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ADVANCED GEOTECHNICAL ENGINEERING

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Lecture No. 30
Module – 4

Lecture - 1 on Stress - strain relationship and Shear strength of soils

Welcome to the lecture series on advanced geotechnical engineering, we are introducing ourselves to module 4 stress-strain relationship and shear strength of soils, so this module 4.

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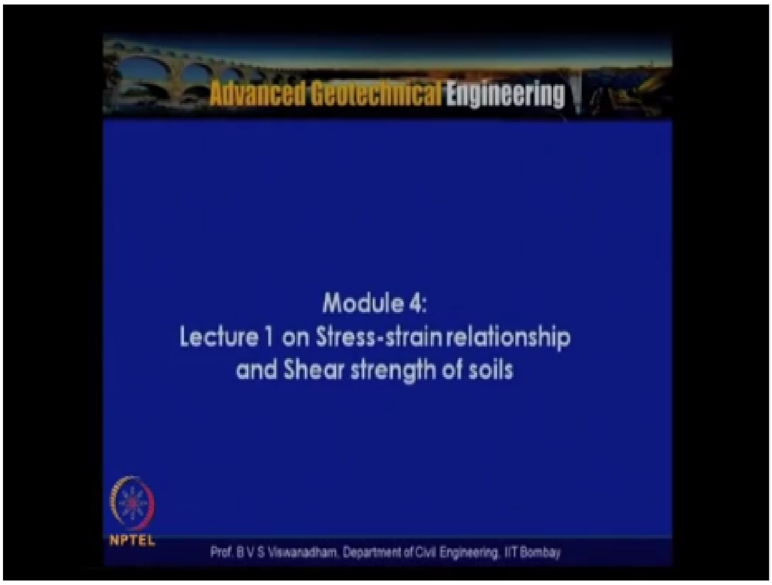

And this is the lecture 1 of module 4 on the stress-strain relationship and shear strength of soils, and we are going to cover the contents of this module in the following way.

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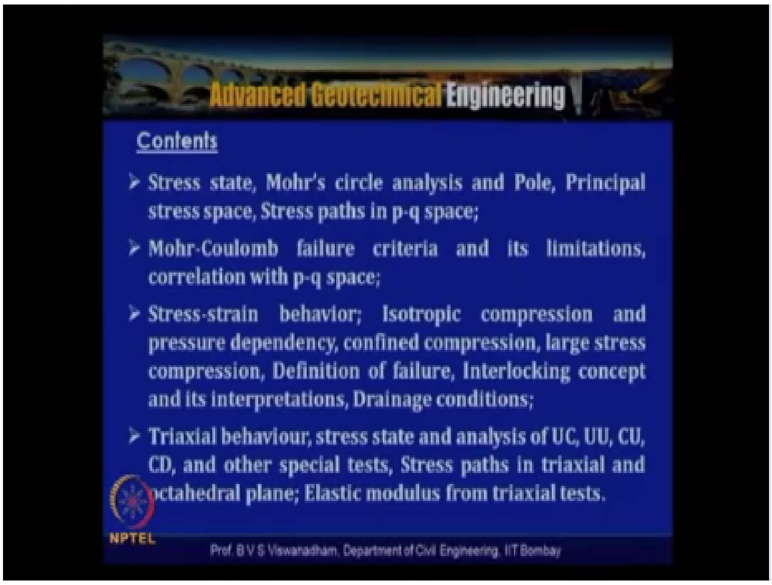

First we will introduce ourselves to stress state and then we will try to understand about the Mohr's circle and Mohr's circle analysis and how to identify a pole and the principal stress space and stress path in p-q space, and then we will try to discuss about the Mohr –Coulomb failure criteria and its limitations and correlation with p-q space. Then it will be followed by the stress and behaviour, isotropic compression and pressure dependency, and confined compression and large stress compression and definition of failure, interlocking concept and its interpretations and drainage conditions.

Then we will introduce ourselves to different types of shear stress which are actually available to determine the shear strength in the laboratory, like you know we have direct shear test and the triaxial tests and the triaxial behaviour we will try to discuss in depth and then the stress state and analysis of unconfined compression, unconsolidated un-drain, consolidated un-drain and consolidated drain attraction conditions with special conditions.

And special tests particularly the extension test for the, for tension and you know will be discussed and there after we will try to concentrate on the stress paths in triaxial and octyohydral plane, elastic modules from triaxial tests and how it can be used further in understanding you know analysis of the geo technical structures. So we have broadly divided this into the following contents where we will first introduce ourselves to Mohr's circles and then we will try to do you know the identification of the pole and principal stress and stress path in p-q state and failure criteria we will introduce and then you know we will try to look into all these contents in a systematic manner.

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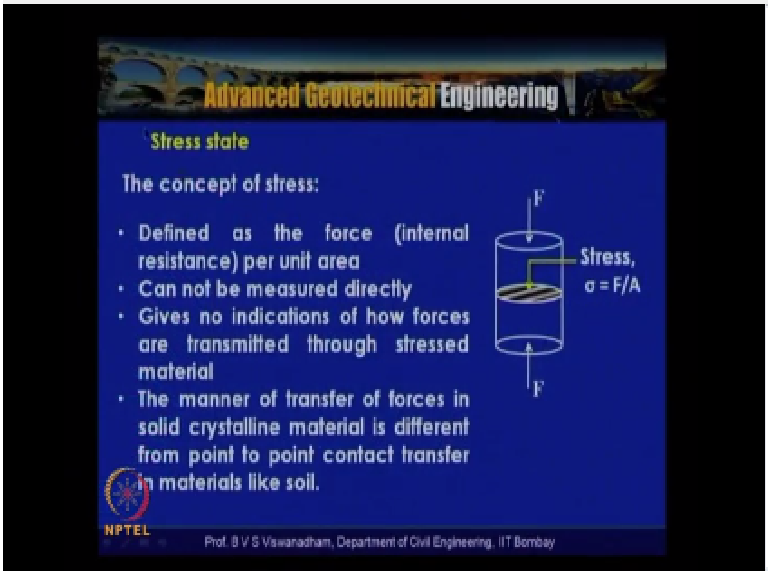

The concept of stress if you look into it, if you take a cylindrical sample and if you consider this is the so called plane then the stress is actually nothing but defined as a force divided by area, so defined as the force, so this area is you know is a collectively from the grain to grain contacts as well as the void area, it is also called as a engineering area. We have discussed this while discussing C page and you know the per ability.

So defined as the force per unit area that is the internal resistance per unit area so cannot be measured directly and gives no indications how forces are transmitted to stressed material, and the manner of transfer of force in solid crystalline material is different from point to point contact transfer in materials like soil. So the manner of transfer of forces in solid crystalline material is different from point to point grain contact transfer in materials like soil. Now consider here a simple actual stress which actually has been applied with a compressive force F.

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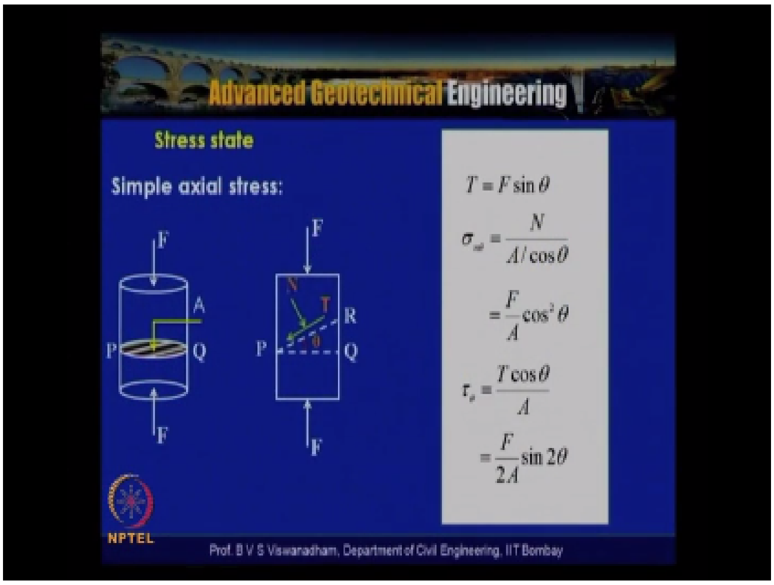

Acting on plane p-q, in another figure which is placed on the right hand side wherein the plane p-r which is inclined at an angle θ. Now we can actually divide, we have said that un plane p-q the stress is nothing but $\sigma = F/A$ but on plane p-r if we look in to it we get you know $T = F\sin\theta$, that means you resolve F components into horizontal components and vertical component, then we get $T = F\sin\theta$.

And then with that we actually get the normal force $\sigma n\theta = n/a/\cos\theta$ because $a/\cos\theta$ is nothing but the area of the plane p r because it is inclined at an angle θ so that if A is that area the plane is p r so $a/\cos\theta$. So with that we can write that $\sigma n\theta = F/a\cos 2\theta$ and $\tau\theta = T\cos\theta$, that is the shear force acting on the area a, with that we can write $F/2A \sin 2\theta$. So with this we actually got stress on the plane pr $\sigma n\theta$ and $\tau\theta$ and.

