Ultrafast Optics and Spectroscopy
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Lecture - 04

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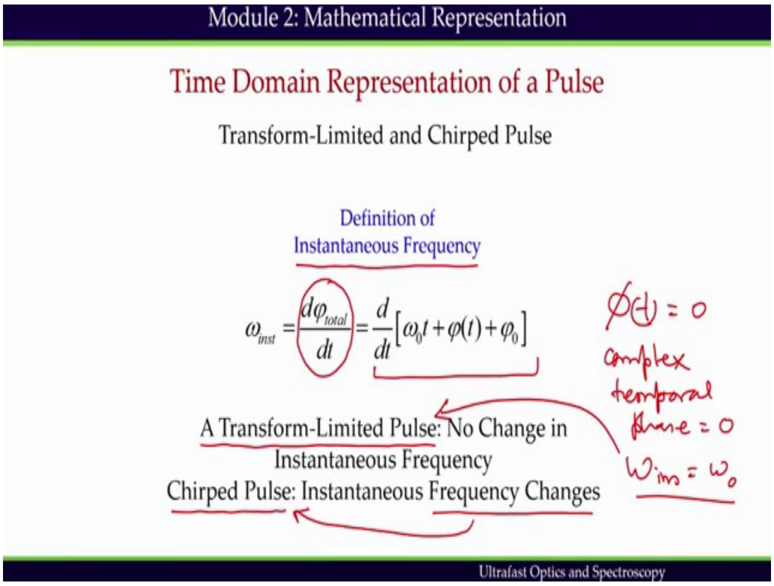

Now, we will define instantaneous frequency of a pulse. This is an important concept. In instantaneous frequency of a pulse is defined by the time derivative of the total temporal phase which is nothing, but this one phi t, this phi total is a total temporal phase of the pulse. If instantaneous frequency does not change over the time, then the pulse is called transform limited pulse and if instantaneous frequency changes over the time then the pulse is called chirped pulse.

So, we have given two different definitions of pulses one of them is transform limited pulse and other one is chirped pulse. For an ultrafast pulse with phi t equals 0; if phi t is 0 that is the complex temporal phase is 0, then omega instantaneous is always going to be omega naught that is omega instantaneous is always constant and that happens for transform limited pulse. Otherwise the frequency will change and if frequency is changing in a pulse for the duration of the pulse it is called chirped pulse.

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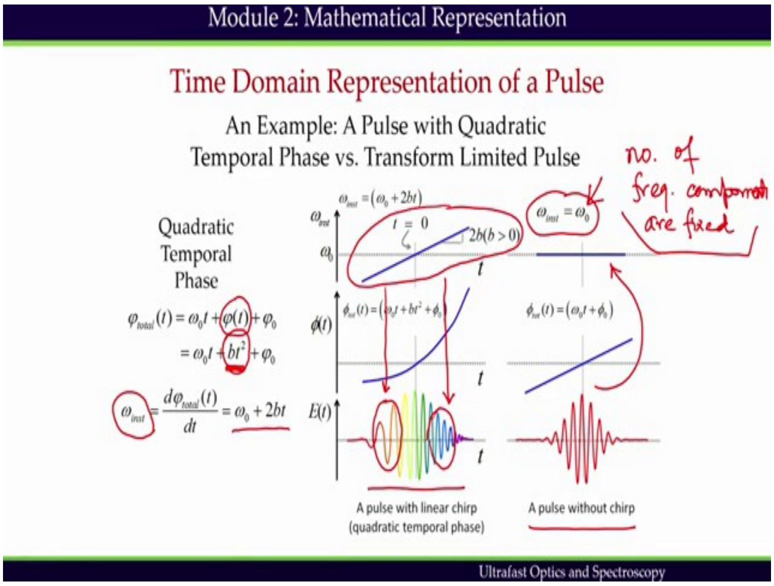

What does it mean? To understand the meaning, we will take a look at a pulse with quadratic temporal phase. We said that phi t can be a function of time. Here we are taking an example of bt square, that is also a function of time; a quadratic function bt square, b is constant. If somehow we introduce this kind of temporal phase in a pulse then instantaneous frequency is represented by omega naught plus 2 bt; that means, instantaneous frequency will change as a function of time here it is linearly changing. <u>(Please look at the slides for mathematical expressions)</u>

So, I have shown instantaneous frequency plot as a function of time, it is changing linearly here and because it is changing linearly what we expect a pulse having different frequency at different time. In this regime I have lower frequency and in this regime I have higher frequency. So, this figure depicts a pulse with quadratic temporal phase. The variation of phi t[φ(t)] and omega instantaneous with respect to time in the pulse is also shown here. A pulse with quadratic temporal phase is call to be linear chirp.

