NPTEL

NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

CDEEP
IIT BOMBAY

ADVANCED GEOTECHNICAL ENGINEERING

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Lecture No. 21

Module-3

Lecture-3 on Compressibility and Consolidation

Welcome to lecture series on advanced geotechnical engineering course we are in module 3.

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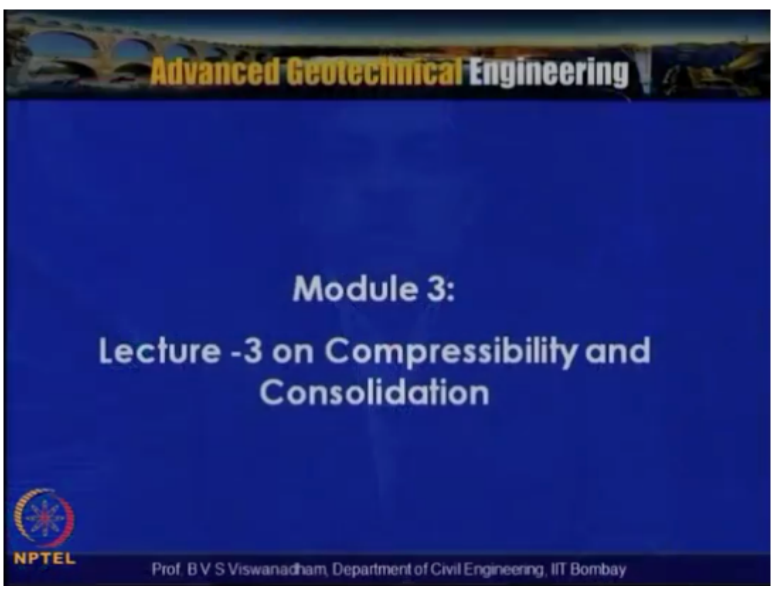

This model is one the compressibility and consolidation and lecture 3 of module 3 so in the previous lectures we had discussed about how the loads which are actually applied on the surface can cost assess and then so called displacements now in this lecture.

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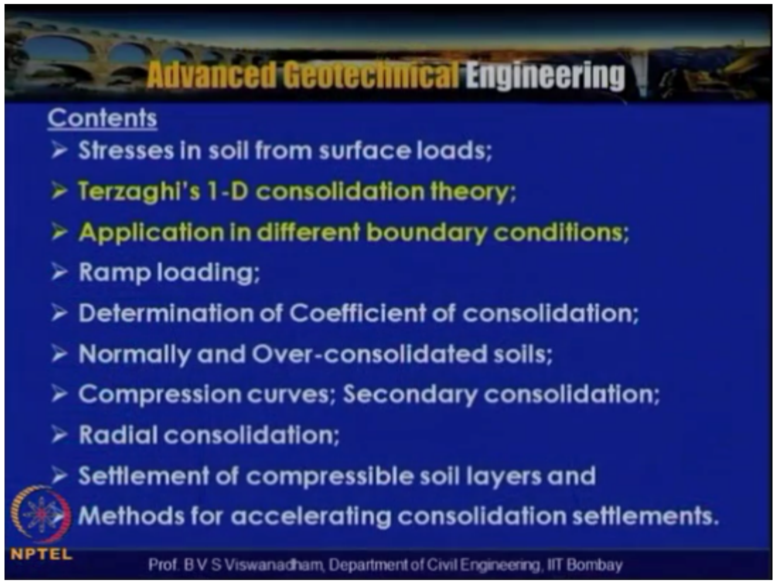

We are going to look into how these stresses can cost settlements particularly for find soils like clay soils so in this connection will try to look into the, the concepts of consolidation and Terzaghi's 1-D consolidation theory and application in different boundary conditions subsequently will discuss about the ramp loading and how we can actually determine the consolidation characteristics of a soil.

And then that is a coefficient of consolidation they are two methods are there and then how we can distinguish between normally consolidated and over consolidated soils and there we look into the several aspects like compression curves and secondary consolidation in the subsequent lectures so in this lecture we are going to discuss about the Terzaghi's 1-Dienisonal consolidation theory.

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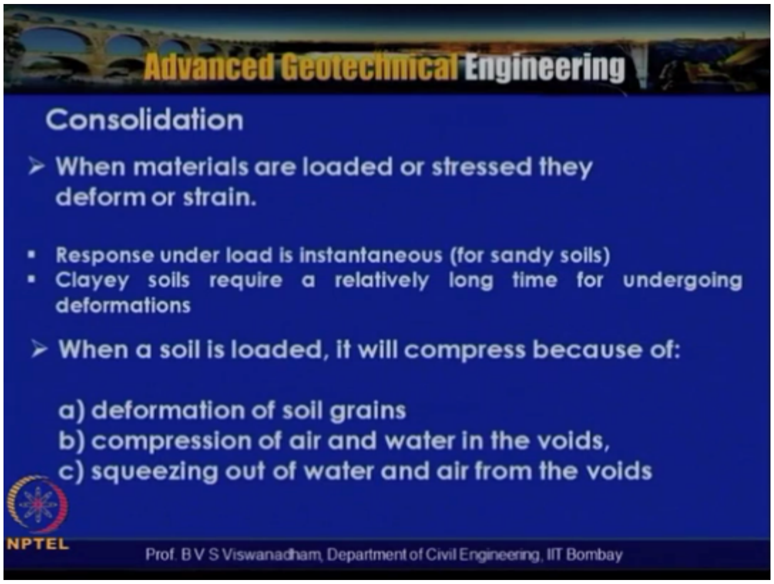

So consolidation basically when the materials are loaded or stressed they deform or strain so that we have actually seen in the previous lectures when the materials are loaded the stresses are actullys subjected and transferred to the ground and response of the under load is instantaneous for sandy soils for certain soils like granular soils or sandy soils where the primilities very high. The response under the load is very high but fine grained soils like clay soils require a relatively long time for undergoing deformations so we have to you know demarked here the response under the load is instructed in your sandy soils and clay soils require a relatively long time for undergoing deformations.

So when a soil is loaded it will compress because of the following reasons one is the deformation of these oil grains and compression of air and water in the voids so that means that the air which is actually within the voids and water within the voids will get compress and the squeezing out of water and air from the voids.

So because of the deformation of the soil grains nd compression of air and water in the voids and squeezing out of water and air from the voids is one of the causes for these are the causes for the you know the soil why the soil undergoes compression.

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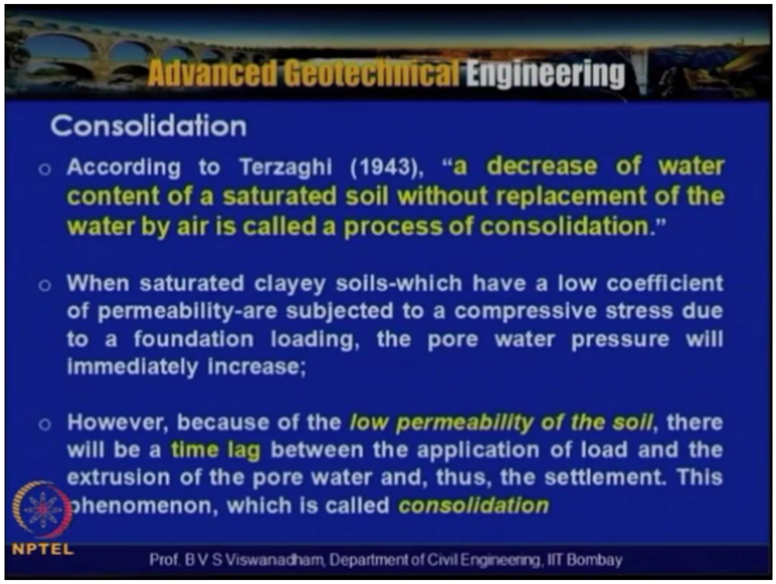

So according to Terzaghi's 1943 he defined consolidation as a decrease of water content of the saturated soil without replacement of the water by air is called a process of consolidation so with the expression of water of an saturate soil upon loading and without replacement of water by air is called the process of consolidation so when a saturated soil are you know which have low coefficient of permeability are subjected to a compressive stresses due to a foundation loading or due to any structure loading the pore water pressures were immediately increased then it takes some time for the pore water pressure to dissipate.

So because of the low permeability of the soils there will be a time lack between the application of the load and a the expression of pore water nd thus the settlement so this phenomenon is also called as consolidation so because of the low permeability of the soil particularly a fiend grained soil like clay soil there will be a time lack between the application of the load and the expression of the pore water and does the settlement so this phenomenon is called consolidation.

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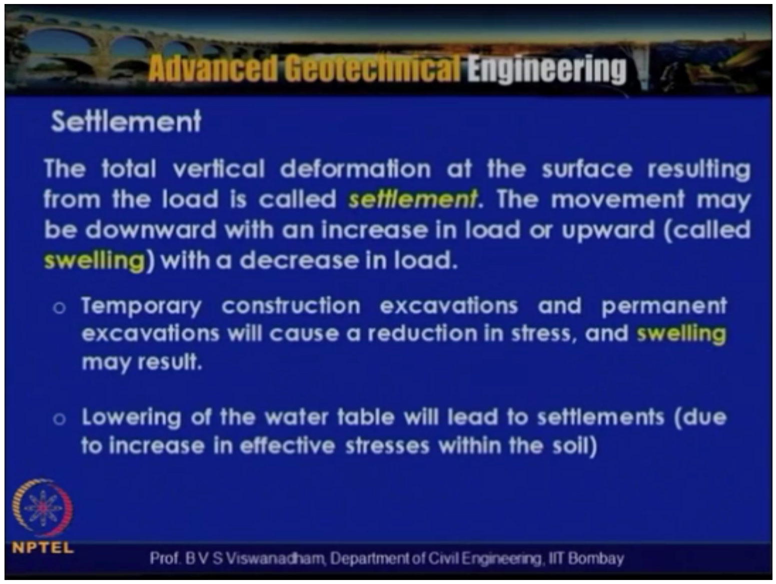

So the total vertical deformation at the surface resulting from the load is called settlement so we have to understand when the when this case particularly the total vertical deformation at the surface resulting from the application of the load is called settlement so that means that if you got a footing then the footing of particular nature is undergoes settlement.

So the movement may be downward with an increase in load or upward with the decrease in the load so sometimes you know when on the you know the when the load is actually decreasing they can be a possibility that the swelling can take place so the downward movement is actually called as settlement the total vertical deformation of the surface resulting from the load is aced settlement.