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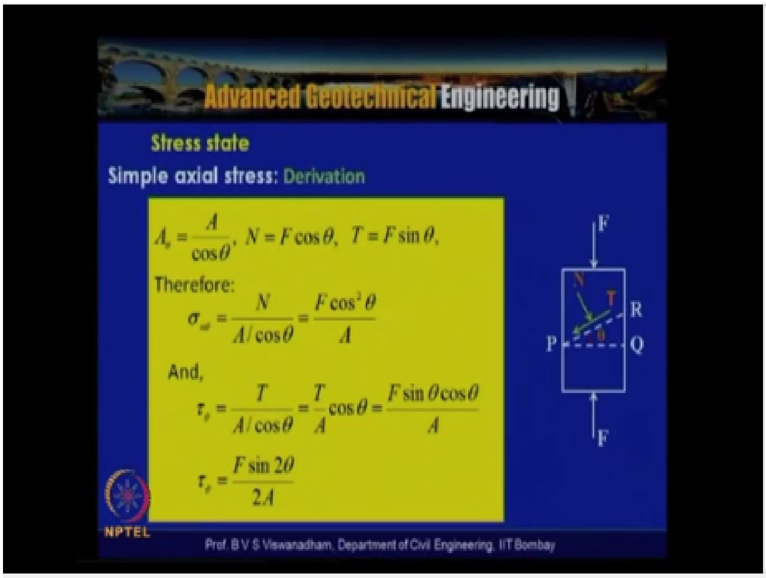

Wherein the derivation is worked out like this you know aθ is nothing but a/cosθ and N-= F cosθ and T = Fsinθ that is F is resolved with an angle θ in horizontal component and vertical component and therefore σnθ = F/A area is nothing but A/cosθ n/A/cosθ is nothing but N= F cosθ, if you substitute that we get $F\cos^2\theta/A$. Similarly τθ = t/a/cos θ so T = F/sin θ so we can write F sinθ cosθ/A with that we can write F sin2θ/2A.

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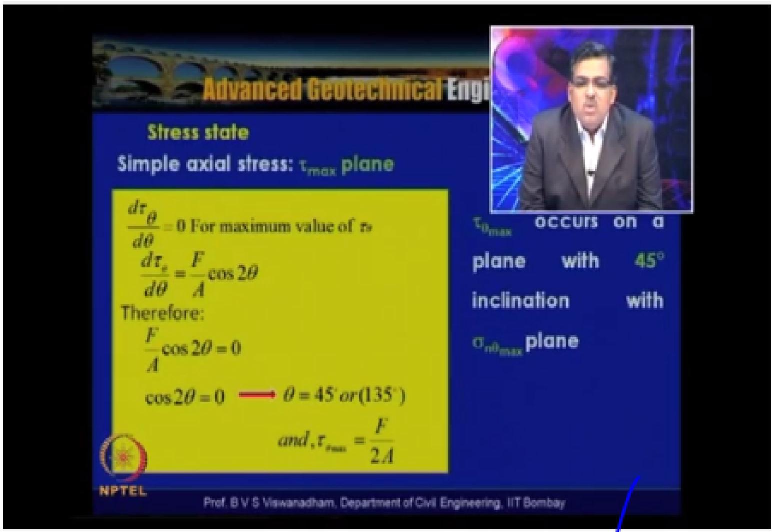

Now in order to locate the maximum shear stress and we can actually find out the angle of the plane of which this shear stress is maximum, so that we obtain by differentiating $d\tau\theta/d\theta = 0$ for maximum value of $\tau\theta$ we get $d\tau\theta/d\theta = f/a\cos 2\theta$ now for =0 $f/a\cos 2\theta = 0$. For $\theta = 45^\circ$ or 135° and $\tau\theta = F/2A$ so $\tau\theta$ will be maximum either at 45 degrees angle or there will be another that will be orthogonal to that and that is 45 and the + 90 is 135.

So $\tau\theta$ max occurs on a plane with 45 degrees inclination with σ and θ plane, so the maximum shear stress occurs on a plane which is inclined at 45 degrees, that is obtained by differentiating $\tau\theta$ and equating to 0 and then for conditions for $\cos\theta = 0$ we have got $\theta = 45$ degrees or 135 degrees, with that $\tau\theta$ max is nothing but f by 2A.

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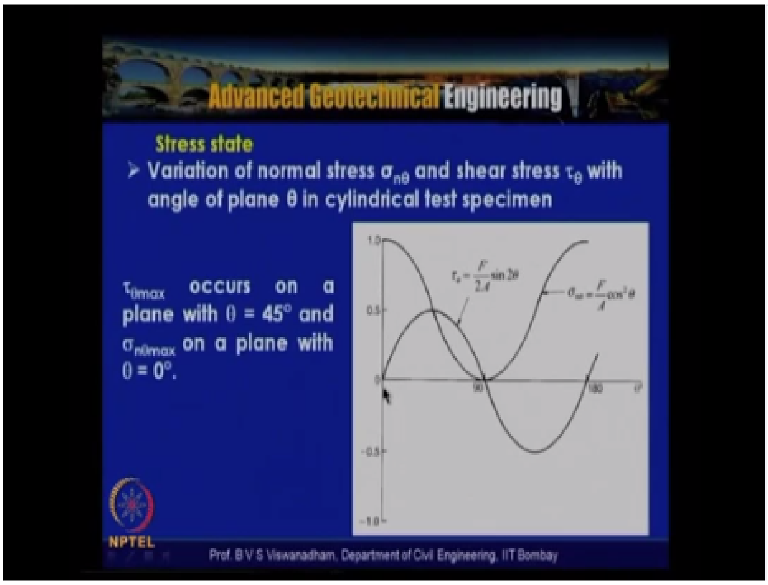

So this variation of the normal stress σ nθ and the shear stress $\tau\theta$ with angle of plane θ in cylindrical test specimen is actually brought here and this is the normal stress or normaliser here so we can see that σ n $_\theta$ is maximum and θ = 0, σ n θ=1 for, this is the normalized value which is equal to 1, 0 at 90^0 and again you know subsequently it is maximum at 180° whereas this is actually variation for how the σnθ varies.

So based on this based on the equation σnθ = F/Acosθ and this is obtained by using the equation σθ=f/2a sin 2θ where in σθ is maximum it occurs when θ= 45°, so you can see that this θ= 45 degrees which means the F/2A is maximum here that is F/A, so the τθ is maximum which occurs on a plane where the θ= 45° and σ, τnθ maximum on a plane where θ = 0 degrees.

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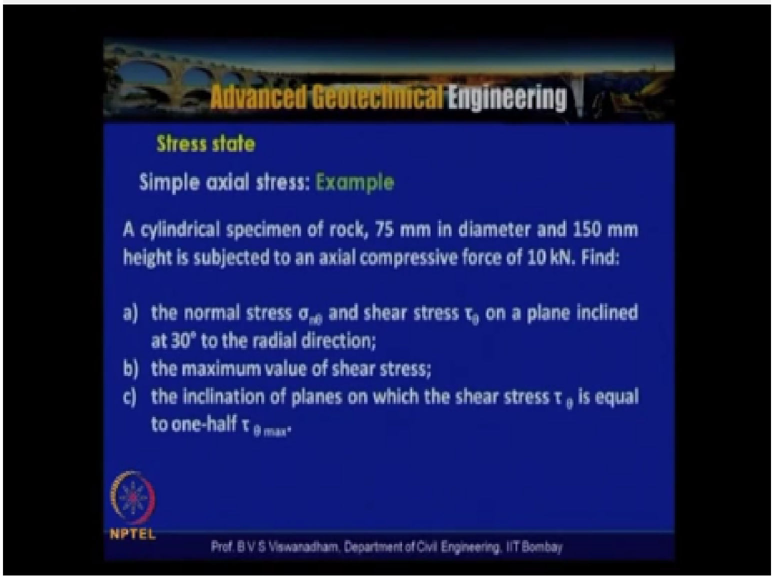

Now let us do a small example for the simple axial stress, a cylindrical specimen of rock having 75 mm in diameter and 150 mm height is subjected to an axial compressive force of 10 kn. So we need to find out the normal stress σnθ and shear stress τθ on a plane inclined at 30° to the radial direction, and the maximum value of the shear stress and the inclination of planes on which the shear stress τθ is equal to one- half τθ max[50].

