Ultrafast Optics and Spectroscopy
Dr. Atanu Bhattacharya
Department of Inorganic and Physical Chemistry
Indian Institute of Science, Bengaluru

Lecture - 06

Welcome back, we are continuing module 2. In this module, we have emphasized that electronics are way too slow to measure both fast electromagnetic oscillation as well as intensity profile.

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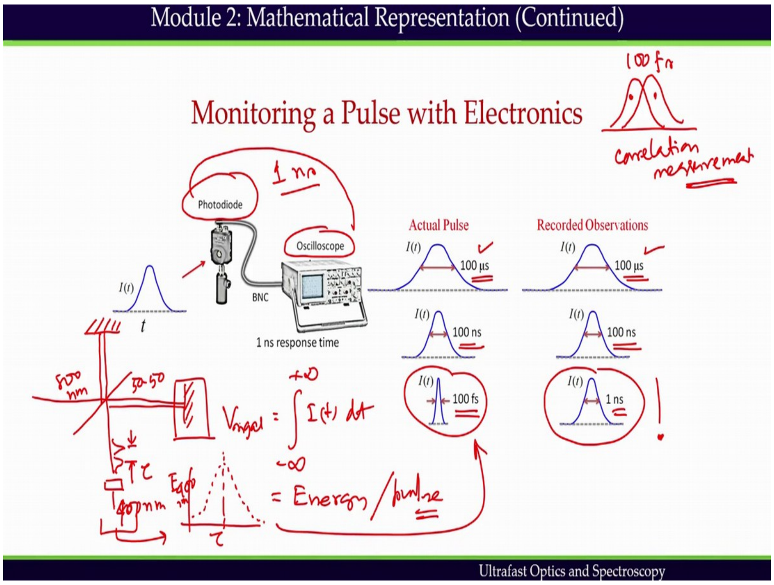

If we try to measure a pulse with the help of electronics, then what might happen a time sensitive detector emits electrons in response to photon.

Let us consider a photodiode whenever I shine light on it, it will give me an electrical response and that electrical response can be monitored with the help of an oscilloscope. Time sensitive detectors include photodiodes including photodiodes and photomultiplier tubes; they are way too slow that we have already pointed out. They have very slow rise and fall times and that gives you the response time which is nanosecond.

So, the whole procedure how long does it take to capture the light from here and plot it in oscilloscope, this whole time scale is going to be 1 nanosecond. Let us assume there is 1 is nanosecond. Therefore, if I have a pulse of 100 microsecond duration then with the help of electronics, I will able to see 100 microsecond pulse I will be able to plot it.

Because for an 100 microsecond pulse electronics is faster and I can plot it for 100 nanosecond pulse I can plot it and I can see 100 nanosecond.

But if I shine femtosecond pulses to the photodiode let us say 100 femtosecond pulse is going to the photodiode then what will happen, photodiode and oscilloscope both will show me a pulse of 1 nanosecond that is all. And this is a big surprise electronics cannot measure the variation of 100 femtosecond variation of the profile; it will just give me a slow response of its own which is nanosecond response. So, any femtosecond pulse we try to monitor with the help of electronics that will become broad electronic response in the oscilloscope.

We cannot just measure the intensity profile with the help of photodiode coupled oscilloscope, but if it is nanosecond pulse I can measure; because the response time is equivalent to the nanosecond. Slow detectors I have already pointed out will measure time integrated intensity. So, this signal if it is a slow detector, then what I will get is a time integrated over minus infinity to pulse infinity I(t) dt this is what I will measure and this is nothing, but energy content of each pulse. What I measure is basically can be connected to the energy pulses. Does detector output voltage is proportional to the pulse energy only. (Please look at the slides for mathematical expression)

The slow response time of available time sensitive detectors does not permit us to make time domain intensity profile measurement of ultrafast pulse. We cannot measure femtosecond or picoseconds pulses with the help of the electronics.

We have to use the pulse intensity profile to measure its own intensity profile and this is this kind of measurement is called intensity auto-correlation measurement. So, let us consider 100 femtosecond pulse, what I have to do? I have to use this profile to measure another 100 femtosecond pulse. I am correlating this pulse with this pulse that is why it is call correlation measurement and this is the only way one can measure an ultrafast pulse with the help of slow detector. Then we do not have any issue we have a slow detector, but we are correlating with two short pulses and get this autocorrelation measurement.