Instantaneous frequency sweeps of frequency changes because instantaneous frequency over the pulse varies linearly with respect to time. For a given spectrum the temporal behavior of a pulse may change depending on the chirp it contents. Presence of chirp will always stretch a pulse in time. So, for a given spectrum; that means, number of frequency components if fixed number of frequency components is fixed. If I have the same number of frequency components or colour components which are interfering and then giving me a pulse, then a transform limited pulse a pulse without chirp will not see any change in frequency with respect to time. That is why instantaneous frequency is always constant which nothing, but carrier frequency is.

But, for the same number of frequency components if I have somehow introduced this kind of temporal phase complex temporal phase in the pulse that can be introduced with the help of dispersion effect in the pulse, then what we will see? We will see a chirped pulse and we can remember this one that a chirped pulse for a given frequency components a chirped pulse will always be longer than a transform limited pulse.

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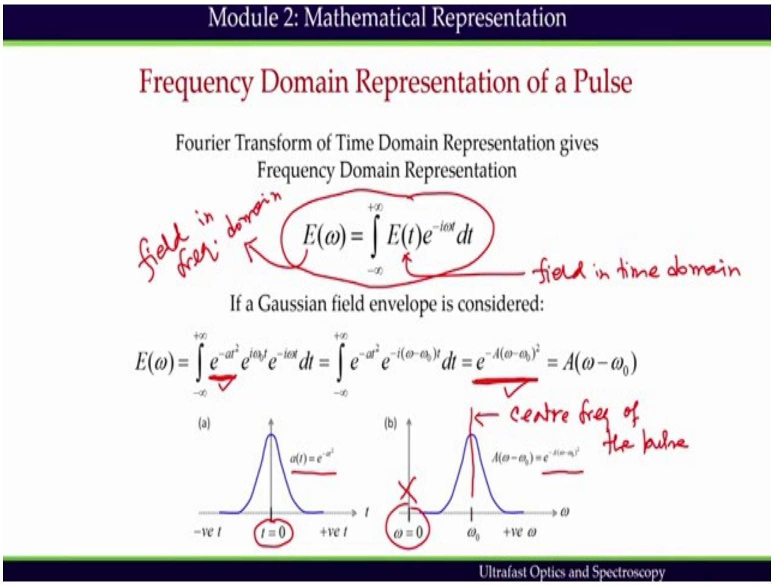

So, that is all about time domain description of a pulse. Now, we look at the frequency domain representation of a pulse. The time domain representation of a pulse can easily be converted to frequency domain using Fourier transform E omega equals this integral. Here E t is the field in time domain and E omega is again the field in frequency domain.

If a Gaussian envelop is assumed for the time domain field that is this one, then by doing this Fourier transform we get another Gaussian function in the frequency domain which is represented by e to the power minus a omega minus omega naught square whole square. But, what is the difference between this Gaussian, the Gaussian in frequency domain and the Gaussian in time domain? In the time domain the field envelope is centered at t equals 0 always and that is why it is represented by e to the power minus at square. (Please look at the slides for mathematical expressions)

But, in frequency domain when we do the Fourier transform we get another Gaussian that is the field envelope in frequency domain, but this field envelope is not centered at omega equals 0, it is centered at omega naught that is the center frequency of the pulse. So, this is the two differences we notice when we convert time domain representation to the frequency domain representation.

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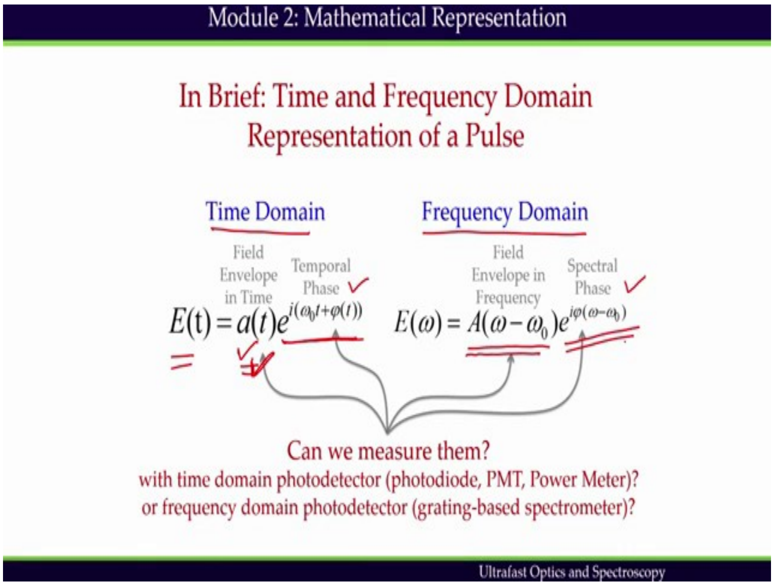

So, in brief whatever we have gone over a pulse has representations in two different domains; one is time domain and other one is frequency domain. In time domain it is expressed with the help of a field envelope function a t and its temporal phase; temporal phase is associated with the carrier wave. And in frequency domain again it has a field envelope and the phase which is related to the spectral phase. We have to remember that this phase is coming in time domain, that is why this temporal phase and this phase is coming in the frequency domain, that is why it is called the spectral phase.