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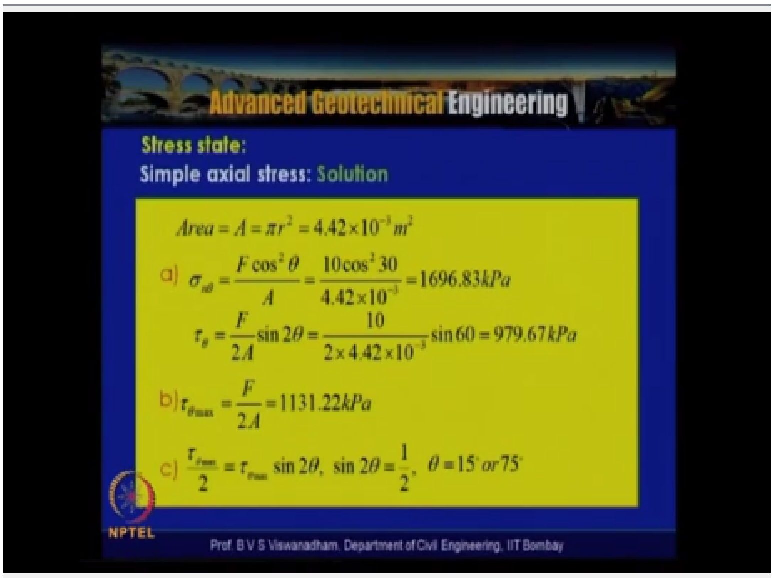

So the area is nothing but 75 mm diameter so with that we can actually get the Πr^2 which is nothing but $4.42 * 10^{-3} m^2$ so σ nθ which is given by f cos $_\theta$ by a, so with this we can write f into f/10 $\cos^2$ and we have been asked to determine on a plane 30 degrees in any direction so f cos/ 30 divided by area, that is 4.4- 3 so we get 396 kilo Pascal's and τθ is nothing but f/2a sin 2θ. That is nothing but 10 by 2x2.4 – 3 x sine 60. So with that 979 Pascal's is the τθ.

But if you look into this the τθ maximum which is nothing but it occurs at 45 degrees if it is horizontal or max=f/2A here F= 10 KN/2*4.42 * $10^{-3}$with that we got 1131Kpa and then we have been asked to find out the inclination to the plane on which the shear stress τθ = 11/2 of τθ max so the inclination of the plane at which actually the shear stress is actually τθ max is half of the τθ max, so τθ max which is τθ so τθ max/2 = τθ max sin 2θ. So with that we actually get sin 2θ=1/2 so which is nothing but θ = 15º n 0r 75° this is the inclination of planes on which the Shear stress τθ = 11/2 of τθ max.

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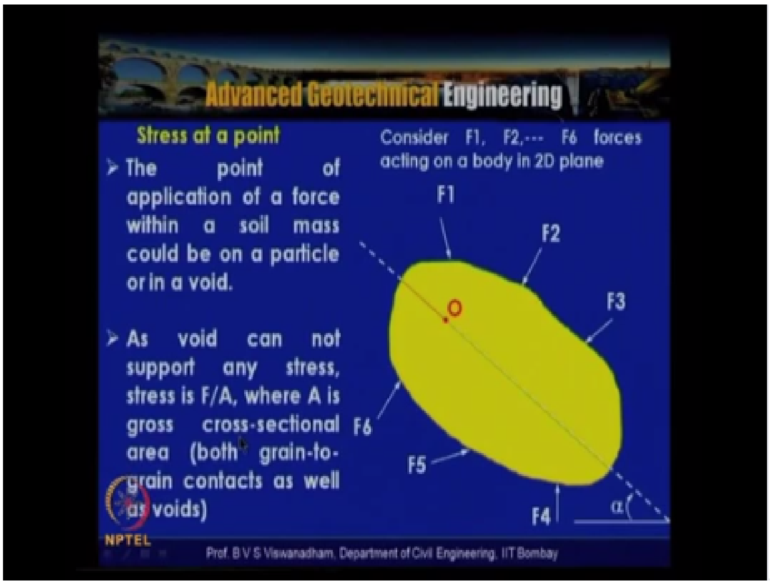

Now let us again consider in order to formulate the Mohr circle, you know consider a body which is subjected to number of forces which is shown here, a body which is having an axis passing through o inclined at an angle ∞ and subjected to forces f1, f2, f3, f4, f5, f6 acting on a body and in a 2d plane. So the point of application of a force within a soil mass could be on a particle or a void.

So the point of application of force within a soil mass could be on a particle or in a void as the void cannot support any stress the stress is nothing but F/A where A is the gross cross sectional area which includes both grain to grain well as voids, so which includes both grain to grain contact as well as voids. So as void cannot actually support any stress the stress is F/A where A is the gross cross sectional area both grain to grain contact as well as voids.

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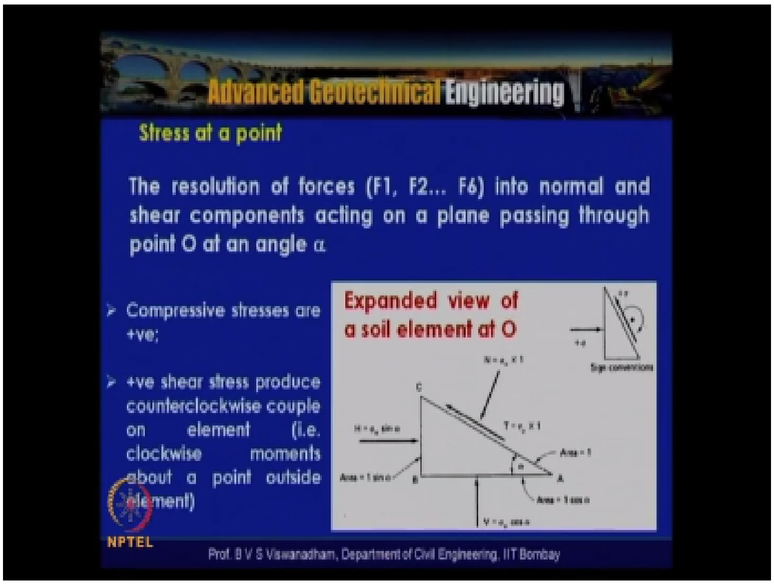

So what we do is that is the resolution of the forces if you do f1, f2, f3 into normal and shear components of acting on a plane passing through point o at an angle so the expanded view of an element at o is actually shown here, the point o is here, and if you consider you know the expanded view of an element and it actually shows like the element A, B, C, where a, b and the plane here the vertical force v is acting and which is nothing but you know, if this area say unit area and this is inclined at α then the area of this one is nothing but the area is = 1cosα, this area is nothing but1sinα so 1cosα is the area of a b and areas of b c and areas of ac is 1.

So with that what will happen is that the vertical force is nothing but stress * area and horizontal force is nothing but horizontal stress * area. So the horizontal forces are written here and the equivalent stresses are also written here, this is the normal force or normal stress and this is the shear force or shear stress. Now here we need to observe here a particularly here in this the sign conventions need to be followed consistently.

So the competent stresses are positive because in majority of the geotechnical engineering the stresses are in compression in nature so because of that the stresses are computative stresses are positive in nature and the tensile stresses are negative and the positive shear stresses produce counter clockwise couple on the element or clockwise movements about the point outside the element that means. That if you are having a shear force acting like this, this is said to be positive if it is producing a counter clockwise moment about the point outside the element that is that if it is producing a counter clockwise moment.

The clockwise moment about the point outside the element so if it is a clockwise moment about a point outside the element then the shear stress is said to be positive. So if let us consider that the shear stress is actually acting in this direction downward direction then in that case it will be negative so the positive shear stress is produced counter clockwise couple on a element and or clockwise moment about a point outside the element.

So with this what we do is that we try to resolve the forces in horizontal direction and vertical forces and then we try to obtain the equilibrium condition and then we try to simplify and see that what is the deduction which we are going to get. So at equilibrium the sum of forces in any direction must be zero so we resolve in horizontal direction and vertical direction.

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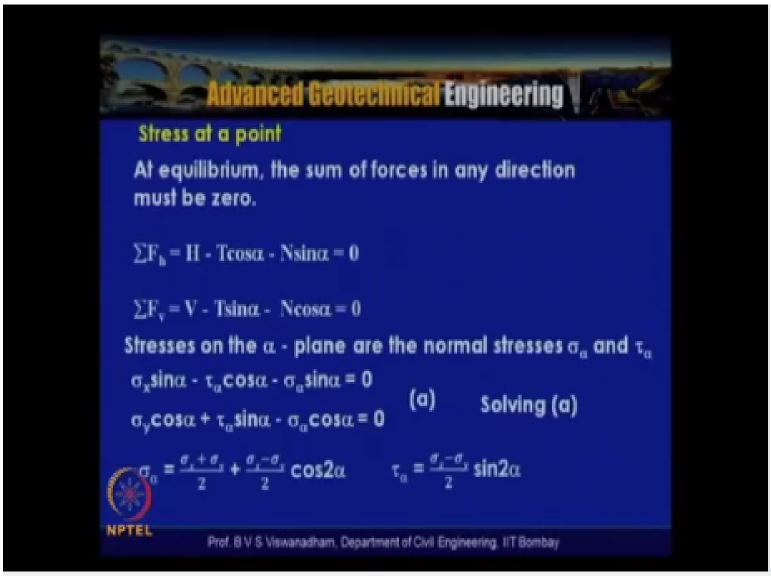

With that ΣF_h = 0 where H- TCosα – Nsinα that means the components have been taken, H in the horizontal direction is positive and TCosα and NSinα that is due to the norm al force on the plane AC. Now similarly ΣFv = 0 where V – Tsinα - NCosα = 0. Now dividing by the respective areas the stresses on the α plane that is the plane AC are the normal stresses σα rα that means that the stresses are here that is σ_α that is the normal stress on the plane inclined at an angle α and τ_α *that* is the plane inclined at angle α with the horizontal that is AB.

