Ultrafast Optics and Spectroscopy
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Lecture – 33
Maxwell's Equations contd

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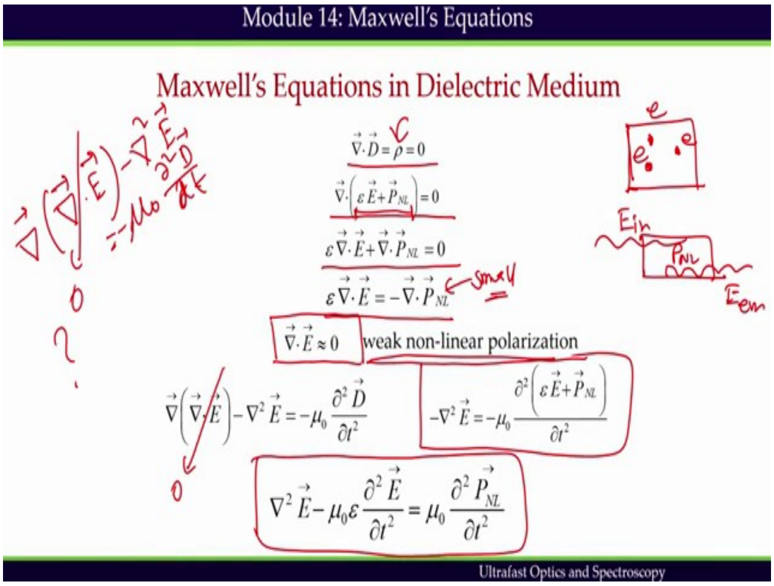

Welcome back to the module 14. We are discussing Maxwell's Equation in the medium. What we have assumed here the medium should not have any free charge in the medium and as why rho which is volume charge density becomes 0. If it is 0 in the medium then I can write down this expression this is nothing, but expression for D that is the relative flux density which includes the vacuum contribution and the non-linear polarization contribution. This is simplified to this equation and finally, I can write this equation.

Now question is in the equation which we have already got this is the equation we have got after getting triple product, we remind our self that this is the equation we have got minus E equals mu naught D d t ok. Now in order to in the in vacuum we have seen that we have made it 0 questions is in the medium can I do that. In the medium it is not necessary that divergence of electric field would be 0 it is not always necessary, but if we consider very weak non-linear polarization this is the perturbative limit.

We are considering that I have a medium input beam is propagating through the medium that is creating this non-linear polarization. This non-linear polarization is very weak and however, this weak non-linear polarization can create another field which is called emitted field. So, if we consider that very weak non-linear polarization then we can assume that this P NL is very small and if it is very small then we can consider to be negligible or 0.

So, under this weak field approximation weak non-linear polarization approximation I can make this field to a divergence of the field to be 0 in that case this become 0. So, this component become 0 and I get this expression. This expression can be written as this. This is called perturbative limit perturbative limit of non-linear effect and under this perturbativelimit we get the expression which we have use previously as fundamental equation of non-linear optics.

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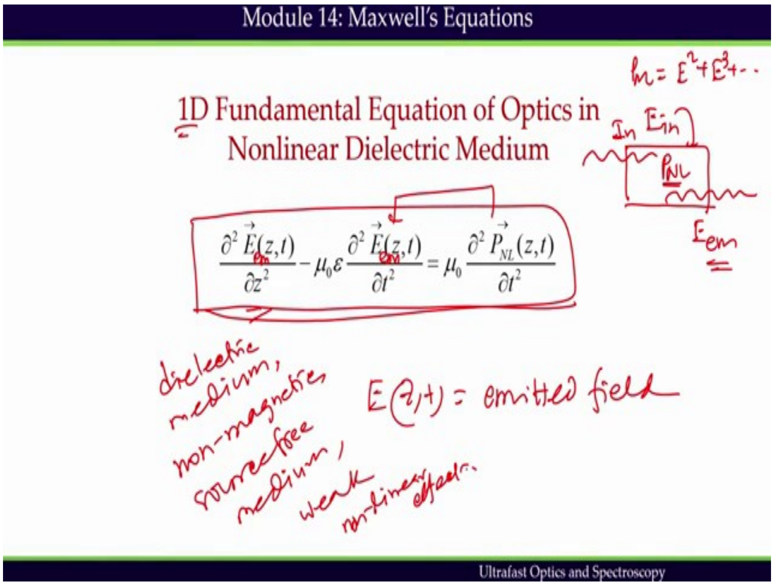

We can reduce it to 1 dimension I will get this expression. This expression was used we remind this expression here e is not emitted E is not input beam E representing emitted beam. There are three step model input beam field is expressed by E input due to preparation of the fundamental beam through the medium I am creating this non-linear polarization.

How they are related? They are related by this Taylor series expansion, then this non-linear polarization is nothing, but oscillatory dipole any oscillatory dipole would be a source of electromagnetic radiation. So, I am creating this new field and how this new field and polarization are related? This is the expression which relates this non-linear polarization and the new field.

To obtain a simplified picture we have considered this propagation along the z direction that is why this z dependent and E z t represents emitted field, this E z t is the emitted field or fundamental beam. P NL[P_{NL}] features a non-linear contribution to the polarization suggesting that non-linear polarization acts as a source for emitted field. (Please look at the slides for mathematical expressions)

This equation enables us to calculate non-linear effects the second and the third order due to propagation of ultrafast pulse in dielectric medium, non magnetic medium source free medium. So, this equation is valid for dielectric medium, non magnetic medium, a source free medium and we have also assumed that we have weak non-linear effect. That is under the perturbative limit.