After understanding these mathematical representations of an ultrafast pulse it is quite instructive now to ask how do we connect these mathematical representations to lab based measurements? Can we measure a t because in order to represent my pulse I need to know either a t, I mean a t and temporal phase or I need to know field envelope and spectral phase? Can you measure them?

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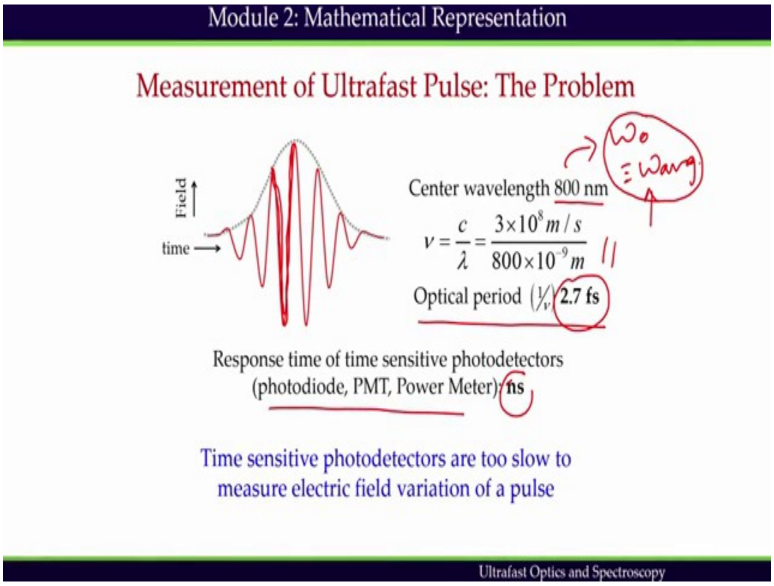

A pulse with the center wavelength 800 nanometer now center wavelength is 800 nanometer which means it is associated with omega naught which is nothing, but omega average which means all different frequencies are interfering, after the interference we get the resultant frequency of the total field and that resultant frequency is omega naught and we have assume that it is transform limited pulse. If it is not a transform limited pulse, then this omega naught would be changing or instantaneous omega will change over the time over the over the pulse.

So, here we have assume that it is a transform limited pulse and the center wavelength is 800 nanometer. Often we call 800 nanometer pulse it means that a pulse with center wavelength 800 nanometer. Now, if you consider the center wave length 800 nanometer then the optical period of this pulse which means the time to take from here to here is going to be 2.7 femtosecond; a simple calculations one can do that very quickly 2.7 femtosecond. On the other hand, the response time of all time sensitive detectors such as photodiode, photomultiplier tube, CCD camera etcetera is mostly nanoseconds.

And, this is why time sensitive detectors are way too slow to measure optical cycles of a pulse. We cannot measure this kind of oscillation, we cannot view this kind of oscillation with the help of time sensitive detector available time sensitive detectors.

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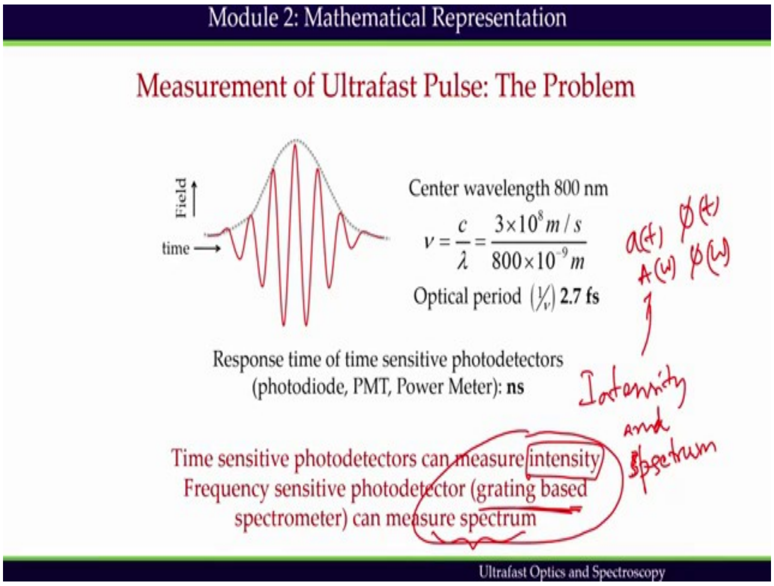

However, we must know that the time sensitive detectors can measure intensity of a pulse this is something which is measurable quantity. And, also grating based spectrometer grating based spectrometer can measure the spectrum. So, these are spectrum and intensity. Spectrum and intensity these are measurable quantity and we have to connect them to a t or A omega[ω] or phi t[φ(t)] and phi omega; spectral phase temporal phase or dose envelope functions. So, that is our next task.