Now the stress on the α plane are the normal stress σ_α τ_α so because that is horizontal force σx sinα - cosα - σα sinα = 0. Similarly σycosα + r_α sinα – σcosα = 0 so this is regarded as (A), by solving (A) we get σ_α = σx σy /2 + σx - σy/2 cos 2 ∞ τα = σx - σy/2 sin2α. So we have got σ_α τ_α stresses by doing the application of the stresses the element is experiencing, and on a plane which is inclined at angle α.

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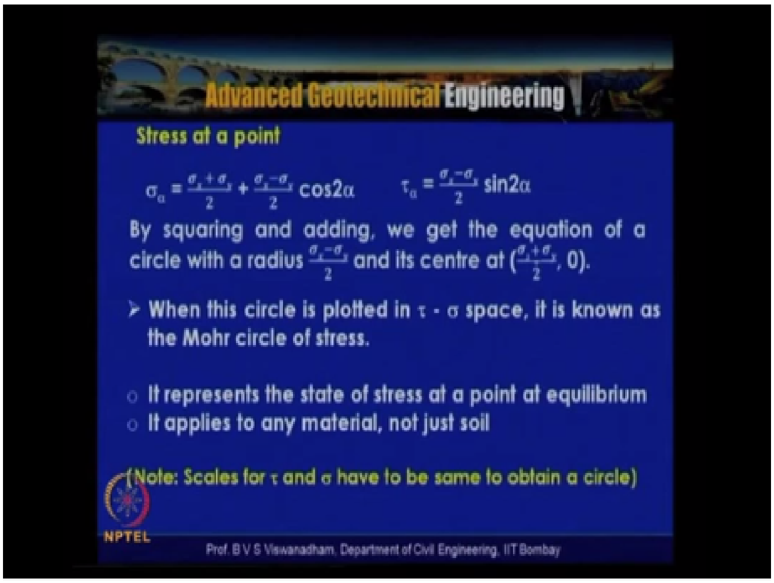

So now $\sigma\alpha = \sigma x + \sigma y /2 + \sigma x - \sigma y /2 \cos 2 \infty$, $\tau\alpha = \sigma x - \sigma y/ 2 \sin 2\alpha$, so by squaring and adding we get the equation of a circle, so by squaring σ_α and τ_α and we get the equation of the circle with the radius σx - $\sigma y /2 / 2$ and it is centred at $(\sigma x + \sigma y /2 , 0)$, so when a circle is plotted in a σ τ space it is known as the Mohr circle of stress, it represents the state of stress at a point at equilibrium and it applies to any material, not just soil. And note that the scales are for the tow and σ have to be same to obtain a circle.

So whatever from the reduced derivation we actually have obtained σ alpha and tow alpha in terms of σ X and σ Y and α and by squaring and adding we actually have got a form of equation which is representing the circle and that circle is having a radius of σ X - σ Y by 2. Σ_X is the stress applied in the X direction. Σ_Y is the stress applied in the Y direction and with a centre at $\sigma_X + \sigma_Y$ by 2, 0. So when this circle is plotted on the tow σ space it is known as the Mohr circle of stress, so it represents the state of stress at a point at equilibrium and it applies to any material, not just a soil. So the Mohr stress circle is also called as 2 dimensional.

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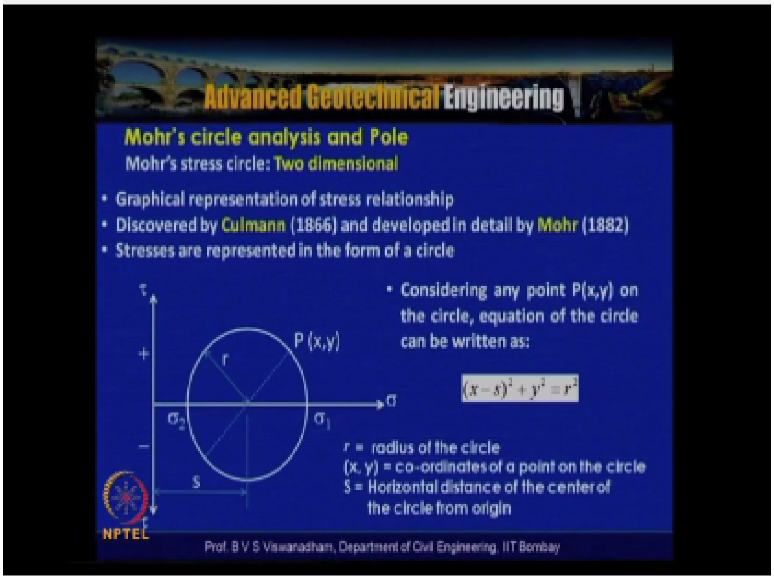

The graphical representation of stress relationship at equilibrium and discovered by Cullman in 1866 and developed in detail by Mohr in 1882. So the development actually goes back into the soil mechanics and then it is used in soil mechanics, so the stresses are represented in the form of a circle, so considering any point, P, X, Y on the circle equation of the circle can be written as (X- S)2 / $Y^2 = R^2$ where R is the radius of the circle and X and Y are the corners of point on the circle.

And S is the horizontal distance of the centre of the circle from the origin, so by getting this you can see that this is the circle where this stress is actually σ 1, so this magnitude from here to here measured is σ 1 and this is on the tow zero space and this ordinate is σ 2 and this is the radius of the circle, so this is called as the Mohr stress circle on tow σ space.

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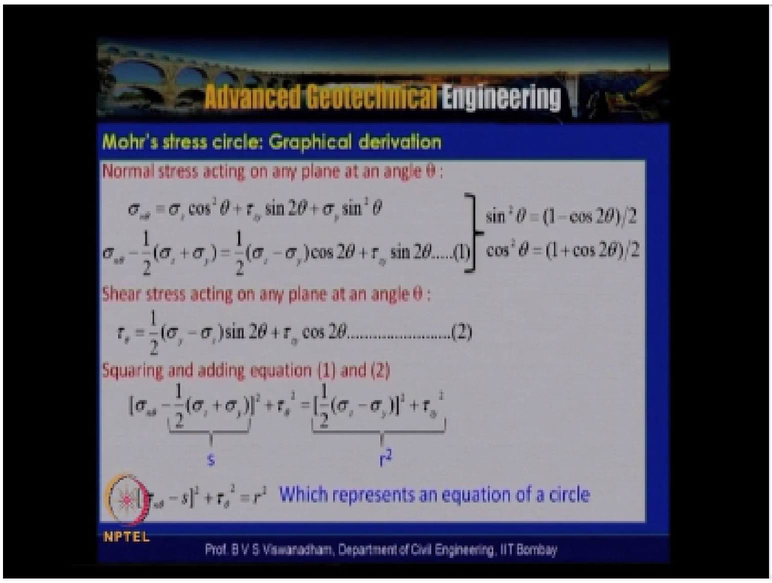

Now this deduction can be obtained like this, the graphical derivation is that once again we can say that normal stress acting on any plane at an angle theta, which is nothing but in terms of σ N θ is equal we can write like σ x cos θ+ tow x y sine θ+ σ y^2 sine ^{2} θ so and using sine square θ is = 1 - cos θ/ 2, cos θ /2 = 1 + cos θ/2 and with that we actually get σ N θ = σ X + σ Y / 2, σ n θ– σ x + σ y/2 = 1/σ x – σ y cos θ+ τxy sine 2θ.

So shear stress acting on any plane at an angle θ is given by τθ = ½ σx – σz sin2θ + τxy sin2θ. So τθ=1/2σy-σx sin2θ+τxy cos 2θ. So squaring and adding the equation, so again this is actually one form of deriving, we already deduce this, but this also says that the equation which is actually reduced to a form of equation of a circle where the $(\sigma n\theta\text{-}s)^2+\tau\theta^2=r^2$ and this represents the equation of a circle. So σ N θ - S that is this one $+\tau\theta^2 = r^2$ this is nothing but the radius turn.

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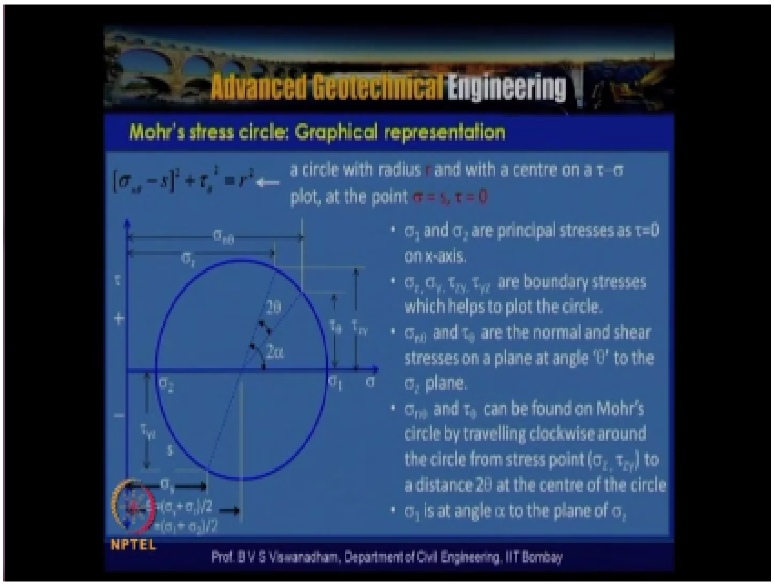

So this is represented in the graphical form like this, the graphical form when you interpret you will get the number of unknown parameters can be deduced, so this is the circle with radius R at with a centre on tow σ plotted at point σ is equal to S and tow is = 0 that is S is equal to σ X + σ Y /2, which is also nothing but σ 1 + σ 2 by 2, so because this σ 1 and this σ 2.