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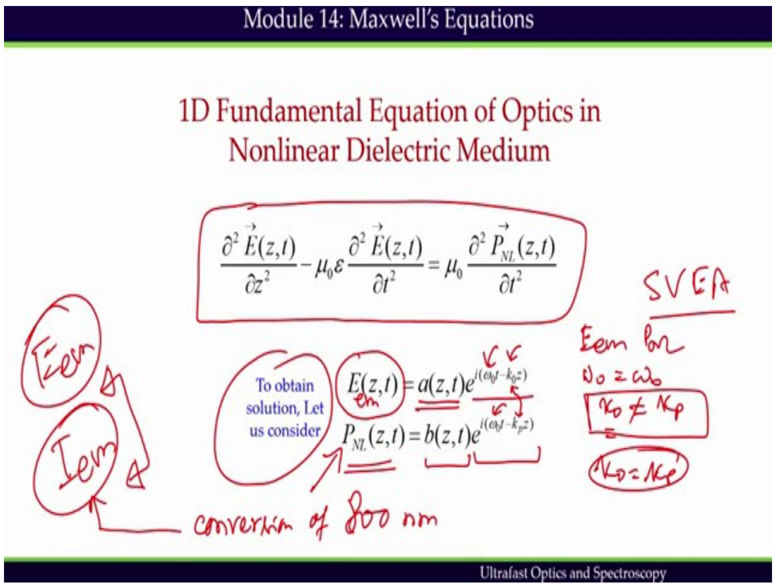

We have seen this equation previously, but we have not done the derivation before. Now to obtain the solution we used slowly varying envelope approximation, we have seen this approximation before and we assumed the form of emitted field as a pulse with the center frequency omega naught and a vector k naught. And non-linear polarization also having an envelope function similar to pulse and its frequency components omega naught.

So, what is the idea? Idea is that oscillatory dipole is nothing, but representing this non-linear polarization and the frequency of this dipoles would be equivalent to the or the same as the frequency of the emitted field. That is why frequencies are the same for polarization and emitted beam.

But it may so, happened that their phase that is k naught and k p they can be different and this is very common if we have k naught equals k p, then we will call it the process is phase matched and if they are not equal, we will call it process is not phase matched. So, here in this to obtain the solution, in the trial solution we have considered the same centre frequency, but different wave vectors k naught and k p for the emitted beam and the non-linear polarization. <u>(Please look at the slides for mathematical expressions)</u>

In addition here we have considered complex notation of the electric field and the non-linear polarization. As the complex notation mixed the mathematic simple; however, we shall remember that in the end of the calculation we need to take the real part E z. So, now, position is how they are related this is something which we need to find out.

In the end I need a solution for E, because electric field if I know emitted electric field, then I know emitted field intensity and I want to find out what are the factors which can control the field intensity of the emitted beam that is exactly what you want to know because this field intensity of the emitted beam is nothing, but the conversion efficiency conversion of 800 nanometer frequency component. So, if I want to find out what is the efficiency of the conversion frequency conversion that will depend on this intensity. So, I need to know this field strength from this equation.

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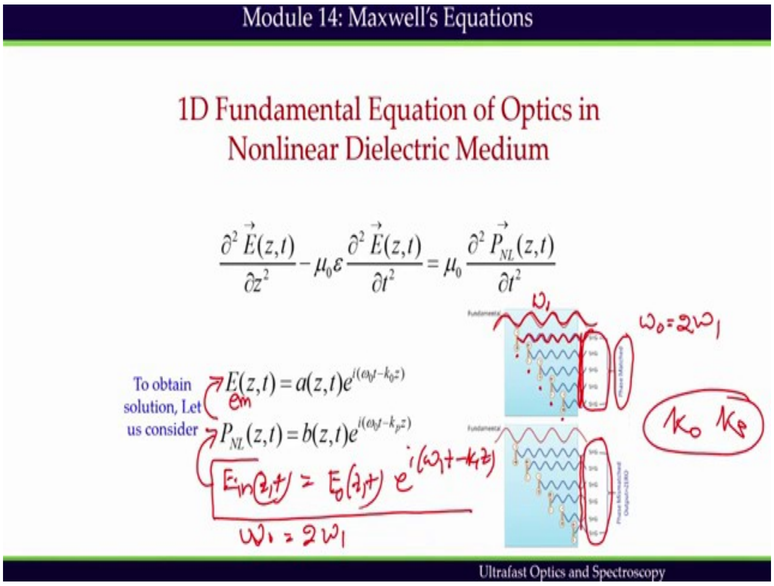

Furthermore here we have considered that due to propagation of input fundamental beam with omega 1 frequency. So, omega 1[ω_1] frequency is the fundamental frequency and polarization is induced in the dielectric nonmagnetic source free medium, the frequency of the induced polarization is omega naught. So, here this is emitted beam and if I need to write down input beam z t, then I have to write down the same.

