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ADVANCED GEOTECHNICAL
ENGINEERING

Prof. B. V. S. Viswanadham

Department of Civil Engineering
IIT Bombay

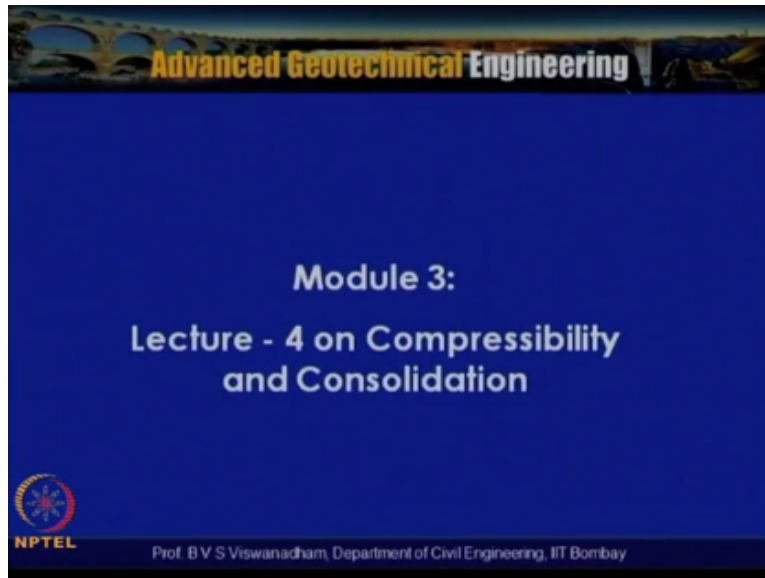
Lecture No. 22

Module – 3

Lecture – 4 on Compressibility
and
Consolidation

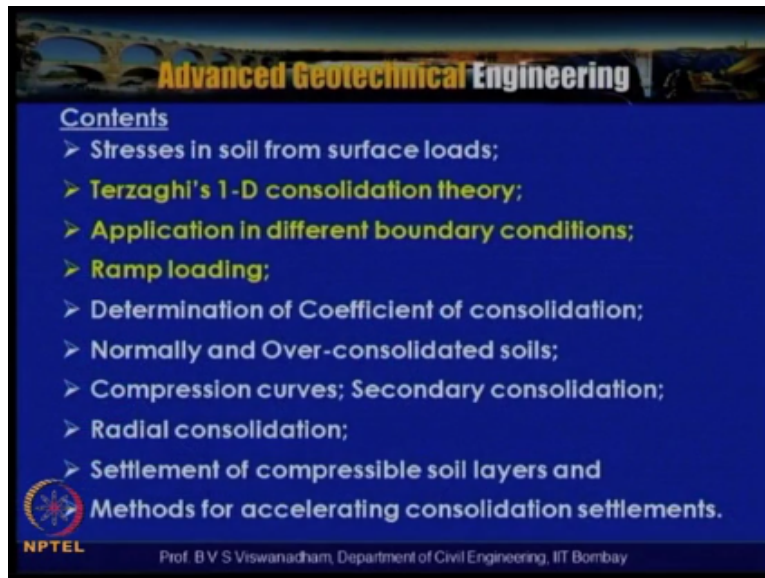
Welcome to lecture series on advanced geotechnical engineering we are actually in module 3 lecture number 4.

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So we are actually discussing about the compressibility and consolidation in module 3 and we introduced ourselves to the Terzaghi's one dimensional consolidation.

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And then in the previous lecture we have reduced the consolidation equation that is $\delta u / \delta t = C_v \delta^2 u / \delta z^2$ and this is for one dimensional consolidation and this one dimensional is assumed to prevalent if a clay layer is subjected to large area loading on its surface and if you are having a strained or a finite dimensions loaded area then there is a possibility that the 3 dimensional consolidation can come.

But however we assumed it like one dimensional consolidation and then we try to solve the calculate the settlements so this example of the possibility of 3 dimensional a 2 dimensional 3 dimensional consolidation can be in a footing resting on a soft clay so in the previous lecture we have deduced an equation $\delta u / \delta t = C_v$.

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Terzaghi's 1-D consolidation theory

Where:

$$m_v = \text{coefficient of volume compressibility} = \frac{a_v}{1 + e}$$

or

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial z^2} = C_v \frac{\partial^2 u}{\partial z^2}$$

Where:

$$C_v = \text{coefficient of consolidation} = \frac{k}{\gamma_w m_v}$$

Basic DE of Terzaghi's 1D consolidation theory and can be solved with proper boundary conditions.

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$\delta^2 u / \delta z^2$ and this C_v is the coefficient of consolidation and it is related if you look into this it is related with permeability and coefficient of volume compressibility so the k the coefficient of permeability or Terzaghi's permeability as the Terzaghi's.

Now assumed to be valid so $k = C_v m_v \gamma_w$ so if you look into this if k is high that means that the permeability is high then the C_v will be high then the time rate of settlements will be very high if k is low like same murrain clay where 1×10^{-9} to 1×10^{-10m} for second that the C_v will be very low and then the consolidation will take long time so this is the basic differential equation of the Terzaghi's one dimensional consolidation and can be solved with proper boundary conditions. So we have solved this by assuming the proper come boundary conditions and this was time.

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Terzaghi's 1-D consolidation theory

To solve the equation, we assume u to be the product of two functions, i.e., the product of a function of z and a function of t , or

$$u = (A_1 \cos Bz + A_2 \sin Bz)A_3 \exp(-B^2 C_v t) \quad \text{--(3)}$$
$$= (A_4 \cos Bz + A_5 \sin Bz) \exp(-B^2 C_v t)$$

where $A_4 = A_1 A_3$ and $A_5 = A_2 A_3$

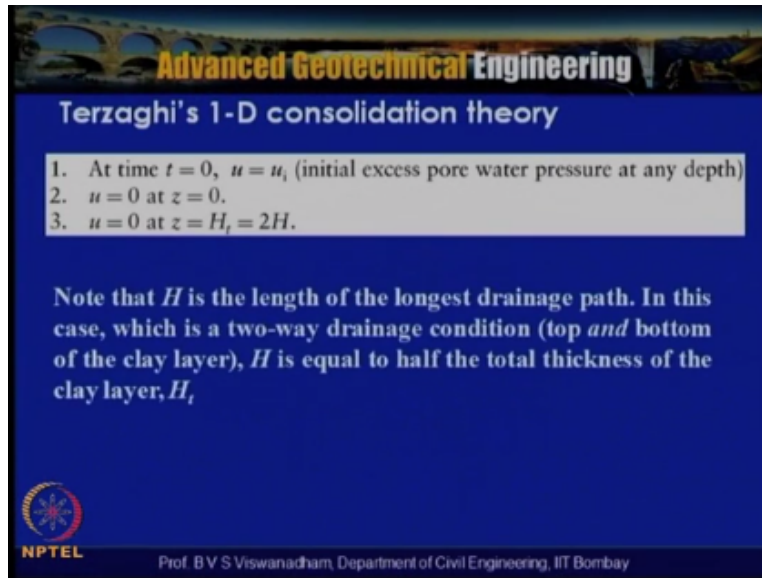
The constants in Eq. (3) can be evaluated from the boundary conditions, which are as follows:



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And further we have assumed these boundary conditions.

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Terzaghi's 1-D consolidation theory

1. At time $t = 0$, $u = u_i$ (initial excess pore water pressure at any depth)
2. $u = 0$ at $z = 0$.
3. $u = 0$ at $z = H_T = 2H$.

Note that H is the length of the longest drainage path. In this case, which is a two-way drainage condition (top *and* bottom of the clay layer), H is equal to half the total thickness of the clay layer, H_T .

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That time $t = 0$ it is assumed that the load is you know instantaneously applied on to the surface of the soil that is at that time the initial axis pore water pressure within the soil is $u = u_i = \Delta \sigma$ is increasing load which is applied on the surface of the soil then $u = 0$ at $z = 0$ and $u = 0$ at the $z = H_T$ that means that once the consolidation commences movement the consolidation comments the pore water pressure at the boundaries reduce it to 0.

This reasons we have discussed in pervious lecture so H is the longest drainage path so if you are having a clay layers and which is between let us say sand layer at top and bottom then if the thickness of the clay is say $2H$ then top portion H will flow toward the upper layer and upper pore the bottom position of H of the clay layer the water in the bottom position of H of the clay layer will flow downward.

So this is called double way drainage that is the double way that is 2 way drainage or you know 2 way drainage or it is also called as you know double open layer in this case the consolidation will be relatively faster and H will be equivalent to H_T if it the thickness then H will be equivalent to drainage parts length $= H/2$ or $H_d/2$ which is you know the effective drainage length let us assume that we are having rock base at the bottom and upper layer is you know the sand layer.