So this is also given as σ 1 + σ 2/2. Now σ 1 σ 2 are the principle stresses as τ =0 in the x axis. So shear less planes are called principle planes. So σ 1 and σ 2 are the principle stresses as τ=0 on the x axis. So σ Y and σ z and τzy τz are the boundary stresses which helps to plot the circle and σ N θ and τ θ are the normal shear stresses on a plane at an angle θ these σ Z plane and θ and τ θ can be found on Mohr circle by travelling clock wise around the circle from the stress point to a distance 2 θ at the centre of the circle and σ 1 is at an angle α to the plane of σ Z. So these are, this is the angle to alpha and this is 2 θ which is actually shown here.

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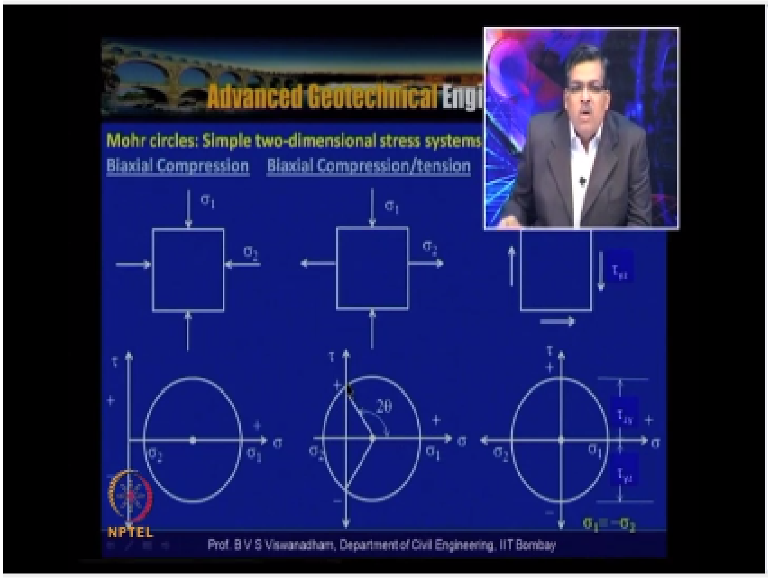

Now let us consider for some examples of the Mohr circles for different conditions of the stresses that is biaxial compression, and tension and biaxial pure shear. So here when you consider this one with σ 1 and σ 2 both are actually compressive stresses acting on element and two dimensional conditions have been selected and the Mohr circle represents in a towards the positive side.

You can see that the centre at this particular point and in case here we have a compression stress and elongation for the element in the horizontal direction. That is the tension in the horizontal direction and here there are no normal stresses only they have, the element is actually subjected to biaxial pure shear. The element is actually subjected to shear like as shown here. So in this case Mohr circle is actually symmetrical about this τ σ axis.

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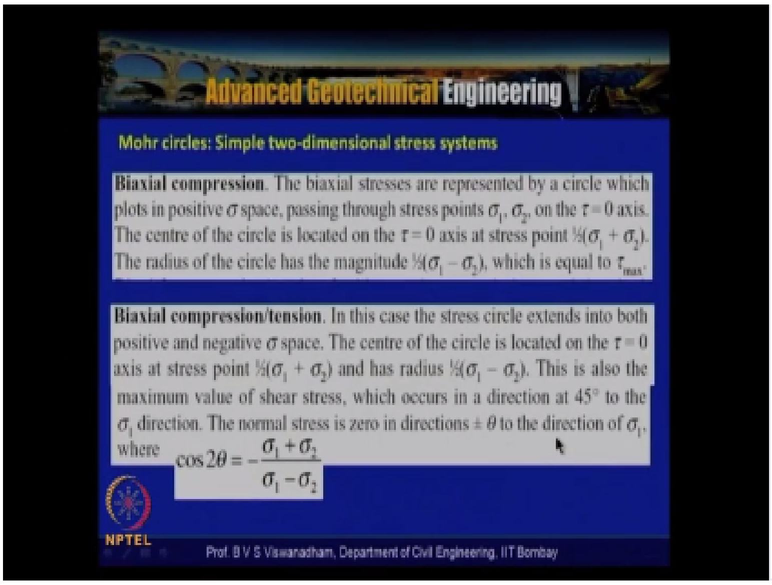

So the biaxial compression, this biaxial stresses are represented by a circle which plots in positive σ space and passing through stress point's σ_1 and σ_2 and on the τ 0 axis. So the biaxial stresses are represented by a circle which plots and positive σ space that is this circle is actually plotted in a positive σ space and with stress point σ_1 and σ_2 on the τ 0 axis these are the points which are actually acting on the τ0 this is the plane on this $\tau=0$ is τ0 axis it is called where the stress here at this point where these meets the σ axis.

That is σ_1 here and where the circle intercepts this point is σ_2. The centre of the circle is located on the τ0 axis at this stress point that is $\sigma_1 - \sigma_2/2$ and the radius of the circle has the magnitude of $\sigma_1 - \sigma_2/2$. So the radius of this circle has the magnitude of nothing but $\sigma_1 - \sigma_2/2$ and the maximum shear stress is also is the $\sigma_1 - \sigma_2/2$. So this is the Mohr circle which is biaxial compression the Mohr circle will be in the on the τ σ space it will be on the positive σ space that is actually called as a positive σ space.

Then the centre is located at the stress point that is $\sigma_1 + \sigma_2/2$. So the centre is actually located at a stress point $\sigma_1 + \sigma_2/2$ and the radius of this circle has the magnitude of $\sigma_1 + \sigma_2/2$ which is the thing, but so the radius of the circle is nothing but the maximum shear stress ordinate which is nothing but the $\sigma_1 - \sigma_2/2$, this is the maximum shear stress located here in the positive side and negative side.

And biaxial compression or tension that means the element is actually subjected to both biaxial compression and tension in this case the stress circle extents into both positive and negative σ space. So the σ space which is towards the positive side is called as a positive σ space and σ space which is actually towards the negative side is called the negative σ space.

Similarly tow that is the tow space which is above called as a positive space and below is called the negative space. The centre of the circle is located on the $\tau 0$ axis at stress point σ 1 +σ 2/2 and has the radius σ 1 +σ 2/2 and this is also the maximum value of the shear stress which occurs at the direction of 45 degrees to the σ 1 direction and the normal stress is zero in direction + or – θ to the direction of σ 1 where cos 2θ = -σ 1 +σ 2/ σ 1 -σ 2. So cos 2θ = -σ 1 +σ 2/ σ 1 -σ 2.

So for the biaxial compression and shear you can see that this is the compression and this is the tension. No shear forces then in that case Mohr circle is actually extends to positive σ space and then negative σ space also and the centre is σ 1 +σ 2/2 and the shear force is maximum shear forces is σ 1 +σ 2/2 and the radius is nothing but σ 1 +σ 2/2 and this also the maximum value of the σ 1 +σ 2/2 also the maximum value of the shear stress which occurs at the directions 45 degrees to σ 1 direction.

And the normal stress is zero indirection + or –θ to the direction of σ 1 where cos θ =- σ 1 +σ 2/ σ 1 +σ 2 and now a case biaxial pure shear. In this case the circle has a radius = τ xy which is equal in magnitude to tow yz but opposite in sign. The centre of this circle is tow 0 and σ 0. The principle stresses are σ 1 σ 2 are equal and magnitude but opposite in sign and they are equal in magnitude to tow zy.

So the directions of σ 1 σ 2 or at 45^{0} to the directions of tow zy and tow yz, so the circle is represented like this where in with that you have got σ 1 the σ 1 here ordinate here and σ 2 here and this is the shear stress that is tow zy and tow yz is represented like this tow zy and tow yz. So this is in case of the biaxial pure shear.