So the temporary construction excavations and permanent excavations will cause a reduction stress and the swelling may result nd lowering of the water table will lead to settlements due to increase in effective stress within the soil so lowering of the water table will also lead to settlements because of the increase in the effective stress within the soil.

So here in this slide what we are trying to determine is the settlement the total vertical deformation at the surface resulting from the load is called settlement and the movement may be downward with an increase in a load or upward with a decrease in the load so if you have a temporary construction excavations are permanent excavations they because of the removal of the material will cause a reduction in the stress nd swelling may result the lowering of the water table will also lead to settlements because of the increase in the effective stress within the soil.

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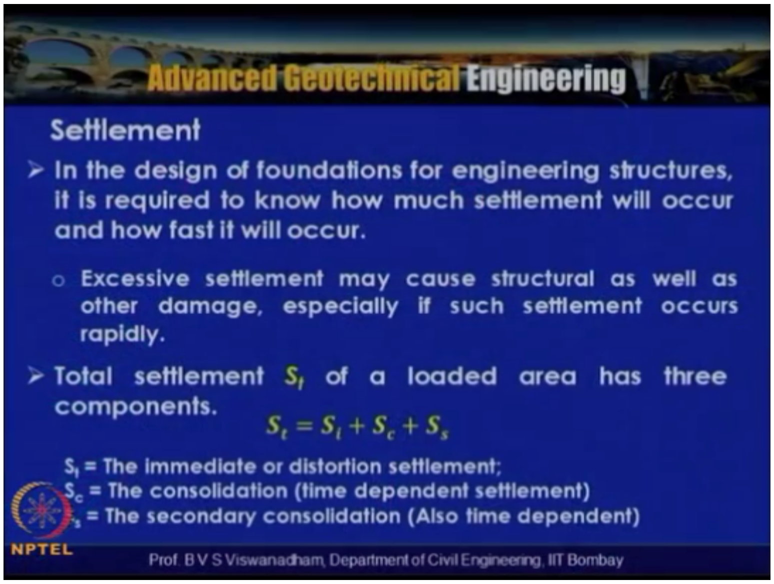

So the design of foundations per engineering structures it is required to know how much settlement will occur and how fast it will occur so for any soil deposit when we are actually trying to construct a structure for a designing foundation we have to calculate what the likelihood of settlements is and how fast they will occur.

So the excessive settlement may cause structural as well as other damages especially if such a settlement occurs rapidly so excessive settlements my cause a structural as well as other damages especially if such settlements occurred rapidly so the total settlement S_t of loaded area has three components so will look into that S_t is you know the is divided broadly into three this things St is nothing but S_i+S_c+S_s and S_i is nothing but the immediate or distortion settlement.

And we also called it has elastic settlement that is the re-edgesment of particle or graveling at the particle with that the immediate distortion settlement will be there and Sc is nothing but the consolidation settlement which is known also known as the time dependent settlement and Ss is called s secondary consolidation or secondary compression settlement and this is also time dependent this is also time dependent but this occurs at the end of the process of consolidations under the constant effective stress.

So that is nothing but the creeping of soil particles will occur and the void ratio will continue to change you know under the constant effective stress so this is the evident for a some example like municipal solid waste which is a man-made you know the solid waste material which will undergo you know very large secondary consolidation settlements.

Because of the its own characteristics because of the choosing mixture of the you know matrix as well as the ongoing bio decomposition which actually take place because of the you know the bio chemical changes which are actually happening in the municipal solid waste with tie. Another example is that the P_t type of soils because of the fibrous nature they undergo this secondary consolidation very high so that is the reason why the Marshall and particularly you know the construction marshal they very highly known for secondarily consolidation settlements.

So the total settlement is actually divided into $S_i+S_c+S_s$ so the immediate or elastic or distortion settlement is S_i then S_c is nothing bit the consolidt6ion settlement which is a time dependent and S_S is nothing but the secondary consolidation this is also a time dependent settlement.

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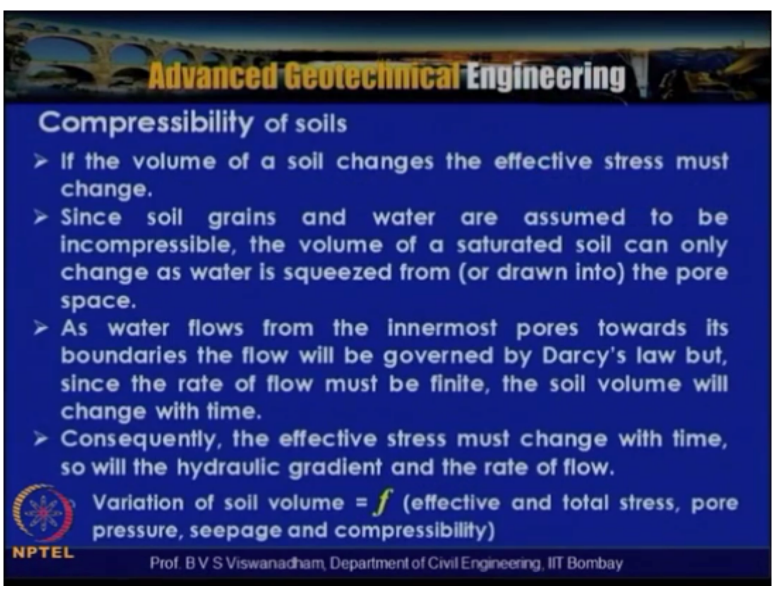

Now let us look into if I volume of the soil changes the effective stress must change since soil grins and water are assumed to be incompressible the volume of the saturated soil can only change as watery squeezed from or drawn into the pore space so since the soil grains and water are assumed to be incompressible the volume of the saturated soil can only change as watery squeezed from or drawn into the pore space.

So as the water flows from the innermost force towards its boundaries flow will be governed by Darcy's law but since the rate of flow must be finite the soil volume will be change with time so consequently the effective stress must change in time so will be hydraulic gradient or data flow so he look into this the variation of the soil volume is a function of effective and total stresses and pore water pressure and seepage and compressibility nature of the soil.

So the seepage and compressibility of the soil so variations in soil volume is the function of effective and total stresses the pore water pressure and seepage and compressibility so the volume of the soil changes the effective stress must change that is if suppose if the water is a expiring out of soil then there will be reduction volume.

Then the effective stress you know will have to change since the soil grains and water are assumed to be incompressible the volume of the saturated soil can only change as watery squeezed from or drawn into the pore space so as the water flows from the innermost force towards its boundaries flow will be governed by Darcy's law but since the rate of flow must be finite the soil volume will be change with time.

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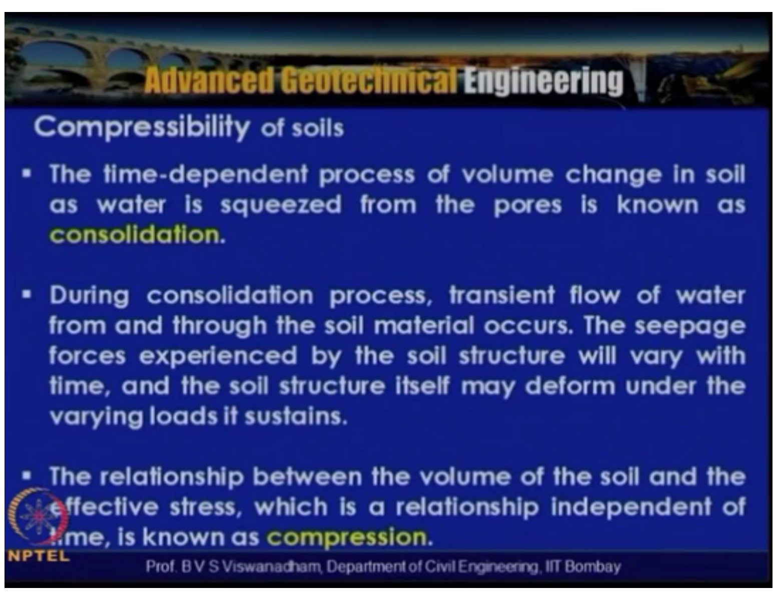

So the time dependent process of volume change in soil as the water is squeezed from the pores is known as consolidation so this also another you know definition which we look into the time dependent process of volume change in the soil and the water is squeezed from the pores is known as consolidation during consolidation process the transient flow of water from and through the soil material occurs.

So during the consolidation process this process is also called the transient because the pore water pressure keeps on changing with time so the seepage pores experienced with the soil structure vary time and the soil structure itself may deform under the varying load sustains so due to consolidation process and this consolidation phenomenon also called a transient flow phenomenon were the transient flow of water from and through the soil material occurs.

The seepage pores experienced with the soil structure vary time and the soil structure itself may deform under the varying load sustains so the relationship between the volume of the soil and the effective stress which is the relationship independent of time is known as compression that is that the relationship between change in volume of solid and the thing but the change in volume of the soil is nothing but change in void ratio because if you assume that the area is A then in that case you know the effective change in void ratio and the effective stress there is which is the relationship of time.

And this is basically known s compression so compression is nothing but relationship between volume of the soil and the effective stress and which is the relationship independent of time is known as compression and the time dependent process of volume change in soil as watery squeezed it from the pores is known as consolidation and the during the consolidation process the transient flow of water from and through the soil material occurs.

And because of this what will happen is that seepage pores experienced of the soil structure will vary with time and the soil structure itself deforms under varying load so if you compare clay verses sand we said that clays is one material were the you know the fine grain nature and the perm ability is very low and the sand were the in example part of you know.