Then all the way water flows for entire thickness so it takes longer time then the $H_T = H$ so that means that here the water flows for take long time to reach the boundary and then it will take longer time if you are having a C_v which is actually very low value come considering let us they are having a low permeable soil.

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Terzaghi's 1-D consolidation theory

From the above, a general solution can be given in the form

$$u = \sum_{n=1}^{n=\infty} A_n \sin \frac{n\pi z}{2H} \exp\left(\frac{-n^2 \pi^2 T_v}{4}\right) \quad \dots(4)$$

To satisfy the first boundary condition, we must have the coefficients of A_n such that

$$u_i = \sum_{n=1}^{n=\infty} A_n \sin \frac{n\pi z}{2H} \quad \dots(5)$$

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So from the above the general solution can be obtained and this general solution is given by u the excess pore water pressure is n^2 , $n = 1, 2, n$ is infinity $A_n \sin n\pi z / 2H$ exponential $- n^2 \pi^2 T_v / 4$ so T_v that turn what we are seeing is called the time factor and which is nothing but T_{cv} / H^2 T is the time required for certain degree of consolidation and C_v is the coefficient of consolidation and $H = H_{dr}$ H_{dr} is the drainage path length.

So to satisfy the 1st boundary condition we must have the coefficient A and such that $U_i = n^2$ $N = 1$ to infinity $A \sin N \pi z / 2H$ because this we have put as.

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Terzaghi's 1-D consolidation theory

Equation (4) is a Fourier sine series, and A_n can be given by

$$A_n = \frac{1}{H} \int_0^{2H} u_i \sin \frac{n\pi z}{2H} dz \quad \text{---(6)}$$

Combining Eqs. (4) and (6),

$$u = \sum_{n=1}^{n=\infty} \left(\frac{1}{H} \int_0^{2H} u_i \sin \frac{n\pi z}{2H} dz \right) \sin \frac{n\pi z}{2H} \exp \left(\frac{-n^2 \pi^2 T_v}{4} \right)$$

where T_v is the nondimensional time factor and is equal to $C_v t / H^2$

So far, no assumptions have been made regarding the variation of u_i with the depth of the clay layer.

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Now equation 4 is a Fourier sine series and A_n can be given by $A_n = 1/H \int_0^{2H} u_i \sin n \pi z / 2H dz$ combining equations previous slide 4 and 6 then we get an expression which is $u = \sum_{n=1}^{n=\infty} \left(\frac{1}{H} \int_0^{2H} u_i \sin \frac{n\pi z}{2H} dz \right) \sin \frac{n\pi z}{2H} \exp \left(\frac{-n^2 \pi^2 T_v}{4} \right)$ SO THE T_v is the non dimensional time factor and is equal to $C_v T / H^2$ that is what we have discussed so far the no assumptions have been made regarding the variation of u_i with the depth of the clay layer.

But there are you know the number of variations which are actually possible for u_i with the depth it can be you know the simplest thing which has been assumed by terzaghi is that the constant variation of u_i with the depth that is mostly used in all the solutions that means that if u_i if $\Delta \sigma$ load is applied then it is assumed that the excess water pressure is assumed to occur uniformly throughout it is depth.

But there are also some assumption like u_i assumed to increase with like parabolic shape or it can be like a 0 at the top and you know at the centre it will be it is approximated like a triangle so the different variations are there but however we are actually discussing on the uniform variation of u_i with depth.
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Terzaghi's 1-D consolidation theory

If u_i is constant with depth—i.e., if $u_i = u_0$

$$\frac{1}{H} \int_0^{2H} u_i \sin \frac{n\pi z}{2H} dz = \frac{2u_0}{n\pi} (1 - \cos n\pi)$$

So,

$$u = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} (1 - \cos n\pi) \sin \frac{n\pi z}{2H} \exp\left(\frac{-n^2 \pi^2 T_v}{4}\right)$$

Note that the term $1 - \cos n\pi$ in the above equation is zero for cases when n is even; therefore u is also zero. For the non-zero terms it is convenient to substitute $n = 2m+1$, where m is an integer.

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So this is actually shown in the slide where $U_i = U_0$ which is actually for a double drainage layer which is actually shown here $H_d = 2H$ so this is for the double drainage layer, so this is you know what the initial axis for the pressure which is actually increased upon loading, so if U_i is constant with that if that is $U_i = U_0$ then we can write $1/H$ to the \int of 0 to $2H =$ into $U_i \sin 2\pi/z / 2h$ into $dz = 2U_0 / n\pi$ into $1 - \cos n\pi = U_0$, so this one will become like $U = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} (1 - \cos n\pi) \sin \frac{n\pi z}{2H} \exp\left(\frac{-n^2 \pi^2 T_v}{4}\right)$.

Here note that the term $1 - \cos n\pi$ is the in the above equation 0 for cases when n is even for $n = 2, 4, 6$ it is 0 so therefore U is also 0 for the non zero terms it is convenient to substitute $n = 2m + 1$ where m is an integer $m = 0, 1, 2, 3$ it will take so $n = 2m + 1$ which what we have assume.

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Terzaghi's 1-D consolidation theory

$$u = \sum_{m=0}^{\infty} \frac{2u_0}{(2m+1)\pi} [1 - \cos(2m+1)\pi] \sin \frac{(2m+1)\pi z}{2H}$$

$$u = \sum_{m=0}^{\infty} \frac{2u_0}{M} \sin \frac{Mz}{H} \exp(-M^2 T_v) \text{ where } M = (2m+1)\pi/2$$

At a given time the degree of consolidation at any depth z is defined as

$$u = u_i (1 - U_z)$$

$$U_z = \frac{\text{excess pore water pressure dissipated}}{\text{initial excess pore water pressure}}$$

$$= \frac{u_i - u}{u_i} = 1 - \frac{u}{u_i} = \frac{\Delta\sigma'}{u_i} = \frac{\Delta\sigma'}{u_0}$$

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So accordingly now what we write is that you know by putting in terms of m then $U = m = 0$ to $m \rightarrow \infty$ $2U_0 / 2m + 1/\pi$ into $1 - \cos 2n + 1$ into $\pi \sin$ to $m1 + \pi$ into $\pi z / 2H$ into exponential of $-2m1$ so we are substituted $n = M + 1$ so $-2m + 1$ whole square $\pi^2 T_v / 4$, so this we can write and simplified like this with $m = 2m1$ into $\pi/2$ so we can write $U = m$ ranger from 0 to ∞ $2U_0 / m \sin mz / H$ exponential of $-m2 T_v$ where $m = T_v$ into $\pi/2$, so at a given time the degree of consolidation at any depth z in within the clay layer that is $z = 0$ and θ and $z = 2H$ at the bottom of the clay layer.

If $Ht = 2H$ then in that case the degree of consolidation at any depth is defined as axis for pore water pressure dissipated to the initial axis pore water pressure, so initial excess pore water pressure is nothing but you writ that is the you know the excess pore water pressure dissipated divided by initial excess pore water pressure, so which is nothing but $U_i - U / U_i$ so w can actually writ like $1 - U/U_i$ which is nothing but $\Delta\sigma' / U_i$ which is nothing but $\Delta\sigma' / U_0$, so this is $\Delta\sigma'$ is nothing but the net you know the stress which is actually the increase in implement area which applied.

And U_0 is the initial excess pore water pressure so by simplifying this one $U_z =$ you know when you write in terms of U then $U = U_i$ into $1 - z$ so from $U_z = 1 - U/U_i$ so if you look into this here by knowing U_i , U_i is nothing but the initial axis pore water pressure that is let us say if you are assuming $\Delta\sigma'$ is increasing effect stator due to consolidation then this $U_i =$ that particular that

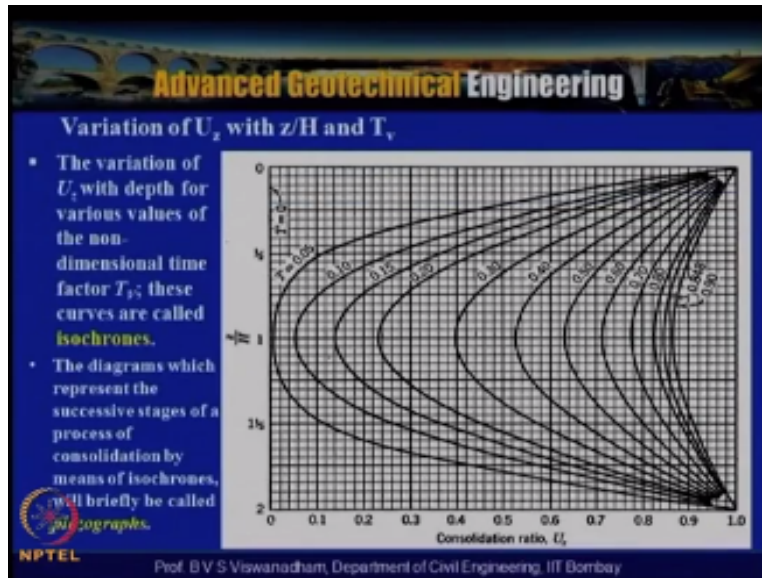
much particular stress, so $U_i =$ let us say if it is if $\Delta\sigma$ of 100KP applied and the constant variation is assumed when $U_i = 100\text{KP}$.