Another field that is e field let say that is E naught field and then e to the power i omega 1 t minus k 1 z this is your input beam. So, there are three steps input beam is creating non-linear polarization, non-linear polarization is creating the input beam emitted beam. If the second harmonic generation is considered then we can write down omega naught equals 2 omega 1 which means oscillating dipole is a source of electromagnetic radiation and that is why the new light is created at the same frequency of the induced polarization. (Please look at the slides for mathematical expressions)

However, the phase of the induced polarization and the emitted field may be different that is why it is shown here. This fundamental frequency omega 1. This omega 1 is propagating through the medium this beam is propagating through the medium taking an example of plane wave at each point of the medium, we are creating non-linear popularization which is oscillatory dipole and each dipole will produce some field that is the emitted field and a omega naught frequency which is nothing, but 2 into omega 1, if I considers second harmonic generation this fields are in phase.

So, if we can create all this fields in phase then we call it phase is matched and we get maximum intensity from SHG process. But if the fields are not matched if the if the fields are destroying each other due to destructive interference then in the end we will not get any conversion for SHG. That is why it is very important to understand how the k naught and k p they are related because these 2 quantities are going to contribute them conversion efficiency.

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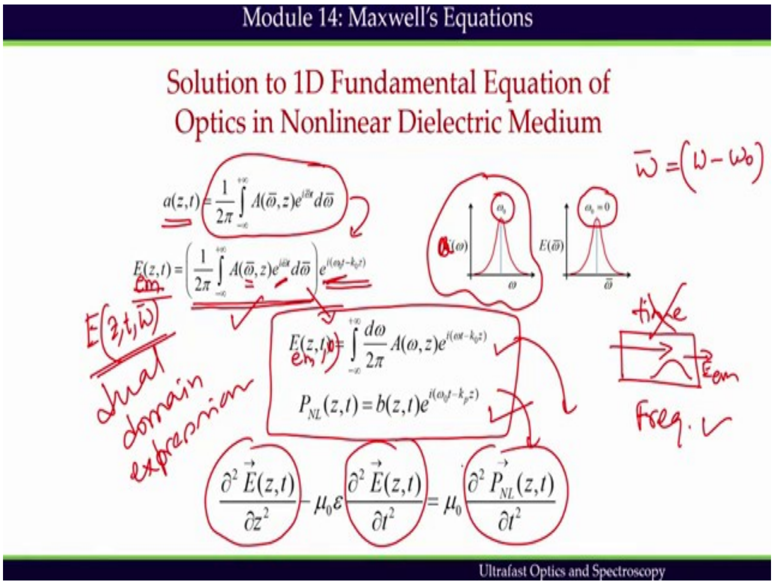

When an ultrafast pulse propagates in a dielectric medium, in addition to non-linear effects the pulse experience is dispersion as ultrafast pulse has broadband with different colour of frequency components experience different refractive indexes while propagating, resulting in dispersion effects which cannot be realized within the time domain representation of the pulse. For that we need to represent the pulse in frequency domain.

In order to incorporate dispersion effect as well, we shall represent a z t in terms of its Fourier transform. So, the basic idea is that, I cannot use when a pulse is propagating through the medium I cannot use its time domain representation, I have to use frequency domain representation. Because frequency domain representation can incorporate the dispersion effect and the pulse will experience dispersion effect now this pulse is the emitted beam that is why A t a z t is expressed in terms of frequency and this is nothing, but the Fourier transform in with Fourier transform and if we express this.

In the field then I can write the field as inverse Fourier transform of the envelope multiplied by carrier wave in the time domain. So, this part in the frequency domain and this omega z omega bar is nothing, but a conversion of the variable E omega it is centered at. So, this is a omega this a omega is centered at omega naught that we know.

So, if we make it centered at omega 0, then I can define this omega bar a new variable which is nothing, but omega minus omega naught this is the new variable. So, this field now I have expressed this field as it depends on z it depends on t and its depends on omega bar as well.

Now, I have incorporated time frequency dual domain expression. This is a dual domain expression I need this dual domain expression because dispersion effect can only be realized in the frequency domain, I should have this frequency domain expression. So, what I get? In the end which can simplify this expression we can multiply this e to the power i omega naught t and e to the power i omega bar t and finally, we get this expression where polarization expression is familiar to us this is the trial solution we have taken already.

But emitted field is expressed in terms of z t and also omega and we will consider that this field is propagating through the medium. So, all we need to do is that we have to plug that in, in this expression finally and we have to get this second derivative with respect to z, second derivative with respect to t and secondary derivative with respect to t of the polarization.

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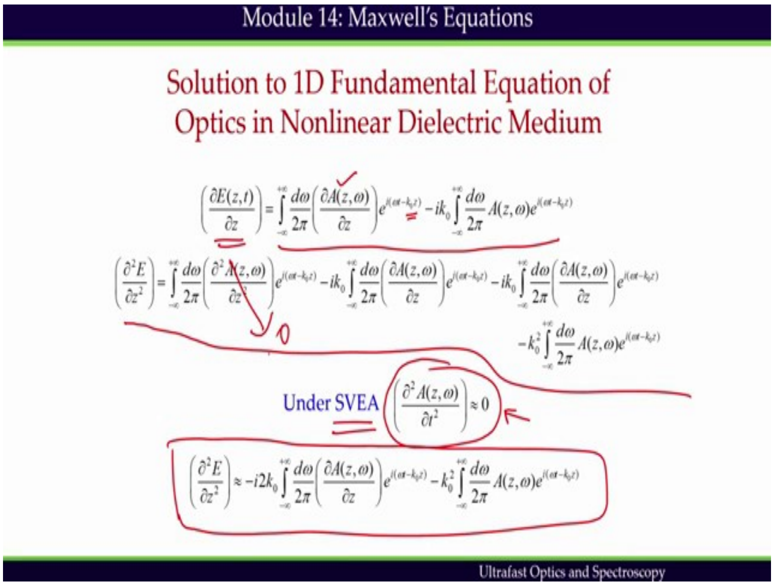

So, this is the terms we need to calculate to plug that in. So, we will take the first derivative of the electric field if we take the first derivative of electric field with respect to z, z dependent term is a is z dependent and also this is z dependent. So, I get these expression the simple derivative second derivative I take and I get this expression.