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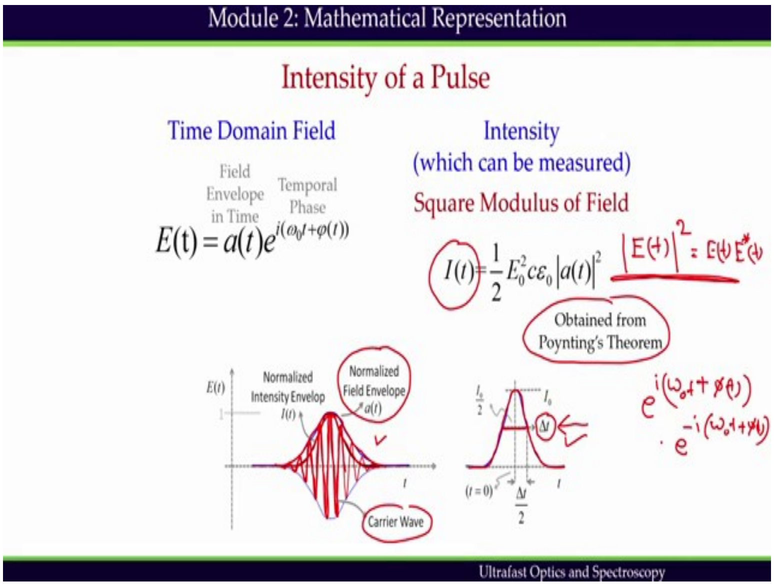

The intensity of electromagnetic radiation is defined as the time average magnitude of Poynting vector. This comes directly from Poynting theorem. From Poynting theorem we find that intensity of a pulse is nothing, but the square modulus of the field. So, what we can measure with time sensitive detector is not the field directly, but the square modulus of the field which is nothing, but E t multiplied by its complex conjugate.

For most of the theoretical calculations and experimental measurements related to ultrafast optics and spectroscopy are good approximation for an ultrafast pulse is the Gaussian pulse which we have shown here. The function is centered at t equals 0 always in time domain. Furthermore, the temporal duration this delta t of an ultrafast pulse is defined in terms of full with at half maximum of its intensity profile.

I repeat this one, one more time. A pulse looks like this and oscillation of an electric field, but this oscillation is modulated by a this is the electric field oscillation. This oscillation is called carrier wave. This oscillation is modulated by a field envelope. This is your field envelope which is represented by a t, but intensity which is represented by the square modulus of the field is here. We get only an envelope function which is also represented here an envelope function representing the intensity.

So, in the representation of the intensity of a pulse we do not have any oscillation because e to the power i omega naught t plus phi t multiplied by e to the power minus i omega naught t plus phi t, they cancels out. And, pulse duration is always defined with respect to full width half max of the intensity profile not the field envelope. So, this is the intensity profile and this one is the field envelope.

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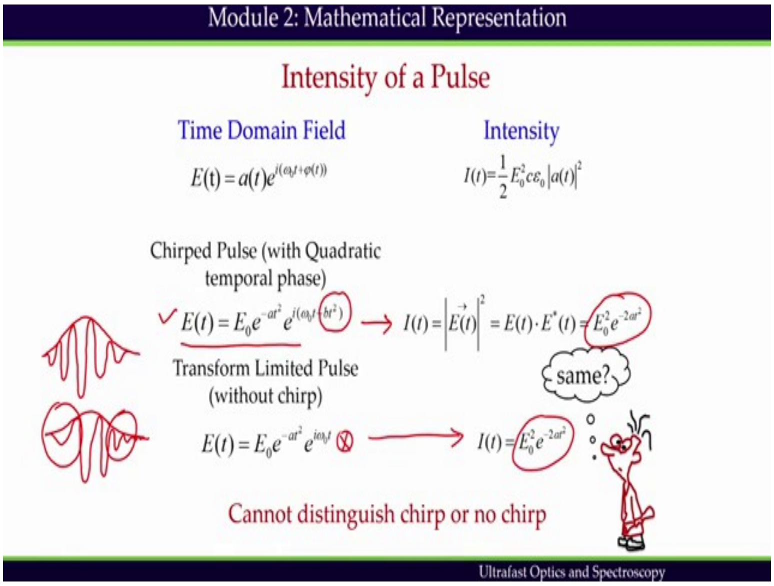

Because the intensity of a pulse is nothing, but the square modulus of the field, only time domain representation of the pulse cannot distinguish pulses with and without chirp. Let us draw this one. I have a pulse. This is without chirp. For the same spectrum I may have a I have a chirp pulse which looks like this. In this regime we have lower frequency in this regime we have higher frequency.