Then U_z is the degree of consolidation at you know that particular in depth within the clay, so if you look into this one where if you take at any time P cyclic greater than $T = 0$ then you know you we can see this T the $U_z = 1$ because you know the moment that the boundaries the moment consolidation comments and the boundaries are the one way respond to the in with the soil actually transfers the you know the water transfers the you know soil, soil grains at the boundaries, so with that the pore pressure will get dissipated like 100% and the in increase in effect to stress will be equal to $\Delta\sigma$.

So at any given time the degree if consolidation is given by U_z excess pore water pressure dissipated to the initial excess pore pressure and $U = U_i$ into $1 - U_z$. Now in most cases if you however we need to obtain the average degree of consolidation for the entire layer and so generally though we have actually defined at U_z but mostly most cases what we do is that we calculate the average degree of consolidation for the entire clay layer thickness. So in most cases we need to obtain the average degree of consolidation for the entire layer and this is given by U_{av} that is the average degree of consolidation is $1/Ht$.

Because Ht is the thickness of the clay layer 0 to Ht $u_i dz - (1/Ht) 0$ to Ht $u dz / (1/Ht) 0$ to Ht $u_i dz$.

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So the solution of this is obtained as this the system of curve which is shown here variation of U_z with z/H and T_v is actually shown here. Here the variation of U_z with depth for various values of non dimensional time factors so this t is nothing but T_v , T_v 0.05 and this is $T_v=1$ here which is you know the degree of consolidation will be you know 1.

That is $T_v=$ you know here which is for degree of consolidation is equal to 100%, so the variation of U_z with depth for various values of non dimensional factors which is shown here and these curves are also called as isochrones and the diagram which represents the successive stages of the process of consolidation by means of isochrones is also briefly called as pico graphs the diagrams which represent the successive stages of process of consolidation by means of isochrones will briefly be called as pico graphs.

That means that if we are actually having let us say you know the so called you know at $T=0$ that initial excess pore water pressure exist to from the hydrostatic pressure the excess pore water pressure increases to say $u_0+\Delta u$ and then it tries to fall in such a way that at the center of the clay layer the pore water pressure will remain at to be dissipated and then at the top and bottom boundaries if it is open layers.

Then you know dissipates rapidly so the system of the migrations of this curve which is the system of successive stages of process of consolidation by means of isochrones is actually called as pico graphs.

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Example problem

Consider the case of an initial excess hydrostatic pore water that is constant with depth, i.e., $u_i = u_0$. For $T_v = 0.3$, determine the degree of consolidation at a depth $H/3$ measured from the top of the layer.

$$u = \sum_{m=0}^{m=\infty} \frac{2u_0}{M} \sin \frac{Mz}{H} \exp(-M^2 T_v)$$

where $M = (2m + 1)\pi/2$

Here $z = H/3$, or $z/H = 1/3$, and $M = (2m + 1)\pi/2$

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So let us consider an example based on the chart we have discussed and in this consider the case of an initial excess pore water, initial excess hydrostatic pore water pressure that is constant with the depth that is what, what we have assumed that $u_i = u_0$ and for $T_v = 0.3$ determine the degree of consolidation at a depth $z/3$ measured from the clay layer that is $z = H/3$. So we have derived you know the equation for the excess pore water pressure as in terms of $u = \sum_{m=0}^{m=\infty} \frac{2u_0}{M} \sin \frac{Mz}{h} \exp(-M^2 T_v)$, where $M = (2m + 1)\pi/2$, so here $z = H/3$ or $z/H = 1/3$ and $M = (2m + 1)\pi/2$ so $M = (2m + 1)\pi/2$ here.

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Example problem For $m = 0, 1$ and 2
 Here $z = H/3$, or $z/H = 1/3$, and $M = (2m+1)\pi/2$

1. z/H	1/3	1/3	1/3	m > 3 is negligible; Hence not required.
2. T_v	0.3	0.3	0.3	
3. m	0	1	2	
4. M	$\pi/2$	$3\pi/2$	$5\pi/2$	
5. Mz/H	$\pi/6$	$\pi/2$	$5\pi/6$	
6. $2/M$	1.273	0.4244	0.2546	
7. $\exp(-M^2 T_v)$	0.4770	0.00128	≈ 0	
8. $\sin(Mz/H)$	0.5	1.0	0.5	
9. $(2/M)[\exp(-M^2 T_v) \sin(Mz/H)]$	0.3036	0.0005	≈ 0	$\Sigma = 0.3041$

Using the value of 0.3041 calculated in step 9(above), the degree of consolidation at depth $H/3$ is $U_{(H/3)} = 1 - 0.3041 = 0.6959 = 69.59\%$

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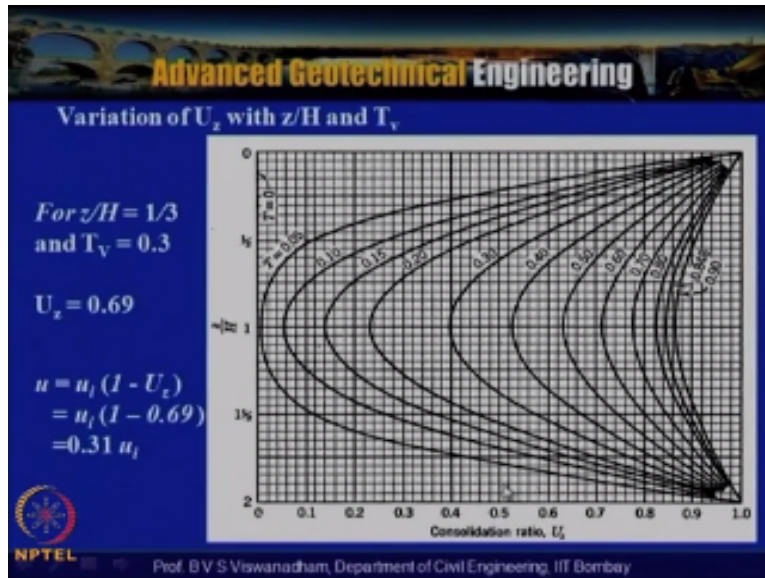
So here what we do is that we have given the solution in terms of the, in terms of a table where $z=H/3$ and $Z/H=1/3$, $M=(2m+1)\pi/2$ so here what we do is that for different values of M . For example if you see $M=0$ then $M=0$ means it is like $3\pi/2$ that $M=0$ here that means that this will become M , M will become yeah, $M=1$ here that $T+1$ $3\pi/2$ is correct, $M=0$ it is $\pi/2$ and $M=3$ it is $M=2$ it is $5\pi/2$.

So this M value is $\pi/2$, $3\pi/2$ and $5\pi/2$ so we what we have given is that we have given 0,1,2,3 like that this we have to see whether where it will be we can actually stop this iteration and the z/H is $1/3$ that is constant, T_v what we wanted is 0.3 and Mz/H after having got this one z/H is $1/3$ so Mz/H is $\pi/6$, $\pi/2$, $5\pi/6$ then $2/M$ 1.273, 0.424 and exponential of $-M^2 T_v$ we have got this $\sin Mz/H$ that is also obtained here.

Then the equation which is gives that 0.3036, 0.0005 and at almost equal to 0, so when you submit these things you know you do this and this summation it comes to 0.3041, so that means that the value of 0.3041 calculated in step 9 above is the degree of consolidation at depth $z/3$. So that means that $u_{H/3}=1-0.3041$ that is 69.59% of consolidation is already occurred.

That means that 69.59% of consolidation is already occurred, so the $u_{H/3}=1-0.3041$ which is nothing but 0.6951 which is nothing but 69.59%. So here what we are actually doing is that we actually have calculated now this we can actually also see from the chart here.

