points like the effective stress circle has the same diameter as the total stress circle and is separated from it by the pore water pressure. The effective stress circle has the same diameter as the total stress circle and is separated by the pore water pressure and the stresses tow – theta and super x theta are effective stresses acting on the plane inclined at angle theta.

By examining the circles we note that tow/theta = tow theta and sigma x theta is = sigma theta – u so what we can actually conclude is that for a given state of total stress changes in pore water pressure have no effect on the effective stresses, they alter only the effective normal stresses.

So an important conclusion which we have deduced here is that for a given state of the stress the change in pore water pressure have no effect on the effective shear stresses, the alter remedy effective in normal stresses. So this is actually represented here tow-theta = tow theta.

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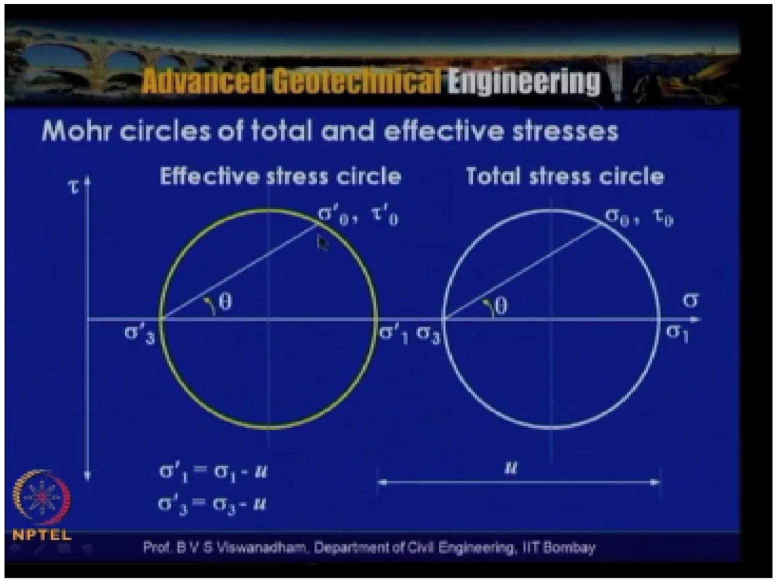

Where you can see that vertical ordinate, this is identical to the though the magnitudes are different, but the vertical ordinate that actually, that is indicated here. And also sigmonoid theta = sigma theta –U.

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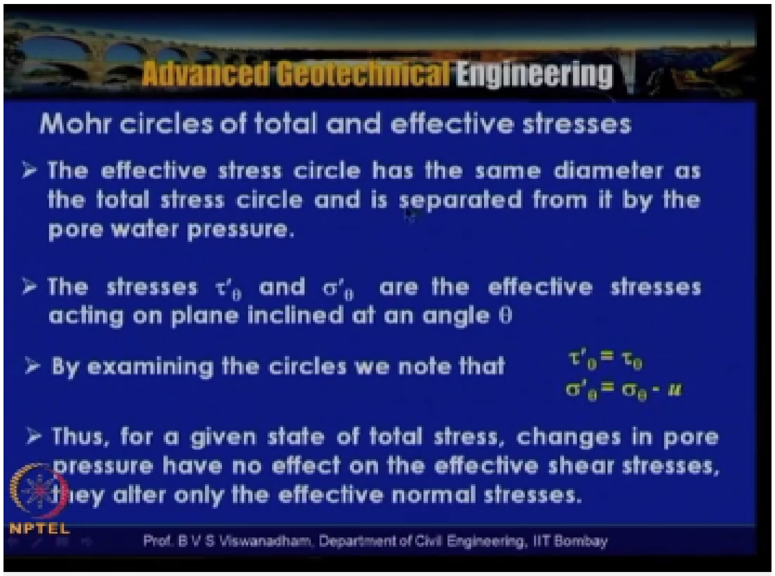

So the effective stress circle has the same diameter as the total stress circle and is separated from it by the pore water pressure and the stresses are tow-theta and sigma-theta which are the effective stresses acting on the plane inclined at theta. So by examining the circles we can see that tow-theta = tow theta and that sigma – theta = sigma-u. So for a given state of the total stress circle changes in the pore water pressure has no effect on the effective normal stresses, they alter only in the effective normal stresses.

So in this particular lecture we actually have tried to understand how the Mohr circles can be drawn and then what are the applications of this and then we try to establish the origin of the planes and the pole point P and then we also have discussed with that how we can actually determine the major principle planes and the minor principle planes, and we can actually say that the principal planes are the once where the shears stresses are zero.

Shear less planes are called principal planes. So while determining the shear strength of a soil if you are able to create these conditions then they are eligible to be called as the shear less planes or principle planes and which is will be discussing in the forth coming lectures by discussing about the partial test.

And then further we have discussed that the Mohr’s circle of the total and effective stresses, the effective normal stresses and total stress circle the diameters are identical and again there is no reason as such there is no tow-theta, so tow-theta = tow because it is equal and only thing what actually gets different is that the sigma- theta will actually change it as sigma-theta is equal to sigma theta –U.

So for a given state of the total stress the changes in pore water pressure have no effect on the effective shear stresses and they are only altered the effective normal stresses. So we have discussed about the Mohr's circle for the total and effective stresses, so we can conclude that for a given state of the total stress, the changes in pore water pressure have no effect on the effective shear stresses.

