Ultrafast Optics and Spectroscopy
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Lecture - 07

Welcome back to Module 2. We are continuing Module 2 to understand the effect of different pulse shape on the time bandwidth product. We have to calculate time bandwidth product depending on what kind of pulse shape we consider.

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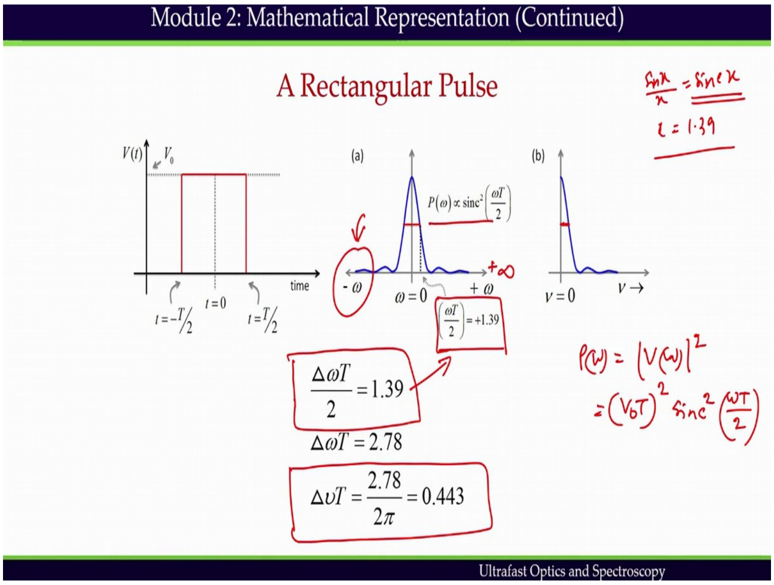

So far we have considered the Gaussian pulse, then we have considered a rectangular pulse.

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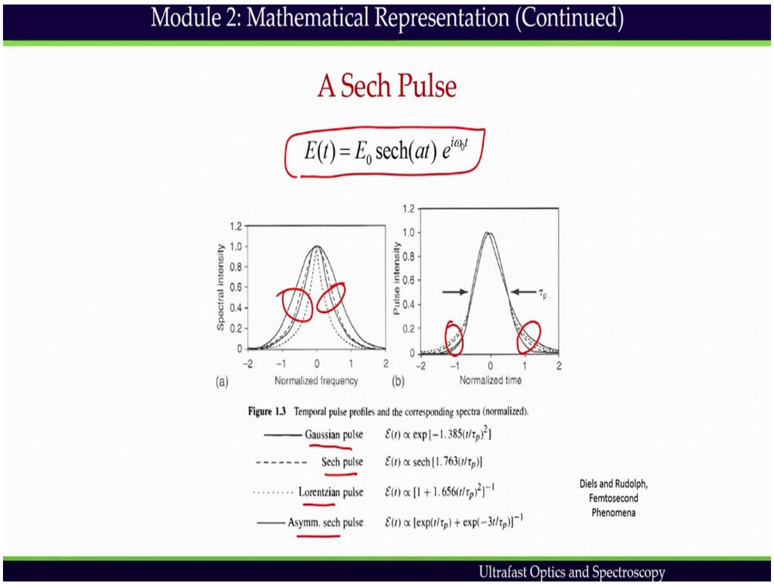

And we can just continue the way we are thinking. We can also assume hyperbolic sech function like this and hyperbolic sech function which is also centered at t equals 0 as an envelope to represent the ultrafast pulse and many other envelop functions can also be used as shown here.

So, the Gaussian pulse envelope, then hyperbolic sech function, lorentzian function, asymptotic sech functions, hyperbolic sech function, many other field envelopes. So, one can assume depending on the field envelope following similar mathematical formulation and get the time bandwidth product for a particular pulse shape. Here, we see that the envelop functions do not differ significantly if we change Gaussian to let us say sech hyperbolic, except for the wing of temporal profile of the pulses. We see a major difference here in this wings but in the spectral profile we see a major difference all over the spectrum.

So, depending on the experimental spectrum and experimentally measured intensity profile, one can assume a certain intensity profile or assume certain field envelop profile to represent the ultrafast pulse.