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So this $z=0$ and $z/H=1$ so $z/H=1/3$ so somewhere here we can actually pick up for $T_v=0.3$ so 0.2 and 0.3 so we can see that you know the value of this the U_z value will be about 0.69 here between this is for 0.3 and this one is about 0.69 so that U is equal to initial excess pore water pressure the excess pore water pressure which is at to be dissipated is equal to $u_i(1-U_z)$ so U_z is the degree of consolidation.

So now if you looking to this suppose let us say if $0.3u_i$ is at to be dissipated that means that 16% of the pore water pressure already got dissipated excess pore water pressure is dissipated and it is transferred to the soil grains and 34% is at to be dissipated at that particular depth $z/H=1/3$. So this is how you know we use this charts for determining this you know the consolidation at any depth.

But for we actually also have you know the average degree of consolidation generally we take but this is an example with determining the consolidation you know this for time factor of T_v that means that after certain time of an application of certain initial u_i then you know we are calculating that as time factor T_v for that we have determine what is the degree of consolidation at a particular depth $z=H/3$.

By using the equation with what we derive and you also tells that if $M>3$ is actually you know this almost like the degree of consolidation which is obtained is negligible hence you know we can actually stop at this particular you know iteration with $M=2$.

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Terzaghi's 1-D consolidation theory

The average degree of consolidation is also the ratio of consolidation settlement at any time to maximum consolidation settlement.

Note, in this case, that $H_t = 2H$ and $u_i = u_0$.

➤ Combining the following equations:

$$u = \sum_{m=0}^{\infty} \frac{2u_0}{M} \sin \frac{Mz}{H} \exp(-M^2 T_v)$$

$$U_{av} = \frac{(1/H_t) \int_0^{H_t} u_i dz - (1/H_t) \int_0^{H_t} u dz}{(1/H_t) \int_0^{H_t} u_i dz}$$

$$U_{av} = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v)$$

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Now the Terzaghi's one dimensional consolidation as we are talking about the average degree of consolidation we let us define that the average degree of consolidation is also the ratio of the consolidation settlement at any time to the maximum consolidation settlement. Note that in this case if $H_t=2H$ $u_i=u_0$ so combining the following equations $u = \sum_{m=0}^{\infty} \frac{2u_0}{M} \sin \frac{Mz}{H} \exp(-M^2 T_v)$ and U_{av} where we have just now defined that $U_{av} = \frac{(1/H_t) \int_0^{H_t} u_i dz - (1/H_t) \int_0^{H_t} u dz}{(1/H_t) \int_0^{H_t} u_i dz}$.

So combining these two equations and then completing the integration we will get the expression for average degree of consolidation as follows, $U_{av} = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v)$ where $M = (2m+1)\pi/2$, where T_v is the time factor which is nothing but T_c/H^2 .

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Terzaghi's 1-D consolidation theory

➤ Terzaghi suggested the following equations for U_{av} to approximate the values from

$$U_{av} = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v)$$

For $U_{av} = 0-53\%$: $T_v = \frac{\pi}{4} \left(\frac{U_{av}\%}{100} \right)^2$

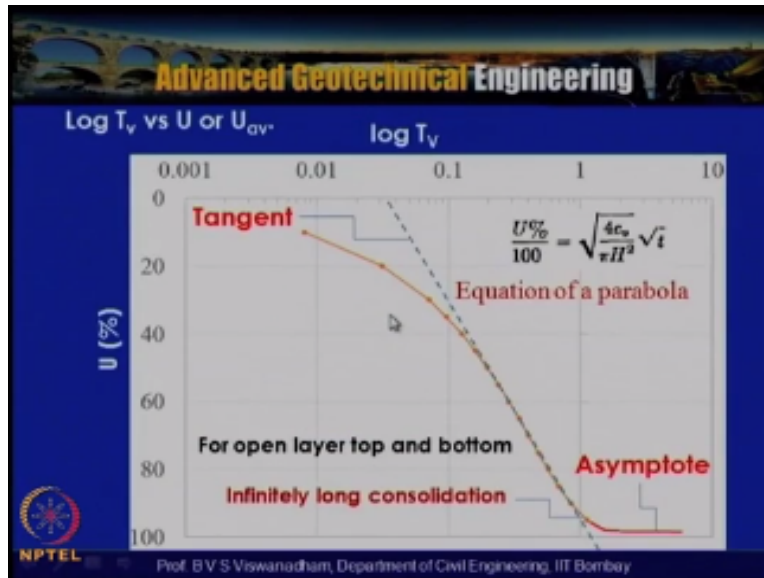
For $U_{av} = 53-100\%$: $T_v = 1.781 - 0.933 [\log(100 - U_{av}\%)]$

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So Terzaghi's suggest at the following equation for U_{av} to the approximate values from the $U_{av}=1-M$ ranges Σ of M ranges from M_0 to ∞ , $2M^2 \exp(-M^2 T_v)$ so for $U_{av}=0$ to 53% there is actually 52.6% it is rounded to 53% in some you know references also say that 60% but let us say according to Terzaghi it is $U_{av}=0$ to 53% , $T_v=5/4 (U_{av}/100)^2$ and then for $U_{av}>53$ or equal to 53 200% that T_{av} is given by $1-1.781-0.933$ within square brackets $[\log(100-U_{av}\%)]$ the parentheses close the square bracket close.

So the U_{av} expression here is given which is $T_v=1.781-0.933$ so there is a reason for giving this so if you plot the logarithmic of you know these T_v the time factor with you know U_{av} then we will get a curve which is shown like this.

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The curve represents the curve goes like this initially the initial part of the curve looks like parabola and then you know it beyond 90%, 95% it will try to you know take the shape of the, it will get asymptote with the horizontal so you can see that this is asymptote which is shown here. So it indicates that you know the consolidation never complete the long consolidation and upto 53% of also if this is the initial portion of the parabola and when you extend the tangent it goes and hit at the negative you know T_v and this resembles the equation of the parabola, that is written by $U/100 = \sqrt{4C_v / \pi h^2}$ so this is the equation of parabola.

And this is the initial part of you can say that the primary for the consolidation of 260% this is actually valid and for this is this graph or this condition is actually for two conditions one is the open layer at the top and bottom and initial excess pour water pressure is constant with the depth. So this particular graph beyond that it actually straightens and beyond that 90% you can see that this get a asymptote.

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- For values of $U\%$ greater than 52.6 the $\log T_v$ vs U curve is almost identical with a curve with the equation

$$T_v = 1.781 - 0.933 [\log(100 - U_{av}\%)]$$
- It should be noted that the radius of curvature of the curve U (%) vs $\log(T_v)$ increases steadily until $U\%$ becomes approximately equal to 50, then decreases once more and assumes a second minimum at about $U\% = 85$.
- The curve thus obtained has a point of inflection at about $U\% = 75$.
- In the vicinity of $U\% = 95$ it flattens rapidly and approaches a horizontal asymptote corresponding to $U\% = 100$.

On overall, the curve represents an equation $\log_{10} T_v = F(U\%)$

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So the explanation for this is given like this by values for $U > 52.6$ which is rounded as 53% of the $\log T_v$ versus U curve is almost identical with the curve with the equation that is for $>$ than that. So that is the reason why this curve was suggested by Terzagh's this particular equation suggested for values > 52.6 the curve is actually is almost identical curve with the equation $T_v = 1.781 - 0.933 \log 100 - U_{av}$.

So it should also be noted that the radius of the curvature of the curve U % versus $\log T_v$ increases steadily until $u = 50$ then decreases once and assumes the 2nd $U = 85$. So the curve that as the point of inflection at about $U = 85$. So the curve that obtained as a point of inflection at about $U = 75$, the point of inflection is about that particular point where you have got one curvature here and you have got another curvature here that is about 75% somewhere here.

And the facility of $U = 95$ it flattens rapidly that is what we are saying approaches corresponded $U = 100\%$. So now on overall the curve represent an equation we can say that $\log T_v =$ function of U %.

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Substituting for $T_v = tc_v/H^2$

$$\log_{10} T_v = \log_{10} t + \log_{10} \frac{c_v}{H^2} = \log_{10} t + \text{const.} = F(U\%)$$

- > If the degree of consolidation of two beds of clay with different values of c_v/H^2 is plotted against the logarithm of time, the time-consolidation curves thus obtained have the same shape but they are separated from each other by a horizontal distance $\log_{10} (c_v/H^2)$.
- > For $T_v/t = 1$ the time-consolidation curve becomes identical with the time factor-consolidation curve.
- > This important property of the semi-logarithmic time-consolidation graph facilitates comparison of empirical consolidation curves with the theoretical standard curve for the purpose of detecting deviations of the real process from the theoretical one. Therefore in many cases the semi-logarithmic plot is preferable to the arithmetic plot.