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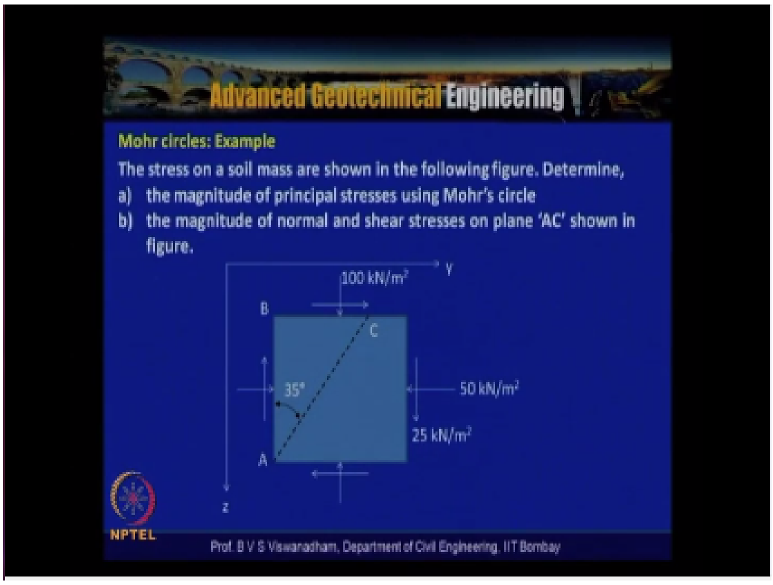

Now consider an example where the stress on a circle on a soil mass as shown in the following figure, we need to determine the principle stresses using Mohr circle and the element is actually subjected so this axis is Y axis and this axis is z axis and this plane is AB and this plane is AC. So this plane is horizontal plane and this plane is vertical plane and this is a link line plane inclined at 45 degrees to plane AB and this is the shear stresses and these are the horizontal stresses.

So here it is given as 50 kg Pascal's in horizontal direction compression and this is also compression 100 kg Pascal's and the shear is 25 kg per meter square. So the magnitude of the normal shear stresses on plane AC as shown in the figure. We need to determine what are the magnitudes of the normal and shear stresses on plane AC which is actually shown in the figure here and the magnitude of the principle stresses using Mohr circle.

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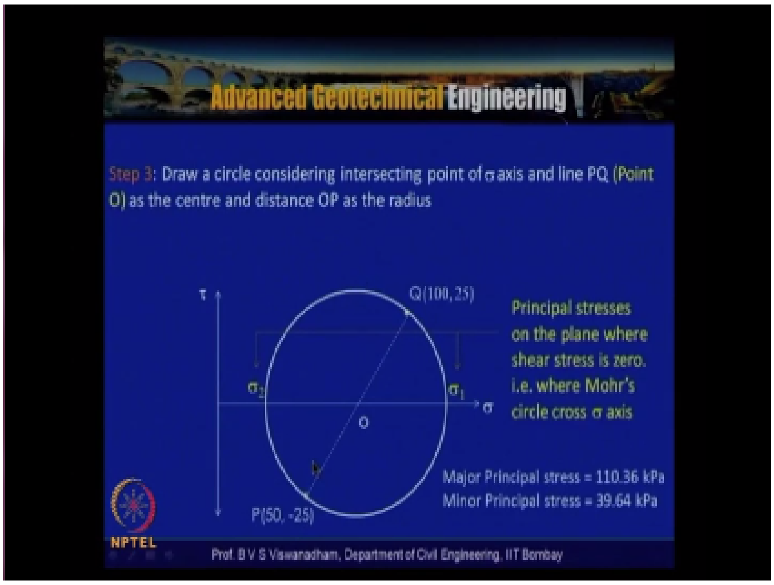

So the available information is that σ Y that is the stress acting on the horizontal direction that is the50 kN/m2in the Y direction and σ Z 100 kN/m2that is the stress in the z direction and then τ yz that is τ yz is 25 kN/m2. The τ yz which is acting the shear acting on this plane is 25 kN/m2, so the step one in constructing the Mohr circle is that mark a point σ Y tow yz and point Q that is σ Z, tow zy on the τ σ coordinate system.

So we have a tow σ coordinate system and we marked actually in the tow σ space, we marked point P and Q, then the next step is that joint P and Q with a straight line then draw the circle. So the draw the circle considering intersecting the point of S axis and line PQ that is the point O as the centre and the distance OP as the radius. So with this what we have got is that we have got a circle now, that is the Mohr circle and this is called as Mohr circle.

Now the principle stresses are nothing but the point where the plane where the shear stress is zero, principle stresses where the plane where the shear stress is zero. That is where the Mohr circle crosses σ axis. So with the, by measuring from the graft we can actually get σ 1 as 110.36 kilo Pascal's here and the σ 2 here as the 39.64 kilo Pascal that is from the measurement of the graft.

Now we can actually further find out angle two alpha that is two times the angle between σ z plane and major principle plane and major principle plane is inclined at 22.5 degrees to the σ z plane and minor principle plane is inclined at 112.5 degrees to the σ z plane. So σ z plane is nothing but the plane on which σ z acts, so after this then plane inclined at 35 degrees to the σ z plane and the stresses on the plane at 35 ^{0}to σ z plane is obtained by the point, at this point.

So we need to get, if you are looking to the problem magnitude of the normal and shear stress on plane AC which is shown in the figure. So this plane is inclined at 35^0, so the 35^0 is actually represented here. 35^0 to the σ z plane and this is actually is 70^0. So $35^0$to σ z plane when you put that this is the point where we get the σ N θ that this horizontal distance ordinate σ and θ and vertical ordinate is tow θ.

So for the measurement of the graphically we can get σ N θ = 42.96 kilo Pascal's and tow θ = 14.94 kilo Pascal's. So this is how what we have got is that from the graphical interpretation we actually have got σ N θ and tow θ for the type of the stresses which are actually given by using the Mohr circle concept.

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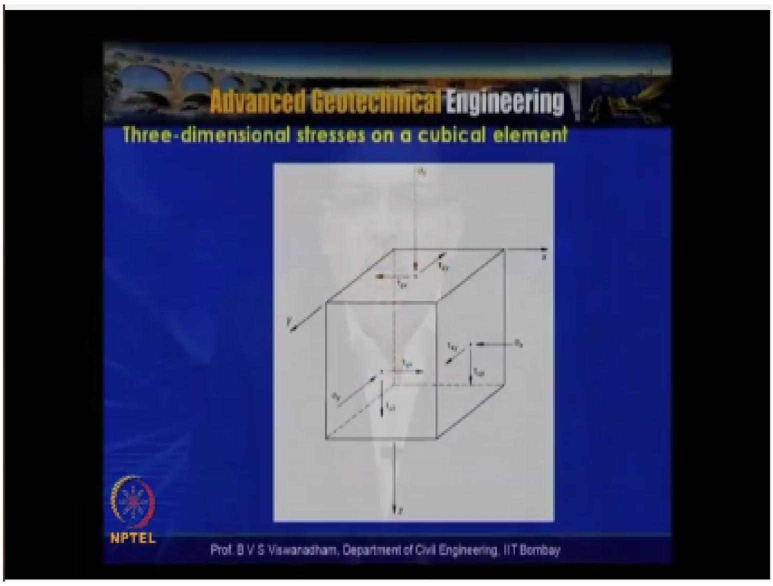

Now the three dimensional stresses on a cubical element, the elements are actually subjected suppose if you consider an element within the soil it is subjected to the following stresses in x and y and z directions and each plane there will be one vertical stress and two shear stresses. You can see that on this xy plane we have vertical stress and shear stress tow xy and the tow zx which is actually acting here. And similarly on this plane σ Y and then these are the stresses shear stresses which are actually acting and similarly on this plane which the stresses are shown on the visible planes clearly.

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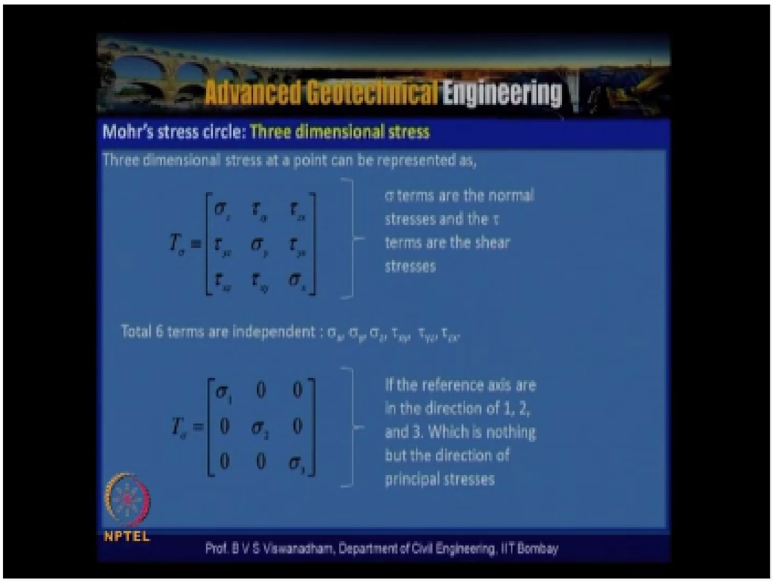

So these are actually represented in the matrix form like this. The three dimension stress at a point can be represented as Tα T σ = σ x τ xy τ zx τ yz σ y τ yx and τ xz τ xy and σ x, σ terms are the normal stresses and tow terms are the shear stresses. So please note that σ terms are the normal stresses tow terms are the shear stresses and total systems are independent and then we have σ x σ y and σ z τ xy tow yz and tow zx.