One of the simplest technique by doing autocorrelation measurement is that we can use 50-50 beam splitter this we will study very soon in the pulse measurement content in the context of pulse measurement. I can have 50 50 beam splitter which will split the beam by 50 percent of its parent intensity and then I can reflect back and then I get this two pulses.

Now depending on the path length difference between these two arm, this is more like a Michelson interferometer and depending on the path length between here to here. This path length difference will decide what is the delay between two pulses? If I use a bbo crystal and produce 800, 400 nanometer from 800 nanometer pulse.

I can check the energy with photometer, I can plot this energy as a function of tau of 400 nanometer beam, and I can get points at different delay and this is a profile which is called autocorrelation profile that can be directly connected to the intensity. So, this is our way to measure the pulse direct measurement. With the help of electronics it is not possible.

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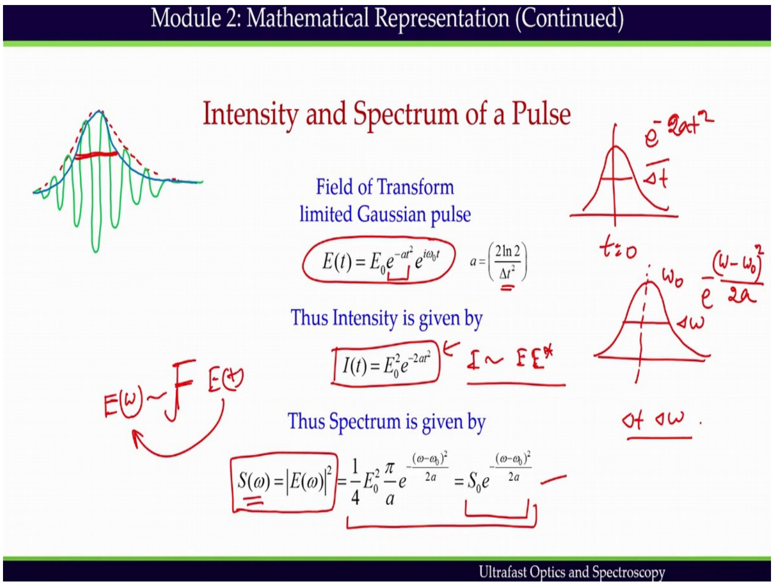

We have seen that a Gaussian envelope can be used to represent an optical pulse field envelope that, we have seen. We will draw one more important concept that we should remember if I plot a pulse I will plot it like the dotted line is the field envelope and this green line is the carrier wave oscillation, and this blue line is representing the intensity profile of the pulse which is similar to the field envelope profile, but they are slightly different. And whenever we say pulse duration, we have to remember it is the full width of max of the intensity profile. This delta t is a pulse duration contain the full width of max of the intensity profile. So, now, this is representing an isolated propagating pulse and this Gaussian function representing the envelope function.

Intensity can be given by this because I which is EE*. We have seen already in this module that this is the expression which is valid for the intensity profile and then we can always get Fourier transform of this time domain pulse to get the frequency domain. So, which means that I can get Fourier transform of this time domain pulse and I get the frequency domain. And once we get the frequency domain field, then I can again take the power spectrum.

So, if you look at this two pulses, the intensity profile is centered at equals 0. This is t equal 0 profile looks like e to the power minus 2 at square. (Please look at the slides for mathematical expression)

On the other hand corresponding spectrum after Fourier transform corresponding spectrum which we measure is S omega[$S(\omega)$] not E omega[$E(\omega)$]. S omega when you measure is also Gaussian. A Gaussian Fourier transform is another Gaussian, but it is centered at omega naught now. We have to remember that the expression for this Gaussian is going to e to the power minus omega minus omega naught whole square by 2 a. So, the expression we get for the spectrum has different positions; one of the spectrum is centered at omega naught and temporal intensity profile is centered at t equal 0. (Please look at the slides for mathematical expression)

Now these are the things which we are already familiar with in this module and we have seen that delta t delta omega are related. This is called time bandwidth product will come back to this time bandwidth product one more time to understand more details of it.