Now, we will employ slowly varying envelope approximation. Slowly varying envelope approximation is suggest that the envelope function varying slowly in time and varying slowly it means that, I can consider the second derivative to be 0. It is varying so, slowly that the variation the gradient of the gradient of the gradient is 0 or approximately can be considered into be 0. So, this slow variation leads us to a reduced expression for this equation where second derivative is taken to be 0.

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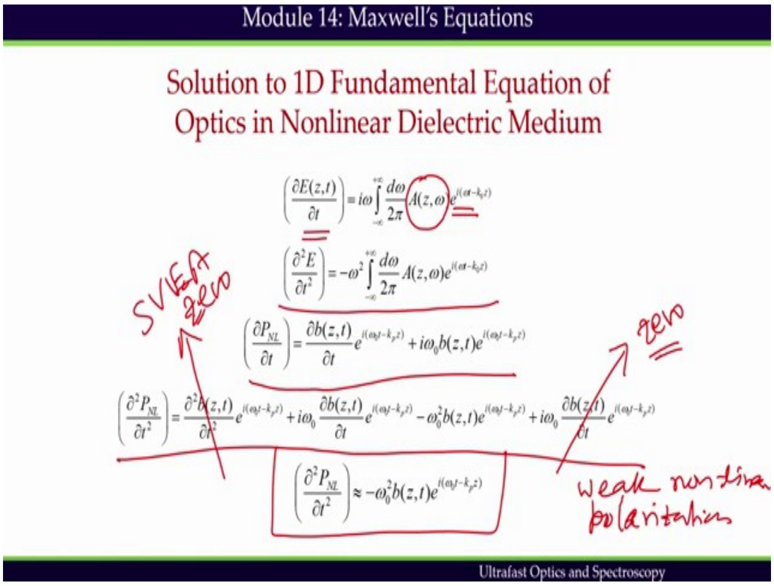

Now, we will take the time derivative. We will consider time derivative and remember A z omega is not time dependent time dependent is part is that. Just this carrier wave that is why this part can be very easily rewritten like this way and another time derivative we have to take polarization expression we will get this finally, polarization expression can be written like this way second derivative and here we have to consider one thing we will consider that under SVEA second derivative of this term is zero. (Please look at the slides for mathematical expressions)

This is SVEA approximation second derivative is zero. In addition to the second derivative I will consider the first derivative to be also zero because we considered weak non-linear polarization. So, polarization nonlinear polarization is so, weak that is first derivative is almost zero. So, approximately I can write down the non-linear polarization expression as like this.

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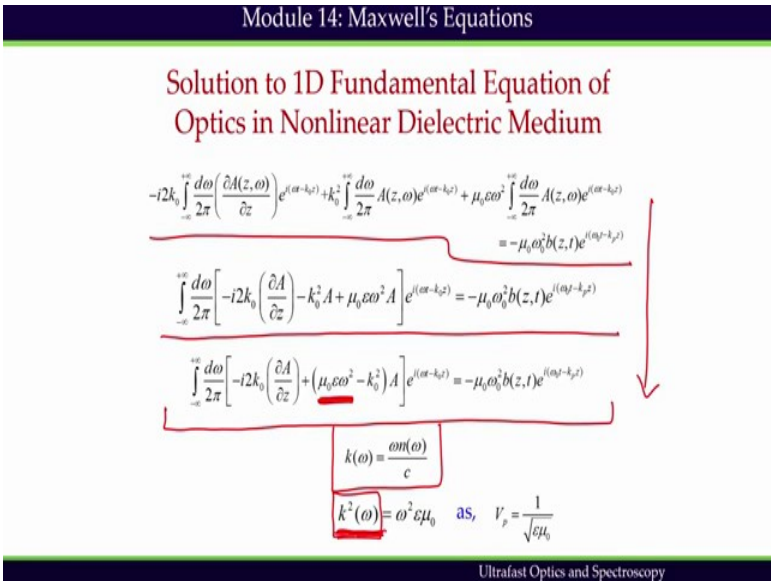

Thus one can insert all the expressions in to the 1dimensional equation of non-linear optics and we get this expression. We can rewrite this expression as this and again we can re write this expression like this steps can be easily obtained. Now at this point we will insert the dispersion relationship. This kind of dispersion relationship we have seen in dispersion effects of non-linear pulse propagation. So, this mu naught epsilon omega naught mu naught epsilon omega square this part can be written as k square and k depends on omega.