Chirp can be introduced with quadratic temporal phase which we have already seen. So, we are taking an example of quadratic temporal phase and we are representing the electric field and this is chirped pulse representation of a chirped pulse and this is transform limited pulse which does not have any complex phase here, no complex phase here.

But, if we take the intensity for each one which is given by the square modulus of the respective electric field we see that the intensity are the same. This means that the time domain representation of the pulse cannot be distinguished. The time domain representation cannot distinguish a chirped pulse and the transform limited pulse. The quadratic temporal phase has dramatic effect on the pulse, but this dramatic effect cannot be judged or felt if we view the pulse in only time domain, frequency domain representation is necessary.

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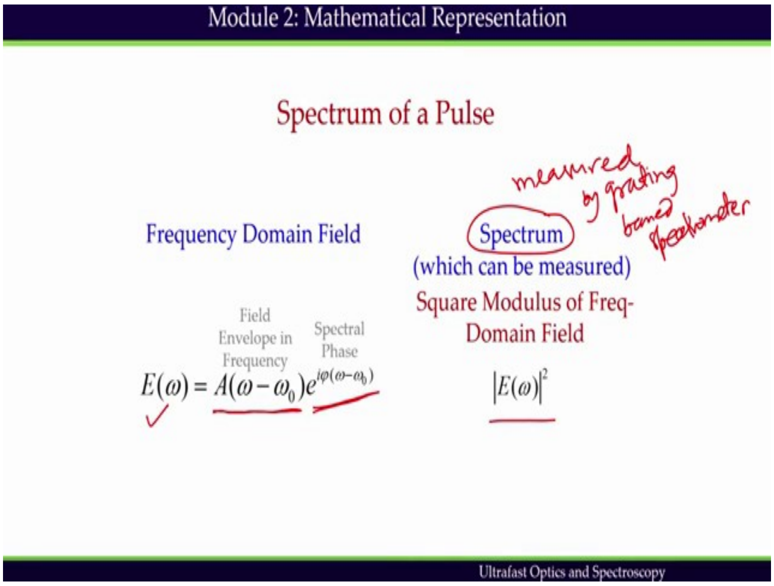

So, in frequency domain we have field envelope in frequency domain and spectral phase. And, again the spectrum which you measure, it is measured by grating based spectrometer that is nothing, but again square modulus of this frequency domain field.

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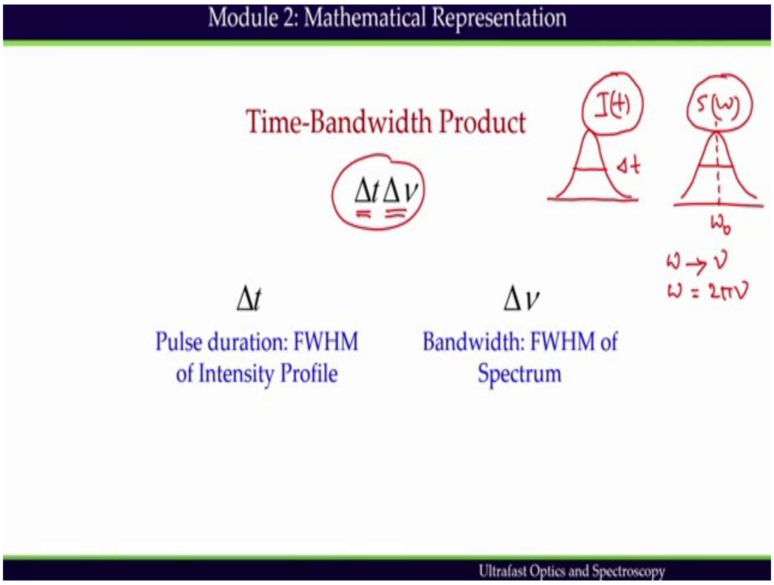

So, thus far we have discussed two different domain representation of pulses, but we should not think here that they are two isolated domains. In fact, they are interconnected. A simple interconnection can be realized through this time bandwidth product, delta t delta nu. Here we have to remember that delta t is representing the intensity full width half max, this is I t not E t and delta nu is representing the spectrum that is S omega not E omega. It is represented by this and this omega is converted to nu by this equation omega equals 2 pi nu.

So, when we talk about time bandwidth product we are looking at the time duration of the pulse and the band width of the pulse. Time duration of the pulse is defined with respect to full width half max of the intensity profile and the bandwidth is represented with respect to full width of max of the spectrum.