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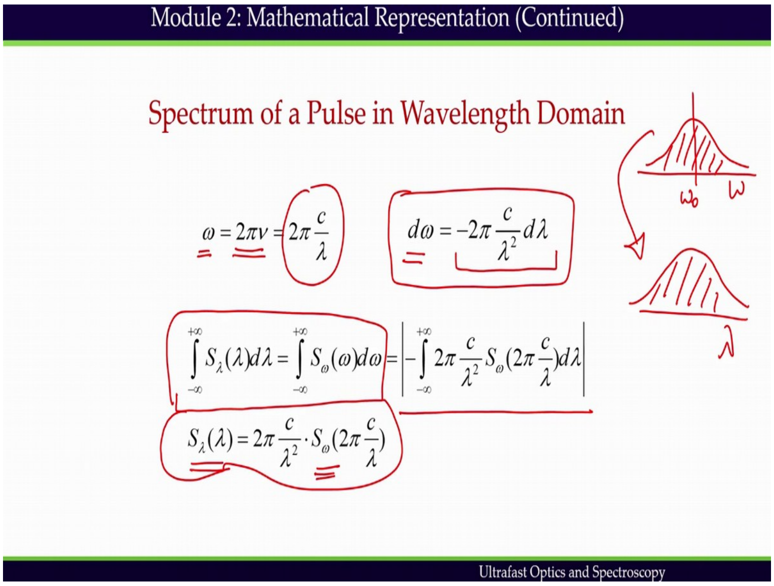

In the previous slide; I have shown that, the spectrum was represented in the omega domain which is centered at omega naught. But often when we talk about spectrum, we represent spectrum in the lambda domain which is the wavelength domain. And it is instructive to understand how to convert this omega domain spectrum to the lambda domain spectrum and that can be done very easily omega which is angular frequency which can be represented by 2pi mu is optical frequency which is nothing, but 2 pi c by lambda. (Please look at the slides for mathematical expression)

Now if take the derivative d omega then we get minus 2 pi c by lambda square d lambda and this is an important realization whenever we are converting frequency domain to the lambda domain, we have to think about along this line only. Now you know that area under the curve would be the same area under the curve representing the probability representing how many such system are contributing so, that will be constant always area under the curve does not matter which domain represent that should be the same. (Please look at the slides for mathematical expression)

So, with this idea, we can make this area under the curve to be the same which is integrated area under the curve for both domains. And once we represent it, then we get S lambda which is spectrum represented in lambda domain can be converted to the spectrum represented in the omega domain or vice versa; both can be converted with the help of this equation. So, this is a simple mathematical trick we need to employed to convert the omega domain spectrum to the very frequently use lambda domain spectrum.

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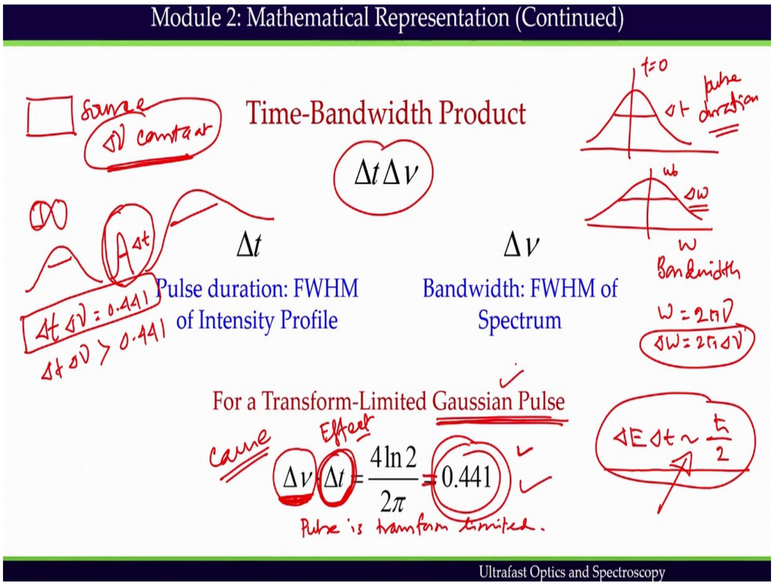

While dealing with pulse intensity profile and the spectrum, we have shown that the intensity profile; this is delta t full width half max that is called pulse duration. And in the omega domain, this is omega naught corresponds to t equal 0 and delta t corresponds to delta omega and this delta omega is related to the bandwidth and delta t is related to the pulse duration which is intensity full width half max or the temporal duration of the pulse.