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So substituting $T_v = H^2 \cdot t$ we can write $\log T_v = \log T_c / H^2$ when you apply the log for both sides we can write $\log T_{10} + \log C_v / H^2$. So we can say that you know as C_v and thickness are constant, so that is constant so we have $\log T_v$ that is the time equate for consolidation = function of U . so the degree of consolidation of the two beds of the clay with the different values of C_v / H^2 is plotted against the logarithm of the time, that time consolidation curves does obtain at the same shape but they are separated from each other by horizontal distance.

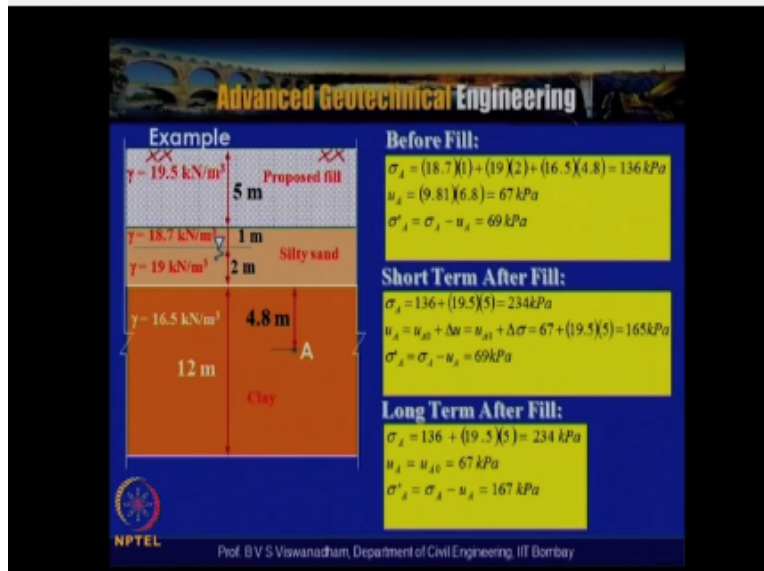
So this equation implies whatever we have written like log to the log $T_{v10} = \log T$ that is time required for consolidation to the base 10 + constant = function of U present it implies that if the degree of the consolidation of two clay beds with the different values for C_v / H^2 is plotted again is the logarithm of time, the time consolidation curves that obtained have the same shape but they are separated by each other with the horizontal distance which is nothing but $\log_{10} C_v / H^2$.

For $T_v/T = 1$ the time that is the $T_v/T = 1$ that time consolidation curve becomes identical with the time factor consolidation curve. So hence because that is the reason why we use this because you can see $T_v/T = 1$ that time consolidation curve becomes identical with the time factor consolidation curve. So this is important property of semi logarithmic time consolidation graph facilitates the comparison of the with standard curves for the purpose of detecting deviation of dual process from the theoretical.

So therefore in many cases the semi logarithmic plot is arithmetic plot, so the important property of this semi logarithmic time consolidation curve facilities the comparison of the empirical

consolidation curve with the theoretical standard curves for the purpose of detecting the deviation of the real process from a point.

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So let us take an example again where we have got a solid stator which is $\gamma 19.5 \text{ k N/m}^3$ that is the unit weight here, the water table is here and this is the purpose film and this is the solid stator and this is the ground surface. On this soil is purposed to be decline for 5m, now we will consider before fill and short term after fill, long term after fill. So we want to calculate what the effective stress at point A.

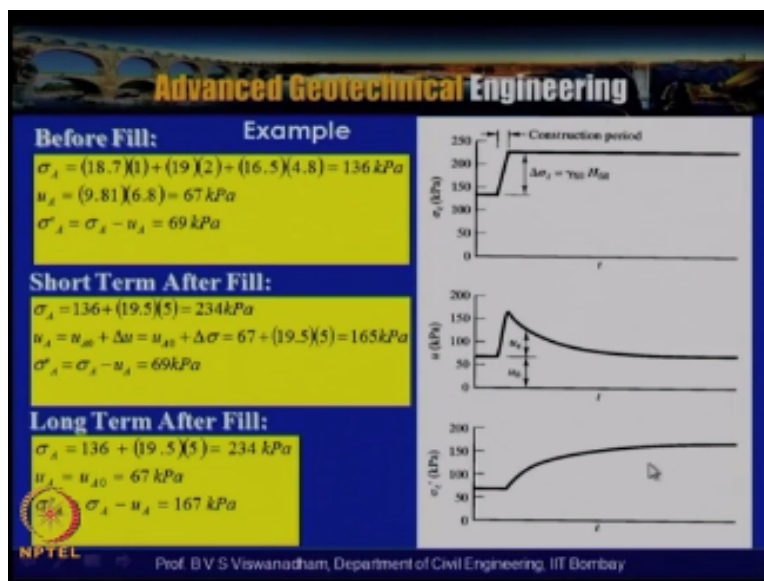
So here this is the silty sand layer and this is water table that is one way below the ground surface and the once the fill is placed then it will become like 6m but then this will become the new ground surface and below the fill that is γ which is given that is saturated 19.5 k N/m^3 and 16.5 k N/m^3 and that below point a is located is 4.8 and before the placement of the fill the total stress can be calculated we know that is 18.7 that is before fill so this will not be there $18.7 \times 1 + 19 \times 2$ that is this thickness + 16.5×4.8 .

So we are actually interested in this point, so that is 136 kPa and the pour water pressure at point A by taking $u \gamma = 9.8 \text{ k}$ that is 67 kPa shortly after placing the fill. Now $\sigma_A = 136 + 19.5 \times 5 = 234 \text{ kPa}$ now the $\Delta \sigma$ is increased by about you know 100 kPa then $U_a = U_{a0} + \Delta u$ so the U_a is which is nothing but $67 +$ shortly after placing the fill enter the pour water pressure is time is started so pour water is 165 kPa .

As the drainage is started the effective stress will remain constant, so because of that $\sigma = \sigma_A = \sigma_A - U$ that is 69k/Pa. now long term after the fill assuming that the clay and rovers consolidation and the complete consolidation is occurred then what will happen is that now the total stress is 234kp and the pore water pressure drops down to 67kp this was provided there is no change in the ground water table levels this $u_a = u_{nr} = 67kp$ now σ' a which is nothing but now $\sigma - u_a$.

Now we can see that now this particular effective stress at this point after a certain after elapsing the time require for the completion of consolidation the soil grain the strength by you know the increasing the enhancement of the effective stress by the amount which is nothing but 136 to 69 to 167 kp. So this is about the effective stress increased by almost two times.

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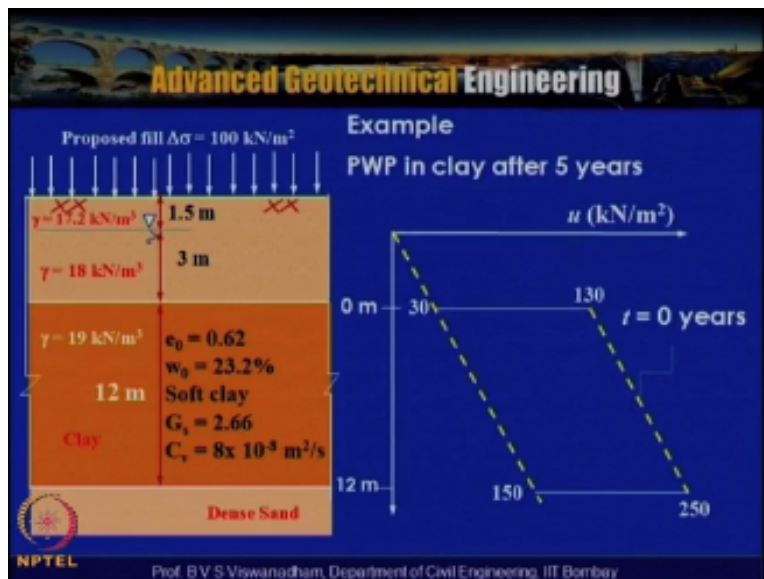


So this is actually given this solution this is the solution but it is actually given in a graphical form where the construction period though instantaneous loading is assumed but the fills are

actually constructed you know over a period of time, so we will be also in this lecture the ramp loading will be considered so this is you know the so-called the construction period and this is the $\Delta \sigma$ which is nothing but you know this maintained constant.

So when we place this one and there is an initial excess pore water pressure and then the consolidation occurs that means that the pore water pressure dissipates, now what we have done is that we actually calculated the effective stress gradually as the excess pore water pressure dissipates the effective stress keeps on increasing, so this is actually whatever before and after and shortly after the fill there is at this point there is at this point and then after a long term after the fill so this is actually at this points. So this indicates this actually shows this example in a picture you will form.