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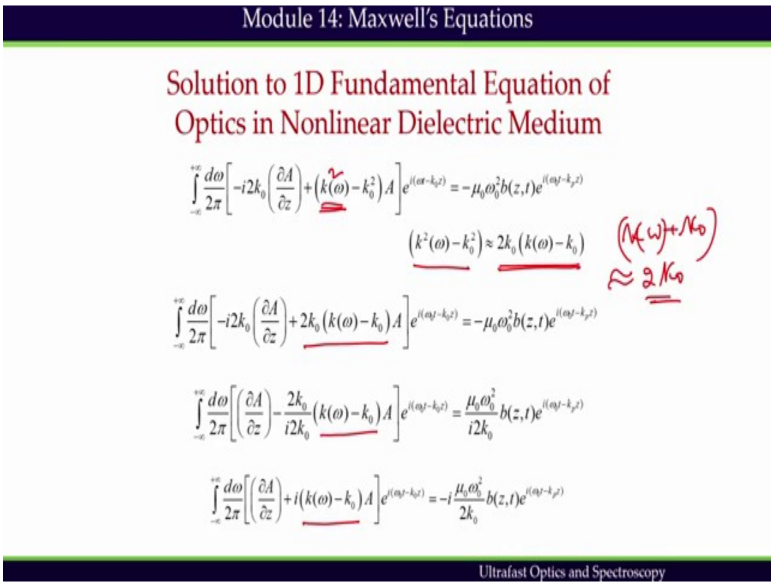

If we write a general expression for k then we get this k omega this is a general expression for k, this is k square remain k square. So, I can write down this k square to be like this because k omega plus k naught this is approximated as 2 k naught. That is why you can write down approximately like this. So, I get this express and rewriting this expression one more time here and we get this expression like this.

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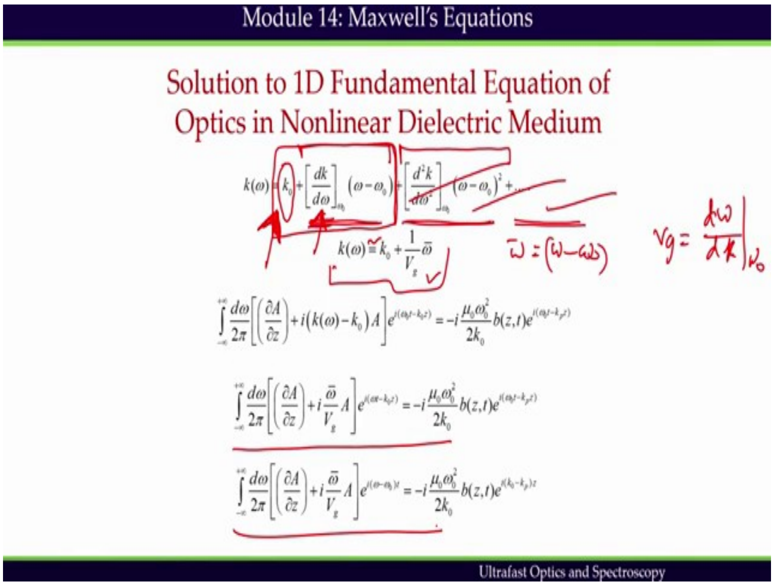

Now, this k omega we do not know the general form, that is why we will be able to express it in terms of Taylor series expansion all we know that the k at omega naught value I know the value of k that is k naught and I know that; I know that derivatives exist. So, that is why we can use the Taylor series expansion to express k and different parts of this expression representing different kind of dispersion that we have seen already in the dispersion effect in non-linear pulse propagation.

For an example, the second term this term is the vacuum term if we do not have any other term if we have this term then this is a vacuum term no dispersion effect. Then the first order term I can introduce first order term representing the group velocity. So, the effect of group velocity can be judged only if we include this first order term.

Then there is a group velocity dispersion term GVD which can be realized with the help of second order term, second order dispersion term. So, what we will do here? We will consider this problem for the first order up to the first order we will neglect every order terms then we can write down that approximately this k omega can be written as like this. We know that omega bar is nothing, but omega minus omega naught that is written here and v g group velocity we have seen previously it is d omega d k at omega naught. So, this is 1 by v g.

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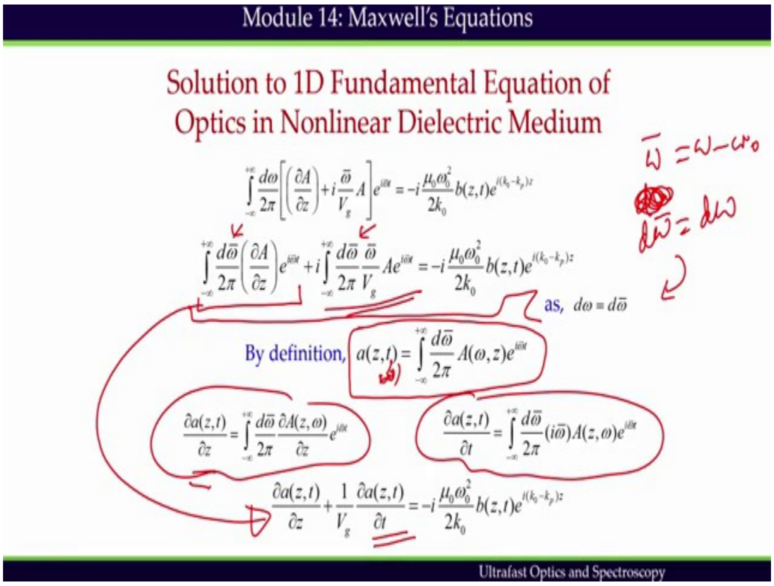

If we insert that then I get this expression rewriting this expression little bit here and finally, we after rewriting this expressions we get this and we have again change the variable because remember d omega; we have expressed omega equals omega minus omega naught. So, d omega sorry omega bar equals omega minus omega naught. So, d omega bar is nothing, but d omega and that is exactly what we have written here.