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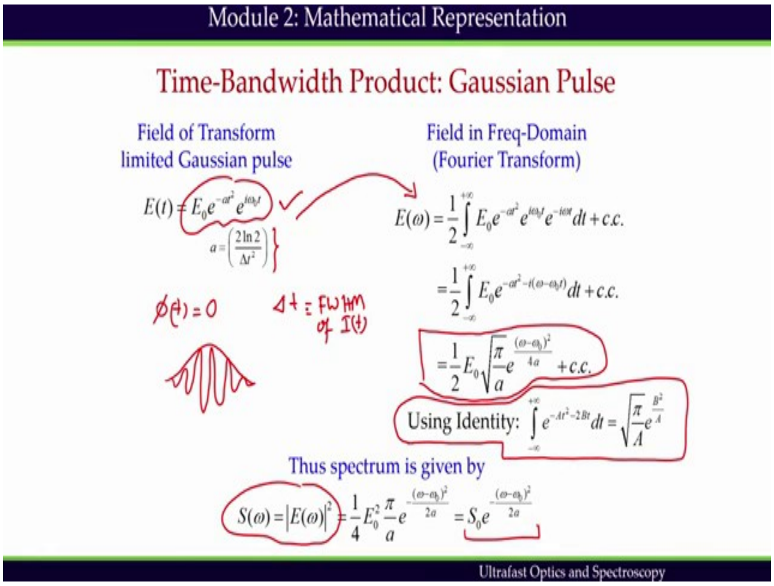

To understand time bandwidth product we will take an example of a Gaussian pulse with and without chirp. First we will closely look at transform limited Gaussian pulse. This means that this is a pulse without chirp. This is given by this equation. Here we remind ourselves that a phi t that was complex temporal phase is 0 and that is why it is transform limited pulse which looks like this. Electric field of a transformative Gaussian pulse in time domain is written here where a is expressed as 2 ln 2 by delta t square; delta t is the full width half max of intensity profile.

So, this time domain representation can be converted to frequency domain with the help of Fourier transform and with the help of this standard Gaussian integral we get the field in frequency domain. And, once we get the field in frequency domain we know that we can get the spectrum by taking square modulus of the frequency domain electric field and we get this simple equation.

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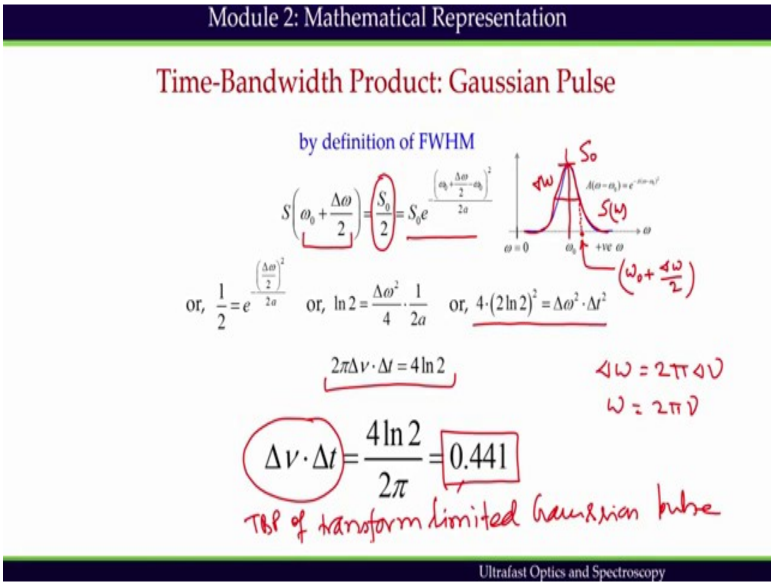

Now, taking the definition of full width half max of the spectrum what does it mean? It means that this is the spectrum which is represented by S omega which is centered at omega naught and if this is delta omega, then this point will be represented as omega naught plus delta omega by 2. At this point intensity would be half of the maximum intensity S naught; S naught is the maximum intensity.

So, S omega naught plus delta omega by 2 is going to be S naught by 2 and if we plug that in we get this equation. Simplify this equation we get this and then again delta omega equals nothing, but 2 pi delta nu because omega equals 2 pi nu and we get this equation and finally, we get delta nu delta t, that is the time bandwidth product of transform limited Gaussian pulse as 0.441. (Please look at the slides for mathematical expressions)

The product of temporal and the spectral width is called the time bandwidth product for a Gaussian pulse TBP which is 0.441, this relationship which is similar to time energy uncertainty principle in quantum mechanics states that shorter pulse requires a broader spectrum. TBP is minimum for a transform limited pulse that is for a pulse width the shortest possible temporal distribution for a given spectrum.