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Now take another example wherein we have a soil strata which is shown here and we have the sand layer at the top so this is the sand layer and this is a sand layer at the bottom then sand layer, so this is called open double open layers that means that two layer system where the clay sand which is between two layers and water table is 1.5m below the ground level and the clay is having 12m thickness so 0 to 12 m is the clay thickness this is the top of the clay this is bottom of the clay and 6m is the mid of the clay layer.


And in this actually proposed fill is about $\Delta \sigma$ is going to 100kp 100km² and the e_0 and initial water content the properties are given but important what we use in this is that c_v coefficient of consolidation is $82 \cdot 10^{-8} \text{ m}^2$ per second so here the pore water pressure in clay after five years is actually can be given like this the pore water pressure that is for in clay after five years you know so if you look in to this so this is the initial excess pore water pressure wherein what we can say is that you know this is the water table so at 0 so $3 + 2 \cdot 150\text{kp}$ will be there.

So this is the initial excess pore water pressure this is provided if there is you know the disturbance of this soil on this particular area then the hydrostatic pore water pressure conditions are prevalent but you know this clay layer is subjected to certain type of fill and randomly and then there can be you know the pore water pressure can be more than the hydrostatic pressures but assuming that if the no such activities then we actually have got this variation then a time $t = 0$ once we apply at you know the pore water pressure increases to 130kp here and here it is because $u_i = \Delta \sigma = 100\text{kp}$ so it increases to 250kp or kn m^2 .

So this is the first sty isochrones at $t = 0$ so what will happen is that the clay the undergoes consolidation over a period of time so we wanted to know what will be the pore water pressure in clay after five years so moment the you know once the you know load is applied and if no drainage is actually taking place then the pore water pressure the upper region is 0 to 30 and then it increase to 130 and then 250 it actually this is the this is at you know immediately after placing the fill.

So now consider this particular portion only so this is will be the final isochrones and this will be the first isochrones this is the first isochrones and this will be the final isochrones which can happen once 100% consolidation which is not possible.

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Advanced Geotechnical Eng

Example

Using $T_v = t c_v / (H_{dr}^2)$

For $t = 5$ years

$C_v = 8 \times 10^{-8} \text{ m}^2/\text{s} = 8 \times 10^{-8} \times 3.1536 \times 10^7 \text{ m}^2/\text{year}$

$H_{dr} = 2H = 12/2 \text{ m (Double drainage)}$

$T_v = 0.35$

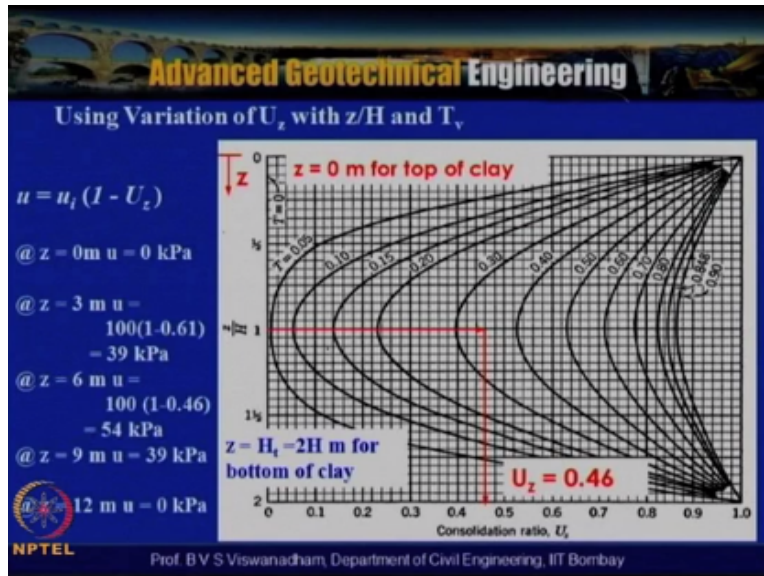
@ $z = 0 \text{ m}$	$z/H = 0$	$U_z = 100 \%$
$z = 3 \text{ m}$	$z/H = 3/6 = 0.5$	$U_z = 61 \%$
$z = 6 \text{ m}$	$z/H = 6/6 = 1.0$	$U_z = 46 \%$
$z = 9 \text{ m}$	$z/H = 9/6 = 1.5$	$U_z = 61 \%$
$z = 12 \text{ m}$	$z/H = 12/6 = 2$	$U_z = 100 \%$

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And if that happens then that is you know going to achieve, now using $t_v = t c_v / h_{dr}^2$ and for $t = 5$ years we can write c_v in terms of m^2 per year when you convert then, so this is $3.1536 \times 10^7 \text{m}^2$ per year and $h_{dr} = 2h / 2$ that is $12/2$ that is double drainage and t_v can be obtained as 0.35. Now what we can actually calculate is that at $z = 0 \text{ m}$ 3 m 6 m 9 m 12 m means at these depths that is 0 3 6 9 and 12 what is the you the degree of consolidation.

So by not assuming as you know the z we will try to find out so this can be obtained from the chart which we have given but $u_z = 100\%$ $u_z = 61$ and $u_z = 46\%$ $u_z = 61\%$ so this is at 3m this is at 9m and these are the top and bottom so you can see that this 100% consolidation is already occurred.

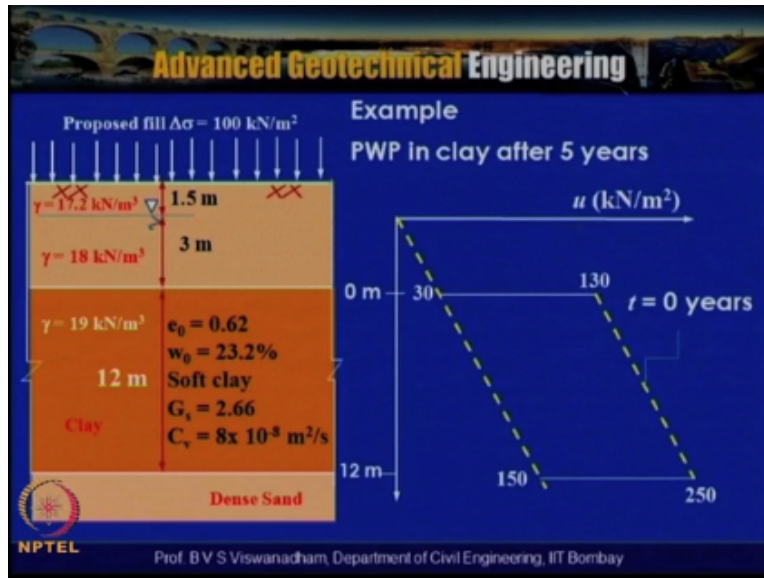
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So these are obtained like this here for this is the top of the clay $Z = 0\text{m}$ for the top of the clay and $z = H/2 = H$ meters below the for the top of the bottom of the clay from this is the bottom of the clay from this the bottom of the clay so we can actually find out here that terzaghi $H= 1$ let us say terzaghi = 1 for $T_v =$ we have got less than 3.5 so for the we can actually obtain $U_z = 0.46$ so for the $z/H = 1$ which is nothing but here $U_z = 0.46$ this is how we have to obtain so by using $u = U_i \times 1 - U_z$ at $z = 0\text{ m}$ $u = 0\text{ kPa}$ and 3m, 6m, 9m and 12m if you look into it.

So what we can actually do is that this 39kPa and 54kPa and 0 you know 0, 39, 54, 39, 0 this is the pore water pressure which is at to be dissipated that is that what we have found is that this excess pore water pressure which is at to dissipated is actually is obtained and this actually after 5 years so we actually calculated the time factor based on the T after applying the search of 100kPa you know what will be the you know the pore water pressure in the soil so this exercise actually has help.

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Plot this isochrones after you know 5 years this is the excess pore water pressure at to be dissipated so you can see that this is at $z/H = 1$ that is you know there this point so this is you know $z/$ you know at 3m and this at 6m 9m and 12m and this is at top so this system of migration of isochrones actually happens here and this portion which actually ached with light blue lines this actually shows that portion is already transferred to the soil grains that means that the water pressure the excess water pressure already transferred to soil grains and the portion within this joule is apt to be dissipated.

So the system of you know this isochrones is actually called as pica graphs and so we can actually draw numeral number of Pasco chromes the isochrones as we actually travels from 0 years to 0 let us say that second years can be here third year can be here and may be after 10 years it can be here so in this example what we have done is that we have tried to calculate.

What is the excess pour water pressure after let us say 5 years of time period by what we done is that we actually have used this in the solution of the differential consolidation differential equation of the consolidation theory and then we tried to use that then try to find you know this you know the pour water access pour water pressure to be dispend after certain period of time.