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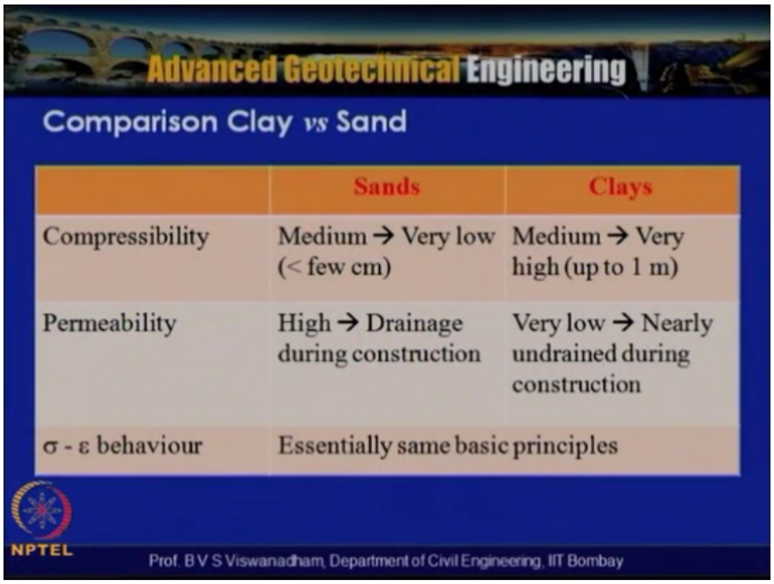

A coarse grain material were with the 0% fines let us say and the compressibility if you look into that is medium to very low and it is less than few centimeters per sands in case of clays is actually medium to very high up to 1 meter so the settlement actually can go the compressibility can actually go up to 1 meter to 1.5 meter in certain type of clays and permeability is high and the drainage is happens during the construction in case of sands permeability is very high.

And the drainage be actually happens during the construction in case of clays very low and nearly undrained during construction that means that in case of clays the permeability is very low and nearly undrained during the construction and if you look into the sustained behavior fro the sands and clays is essentially the same basic principles are apply.

So we have to note here that the sand actually has a high permeability and drainage actually happens during construction and clays actually have very low permeability and they behave like nearly undrained during construction and the compressibility can be very high up to 1 to 1.5 meters for certain type of clays and the it is actually rated has medium to very high per clays and for sands medium to very low and sometimes there is few centimeters compressibility for sands.

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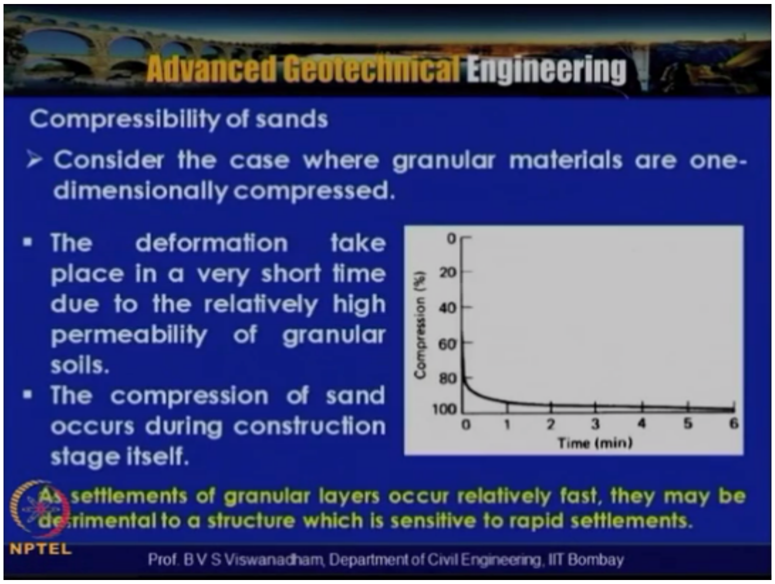

So in this particular slide then the within fraction of few seconds then we can see that the sand actually have gone to 80% of compression so that means that the settlement of the granular soils takes place very, very instanteously so consider the case where if you consider the case where the granular materials are one dimensionally compressed that the deformation takes place in a very short time due to the relatively high permeability of granular soils.

The compression of the sand occurs during the construction stage itself so the compression of the sand occurs during construction stage itself as settlements of granular layers occurs relatively fast they may be detrimental to a structure behavior so compressibility of sands if we look into deformation take place in a very short time due to the relatively high permeability of granular soils and the resistance more of friction in nature and the compressible compression of sand occurs during the construction stage itself.

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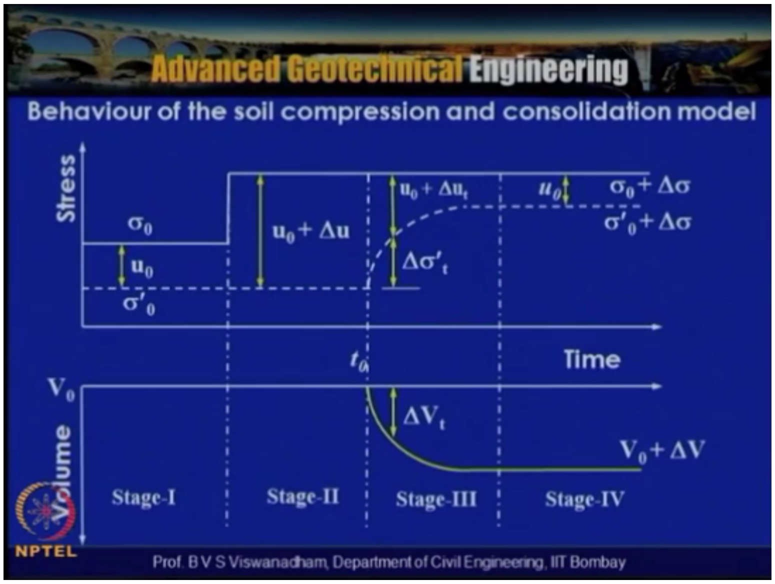

Now let us look into this particular behavior of the soil compression and consolidation model is here we have got a you know graphs which are actually plotted in this slide were the upper one is actually were time verses the stress and the bottom one is that time verses volume and V_o is the initial volume of the soil and this is the this is divided into four stages that is stage 1, stage 2, stage 3 and stage 4.

And this particular stage were in the general hydrostatic water table is there that is the total stress is equal to σ' effective stress hydrostatic water pressure that is $u_{o,}$ so in this particular stage a time you know the changes in volume is 0 and the settlement is 0 the change in volume is 0 and the total stress is equal to σ_o'+u_o were u_o is the hydrostatic pressure because of the prevalent down water table in a given depth now then at time t=0 in stage 2.

Let us assume that this is the initial effective stress stoat is σ_o' and the these oil surface is gain subjected to an increase in load which is actually called as Δu is said as σ the Δ σ is the incremental stress which has actually has been applied and when this is applied the immediately at time t=0 the water in the water within the voids will be subjected to the pressure equivalent to that of the incidence of the pressure which has been applied on the surface of the soil so that is nothing but it becomes Δδ becomes Δu so that is reason why at that particular moment at time t=0 the pore water pressure is u0+Δu and effective stress is σ0.

So in this stage also no volume change occurs no settlement occurs and the effect stress is σ0 but the pore water pressure is u0+Δu now in this stage 3 where the commencement of the dilation of pore water pressure takes place that is that you know once that the Δu starts decreasing that means the soil gains starts you know supporting the or attracting the door in such situation this actually can be you know at a point if we can see that this.

So called $\sigma 0+\Delta\sigma t$ is the stress already you know $\Delta\sigma t$ actually the stress which is already transferred to the gain and this portion of the pore water pressure is at be disputed and where in you have the Δut is actually the yet to be dissipated and further when you go for further and it reaches to so called $u0+\Delta u$ tenses to become to $u0$ that means that this is the portion, this is the time when $t\infty$ where the process of the consolidation takes place.

So in this circumstances what will happen is that the total effective stress now changes to $\sigma 1\Delta\sigma$ that means that once the pore water pressure you know ranges and despites over this particular in this pattern and during this particular time lack there is a volume change will occur that means that the volume change is nothing but the settlement and you know that this process actually is you know told can be set as the you know the consolidation process.

So at the end of the consolidation process $\sigma 0+\Delta\sigma$ is the μ effective stress and the $u0$ is the you know the original hydrostatic more or less the water table remain to remain constant then it is the original hydrostatic pressure conditions are mended so in this way what we are saying is that if you have a soil disposition and if it is subjected to certain time of loading and it takes over a period of time to you know transfer the load to the effective stresses.

And you know the effective stresses tries to pick up that load and then you know gain too or reach to that particular symbol $\Delta\sigma$ at the end of the process of consolidation these also can be explained you know by considering a spring energy model let us assume that we are having a rigid container in which a spring which is actually placed and the container is actually filled with water and we have a piston which is actually connected to the spring.

And the piston actually has a hole and which actually represents the rated at which water flows out of the soil that is nothing but the perimeter of the soil and initially we assume that the you know we have the piston and the spring and the container is within the portion within the container below the piston is filled with water and no stress is applied no $\Delta\sigma$ si applied in that case the total stress is equal to if due to $\sigma 0$=if total stress is actually equivalent to that is applied then $\sigma 0i=\sigma 0+v0$

And where in here the soil gains actually they are represented by the spring and water as thus the water in the cylinder as the pore water now moment to the Δσ is actually applied then you know the pore water pressure the water within the cylinder raises to Δu and moment once the you know the drainage conditions commences that is in stage 3 then you know the slowly the spring starts undergoing compression that indicates that the raise in the effective stresses here you know are nothing.

But the indication of these4 compression of the spring where in the σ0 continues to increase and reaches to σ0+Δσ so you know this particular moment you know the final hydrostatic conditions are actually reached then you know these original hydratic equilibrium conditions are maintained and then you know the spring is now compresses to such a extend that it is stress is now σ0-+Δσ and u0 is the you know the hydratic pore water pressure so this model explains the soil compression and consolidation behavior you know.

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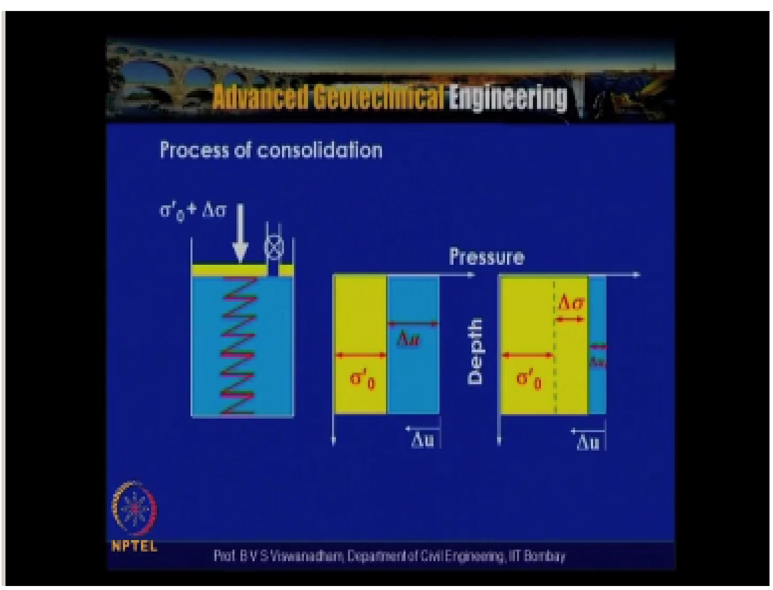

So this is you know this what actually is the process of consolidation explains the process of consolidation the process of consolidation is actually explained here if you are having a spring which is loaded with the piston which is loaded with $\sigma 0$ with $\Delta\sigma$ and the wall then it is actually close then the water pressure which is nothing but $\sigma 0+\Delta u$ that means that initially Δu is 0 here and moment the water pressure.