And if the references axis are in the directions of 1 2 3 and which is nothing but the directions of the principle stresses then in the Joule techniques we are actually, suppose if you are having a symmetrical sample or let us say a cubical sample we have if it is in the major vertical stress directions σ 1, then it is called σ 1 as the major principle stress and σ 2 as the intermittent principle stress and σ 3 as minor principle stress.

For example which is cylindrical in nature being the sample would be having a vertical stress that is σ 1 major principle stress and two these stresses parallel to the acting on the plane that is nothing but σ 2 intermediate principle stress and minor principle stress on the cylindrical sample as σ 2= σ 3. We generally refer it as major principle stress and minor principle stress that is σ 1 as the major principle stress and σ 3 as the minor principle stress and σ 2 as the intermediate principle stress.

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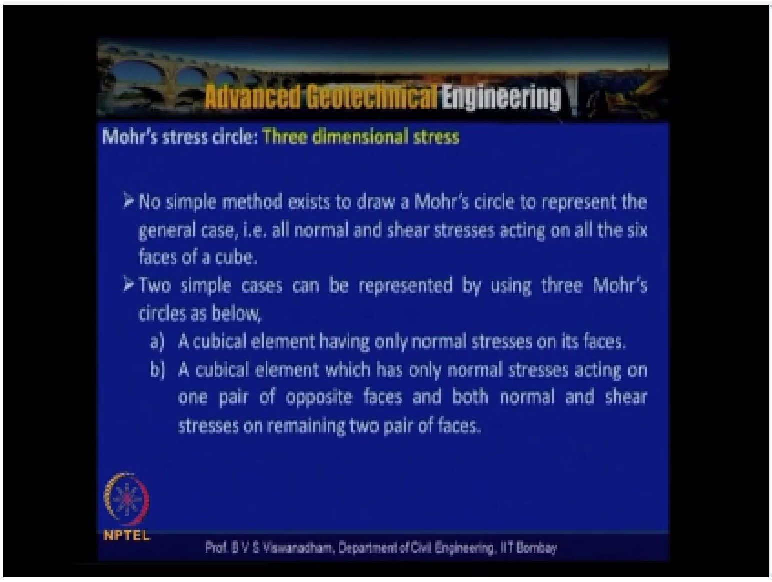

So the Mohr stress circle particularly in the three dimensional stress conditions is a low simple method exist to draw a Mohr circle to represent the general case, all normal shear stress acting on the all the six faces of a cube, so two simple cases can be represented by using the Mohr circle as given below and one is that a cubical element having only normal stresses on its spaces and a cubical element which has only normal stresses acting on pair of opposite faces and both normal and shear stresses on remaining two pairs of faces.

So with this slide what we wanted to convey is that low simple method exist to draw a Mohr circle to represent a general case and all normal and shear stresses acting on all this six faces of a cube. That means that all normal and shear stresses six faces of cube cannot be represented and two simple cases can be represented by using three Mohr circles which is the case a is the cubical element having only normal stresses.

And another case which is cubical element which has only normal stress as acting on pair of opportunity faces and both and normal and shear stresses remaining two pairs of faces. So it can be proved that the stress conditions on any plane within the element must fall within the shaded area, but it usually sufficient to able to determined these stresses on the planes which are perpendicular to atleast one opposite pair of element boundary faces. So stresses on these planes lie on the circle bounding the shaded area.

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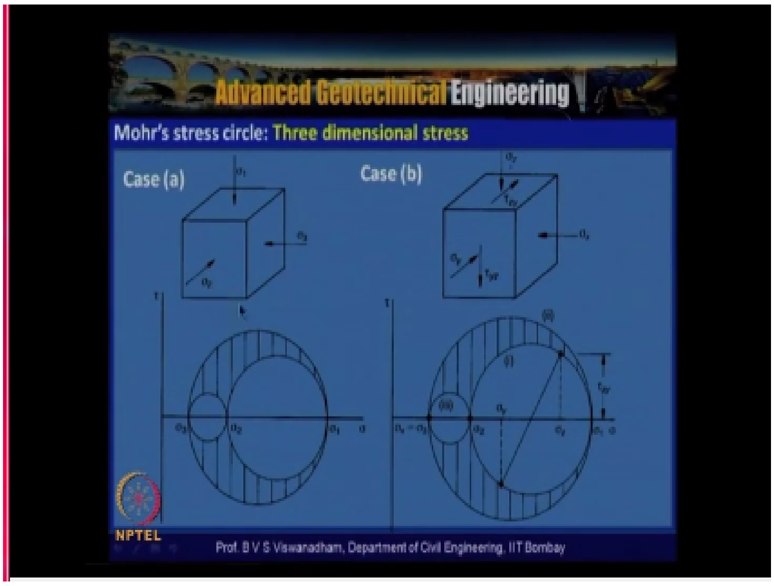

So the stresses for the case A where block which is actually having only normal stresses, you can see the σ 1 σ 2 and σ 3 are shown here. So in this case the Mohr circle is actually represented as this is the circle and with σ 1 and σ 3 and σ 2 is the intermediate stress that is the intermediate principle stress.

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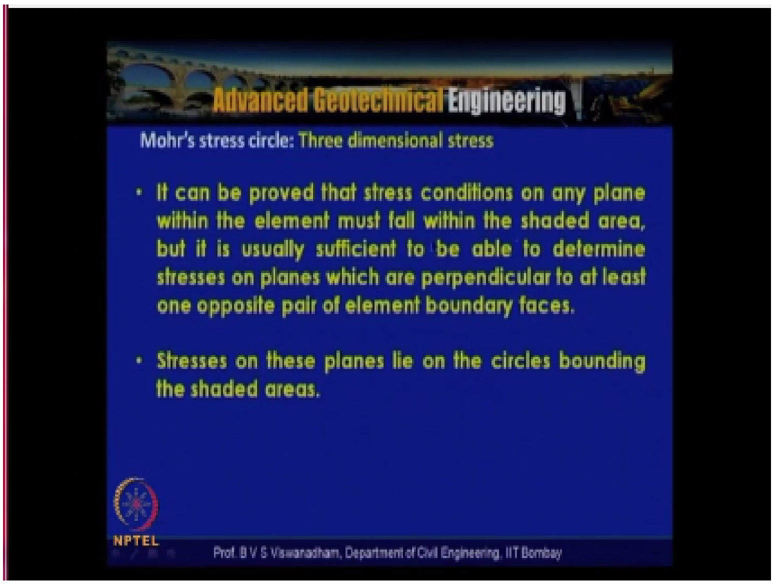

So here in this case A it can be proved that the stress conditions on any plane within the element must fall within the shaded area, that is this shaded area and it is usually sufficient to be able to determine the stresses on planes which are permanent to at least one opposite pair of element boundary faces. So in case B it is represented where you are having normal stresses and atleast two phases are actually having the shear stresses in that case we draw three Mohr circles here.

This is the Mohr circle one and second and third one. So now here first drawing this Mohr circle and then after knowing this as the major principle stress and this as the minor principle stress σ 2 and σ 3, then this is the third Mohr circle.

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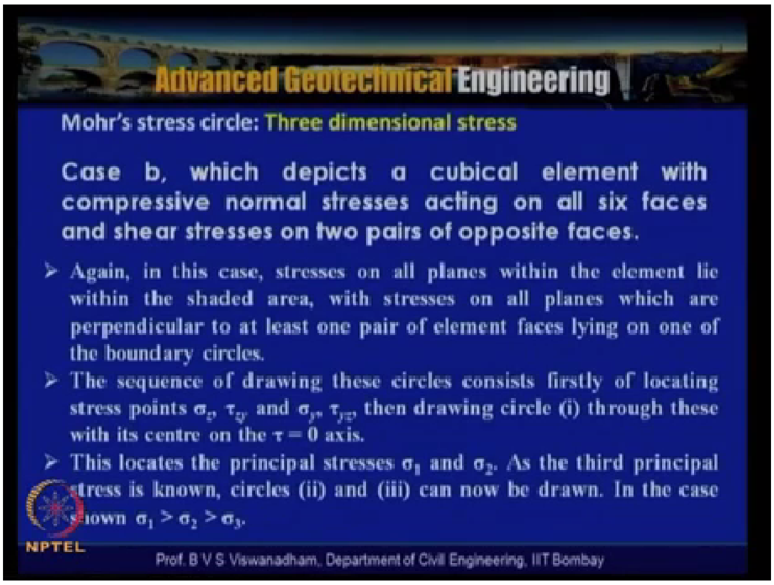

So in case of case b which repeats a cubical element with compressive normal stresses acting on all six faces and shear stress on two face of opposite faces, so again in this case the stresses on all planes within the element lie within the shaded area and within stresses on all planes which are perpendicular to at least one pair element faces lying on one of the boundary circles, the sequence of drawing these circles consists of first drawing of locating this stress point σ z tow zy and σ y and tow z then drawing this circle.