So, the derivative we have changed with respect to d omega bar, here also d omega bar and this was the definition of a z t we have seen in terms of frequency, this is also frequency dependent and if we take the first derivative with respect to z, I get this expression and with respect to t I get this expression. So, what does it mean? It means that this first term is nothing, but the derivative of a is the field envelope of the emitted beam with respect to z and the second term is representing the derivative of the field envelope of the emitted beam with respect to t. (Please look at the slides for mathematical expressions)

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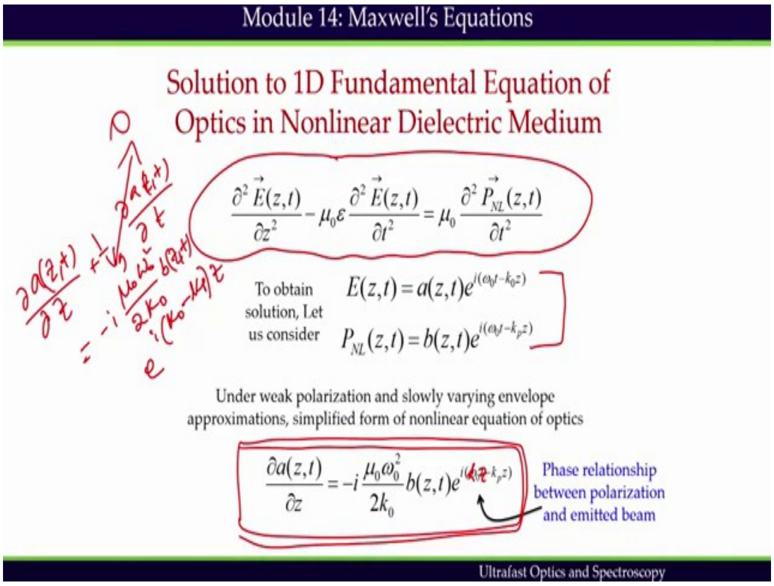

Finally, we can write down this expression. Here what we have done is that, this was the expression we have got from Maxwell's equations and using this trial solutions, we have found a solution was like this plus 1 by v g d t equals minus i omega naught, omega naught square mu naught squared divided by 2 k naught b z t, e to the power i, k naught minus k p z. (Please look at the slides for mathematical expressions)

So, this was the expression we have got finally. Then what we will do is that, we will turn off this time dependent part and we will just look at the space dependent component. This simplification or transformation is like turning of the time dependence to monitor space dependent effects only and this can be practically realized if we transform the time coordinate to be centered on the pulse.

If we do that transformation, then we can turn off this time dependent component and we have the phase dependent component only. This simplification will ultimately lead us to the understanding of an important concept which is called phase matching in non-linear medium.

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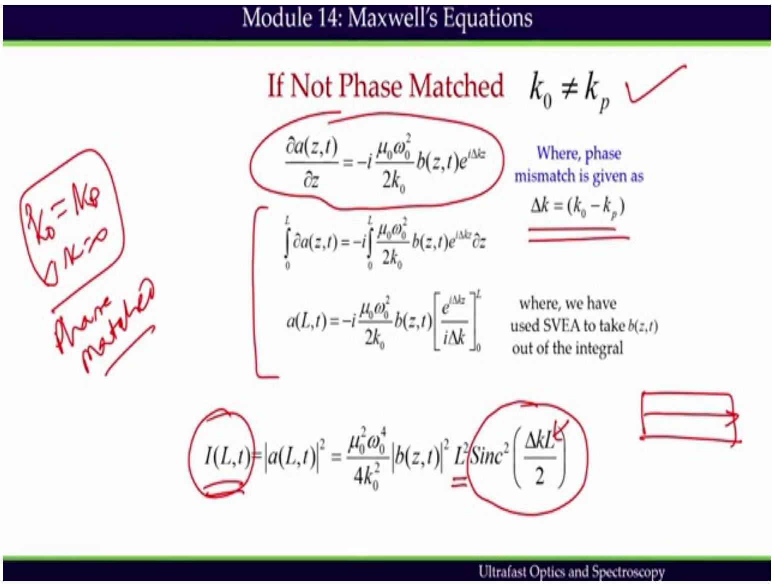

When a non-linear process holds that is k naught equals k p then I have this expression delta k is nothing, but k naught minus k p and this delta k if it is 0 then the process is called phase match. So, if I have k naught minus k p, k naught equals k p, then delta k becomes 0 and we call it a phase matched. (Please look at the slides for mathematical expressions)

But if we do not have phase matching condition fulfilled, then we have to integrate this expression to get the intensity final intensity because we are interested in final intensity how much intensity I can get? We see that the final intensity depends on L square that is the medium length definitely if we use thicker length, if we use thicker crystal we will get higher yield, but we also have to remember that it depends on this also which having L contribution this is call cardinal sine function.