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Advanced Geotechnical Engineering

Degree of consolidation under time-dependent Loading (Ramp loading)

➤ Olson (1977) presented a mathematical solution for one-dimensional consolidation due to a single ramp load.

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Now consider now as it has been told her that the loading which is you know we assume that suppose if we are having a clay layer of certain thickness $ht=2h$ and if you are having a double layer at both top and bottom that is forest layer and then ground water level as this point here and if you are actually trying to put the field it is not possible to instantaneously which can actually can happen time $t=0$.

So there is you know in fact in this particular you know 0 to t_c that is g_c is the time required for construction so for filling up to height on particularly if the soil is very soft it is not possible to load you know the entire these filled in a one row you have to do in different stages so in a way what will happen is that it requires certain time period of construction that means that what we can do is that what one need to do is that you have to fill certain height h_1

And weight for certain time and again fill for h_2 weight for certain time so that weighting period is called weighting period of you know for you know disporting the access pour water pressure which actually will occur for a given weight so for a given fill height so once we do that incremental as stage wise construction then we can actually achieve that so the 1977 presented a mathematical solution for one dimensional consolation due to a single ramp load.

But if you are actually assuming you know the instantaneously load then you know we will be actually looking into that where in it actually has got a you know the deviation from the you know the instantaneously load conditions so Olsen 1977 given a mathematical solution for one

consolidation due to a single ramp load where stage way loading has not been considered wherein a single ramp has been constructed with time t_c .

But when you look into this it can be approximately when you have got multiple stages let us say that the q_c is reached into q_{c1} , q_{c2} , and q_{c3} and where final $q_{c3} + q_{c1} + q_{c2} = q_c$ then you know if you take this same slope and that can be approximated but this is for single ramp load where t_c is the construction time for placing a fill or a training q_c .

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Ramp loading

The expression for the excess pore water pressure for the case where $u_i = u_0$ is given by:

As stated above, the applied load is a function of time: $q = f(t_a)$ where t_a is the time of application of any load.

For a differential load dq applied at time t_a , the instantaneous pore pressure increase will be $du_i = dq$. At time t the remaining excess pore water pressure du at a depth z can be given by the expression:

$$du = \sum_{m=0}^{\infty} \frac{2dq}{M} \sin \frac{Mz}{H} \exp \left[\frac{-M^2 C_v (t - t_a)}{H^2} \right]$$

$$= \sum_{m=0}^{\infty} \frac{2dq}{M} \sin \frac{Mz}{H} \exp \left[\frac{-M^2 C_v (t - t_a)}{H^2} \right] \quad \dots (a)$$

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So the expression for the excess pore water pressure for a case where $u_i = u_0$ is given by that what we have derived where $u = m=0$ to ∞ $u_0 / m \sin m z / h$ usually $-m^2$ where $m = q_m + 1 \times \pi / 2$ as stated above the applied load is a function of I and $q =$ function of time where t_a is the time application of any load so for differential load dq applied at any time t_a where instantaneous pressure increase will be $du_i = dq$ whatever is been applied that will be mobilized that is $du = dq$.

So at time t the remaining excess pore water pressure du at depth z can be calculated and given by the expression $du = u_0 \sin \frac{mz}{h} \exp(-m^2 cvt - ta/h^2)$ so t_i is the time of application of the load so now you know by simplifying this we can get $m=0$ to ∞ to substituting for $du = dq$ so $2dq/m \sin mz/h \exp(-m^2 cvt - ta/h^2)$ and this is termed as equation a

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Ramp loading
The average degree of consolidation can be defined as:

$$u_{av} = \frac{\alpha q_c - (1/H_c) \int_0^{H_c} u \, dz}{q_c} = \frac{\text{settlement at time } t}{\text{settlement at time } t = \infty} \dots (b)$$

where αq_c is the total load per unit area applied at the time of the analysis. The settlement at time $t = \infty$ is, of course, the ultimate settlement.

Note that the term q_c in the denominator = instantaneous EPWP $u_i = q_c$ that might have been generated throughout the clay layer had the stress q_c been applied instantaneously.

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Then the equation b is obtained by calculating the average degree of consolidation which is settlement at time t is equal to settlement at time $t = \infty$ so $u_{av} = \frac{\alpha q_c - (1/H_c) \int_0^{H_c} u \, dz}{q_c}$ where the αq_c is the total load per unit area applied at the time of the analysis and the settlement at time $t = \infty$ is of the course the ultimate settlement so $t = \infty$ means the ultimate settlement.

And settlement at time t is the average dvf consolidation so note that the term q_c in the denominator is instantaneous excess pore water pressure $u_i = q_c$ that might have been generated throughout the clay layer had the stress q_c has been applied instantaneously so that is the you know the q_c is the αq_c actually accounts that the partially you know generated you know the factor α indicates the partial generation of the you know applied pore water pressure in the soil so for the proper integration of equations of a and b what we get is that for t_v .

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Ramp loading

Proper integration of Eqs. (a) and (b) gives the following:

For $T_c \leq t_v$

$$u = \sum_{m=0}^{\infty} \frac{2q}{M^2 T_c} \sin \frac{Mz}{H} [1 - \exp(-M^2 T_c)]$$

and

$$U_{av} = \frac{T_c}{t_v} \left[1 - \frac{2}{T_c} \sum_{m=0}^{\infty} \frac{1}{M^2} [1 - \exp(-M^2 T_c)] \right]$$

For $T_c \geq t_v$

$$u = \sum_{m=0}^{\infty} \frac{2q}{M^2 T_c} [\exp(M^2 T_c) - 1] \sin \frac{Mz}{H} \exp(-M^2 T_c)$$

and

$$U_{av} = 1 - \frac{2}{T_c} \sum_{m=0}^{\infty} \frac{1}{M^2} [\exp(M^2 T_c) - 1] \exp(-M^2 T_c)$$

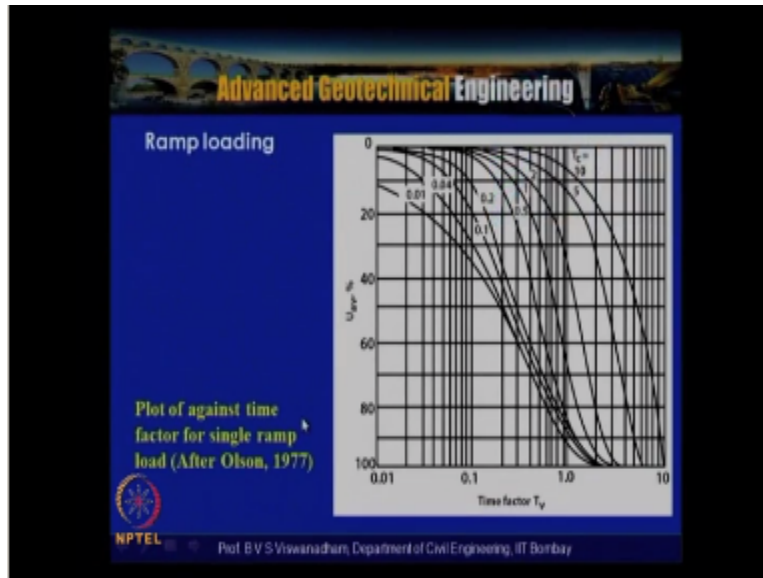
where

$$T_c = \frac{C_v t_v}{H^2}$$

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Time factor $\geq t_c$ then u is nothing but 0 to ∞ $2q_c/mqtc \sin mz$ is exponential of 1 -exponential of $-m^2 t_v$ and u_{av} can be obtained as t_v/t_c $1 - 2/t_v \sum_{m=0}^{\infty} 1/m^2$ 1 -exponential $-m^2 t_v$ and similarly for $t_v > t_c$ and which is actually given here so where $t_v = t_c$ that is time required for the construction of particular field of having in this $dqc/c/h^2$

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Now this is actually given in the form of a charts here for plot against the time factor for single ramp loading and this plot is for t_v that is time factor and for different t_c so $t_c = \text{nothing}$ but t_c/h^2 so if you know the t_v and then by knowing the construction p_n then you can actually calculate what is the average degree of consolidation. So here, you know the construction time actually has been accounted here.