So, this is pulse duration and this product has a particular meaning for a particular pulse delta t delta nu instead of omega we can write down delta n u; we can convert omega to delta nu very easily by 2 pi mu[μ]. So, delta omega is going to be always 2 pi mu that is all we can convert that very easily. So, this 2 delta t pulse duration and delta nu band width are related by time bandwidth product. (Please look at the slides for mathematical expression)

This concept is very important concept for the ultrafast optics and spectroscopy. The product of the temporal and the spectral width is called time bandwidth product we have seen that for a transform limited Gaussian pulse the time bandwidth product is going to be 0.441, this is the time bandwidth product for a Gaussian pulse.

This relationship which is similar to time energy uncertainty principle in quantum mechanics; time energy uncertainty principle in quantum mechanics looks like delta E delta t is going to be h cut by 2. So, it is quite similar to this uncertainty principle, which suggest that the shorter pulse requires a broader spectrum. Time bandwidth product is minimum for a transform limited pulse. (Please look at the slides for mathematical expression)

What does it mean to have a transform limited pulse? That for a given spectrum this is the shortest duration pulse. I have often seen student will confuse in this time bandwidth product concept when they are thinking about time bandwidth product; we have to remember that delta nu is the cause, I need the bandwidth to produce a pulse and this is called effect. So, the right terminology is always to say that in order to get a short pulse I need large bandwidth sometimes by mistake we say that.

If I can reduce the pulse duration the bandwidth will change; I cannot change the bandwidth. Bandwidth is something which is a characteristic of the source. So, for a particular source if I have a source for a particular source delta nu is constant with this delta nu I can have infinite possibilities infinite possibilities of pulses I can have a pulse like this, I can have a pulse like this, I can have pulse like this many other possibilities, but I will have one possibility one pulse having the shortest duration.

And in that case only delta t delta nu is going to be 0.441 otherwise delta t delta nu is going to be greater than 0.441. So, for a given source for a given bandwidth when I have this delta t delta nu equals 0.441 it means that I have been able to obtain one shortest pulse for a given source. If I do not get the shortest pulse, then the pulse duration always be longer and that is the consequence of this time bandwidth product.

But question is this derivation we have seen in this module already and the question may come whether we always need to use a Gaussian pulse to represent the intensity profile because this equation is valid only for Gaussian pulse, this equation is not valid for any other pulse. And one more point I would like to make here is that all though this equation is similar to the Heisenberg time energy uncertainty principle in quantum mechanics, but we cannot get this number from this equation. We have to use the derivation which we have shown in this module for the Gaussian pulse.

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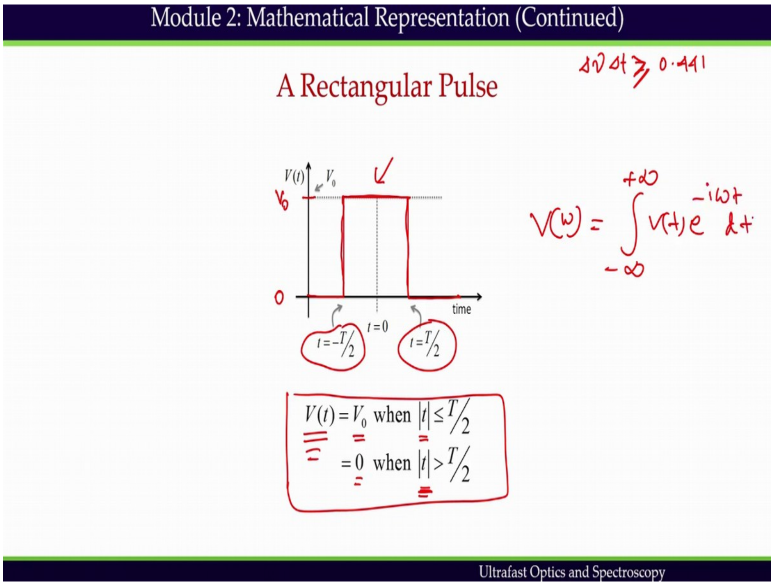

And question is it always necessary to use a Gaussian pulse to represent the experimental ultrafast pulse. So, far we have seen the Gaussian pulse envelop is called when we consider Gaussian pulse envelop the time bandwidth product becomes delta nu delta t equals greater than equals 0.441 condition obtained for the shortest duration pulse for a given spectrum.