The load is actually $\Delta\sigma$ is applied the $\Delta u+\Delta\sigma$ is actually is born by the water now what will happen is that once this value is opened then this yellow portion starts you know covering the blue portion that means that that is nothing but, $\sigma 0$ for $\Delta\sigma t$ any time if this happens then what will happen is that this is the portion which is actually transferred to the soil gain and this portion is yet to be displayed moment this reaches to this level this point.

And it is called as the so called the completion of consolidation and that is called $\sigma 0+\Delta\sigma$ so in this case if you look into this the variation of the pour water pressure with time and with depth is actually also called as the isochronal so the moment at time v=0 when instanteously when the pressure is actually applied and this is actually called as the you know the first isochronal and moment you know the as the you know dissipation of the pour water pressure that is as the water actually leaking out of the cylinder.

Then there is the possibility that the isochrones actually migrate towards this and this is the called as the final isochrones so this is at the time t where this is the isochronal at this particular time and this is at time $t\infty$ that is at the end of the consolidation and this is the final isochronal so the isochrones are nothing but you know identical pore water pressures at particular time so identical pore water pressures for identical times so if you are having the you know that is for single a small clay layer where we have spring analysis but the same spring analysis can be extended.

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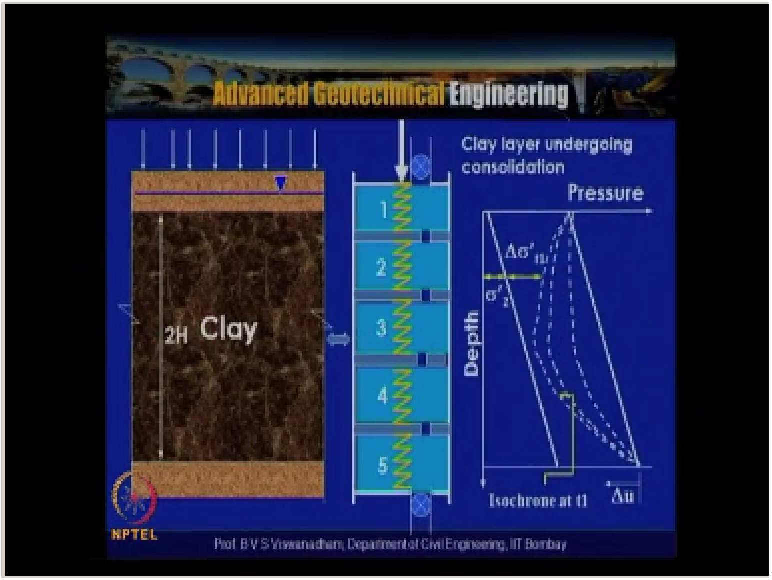

If you are having a clay layer which is you know have got double open layers both the top and bottom and clay layer is actually saturated and clay which is actually having a thickness 2h and if this is clay is actually completely saturated and where you have got you know the upper layer and the bottom layer are hence you know the open layers were also called as you know the drainage layers.

So this is actually is indicated by spring analysis with multiple springs connected in the series where in 1,2,3,4,5 these springs are actually placed like this and been each and every complacent is connected and the upper and the bottom suppose let us say that both the layers are you know sand layers then the both these walls are actually opened simultaneously or if the one layer let us say that if you are having the rock layer at the interface layer at the base.

Then this wall kept as close then you know the water will have to escape through you know this particular upper value only so if you are actually having if this walls are actually you know let us assume that open simultaneously simulating the double open layer double drainage condition then in that case what will happen is that first the spring 1 and 5 will undergo compression then subsequently spring 2 and 4 will come under compression.

And finally the last spring to undergo compression is 3 that means that the springs which are actually closed to the wall they actually attack the you know the effective stress you know very rapidly and flowed by the springs which are actually far away from the drainage phase so this is you know indicates so for example this particular is indicated by a pressure variation with the depth so this is the initial effective stress variation in case of spring analysis it can indicated like a uniform pressure.

But for conveniences for the clay layer this is indicated with certain and this is the effective stress if this is the effective stress then what will happen is that the time $t=0$ when it is subjected to a increase in load due to some $\Delta\sigma$ then the water pressure that means this is it will happen that the first is chrome will develop here and then if you notice here at time $t<t=0$ that means $t1,t2,t3$ when it actually happens then what will happens is that immediately the pressure at the drainage phases are actually drops down 0.

And that means that the soil which is actually close to the clay soil which is actually close to the drainage phase is the first one you know transfer these stresses attract these stresses and the soil water transfer the stresses to the soil rapidly and you know the effective stress here is nothing but $\Delta\sigma$ and here also it is $\Delta\sigma$ but when it comes to central point here and you can see that the being at the mid depth of the clay earth.

And also the you know the hydraulic gradient which is actually there at the center is very, very low and also $\Delta u/\Delta h$ is actually 0 so because of that what will happen is that the flow is virtually 0 but in the case when you look into this particular portion that Δu variation with the you know Δz the hydraulic gradient is actually ∞ so because of the you know because of this you know sharp hydraulic gradient changes.

The rapid flow actually happens in the drainage bases and the transfer of the effective stresses becomes very, very you know rapid at the boundaries of the clay layer so in case if the clay layer is having let us say that half closed layer that is at the rock clay layer is bottom base that means that you know the peak will be at this center that means approximately the this will become the you know the drainage part the water raise to flow.

And then you know water pressure escapes only in this direction provided if we are having only one dimensional consolidation so this is you know the first isochrones at time $t=0$ that is $t=0$, $t=t0$ is the time at the moment the application of the load has been applied but once $t1,t2,t3$, types are there this is the isochrones and $t2 >t1$ $t3>t4$ and this is for $t\infty$ so what will happen is that this if you look into this at this particular depth this is the $\sigma 1z+\Delta\sigma d1$

So this much portion of the effective stress is already transferred to the soil but at this portion here you can see that the soil is still having the effective stress which is actually at $\sigma 0$ whatever the initial effective stress is there and that is still provides at the center so here we look into just see here the migration the moment of the is chrome actually happens here ultimately and this process this is actually the time which is required for the dilation of the pore water pressure that is also called as the time lag and which is you know depend upon the premibilty of the soil.

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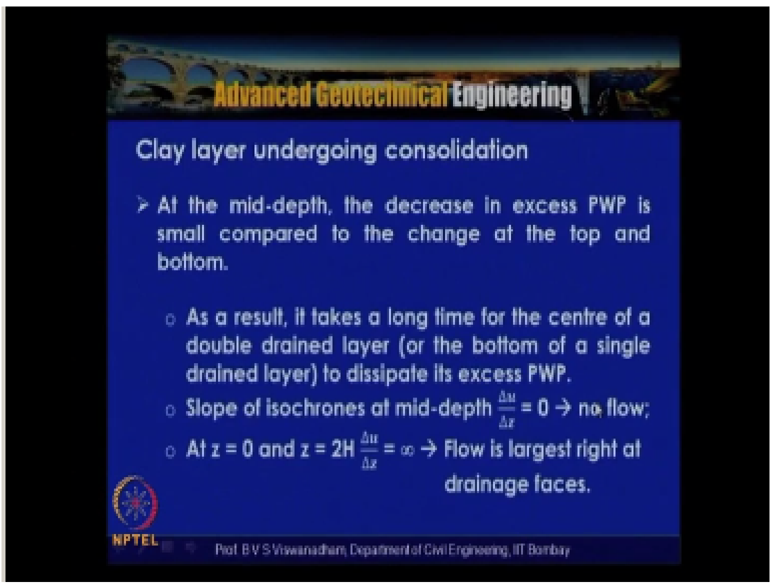

So at the mid that the decrease in axis for pore water pressure is small compared to the change in change at the top and bottom so a result what we are trying to explain is that it takes a long time from the center of a double train layer or the bottom of the single drain layer to despite the excess for pore water pressure.

And slope of isochrones that with that is almost like $\Delta u/\Delta z=0$ and no flow condition actually occurs at $z=0$ and $z=2h$ that is at the both top and bottom the $\Delta u/\Delta z$ is infinite and the flow is large rest right at the so because of the because the flow is actually largest at the right and bases that means that because of that there is the possibility that you know the soil undergoes you can say that consolidation very, very rapidly at the boundaries.

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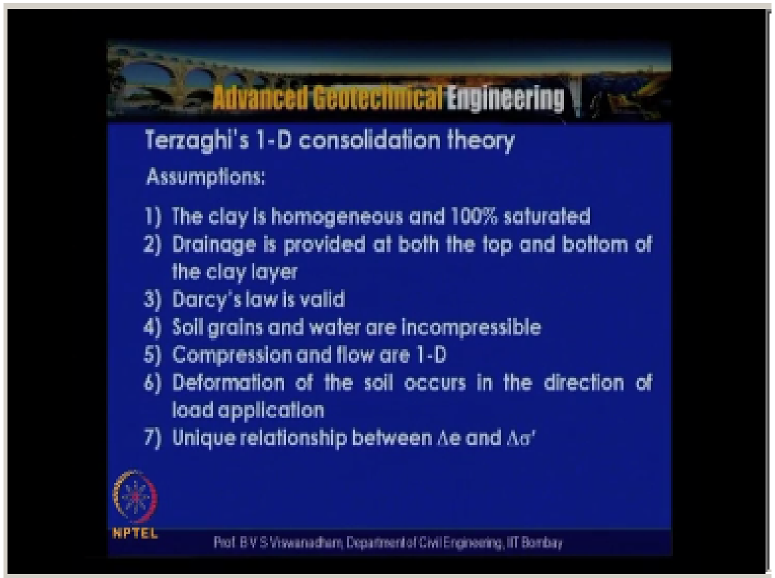

So the terzaghi's 1dimesional consolidation theory the basic the assumptions which are actually involved the clay is homogenous and 100% saturated and drainage is provided at both the top and bottom of the clay layer and Darcy's law valid and this is what it has been assumed but after wards the theory which can be derived for different boundary conditions and Darcy's law is assumed to be valid and soil grains.