That means that first we have to locate these σ z and tow zy and σ y and tow yz and drawing the circle one that first one is the drawing this circle one locating these two points and then dropping here perpendicular and these are the σ z axis that is measured from here and this is σ y. So drawing these points then drawing the circle through with the centre on the tow zero axis, so this is the first Mohr circle which is drawn, the second one locates the principle stresses σ 1 σ 2.

So once we draw the circle one, we have got the opportunity to identify σ 1 and σ 2, and then as the third principle stress is known now the circles 2 and 3 can be and then finally the third circle with intermediate principle stress σ 2 can be drawn. So this locates the principle stresses σ 1 σ 2 and the as the third principle stresses known the circle 2 and 3 are drawn subsequently, so in this case σ 1 is greater than σ 2 and then σ 3.
So as it has been in told in Joule's case it is actually convenient to use σ 1 as the major principle stress, σ 2 as the intermediate principle stress, σ 3 as the minor principle stress.

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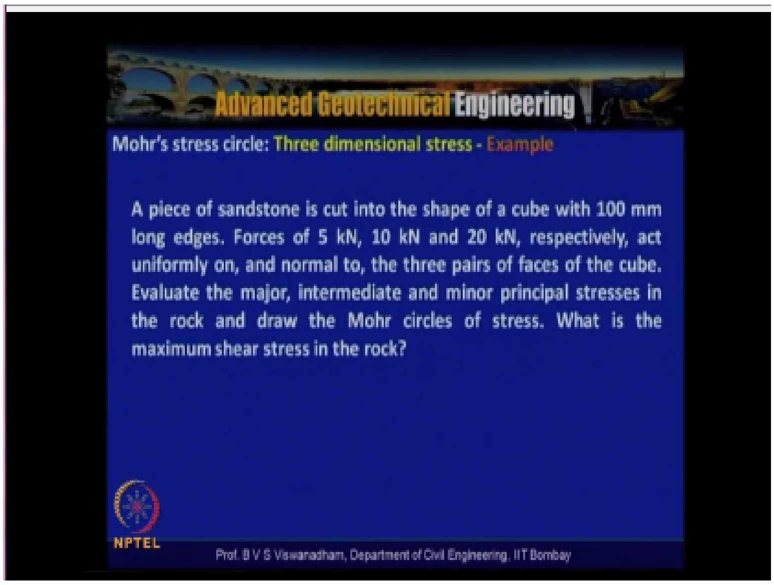

So let us look into a problem where in a piece of sand stone is cut into the shape of a cube with 100mm long edges and the forces of 5kg Newton and 10 kg Newton and 20 kg Newton are applied respectively and uniformly and are normal to the three pairs of the faces of the cube. So a piece of sand stone is cut into the shape of 10 cm in size, having the long edges of size of the edge as 10 cm and forces of 5 kn 10 kn and 20 kn are applied uniformly and normal to the three pairs of faces of the cube.

And evaluate the major intermediate and minor principle stresses in the rock and subsequently draw the Mohr circles of the stress and we need to also find out what is the maximum shear stress in the rock? So by knowing the forces the element is actually subjected. Then we can actually calculate what are the maximum shear stress within the rock and what are the major and intermediate and minor principle stresses the element which is the sand stone element are a piece of rock subjected. Now for this what we take is that the area of the each face.

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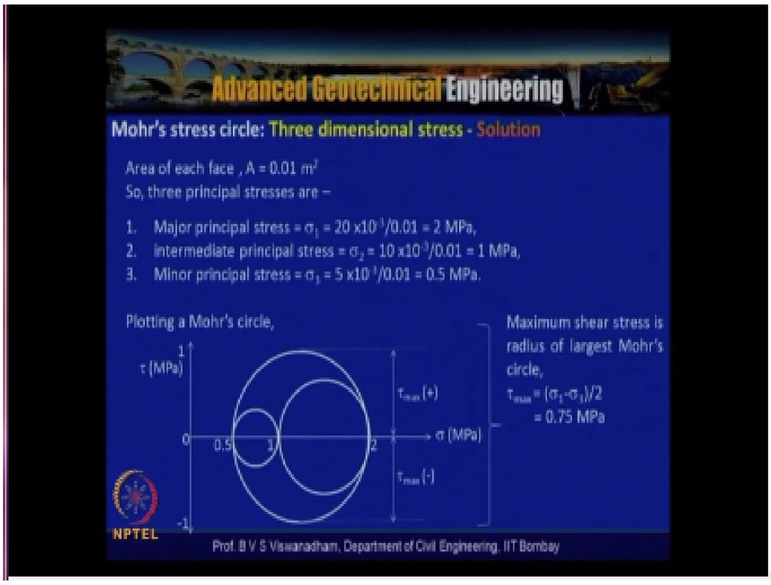

A = 0.01 meter square wherein there is nothing but 10cm x 10 cm, 100 cm2 in terms of meter it is 0.01 m2 hence three principle stresses major principle stress σ 1 = 20 x 10 -3 /0.01, that is 2 mega Pascal's and intermediate principle stress which is nothing but σ 2 which is 10 x 10 -3 /0.01 is one mega Pascal and minor principle stress σ 3 = 5 x 10 -3 /0.01 that is 0.5 mega Pascal's.

Now we have got σ 1, σ 2 and σ 3 the stresses, this we have calculated based on the sandstone which is nothing but the force which has been subjected divided that area of the face of that particular sandstone rock piece and with that we have got these major principle stress two mega Pascal's and intermittent principle stresses one mega Pascal and minor principle stress σ 3 0.5 mega Pascal's, and with this what we have got is that now on the τ σ space where the tow there is scale which is actually here is on the equal scale when you represent and we can write that on the σ which is here.

So the major principle stress being the σ with this as σ 1 and σ 2 that is intermediate principle stress 1 mega Pascal. So with one mega Pascal's /2 that is 0.5 mega Pascal's as the radius, you can draw a circle so with that we get the this first circle then once we get this one then with this has σ 2 that is one mega Pascal and σ 3 as 0.5 mega Pascal.

So 1 -0.5 that is 0.25 mega Pascal's as radius we can draw another circle that is the this one and then one more circle is actually possible that is σ 1 and that is the major principle stress and minimum principle stress that is 0.5 that means that 1.5 mega Pascal as radius. So we have drawn circle one circle 2 and circle 3. So with this we can actually see that the maximum shear stress in the element which is actually subjected which is the radius of the largest Mohr circle.

So the maximum shear stress is nothing but the radius of the largest Mohr circle. So if you look into this the maximum shear stress which is actually given by this circle is nothing but 0.25 mega Pascal and here this one is nothing but 0.5 mega Pascal, but however if you look into this circle which is having 0.75 mega Pascal as the radius, so the maximum shear stress is the element is actually subjected is about 0.75 mega Pascal's.

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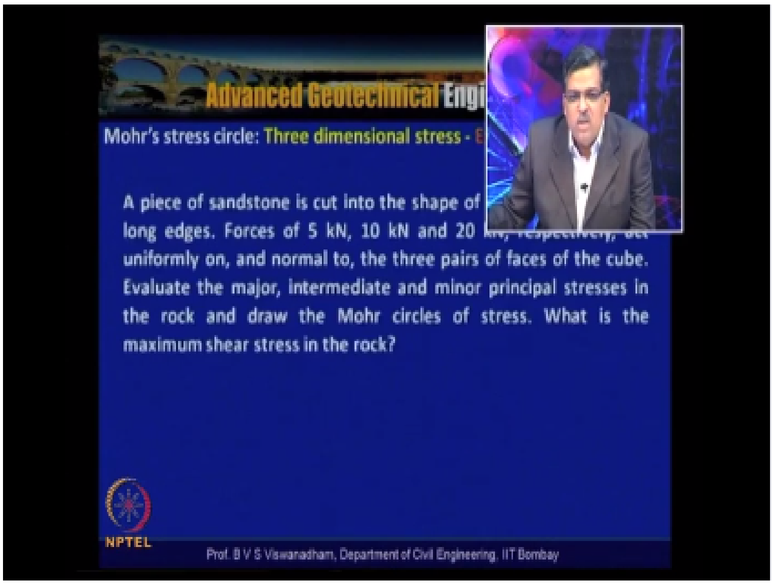

So what we have done in this particular problem is that we actually by knowing the forces subjected to the cubical shape sandstone piece, we actually have calculated what are the stresses acting on the major and intermediate and minor principle stresses and then we have drawn the Mohr circle of stress and then we also find out what is the maximum shear stress in the that rock piece.

So for this what we have done is that we actually have calculated the stresses with force by area and then we plotted at these stresses on the tow σ space with drawing three more circles wherein order to reduce the maximum shear stress here the tow maximum is actually here indicated and this is the circle actually the maximum shear stress. So further type of forces which has been subjected the rock piece actually has been subjected to shear stress of 0.75 mega Pascal's.

So if we are having this much shear stress and in order to favour along that particular plane then you know the material should have adequate shear strength, so the shear strength which is actually count as the shear stress which is actually resulted due to the external loading system. So in our further lectures what we try to look into is that how further we discuss about on this concept of this Mohr circles and the how to locate a pole and then how this different element conditions can be used in understanding about this stress states in a, for the elements and then we deduce, we connect ourselves to the stress paths in the PQ space.

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