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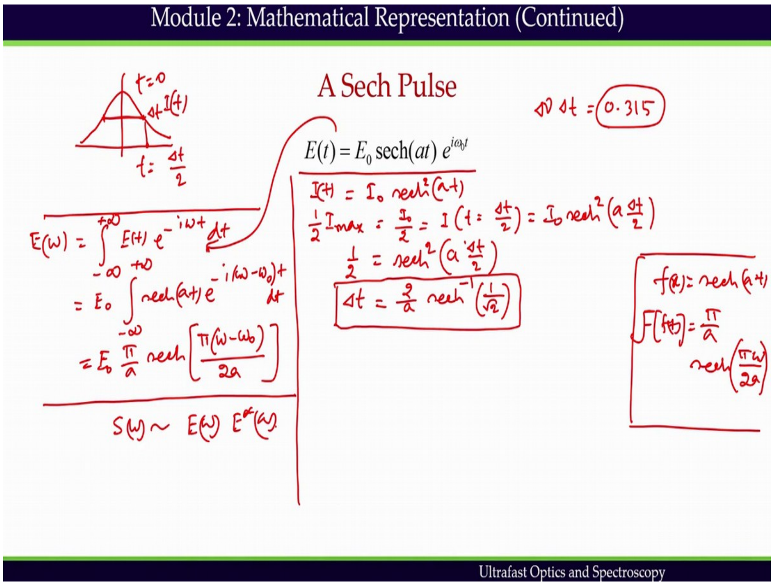

We will continue for the sech hyperbolic function and which can be represented by this and it can be shown easily that the time bandwidth product for transform limited sech hyperbolic pulse, it can be given by delta nu delta t equals 0.315.

So, we will try to find out how we get this number. We know that this sech hyperbolic function is centered at t equals 0. So, if this electric field looks like this, then intensity profile can be written as shown in slide and from the definition of the pulse width we know that is intensity profile, then I get the half maxima when t equals delta t by 2. (Please look at the slides for mathematical expressions)

So, from the definition of full width half max of the intensity profile I get I max equals I naught divided by 2 which is the maximum intensity divided by 2 which is nothing, but I at t equals delta t by 2. If I plug that in I get I naught sech hyperbolic square a delta t by 2 or in other words, half equals sech hyperbolic square a delta t by 2. (Please look at the slides for mathematical expressions)

Or I can write down delta t equals 2 by a inverse of the sech hyperbolic function 1 by square root of 2. So, this expression will keep it for future will write it down. On the other hand if we have time domain field, I can also express frequency domain field in a following way, but the Fourier transform of the time domain field is shown in the slide. Procedure is the same. Mathematics can be little different. (Please look at the slides for mathematical expressions)

The standard Fourier transform goes like this. If I have f(x) a hyperbolic function; then the Fourier transform is shown in the slide.

So, we can use the standard integral here. So, if we get the field in the frequency domain, then I can get the power spectrum in the frequency domain which is S omega nothing, but equals E omega multiplied by E star omega that we have to calculate. (Please look at the slides for mathematical expressions)

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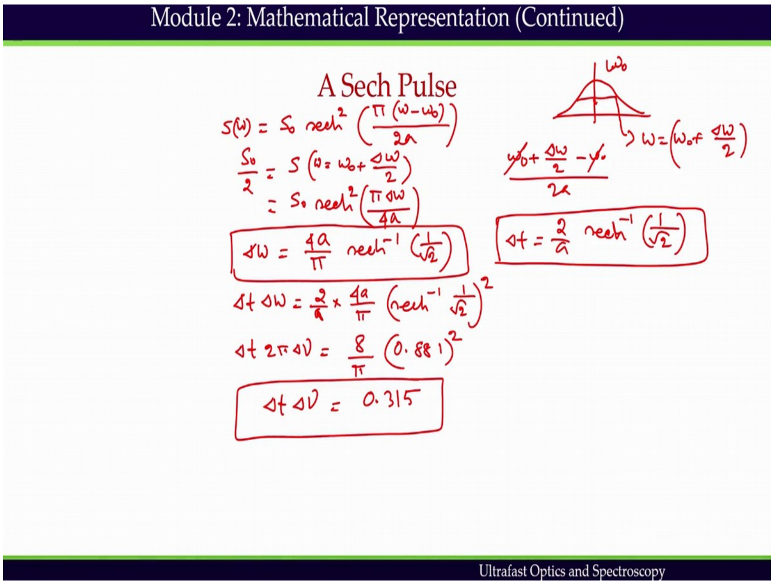

And if we calculate that, then we get S omega. (Please look at the slides for mathematical expressions)

Again this is centered at omega naught just like a Gaussian function. So, from full with half-max definition, we know that we can get the half of its maxima where omega equals omega naught plus delta omega by 2. So, I can write down S naught divided by two-half of its maxima. We obtain when S omega equals omega naught plus delta omega by 2 which is nothing, but S naught square of this hyperbolic function pi. (Please look at the slides for mathematical expressions)

Now, omega I have to plug in this one omega naught plus delta omega by 2, that is my omega minus omega naught divided by 2 a. So, this omega naught cancels out. So, finally I get pi delta omega divided by 4 a or in other words, delta omega if I rearrange this equation, we get 4 pi sorry 4 a by pi sech hyperbolic inverse 1 by square root of 2.