And water are assumed to be incompressible and compression and flow are 1 dimensional that means z here in this case 1 dimensional is assumed but in the later part of this lectures discussing the 3 dimensional that is this one dimensional as well as the two dimensional flow if that actually happens there is the possibility that the consolidation can be acierated and the deformation of the soil occurs in the direction of the load application.

So here the deformation of the soil occurs in the direction of load application at the unique relationship between Δe and Δσ so here during the process of consolidation that is during the process of consolidation there is a neat relationship between Δe and Δσ so the deformation of the soil occurs in the direction of the load application and compression and flow are one dimensional nature and soil grains are water are incompressible and Darcy's law is valid so here the relationship between Δe and Δσ are given.

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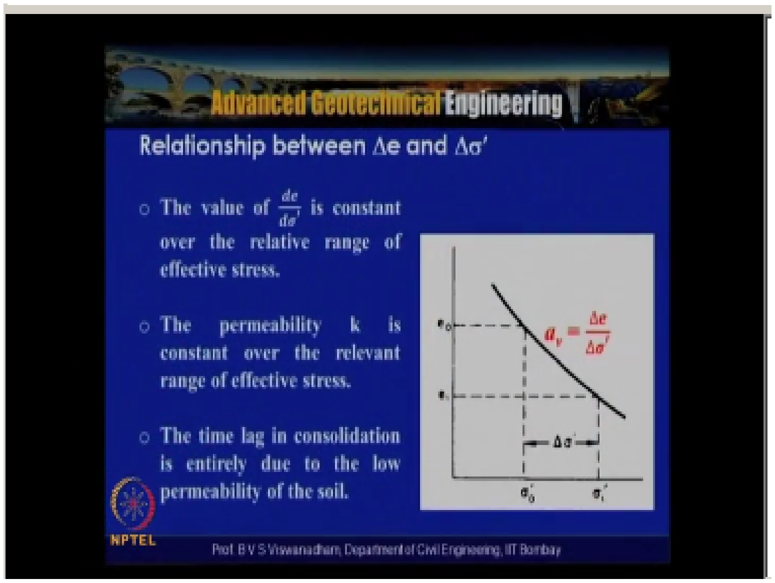

So where the value of the d/dσ that the change in the effective stress and the change in the valid ratio so initially if you are having a valid ration so say eo and at σ r- moment at the Δσ is applied the σ1- is now nothing but σ1 but σ01+Δσ so because of the increase in effective stress in soil there is a reduction in void ratio from e0 to e1

So the e1-e0-e1 that is nothing but the be or Δe so the value of d/dσ1 is constant for two range of effective stresses and the permeability k is also assume to be constant for the range of effective stress and the timer lag in consolidation is entirely due to the low permeability of the soil the time lag in the consolidation is entirely due to the low permeability of the soil.

The moment the load actually has been applied instantaneously the initial excess pore water pressure at any depth is you know is taken and at z=0 that is u=0 and at z=ht=2h bottom of the clay layer the pore water pressure is again 0, the increase in pore water pressure is 0. So not that h is the length of the longest drainage path and in this case which is the two way drainage condition top and bottom of the clay layer h=1/2 of the total thickness of the clay layer, so water actually half of the portion way water in the voids actually flows upwards and half of the portion of the water flows actually downward to the bottom drainage layer.

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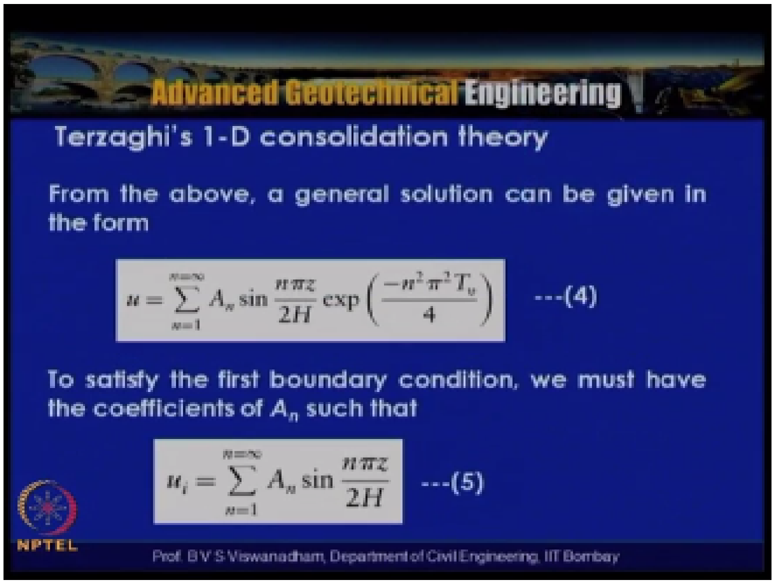

So here from the above a general solutions can be obtained where u=n integer is equal to 1 to ∞ Σ an sin and ϕz/2h exponential of $-n^2/\pi^2$ Tv/4 where Tv is the time factor, Tv is the time factor okay, to satisfy the first boundary condition we must have the coefficients a and such that u1=n is equal to 1 to ∞ where An is sin and πz/2h and this is regenerated as ϕ.

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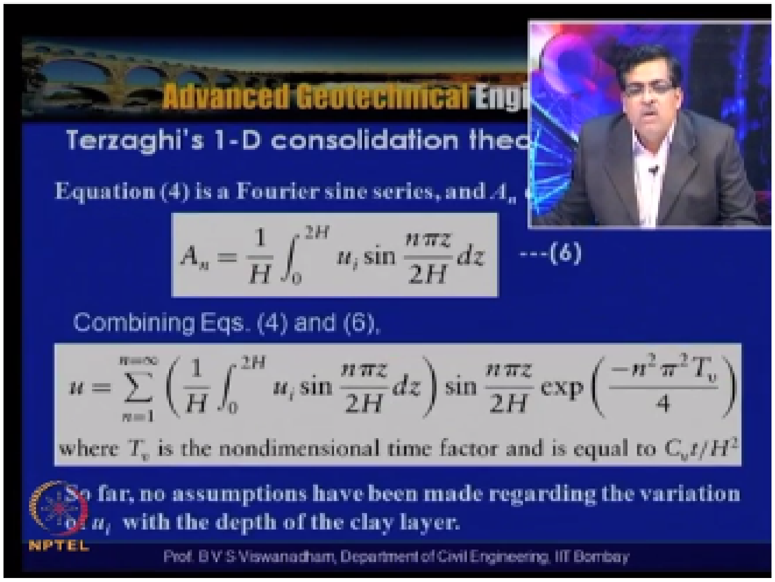

And with that equation 4 is a Fourier series and there is An can be obtained by An=1/h 0 to 2h that is the total thickness of the clay layer, ui sin and πz/2h dz and combining equation 4 and 6 we get u=n to 1 to ∞, 1/h 0 to 2h ui sin and nπz/2h dz into sin nπz/2H $\exp(-n^2\pi^2 Tv/4)$, where Tv is the non dimensional time factor and which is equivalent to Tcv/H^2 so H is nothing but here the drainage path in case of a clay having double drainage layer which is actually is H/2 in case of a single layer the H=H.

So far no assumption have been made regarding the variation of ui with the depth of the clay layer, but there can be number of variations can be assumed like assuming that if you are having a uniform variation if it is ui is actually constant with the depth.

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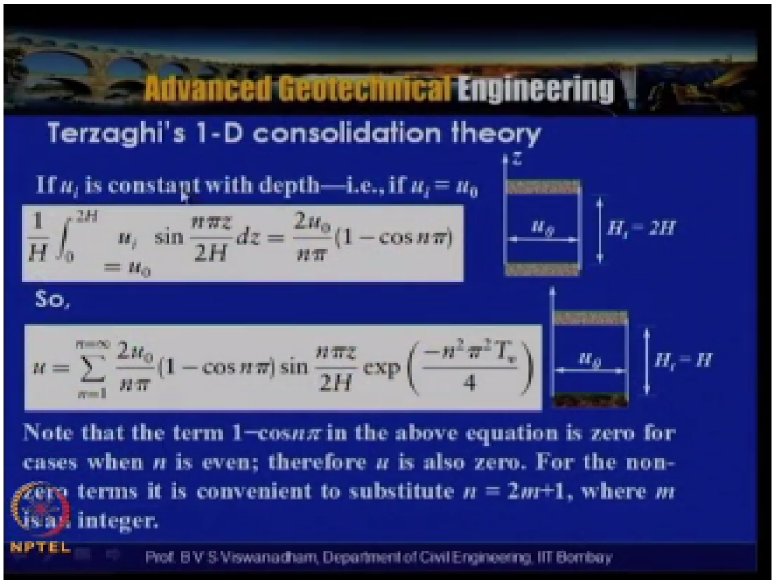

And if you are having a double drainage layer then Hd=2h in this case when we look into this and this converts into u=n to 1 2u0/nπ cos 1-cos nϕ/ sin nπz/2h exp($-n^2\pi^2$Tv/4). So here this n=2m+1 where m is an integer when you put into this and this is for the one way drainage and this is for you know two way drainage and this is actually is assumed to be uniform variation, but there is also another case where upper layer is actually impermeable bottom layer is permeable but that situation may not actually arise in practical terms.

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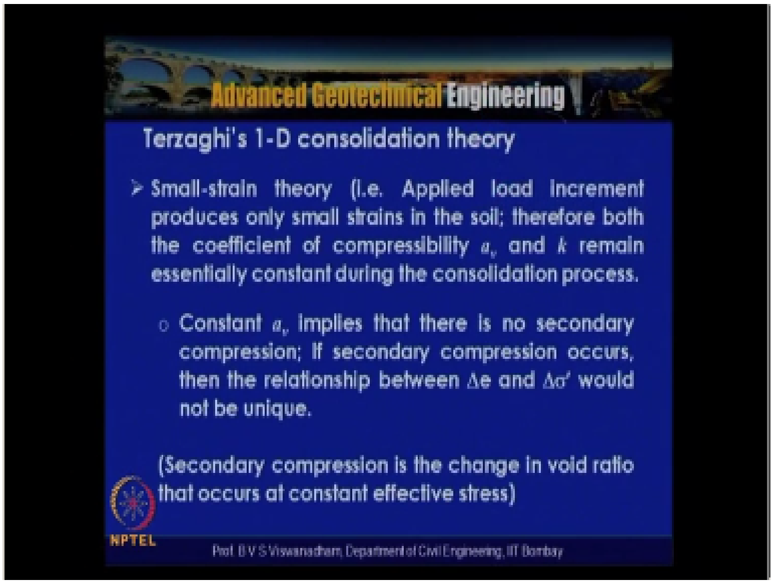

Terzaghi's one dimensional consolidation theory is essentially as small strain theory that is the applied load increment produces only small strains in the soil and therefore both the coefficient of compressibility av and k remain essentially constant during the consolidation process so the applied load increment produces only small drains in the soil and therefore the both the quotient of compression that is av which is nothing.