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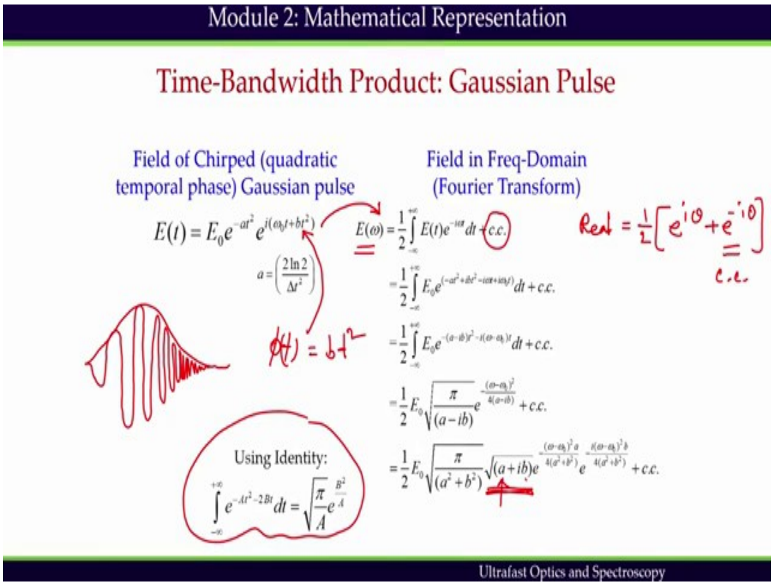

Let us now consider a pulse width quadratic temporal phase. Note that here we have introduced a complex phase phi t is now b t square I have included and in this case pulse would look like this. It should start with this and slowly it will move to higher frequency. So, if we have a pulse, a Gaussian pulse with linear chirp we can express the time domain feel to be like this and we can convert into frequency domain with the help of Fourier transform again.

And, what we get here the c c represents complex conjugate because we are taking the real field. So, the real field is always represented by half of e to the power i theta plus e to the power minus i theta, this part is represented as complex conjugate. So, we plug that in and finally, again using the standard Gaussian integral, we get these equations and this is a complex number. We do not need to know what is a mean by a and b.

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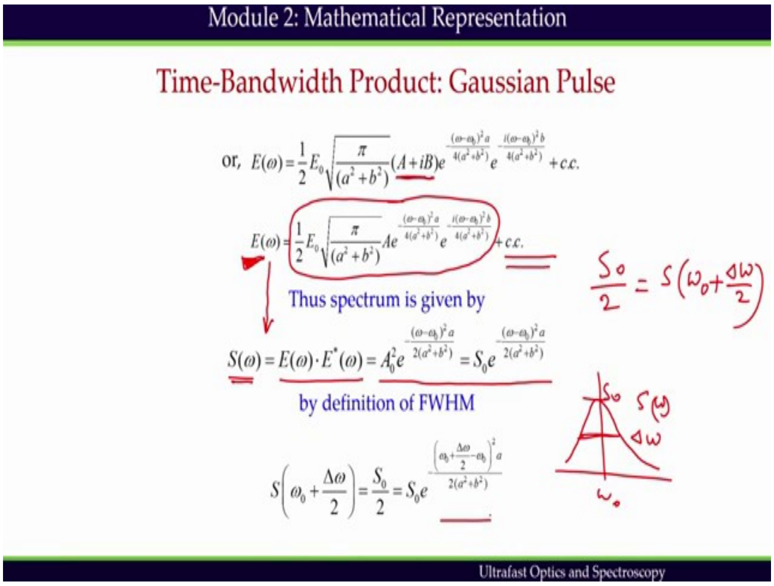

We can represent it with the different complex number A plus i B capital A plus i capital I B and then we can separate real part and the imaginary part. This is purely real part and rest of them is imaginary part and this is the spectrum. So, this is the electric field we get and from this electric field in frequency domain we get spectrum S omega as square modulus of the electric field in frequency domain and we get finally, the expression for spectrum. (Please look at the slides for mathematical expressions)

And, again taking the definition of full width half max which means I have a spectrum S omega which is centered at omega naught and full width half max is delta omega. So, S naught by 2 I get half of the maximum intensity I get when I have omega naught plus delta omega by 2 and that is exactly what we have used.

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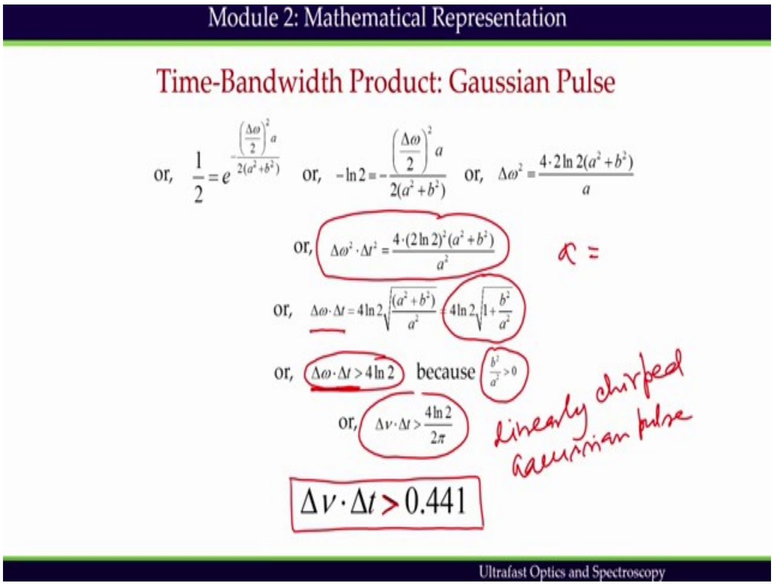