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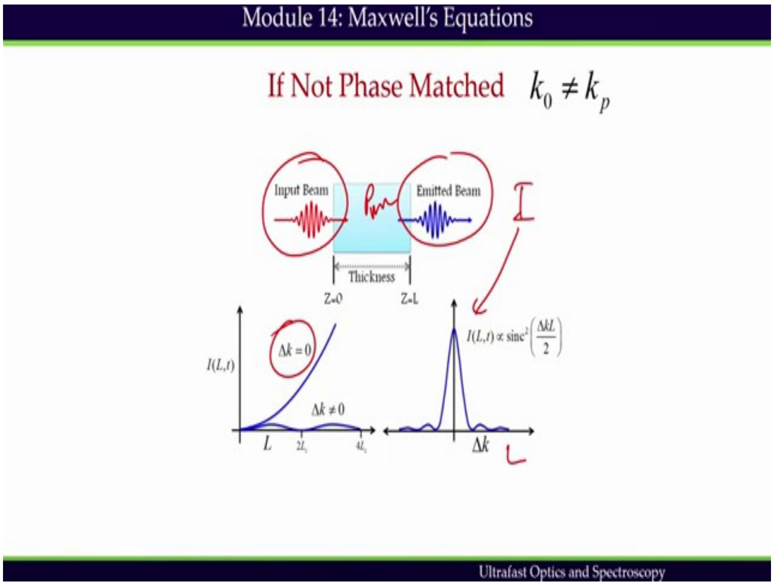

So, it depends on the behavior of a cardinal sine function, I have an input beam now I have emitted beam due to non-linear polarization. Now this emitted beam intensity I depends on this cardinal sine function. If it is phase matched then always the intensity will grow, but if it is not phase matched then intensity will dropdown very quickly as a function of the length. If we increase the length it will dropdown immediately, we have seen all these behavior in module 3 already.

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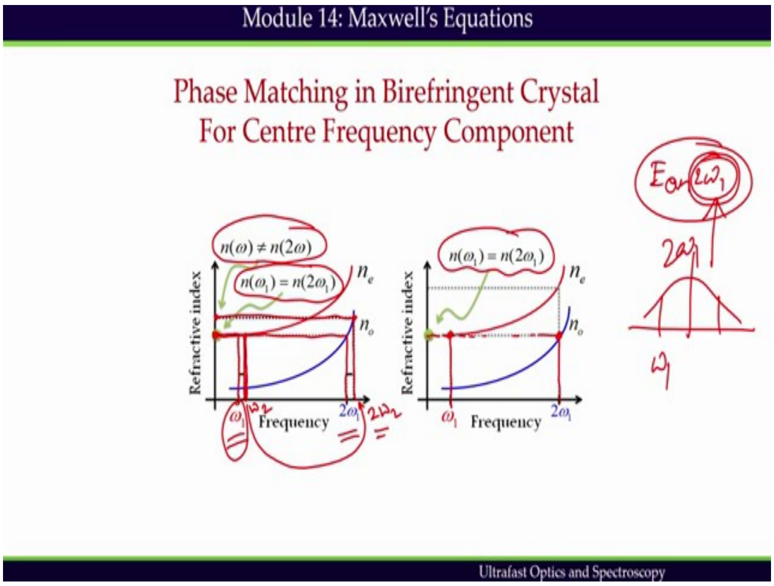

So, to achieve phase matching for high harmonic generation, it is necessary to find out non-linear medium for which refractive indexes at omega 1 and 2 omega 1 are the same. Unfortunately the index of refraction varies with frequency of radiation, this is a phenomenon called dispersion effect therefore, phase matching condition cannot be fulfilled in any medium the solution is to use birefringent crystal.

This is also we have seen before in the birefringent crystal we have ordinary and extraordinary beams and one beam polarization can be in phase with the emitted beam by the other field and that is the basic idea how we get we achieve phase matching in birefringent crystal. In birefringent crystal when like propagates it is decompose in to 2 beams with mutually orthogonal polarizations, this beams are called ordinary beam and extraordinary beam.

Refractive indexes are different for ordinary and extraordinary beams and therefore, it is possible to achieve phase matching for centre frequency component that is shown here by omega 1 and 2 omega 1. This is an analysis of phase matching condition taking one frequency component only. So, one can say that for omega 1 this is omega 1, this refractive index can be same at omega 2 omega 1 and they can be the same. This is possible and we can say that phase matching condition can be fulfilled. <u>(Please look at the slides for mathematical expressions)</u>

But you have to remember that even at the centre frequency component if we have phase matching let say this is these 2 points they are having phase matching because their refractive indexes are the same. But if I have a with other frequency component for an example, this frequency component.

This frequency component having refractive index here and corresponding this is I call it omega 2, then this is 2 omega 2 and 2 omega 2 I have refractive index different which means that at the centre frequency component I can have phase matching, but other frequency components I may not have phase matching and we have to remember that a pulse will exist only when all frequency components should be present or many frequency component should be present. So, what might happen in a non-linear medium?