But $\Delta e/\Delta\sigma 1$ which is actually shown in the previous slide where $av=\Delta e/\Delta\sigma$ the slope of the line is defined as quotient of compressibility which is nothing but $\Delta e/\Delta\sigma 1$ and k the position of compressibility av and k remain essentially constant during the consolidation process and constant av implies that there is no secondary constant compression if the secondary compression occurs that the relationship between Δa and $\Delta\sigma$ would not be linked.

So secondary compression has we defined earlier which is the change in the void ratio that occurs at constant effective stress so the terzaghi's one dimensional consolidation theory is essentially the small strain theory and this means that the applied load increment producers only small drains in the soil and therefore the quotient of permeability and the quotient of compressibility remain constant during the consolidation process and constant of compressibility means that there is no secondary compression.

During the process of consolidation which we what we call is the primary consolidation if the secondary consolidation or secondary compression occurs that the relationship between Δa and $\Delta\sigma 0$ σ would not be unique so now we will try to derive so that terzaghi's one dimensional consolidation equation when now consider when the c lay is subjected to increase in vertical pressure Δσ and then pour water pressure.

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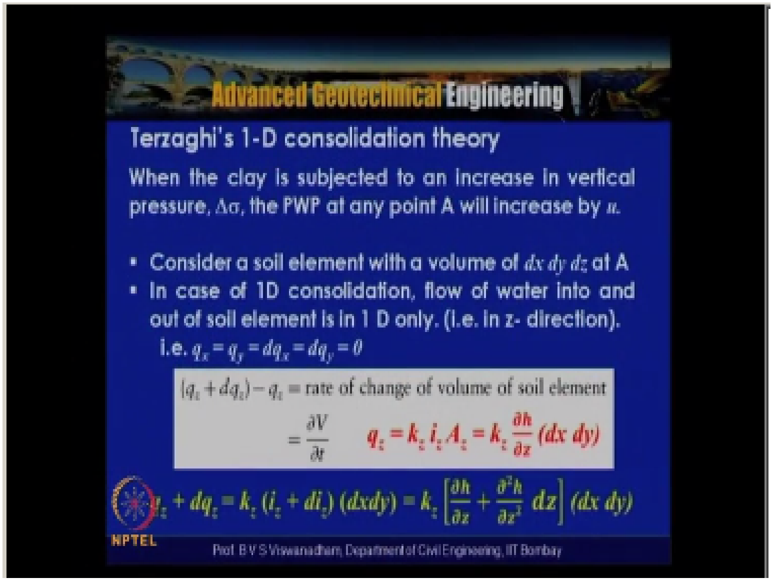

At any point a will increase by u so consider a soil data.

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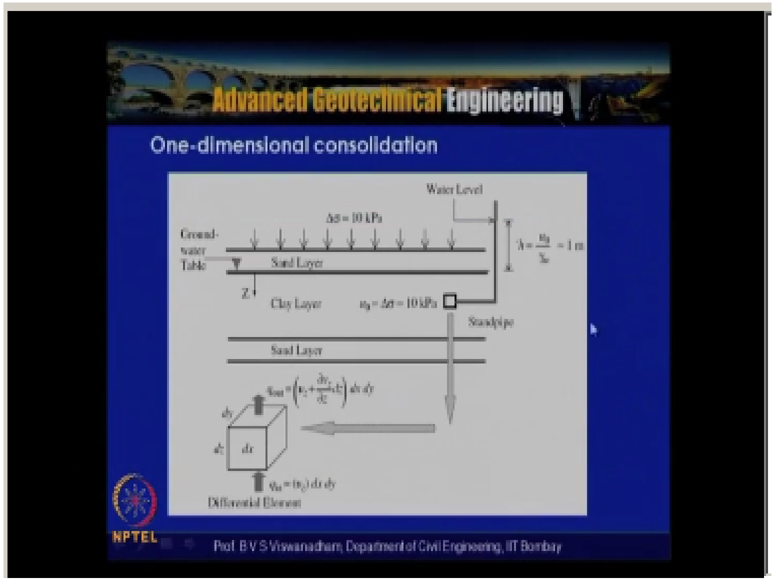

Where here you know for example the Δσ=10 kilo Pascal's is been shown here but assume that you have got a clay and the water table at this point and it is w drain and clay is the portion which is sand between two sand layers and on the top the ground surface there is a Δσ which actually been applied now consider a small element having volume dx,dy, dz and x is this direction so x and y directions are z is this direction.

And the plan perpendicular to this z direction is the x and y so in this x and y direction the flow actually happens that means that the water enters the element and water leaves out the element so the water enters into the element which is actually nothing but q=av continuity equation where in q in =dxdy it is nothing but the element area into vz at q out is nothing but that vz/∂vz/∂z.dz.dx.dy that is the because of the pore water pressure change.

So consider we have what we have discussing is that consider a small element having a volume of dx,dy dz so v=dx,dy,dz at the result point a so in case of one dimensional consolidation the flow water into and out of the soil limit is one dimensional only that is in the z direction so in that case qx=qy=change in flow in x and y direction 0 because this one dimensional consolidation is valid if you are having a soil deposit where it is subjected to large area loadings the area loading is such that it is assume that it is you know to the for the convenience assume that it is instead in nature.

And it is extending large area extend in that case what will happen is that one dimensional consolidation only predominantly takes place if you are having a finite area which is actually loaded there is the possibility the three dimensional flow in by direction x direction as well as the you know so called z direction it can happen so but in case with the large area loading the one dimensional consolidations is actually possible.

Let us assume that for example of one dimensional consolidation is that if you are having a soft clay when it is actually loaded with is area fill which is about say 102 meters extending over the entire area then that case you know the area extend of the area is much larger than the thickness of the clay area in that case what will happen is that the one dimensional consolidation only predominantly happens now here in this slide the change in flow that is outflow to in flow that is difference qz+2qz-qz is given as rate of change of volume of soil element which is nothing but $\partial v/\partial t$.

So qz is defined as we have written in the next slide as vz.az which is nothing but kz,iz.az and kz is the quotient of the permeability in the vertical direction that is z direction and iz is $\partial h/\partial z$ and flow is actually happening in dxdy perpendicular to dx dy so, az=axay now qz+dqz=kz+iz+diz.dxdy so by writing kz.$\partial h/\partial z+\partial^2 h/\partial z^2$.dz.dxdz so by taking the subtraction with qz+qz-qz what will get is that kz.$\partial^2 h/\partial z^2$.dxdydz.

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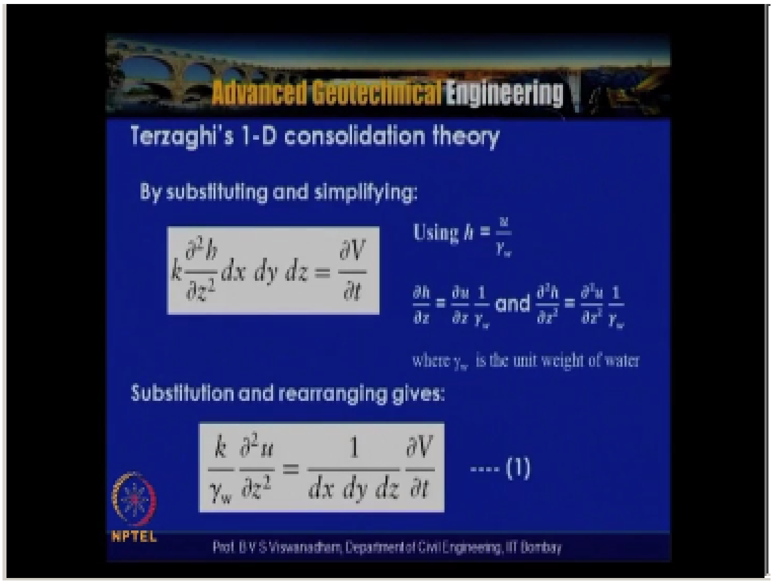

So $\partial v/\partial t$ is now nothing but $\partial v/\partial t$ by substituting and simplifying $\partial v/\partial t$ is nothing but k. k is indicated as now here kz indicated as k $\partial^2 h/\partial z^2 dxdydz$ so using $h=u/\gamma w$ so we can write differentiate this partially $\partial h/\partial z=\partial u/\partial z.1/\gamma w$ where γw is unit weight of water and differentiating one second $\partial^2 h/\partial z^2 \partial^2 u/\partial z^2.1/\gamma w$ so this is $\partial^2 h/\partial z^2$ so the substituting for $\partial^2 h/\partial z^2$ here it becomes $k/\gamma w.\ \partial^2 u/\partial z^2 =1/dxdydv/dt$ so here this particular equation now turned out to be $k/w.\ \partial^2 u/\partial z^2 =1/dxdydz.\partial v/\partial t$

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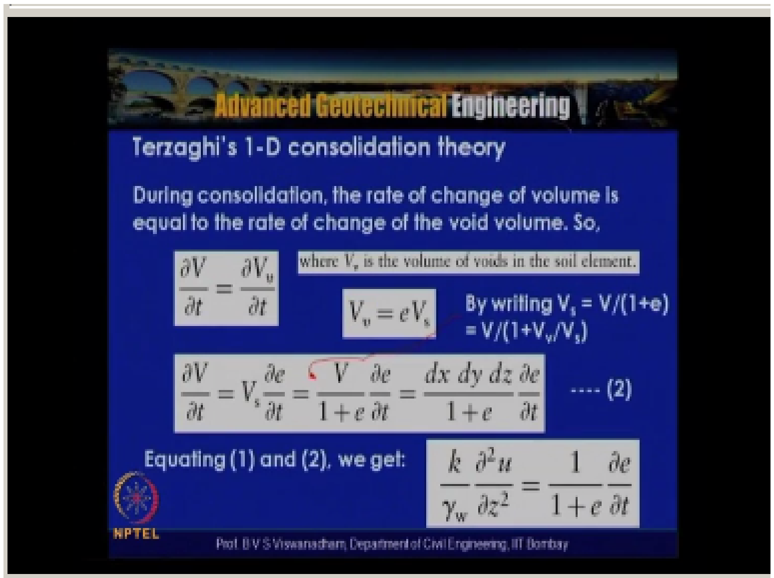