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Example
 Based on one-dimensional consolidation test results on a clay, the coefficient of consolidation for a given pressure range was obtained as $8 \times 10^{-3} \text{mm}^2/\text{s}$. In the field there is a 2-m-thick layer of the same clay with two-way drainage. Based on the assumption that a uniform surcharge of 70kN/m^2 was to be applied instantaneously, the total consolidation settlement was estimated to be 150 mm. However, during the construction, the loading was gradual; the resulting surcharge can be approximated as:

$$q \text{ (kN/m}^2\text{)} = \frac{70}{60} t \text{ (days)}$$

for $t \leq 60$ days and $q = 70 \text{ kN/m}^2$ for $t \geq 60$ days

Estimate the settlement at $t = 30$ and 120 days after beginning of construction (Given $t_c = 60$ days)

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So we look into an example here, wherein we can say that based on the one dimensional consolidation test results on a clay, the coefficient of consolidation for the pressure range was obtained as $8 \times 10^{-3} \text{mm}^2/\text{sec}$. In the field there is a two meter thick layer of same clay with the two-way drainage. So based on the assumption that a uniform surcharge of 70kN/m^2 was to be applied instantaneously, the total consolidation settlement was calculated as 150mm.

However, during the construction, the loading was gradual, and the result is surcharge can be approximated as $q(\text{kN/m}^2) = 70/60$ with time T, and time T measured in days. So estimate the settlement at time T=30 days and 120 days after beginning of construction, that means that after beginning of construction. So in this if you see that T_c is given as the time of construction is actually given as 60 days.

So based on the one dimensional consolidation test on a clay, where here in this case the load is not actually placed instantaneously, if the load would have been placed instantaneously the settlement was actually calculated as 150mm. But if the, for the given properties of the soil which is actually considered, but if the load is actually placed over a period of time and that is actually approximated as $70/60(T)$, where T measured in T in dimensions are in days.

Then we can actually, what is actually has been asked is that, the settlement at 30 and 120 days after beginning of the construction.

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Solution:

$$T_c = \frac{C_v t_c}{H^2}$$

Now, $t_c = 60 \text{ days} = 60 \times 24 \times 60 \times 60 \text{ s}$; also, $H_t = 2 \text{ m} = 2H$ (two-way drainage), and so $H = 1 \text{ m} = 1000 \text{ mm}$. Hence,

$$T_c = \frac{(8 \times 10^{-3})(60 \times 24 \times 60 \times 60)}{(1000)^2} = 0.0414$$

At $t = 30 \text{ days}$,

$$T_v = \frac{C_v t}{H^2} = \frac{(8 \times 10^{-3})(30 \times 24 \times 60 \times 60)}{(1000)^2} = 0.0207$$

for $T_v = 0.0207$ and $T_c = 0.0414$, $U_{av} \approx 5\%$. So,
Settlement = $(0.05)(150) = 7.5 \text{ mm}$

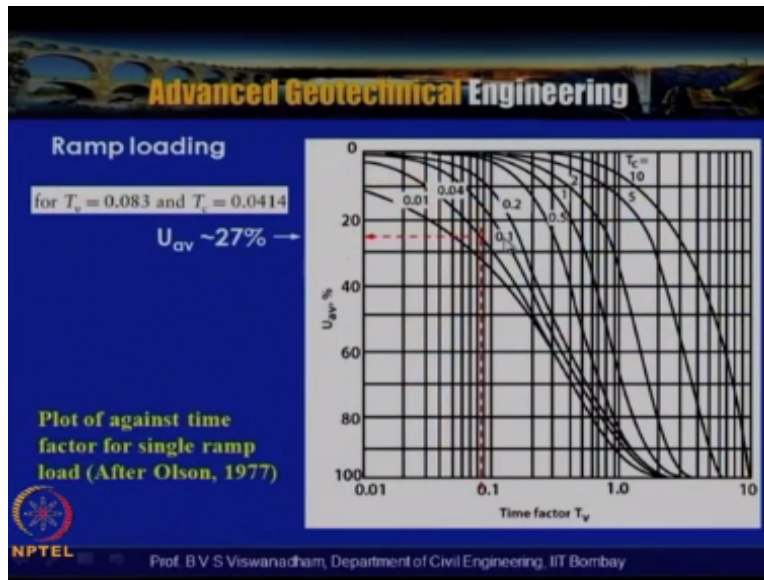
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So here $T_c = C_v t_c / H^2$, so where T_c is the time required for the time of construction of a ramp load and now by taking $T_c = 60$ days which is nothing but $60 \times 24 \times 60$, then it is nothing but, you know we actually converted into this time of construction into seconds. And H_t the thickness, the clay is very shallow in thickness, $H_t = 2 \text{ m}$, it is 2A drainage. And $H = 1 \text{ m}$ that is 1000 mm . So by substituting we can actually get $T_c = 0.0414$, and time T as 30 days.

So for time $T = 30$ days, you know what we get is that T_v is obtained as 0.0207 . So time factor for which accounts the construction time of an ramp load is T suffix C which is for a time of construction of the ramp is 60 days. So that is being calculated, and time T of 30 days and by time conventional time factor is actually obtained as 0.0207 . Now what we have to do is that from the chart which you have obtained for T_v of 0.0207 and T_c of 0.0414 what is the average degree of consolidation.

Once you know the average degree of consolidation then ultimate settlement is what we have defined in this case also is 150 mm , that means that up to 30 days only 7.5 mm of settlement takes place, up to 30 days of as the construction is actually proceeding half way of the construction period only 7.5 mm of settlement takes place.

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So how it is have been calculated, for let us say that we will see for 60 days if you look into it, for $T_v=0.083$ and $T_c=0.14$, we can see that we will get this $U_{av}=27\%$, that is actually here. So this chart is for T_v and T_c different curves are here, so this is actually valid for this is after Olson, 1977 this is the plot against the time factor of the single ramp loading, the ramp loading, single ramp loading in the sense that, the ramp loading actually has taken place in a single phase.

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Solution:

At $t = 120$ days,

$$T_v = \frac{(8 \times 10^{-3})(120 \times 24 \times 60 \times 60)}{(1000)^2} = 0.083$$

for $T_v = 0.083$ and $T_c = 0.0414$, $U_{av} \approx 27\%$. So,
 Settlement = $(0.27)(150) = 40.5$ mm


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So this is for 120 days, similarly what we have done is that for 120 days we have calculated T_v and then for, we have determined T_c that is for the, we actually have got initially that is from here for 60 days of construction we have got 0.414, then for T_v of 0.803 and T_c of 0.414 we can say that this is the average degree of consolidation about 27%. And once you know the average degree of consolidation, so that means that after 120 days of construction the field, it is actually has got 40.5mm of settlement only.

So this shows the market different of assumption of instantaneous loading as well as the so called the ramp loading conditions.

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Limitations of 1D consolidation

- In the derivation of 1D equation the permeability (K_z) and coefficient of volume compressibility (m_v) are assumed constant, but as consolidation progresses void spaces decrease and this results in decrease of permeability and therefore permeability is not constant. The coefficient of volume compressibility also changes with stress level. Therefore C_v is not constant.
- The flow is assumed to be 1D but in reality flow is three dimensional
- The application of external load is assumed to produce excess pore water pressure over the entire soil stratum but in some cases the excess pore water pressure does not develop over the entire clay stratum.

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So once having looked into that the limitations of one dimensional consolidation can be seen in the derivation of one dimensional equation, the permeability of K_z and coefficient of volume compressibility are assumed to be constant. But as the consolidation progresses void spaces decrease and these results in the decrease of permeability. Therefore, the permeability is not constant.

So coefficient of volume compressibility changes with the stress level, therefore C_v is not constant. Similarly, the flow is assumed to be one dimensional, but in reality flow is three dimensional. And the application of external load is assumed to be produced excess pore water pressure over the entire soil stratum, but in some cases the excess pore water pressure does not develop over the entire clay stratum.

So what we have done in this particular lecture is that, we continued the theory of Terzaghi's theory of one dimensional consolidation, and we have seen then ramp loading. And we deduced the equations and then the relevant equations. And we also have solved the couple of example problems as an application for what we studied. And then finally we have looked into the limitations of one dimensional consolidation theory.

Wherein we said that some parameters which are actually assumed like permeability coefficient of compressibility are assumed to be constant, but as the consolidation progress is actually happening void spaces decrease and the result in decrease in permeability. Therefore, the

permeability is not constant, and similarly the value of the C_v is actually not constant. So further we will actually look into the different aspects of consolidation in subsequent lecture.

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**NPTEL
Principal Investigator
IIT Bombay
Prof. R. K. Shevgaonkar
Prof. A. N. Chandorkar**

**Head CDEEP
Prof. V. M. Gadre**

**Producer
Arun Kalwankar**

**Project Manager
Sangeeta Shrivastava**

**Online Editor/ Digital Video Editor
Tushar Deshpande**

**Digital Video Cameraman
Amin Shaikh**

**Jr. Technical Assistance
Vijay Kedare**

**Project Attendant
Ravi Paswan
Vinayak Raut**

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