So, I have delta omega expression, in the previous slide I had the delta t expression which was 2 by a sech hyperbolic inverse this. So, to get time bandwidth product I just multiply delta t delta omega I get 2 by a multiplied by 4 a by pi 1 by square root 2 whole square. (Please look at the slides for mathematical expressions)

If we simplify, then I get delta t 2 pi delta nu which is nothing, but 8 by pi. So, after little bit of calculations as shown in slides, we get hyperbolic sech function time bandwidth product is going to be 0.315, which is different from a Gaussian pulse or a rectangular pulse.

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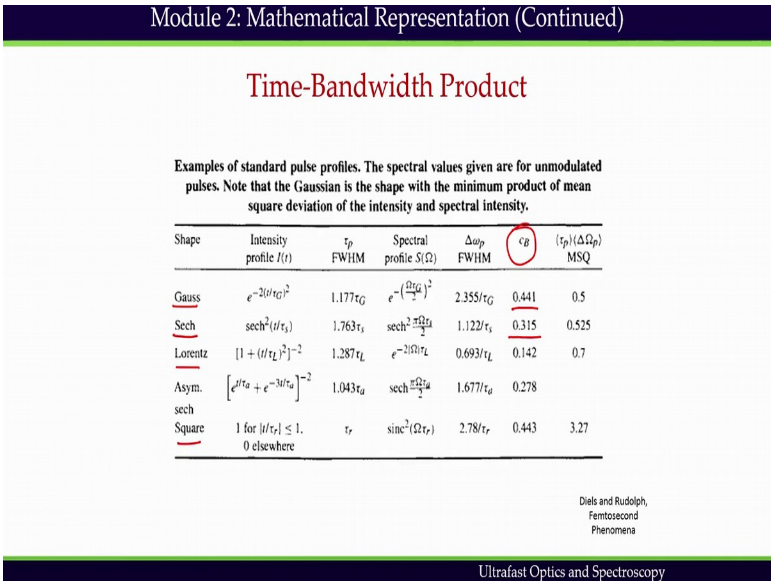

This table here is showing time bandwidth product, this C_B is time bandwidth product of different pulses. If it is a Gaussian pulse, it is 0.441, if it is a sech hyperbolic function, it is 0.315 and for many other line shapes we have different values. We have to remember that we have to assume a pulse shape first, then corresponding time bandwidth product. We should not mix these numbers.

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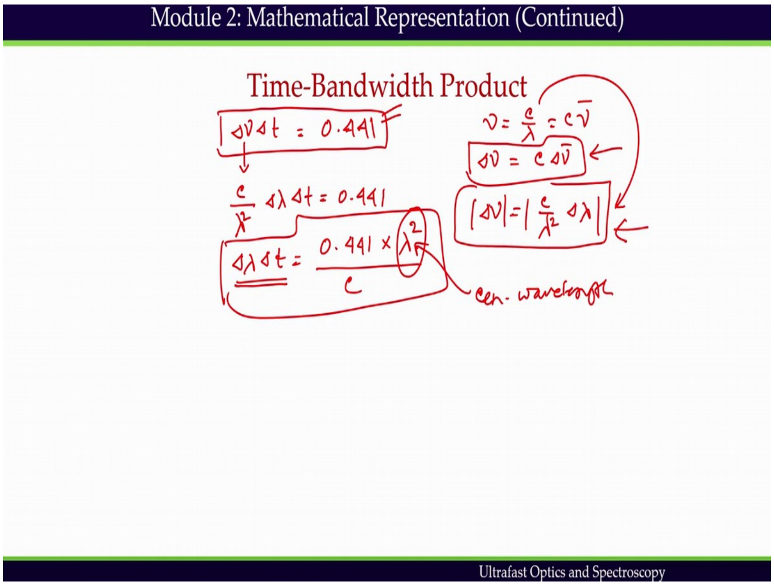