Now here during the consolidation the rate of change of volume is equal to the rate of change of the void volume so the $\partial v/\partial t=\partial v/\partial t$ so vv is the volume of the void in the soil element so we can write vv=evs and now what we can do is that the particular equation we can write in terms of $\partial v/\partial t$ we can write like $\partial v/\partial t$= you know this by differentiating this we get $\partial v/\partial t$ vs.$\partial u/\partial t$ so for $\partial v/\partial t$ if you substitute vs.$\partial v/\partial t$

So further vs is rearranged as vs=v/1+c and with that our e =vv/vs with that what we can write is that v/1+e we can write so this is nothing but vs the volume of the solids is written in terms of v/1+e.$\partial u/\partial t$ and this is equal to dx, dz,dy which is nothing but v is nothing ax,by,dz /1+e.$\partial u/\partial t$ now equating this equation 2 with 1 that is $k/\gamma w\partial^2\partial\ z^2$ with this one what we get is that the dx,dy,dz will get cancelled

And what we will have is that $k/\gamma w.\partial^2u/\partial z^2=1/1+e.\partial u/\partial t$ so here what we have is that by equating one and two we have got now $k/\gamma w.\partial^2u/\partial z^2=1/1+e.\partial u/\partial t$ now further change in void ratio that is reduction in void ration when it is it actually happening from e0 to say e1 due to the increase in effective stress or increase in effective stress and also we have to find note that the decrease in the axis for water pressure increase in their effect stress which is preceded by the you know the decrease in the access for water pressure.

So assuming that there are they are linearly related so we can say that $\partial e=-av.\partial\sigma/a-av.av$ is nothing but quotient of compressibility $\partial.\partial\sigma$ so where a is the coefficient of compressibility again the increase in effective stress is due to the decrease in access for water pressure so we can write

$\partial e = av\Delta u$ so by you know substituting this $\partial e/\partial t = av\Delta e$ so we can write the previous equation that $k/\gamma\ \partial^2 u/\partial z^2 = 1/1+u/\partial u/\partial t$

So what we have done is that we substitute for $\partial u/\partial t = av.\partial u/\partial t$ so $k/\gamma w \partial^2 u/\partial z^2 = av/1+e.\partial u/\partial t$ so this av/1+e is nothing but defined as mv and that is the quotient of volume compressibility so quotient of volume compressibility is a parameter and where the units of this are nothing but meter square per kilometer av/1+t0 because quotient of compressibility units are meters square per kilometer so this is n actually followed exactly the same and $\partial u/\partial t$ so $k/\gamma w.\partial^2 u/\partial z^2 = av/1+e.\partial u/\partial t$ which is nothing but $mv = \partial u/\partial t$.

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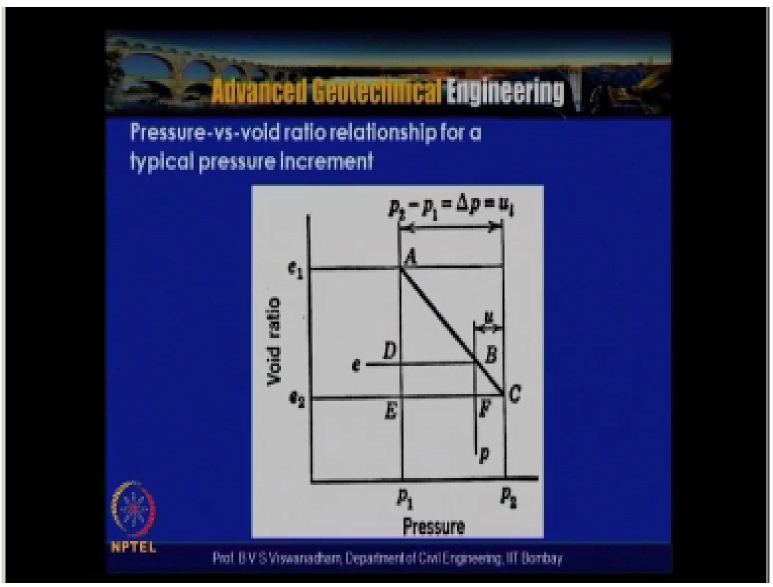

So in this figure you know the pressure void ratio relationship is typical pressure here suppose p1 which is nothing but σ0 in our case and if that happens Δσ and this is σ1 that is σ1 is nothing but σ0+Δσ this is end of the process of consolidation so this portion if there is increase in the pore water pressure this is the pore water, water pressure increase this is the void ratio which is actually decrease at any time t.

And this is the pore water increase in the effective stress and this is the pore water pressure at to be disputed so the using that relationship this is what actually this explains that increase in effective stress is due to the decrease in excess pore water pressure so because of that you know what we have written is that $\partial u = av\Delta u, av\partial u$ and which by writing $\partial u/\partial t = av/av.\partial v/\partial t$ so that is how we have got $k/\gamma w.\partial^2 u/\partial z^2 = av/1+e.\partial u/\partial t$

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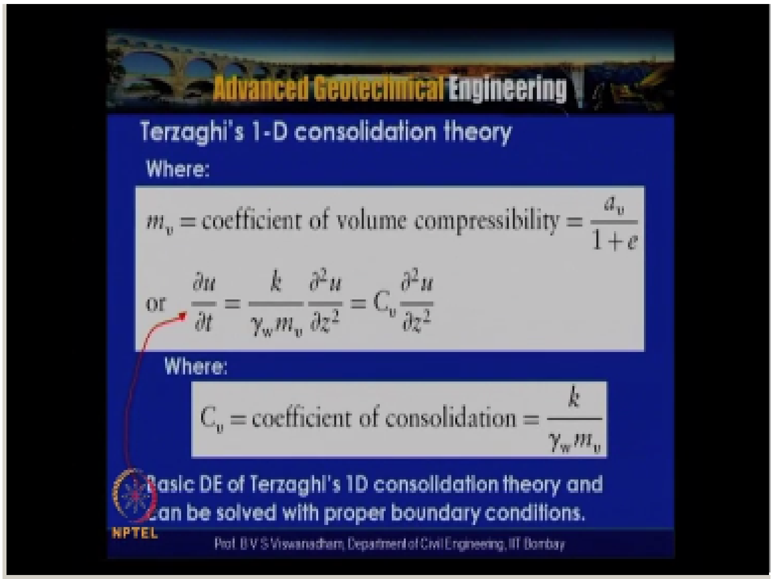

So now further continuing we can say that now ∂u/∂t =k/γw fv∂^{2}u/∂z^2 so this particular equation is called as ∂u/∂t=cv.∂^{2}u.∂z^2 is the basic differential equation of the terzaghis one dimensional consolidation theory and we can be solved with proper boundary conditions that means that we have to see whether you know the pour water pressure whether it is having open layer at the bottom top and what is the variation of the pour water pressure.

So for the different variation for the pour water pressure within the soil and this can be solved so the ∂u/∂t=k/γwmv so if you look into this here k=quotient of permeability is related as cvγw.mv so quotient of permeability k=cvγw mv is the quotient of volume consolidation and γw is water and difference basic differential equation ∂u/∂t=cv is nothing but the consolidation the units of this consolidation are the meters per second.

So if the cv value is high that means that looking at ck if the k is actually high the cv value is high and k is value is say low very soil the cv value will be low so the larger the cv value the faster is the settlements when actually taking place and lower is the cv value you know that much time that much delay will actually happen in happening the settlements this cv actually indicates the you know the time rate of rate of settlements.

Basically if the parameter that is the only parameter soil parameter which is actually there in the terzaghis one dimensional consolidation equation so the cv the quotient of consolidation which we discussed further how to determine this in the laboratory and how this is actually found to be dependent on the effective stress where for the range of the loading when can actually can happen is subjected in the laboratory where cv is k/γw can be so this is the basic differential equation one dimensional consolidation theory and this can be solved with proper boundary conditions.

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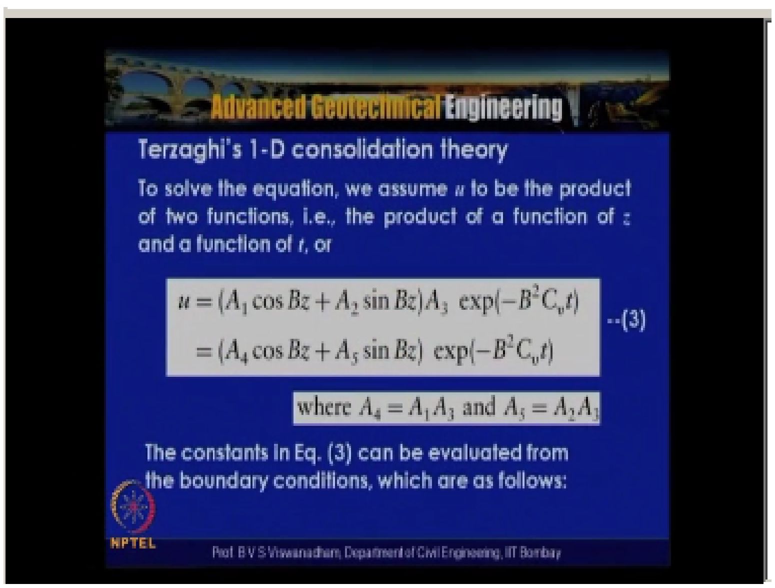

So the we need to solve this equation we assume that u basically the pore water pressure into be the product of 2 functions and the product of a function of a z and function t are they given as u=a1cosbz+a2sinbza3 and to the expontial of e-b2cvt=a4cosbz+a5sinbj expontial –b2cvt where a4 =a1,a3 and a5=a2,a3 so the constant is equation 3 can be evaluated from the boundary conditions as small as so by considering the proper boundary conditions.