Now, Gaussian envelops are most commonly used in ultrafast optics because subsequent analytical math becomes very simple and closely represent an experimental ultrafast pulse; however, it is not necessary that we have to consider a Gaussian field envelop. One can assume many other pulse envelop which may closely represent an experimental pulse. Selection of the appropriate field envelop depends on the experimental observation. For example one can say that a rectangular pulse can also be another way to represent ultrafast pulse.

I have nothing 0 then suddenly I have this V naught value then again coming down 0. So, this is your 0 and this is your V naught. A rectangular pulse is commonly used in electronics and in signal transmission lines. We can also consider this kind of rectangular pulse to represent an ultrafast pulse theoretically; we can take a look at the time bandwidth product for this rectangular pulse. Procedure is known to us now a rectangular pulse is depicted here and V(t) is represented as the temporal profile of the pulse. It is centered at t equals 0 and this pulse can be represented by this two equations. (Please look at the slides for mathematical expression)

From theory of interference of plane waves it is already known that a pulse is originated due to superposition of many pure waves frequency components. So, that is true for even rectangular pulse which is shown here.

So, in order to produce this kind of rectangular pulse in femtosecond domain let us say, it is an hypothetical idea we are discussing here. We are producing this kind of rectangular pulse in femtosecond domain then we have to consider where this pulse was produced with the help of many frequency components because only optical way one can produce pulse is the interference. And we can immediately convert this time domain pulse to the frequency domain by Fourier transform which would look like minus infinity to plus infinity, time domain representation multiplied by e to the power minus I omega t dt. So, I can get this frequency domain representation. (Please look at the slides for mathematical expression)

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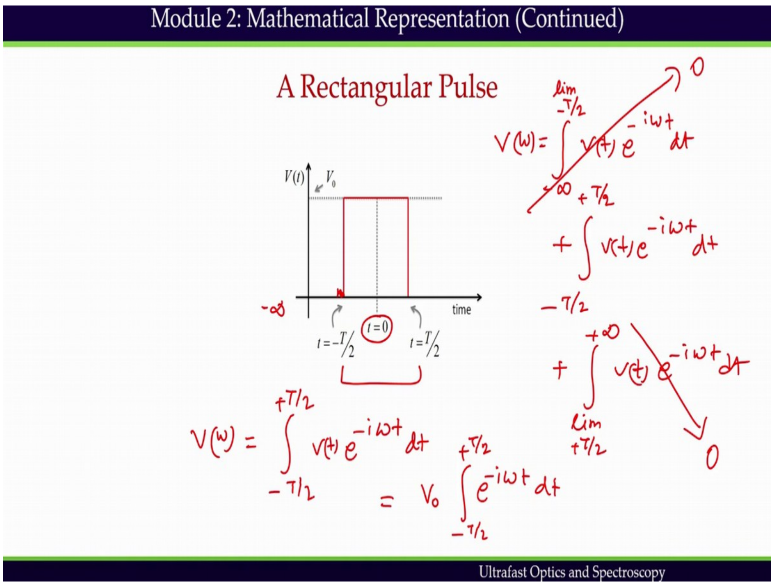

As a representative example, we have considered here an ideal rectangular pulse which is centered at t equals 0. So, Fourier transform of this function will get V(ω) as shown in slide. (Please look at the slides for mathematical expressions)