And finally, after doing little bit of math which can be followed very easily we get this expression and we can insert a; a is nothing, but previously we have already shown expression we had for a we get delta omega delta t. Delta omega delta t equals this. So, the product looks like this. Now, we do not need to know what is it what is the mean by b and a, but we can always say that because this is a square term is always greater than 0, and because it is greater than 0 delta omega delta t product is always going to be greater than 4 ln 2. (Please look at the slides for mathematical expressions)

And, if it is greater than 4 ln 2, then we will be able to find out the product of delta nu delta t which is greater than always 4 ln 2 by 2 pi. Which gives us that delta nu delta t time bandwidth product of a linearly chirped a Gaussian pulse linearly chirped Gaussian pulse is always going to be greater than 0.441. (Please look at the slides for mathematical expressions)

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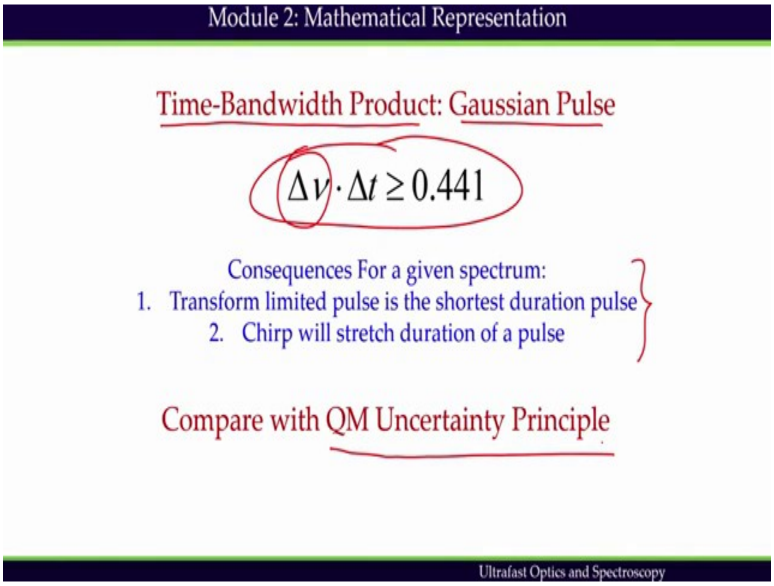

So, what is the meaning of this equation? Finally, we are getting a general expression for the time bandwidth product of a Gaussian pulse as delta nu multiplied by delta t greater than equals 0.441. This means that for a given source when I have a source is fixed; that means, number of frequency component is fixed, spectrum is constant. How many frequency components we have is constant. For that given source or for that given bandwidth delta nu is fixed.

For a given bandwidth transform limited pulse represents the shortest duration pulse; for the same given band width chirp will always broaden the pulse in the time domain. So, this is a two consequences we find from time bandwidth product and this idea is comparable to the quantum mechanical uncertainty principle.

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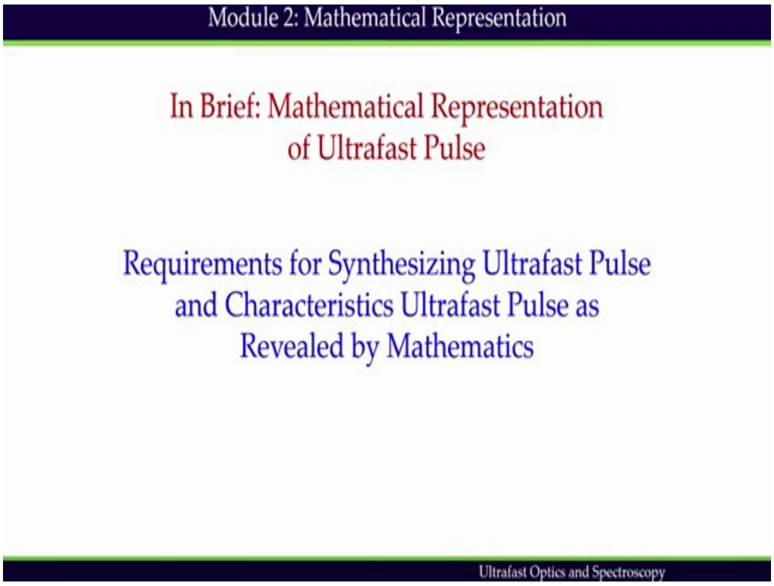

With this we come to the end of this module. In this module we have learnt many requirements for the synthesis of ultrafast pulses and some of the characteristics of ultrafast pulses and they are revealed by mathematics. We will meet again for the next module.

Thank you.