For delta nu, delta t if we assume that it is a Gaussian pulse, then I can write down this time bandwidth product for a transform limited pulse and often we need to convert this equation to the wavelength or wave number and we would like to know how we can convert it. So, this delta nu can be written we knew that nu equals c by lambda which is nothing, but c nu bar which is the wave number. (Please look at the slides for mathematical expressions)

So, delta nu can be converted to wave number with the help of this expression. On the other hand if we have to convert it to wavelength, then we have to write down delta nu equals c by lambda square delta lambda is coming from this expression. There is a negative sign here, but we are omitting it. We are just considering the magnitude of this. So, these are the Conversions. We should remember if we want to convert delta nu to its wave number, this is the equation and if we have to convert delta nu to delta lambda, then this is going to be the expression. (Please look at the slides for mathematical expressions)

So, now we plug that in and we convert it to wave length as shown in slide. This lambda in this equation is going to be center wavelength, we call it 800 nanometer, pulse is going to be 800 nanometer.

So, what we see is that delta lambda delta t will depend on its center wavelength which we have not seen before in the time bandwidth product.

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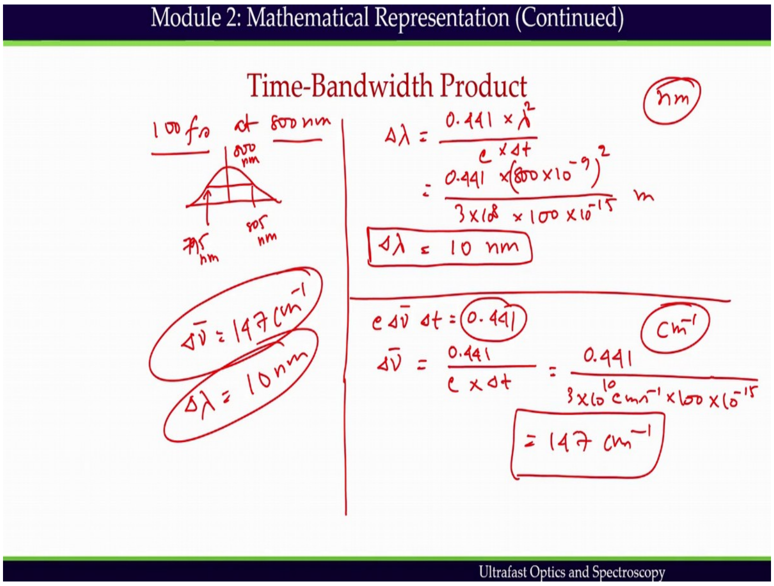

So, we can use these expressions and we can take an example of 100 femtosecond pulse. We can consider 100 femtosecond pulse at 800 nanometer. We know the meaning of it. Now, 100 femtosecond pulse it means that it is the full width half max of the intensity profile of the pulse center wavelength is going to be 800 nanometer.

So, if it is 100 femtosecond pulse at 800 nanometer centroid, 800 nanometer, then delta lambda can be calculated with the help of that expression 0.441 multiplied by lambda square divided by c which is nothing, but 0.441 multiplied by 800 multiplied by 10 to the power minus 9 square divided by 3 into 10 to the power 8 meter per second multiplied by delta t. (Please look at the slides for mathematical expressions)

So, finally this will give me a meter and if we use the calculator, we will be able to quickly get 10 nanometer. Please check with the calculator whether these numbers are correct. So, delta lambda the bandwidth in the wave length for a 100 femtosecond pulse at 800 nanometer is going to be 10 nanometer.

So, what does it mean? It means that I have center wavelength 800 nanometer, but its width is plus minus 5 nanometers. So, this is going to be 805 nanometer and this is going to be 795 nanometer. So, this is the spread of the spectrum which we see. On the other hand for the same pulse if we have to express this is in nanometer. Now we want to express in wave numbers centimeter inverse in many occasions we have to use these conversions to understand the spread. That is why it is instructive to take an example.

So, we know that this same expression can be re-written as delta nu bar delta t equals 0.441. We are assuming that we have Gaussian pulse. If if we assume that it is a hyperbolic sech function, then this number will be different, but if we assume Gaussian pulse, then this delta nu bar is going to be 0.441 divided by c multiplied by delta t. We can plug that in all the numbers 0.441 divided by 3 into 10 to the power 10. (Please look at the slides for mathematical expressions)

So, what I get is 147 wave number. Please check that number with a calculator. So, what we see is that now if we plot it in the wave number regime, then delta nu bar thus the width of the spectrum is going to be 147 wave number and if it is delta lambda, then it is going to be 10 nanometer. So, with this one can convert to different representations for the time bandwidth product.