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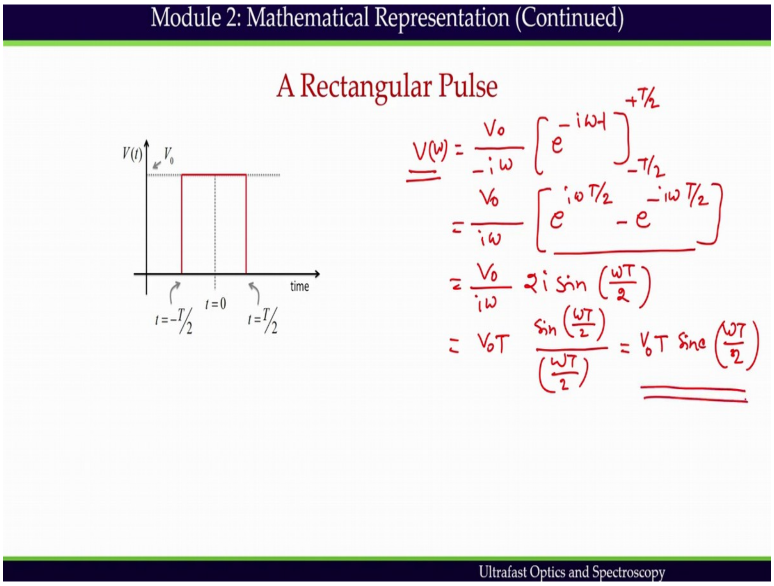

And further we can write down this integration we can simplify this. So, V(ω) can be solved step by step as shown in the slides. (Please look at the slides for mathematical expressions)

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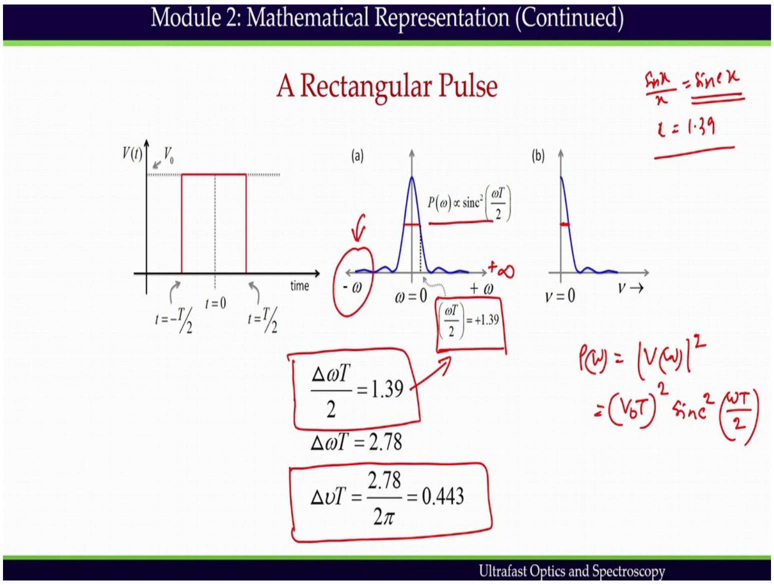

And if you look at the cardinal sin function the power spectrum is going to be the power spectrum can be represented by this P(ω) which is nothing, but V(ω) square modulus and of V(ω) which is nothing, but V naught T whole square cardinal sin function square omega T by 2. (Please look at the slides for mathematical expressions)

The power spectrum plotted here and this is the plot we have and it is evident that V(t) is the rectangular pulse contains many frequency components all these frequency components extended up to plus infinity. So, theoretical from 0 to plus infinity all this frequency components we have, the amplitude of a cardinal sin square function decreases by a factor of half when this omega T by 2 becomes 1.39. This is very well known number we will use this number in non-linear optics as well any cardinal sin function when you write down this sin x by x which is nothing, but cardinal sin function. (Please look at the slides for mathematical expressions)

This cardinal sin function will drop to its maximum intensity when x is 1.39. So, we can use this only positive component of the frequency see negative frequency is nothing, but mathematical artifact is just as given to me because we have taken both complex conjugate and the complex number. So, we have to avoid considering this negative frequency component which does not mean anything. We have to consider only positive frequency component and we would like to find out what is the full width of max for this and for that we are saying that it is delta omega T by 2.

That is nothing, but 1.39 coming from this characteristic of cardinal sin function and from that we get the time bandwidth product of an rectangular pulse which is nothing, but 0.443 which is which is quite similar to the ultrafast pulse for a Gaussian pulse. We will stop here and we will continue this module in our next lecture.