They alter only the effective normal stresses because from the demonstration here tow-theta =tow theta and sigma – theta = sigma –u. Further we can actually look that uses of pole construction on the effective Mohr's circle.

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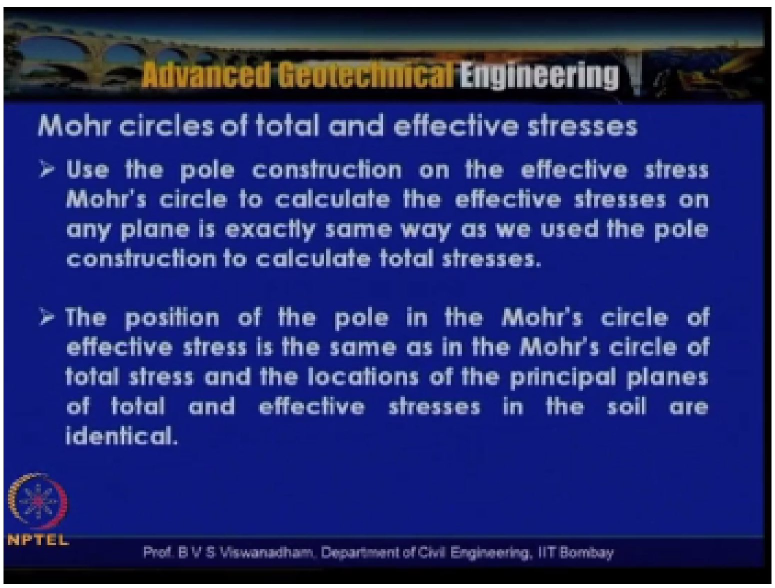

To calculate the effective stress on the any plane is exactly same way as we use the pole construction to calculate the total stresses. So if you are having the effective stress conditions are effective normal stresses circle also the same way we can actually use the pole construction method and another thing is that the position of the pole in the Mohr's circle of the effective stress is same as in the Mohr's circle of the total stress and the location of the principle planes on the total and effective stresses in soil are identical.

So when we have got the total stress conditions and effective stress condition then what we said is that the pole construction which is actually the procedure is same and the principal planes in the effective stress condition and total stress condition remain same. That means that the position of the pole in major circle of effective stress is same as the Mohr's circle of total stress and the location of the principle planes of the total and effective stresses in soil are identical.

So the positions of the pole in Mohr's circle of effective stress is same at the in the Mohr's circle of the total stress and the locations of the principle planes of total and effective stresses in the soil are identical. So that can be seen from the diagram here.

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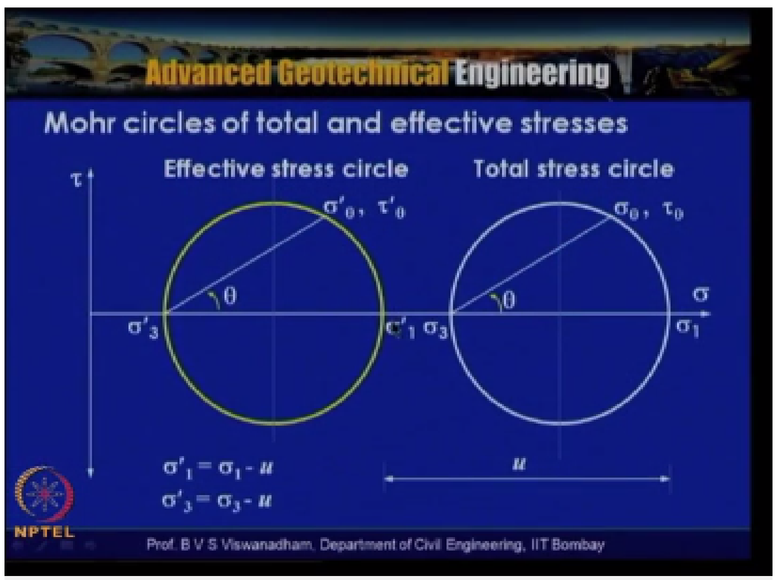

When you look at the major principle plane is here and in the effective stress conditions and minor principle plane is here, it does not change, here you can see that this is the major principle plane even in the total stress conditions and here this is the minor principal plane that is the perpendicular one in this case for the total stress condition also.

And the pole is here from the total stress condition and here also the pole is here so in a way what is actually changing is that sigma – theta which is actually nothing but sigma theta –U. So the U is that pore water pressure which is there is separating for the normal stress only, but where as you can see that the tow=theta = tow theta, that means that it is not effective shear stresses.

So with this we actually have discussed in length about the Mohr's circles for and then pole interpretations and we connected to Mohr's circles of the total and effective stresses.

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NPTEL
Principal Investigator
IIT Bombay
Prof. R. K. Shevgaonkar
Prof. A. N. Chandorkar

Head CDEEP
Prof. V.M. Gadre

Producer
Arun Kalwankar

Project Manager
Sangeeta Shrivastava

Online Editor/Digital Video Editor
Tushar Deshpande

Digital Video Cameraman
Amin Shaikh

Jr. Technical Assistant
Vijay Kedare

Project Attendant
Ravi Paswan
Vinayak Raut