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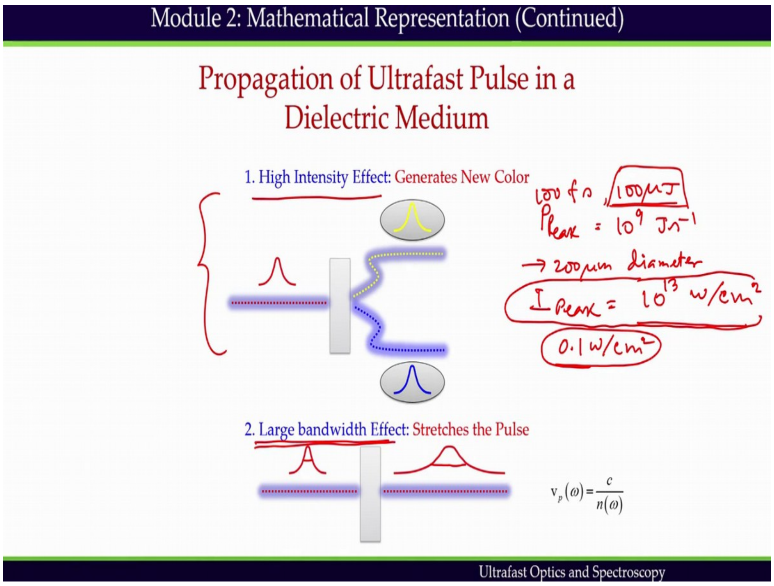

So, what we have seen so far is that we have discussed different characteristics of an ultrafast pulse. In this module we have discussed how to calculate the frequency spread for a particular pulse or frequency content of a particular pulse from the temporal duration or in other words, if I want to achieve a certain pulse duration what are the frequency components we need that we have discussed and when we have discussed all these characteristics of the pulse, we have always considered that I have an isolated pulse.

Now, isolated pulse in any experiment will propagate through the medium. We use lens, we use crystal, we use wavelet, all of these materials are of dielectric nature. So, we should understand the propagation of the pulse through the dielectric medium. Mostly two effects we observed. When an ultrafast pulse propagates through a dielectric medium, the high intensity effect and large bandwidth effect. When you say high intensity effect we can consider 100 femtosecond pulse again.

And each pulse contents let us say 100 micro joule energy. So, with this 100 micro joule energy, now we can calculate peak power. The peak power is going to be the peak power is going to be can be written as 10 to the power 9 joule per second. One can calculate it quickly with the help of peak power equation and if it is focused to let us say 200 micron diameter spot size, if it is focused to that, then what I get is peak intensity at the point where it is focused that peak intensity is going to be almost 10 to the power 13 Watt per centimeter square.

This high peak intensity if we considered sunlight peak intensity in a brightest day is going to be point let us say less than 0.1 Watt per centimeter square. So, we can compare sunlights peak intensity and the peak intensity we one can achieve from an ultrafast pulse with a moderate energy. This 100 micro joule is pretty low energy, but very frequently used in the ultrafast laser spectroscopy lab.

So, which suggests that if we have a moderate energy pulse, that pulse can give me peak intensity of the (Refer Time: 22:11) 10 to the power 13 Watt per centimeter square. It is a huge intensity and because of that intensity we get polarization, we get non-linear effects. So, these are all you know related to the non-linear effects. On the other hand, if we look at the large bandwidth effect we know that there are many frequency components traveling through the medium and it may so happen that the red component are facing different velocity than blue component and that is why they will spread out. So, always due to large bandwidth, due to dispersion the pulse duration will always elongate as compared to the due to that dispersion.

So, these are the two effects which will experience when the pulses are propagating through the medium. We have come to the end of this module and will discuss all this high intensity effects and dispersion effects in the next modules. What we have studied here in this module is to how to represent an ultrafast pulse mathematically and how to get different characteristics of the ultrafast pulse with the help of mathematical formulation. These mathematical formulations defines different terminologies like bandwidth products like pulse duration, bandwidth, Gaussian pulse hyperbolic sech functions all these terminologies will be frequently used in this course. We will meet again for the next module.