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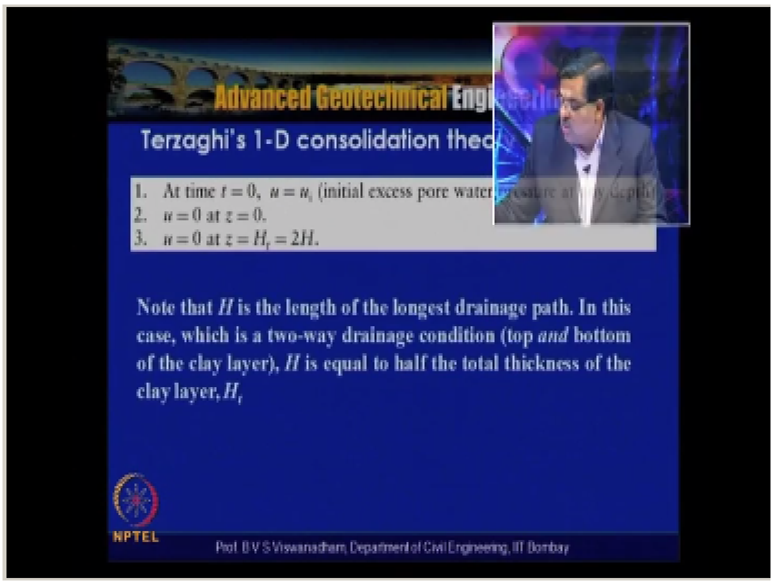

Can be solved so time t=0, u=ei that is u=a means that time t=0 the moment the load actually has been applied instansteously by initial access for pore water pressure any depth is you know is taken and at z=0 that u=0 and at z=ht=2h bottom of the clay layer.

The pore water pressure is again 0, the increase in pore water pressure is 0. So not that h is the length of the longest drainage path and in this case which is the two way drainage condition top and bottom of the clay layer h=1/2 of the total thickness of the clay layer, so water actually half of the portion way water in the voids actually flows upwards and half of the portion of the water flows actually downward to the bottom drainage layer.

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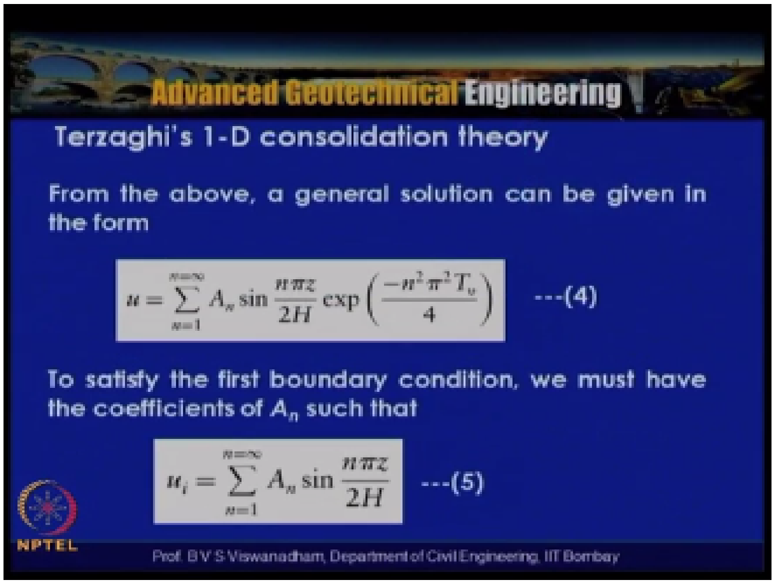

So here from the above a general solutions can be obtained where u=n integer is equal to 1 to ∞ Σ an sin and ϕz/2h exponential of $–n^2/\pi^2$ Tv/4 where Tv is the time factor, Tv is the time factor okay, to satisfy the first boundary condition we must have the coefficients a and such that u1=n is equal to 1 to ∞ where An is sin and πz/2h and this is regenerated as ϕ.

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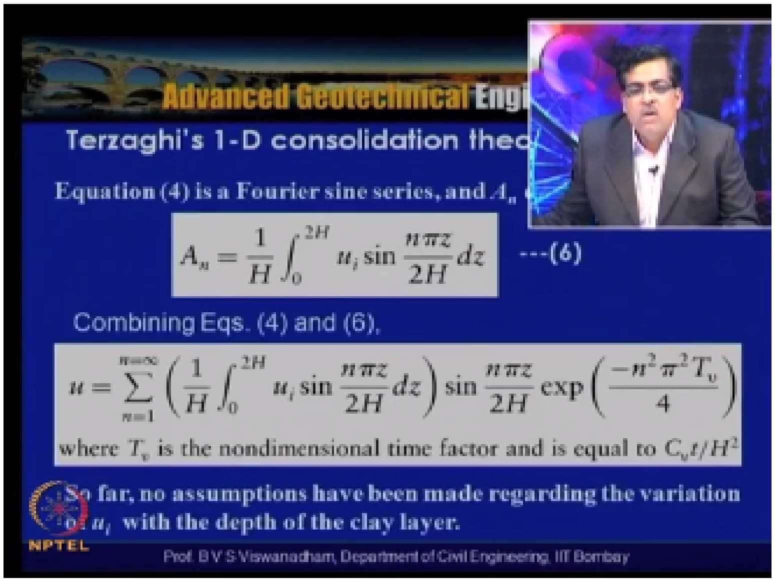

And with that equation 4 is a Fourier series and there is An can be obtained by An=1/h 0 to 2h that is the total thickness of the clay layer, ui sin and πz/2h dz and combining equation 4 and 6 we get u=n to 1 to ∞, 1/h 0 to 2h ui sin and nπz/2h dz into sin nπz/2H exp($-n^2\pi^2$Tv/4), where Tv is the non dimensional time factor and which is equivalent to Tcv/H^2 so H is nothing but here the drainage path in case of a clay having double drainage layer which is actually is H/2 in case of a single layer the H=H.

So far no assumption have been made regarding the variation of ui with the depth of the clay layer, but there can be number of variations can be assumed like assuming that if you are having a uniform variation if it is ui is actually constant with the depth.

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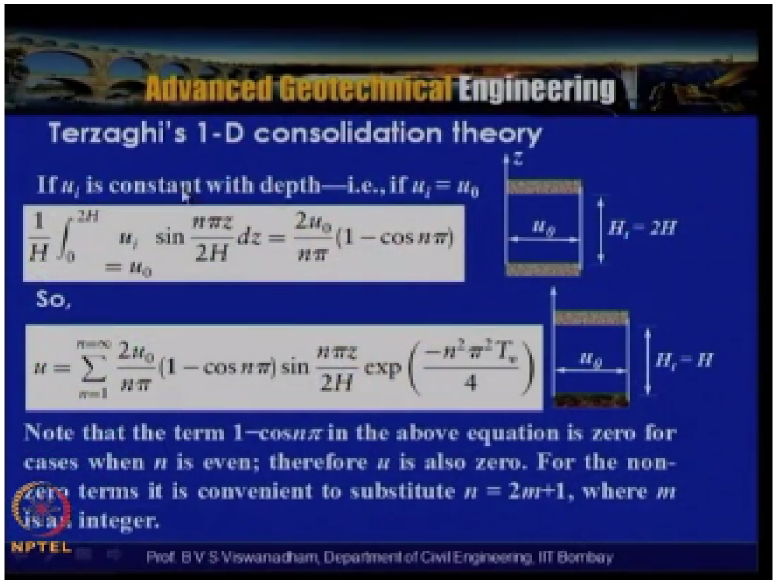

And if you are having a double drainage layer then Hd=2h in this case when we look into this and this converts into u=n to 1 2u0/nπ cos 1-cos nϕ/ sin nπz/2h exp($-n^2\pi^2$Tv/4). So here this n=2m+1 where m is an integer when you put into this and this is for the one way drainage and this is for you know two way drainage and this is actually is assumed to be uniform variation, but there is also another case where upper layer is actually impermeable bottom layer is permeable but that situation may not actually arise in practical terms if you are having a impervious soil surface or impervious rock here then it is actually one dimensional flow only happens here.

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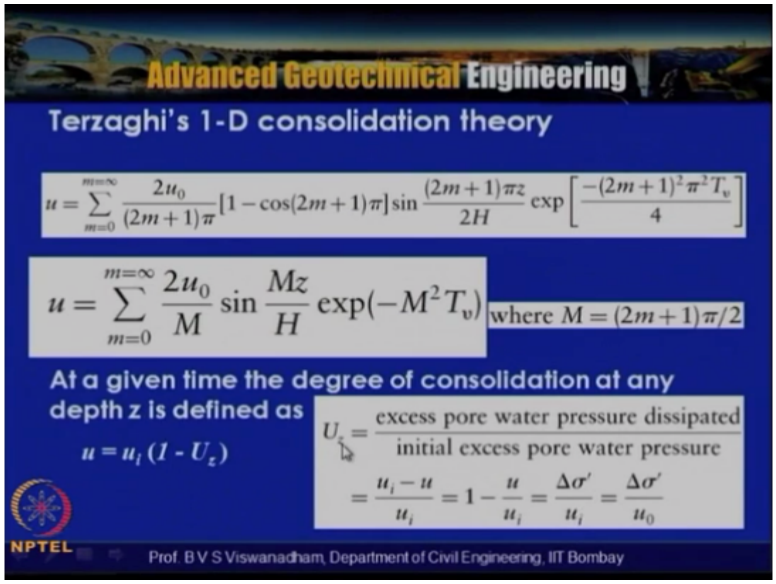

So further this actually gets simplified to this one were the Terzaghi's one dimensional consolidation theory were we actually finally we narrow down to u=m=0 $2u_o$/m/sin m z capital MZ where capital MZ is nothing but 2M1*2M1 within brackets into π/2 exponential –m^2tv so at a given time the degree of the consolidation at any depth can be determined so u_z is degree of consolidation at any depth z within the thickness of the clear layer is nothing but the access pore water pressure dissipated by the initial excess pore water pressure.

So excess pore water pressure dissipated that is u_i-u, u is actually a pore water pressure at to be dissipated so u_i-u is you know is the excess spore water pressure dissipated to the initial excess spore water pressure now this is nothing but 1-u/u_i and which is nothing but Δ σ'/u_i is Δ σ' so by using this u by knowing u_z at any point of any, any level you can actually construct the isochrones that is for given time if you are having u_z then in u_i into 1-uz we can actually find out u=ui*1-uz so in this particular lecture we try to understand about the definition of the consolidation nd consolidation phenomenon we understood that is transient condition and where in we actually have discussed and introduced about the Terzaghi's 1-dimenisonal consolidation theory.

The further we will actually apply how this Terzaghi's 1-dimensional consolidation theory can be applied to the different boundary conditions and also further extend to ramp loading and other conditions.

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