For a particular frequency component I can achieve phase matching and that is why I can have emitted beam intensity for that frequency component 2 omega 1. But other frequency component because they are destroyed because phase matching is not achieved in the end, I do not have a pulse because in order to exist in order to obtain a pulse at 2 omega 1 frequency centre frequency component I need all the frequency component to be present. (Please look at the slides for mathematical expressions)

So, this is 2 omega 1 there are many other frequency components should be there here that is why phase matching bandwidth is very important, we shall for simplicity considered SIG process here with an ultrafast pulse. (Please look at the slides for mathematical expressions)

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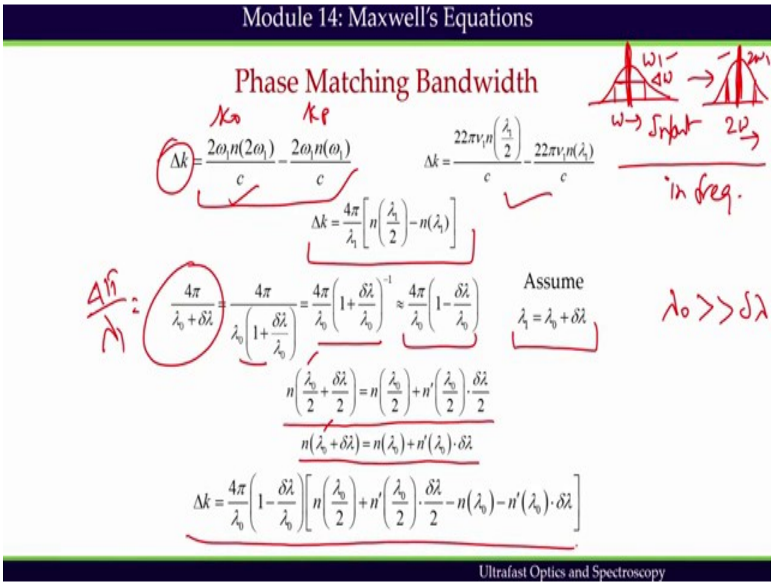

The range of frequency components or wavelength components that achieve phase matching is called phase matching bandwidth. So, question is if I have this is your omega 1 input beam. This will be converted to 2 omega 1 2 omega this is omega. So, this is nothing, but non-linear frequency conversion in frequency domain. I am creating 800 pulse from 800 pulse I am creating 400 pulse 400 nanometer pulse by in this conversion process we have to remember that very easily I can get a phase matching condition fulfilled for this frequency component. (Please look at the slides for mathematical expressions)

But it is not necessary that I will be able to achieve phase matching condition for other frequency components as well. And if I cannot achieve phase matching condition for other frequency component, the pulse will break down and I will not have any pulse anymore at the higher frequency. So, phase matching bandwidth is very important question is how many such frequencies can be phase matched?

Delta omega that is called phase matching bandwidth. So, what we are going to express is that, we are interested in delta k. So, delta k can be expressed as this, this is an expression which we have seen previously this is nothing, but k naught and this is nothing, but k p. Here second harmonic generation we have considered. <u>(Please look at the slides for mathematical expressions)</u>

So, k p is expressed like this way and k naught is expressed like this way in terms of wavelength we can immediately express like this way. So, this two this does not exist here, no they should exist here they should be there here ok. So, finally, what we see is the delta k in terms of lambda, we can express like this way and we will assume that this lambda 1 is nothing, but lambda naught plus delta lambda. <u>(Please look at the slides for mathematical expressions)</u>

So, if we express like this way then 4 by lambda 1 is 4 pi by lambda 1 can be expressed as this which is nothing, but this and then finally, expressed this and one can write down this because it is just because lambda naught is much greater than delta lambda. So, finally, if what we get is, an expression like this and what we are expressing here is refractive index in terms of Taylor series expansion at centering this wavelength and this wavelength. <u>(Please look at the slides for mathematical expressions)</u>

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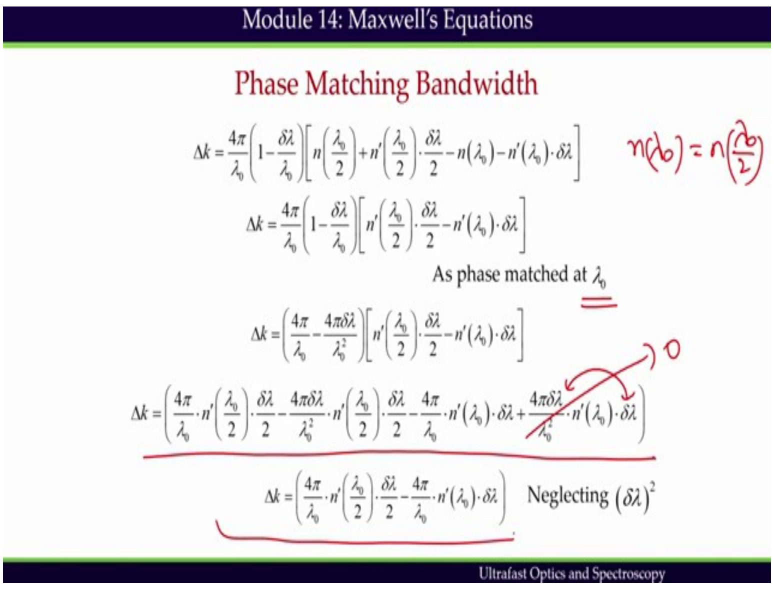

So, we plug that in finally, we get this expression and because it is phase matched at lambda naught, we can write down that n lambda naught equals n lambda naught by 2 this is what we can write down and that is why we can reduce this expression and finally, we get an expression like this, where we are neglecting square terms delta lambda square terms. So, this term has been neglected because we have a square term. So, this is considered to be 0. (Please look at the slides for mathematical expressions)

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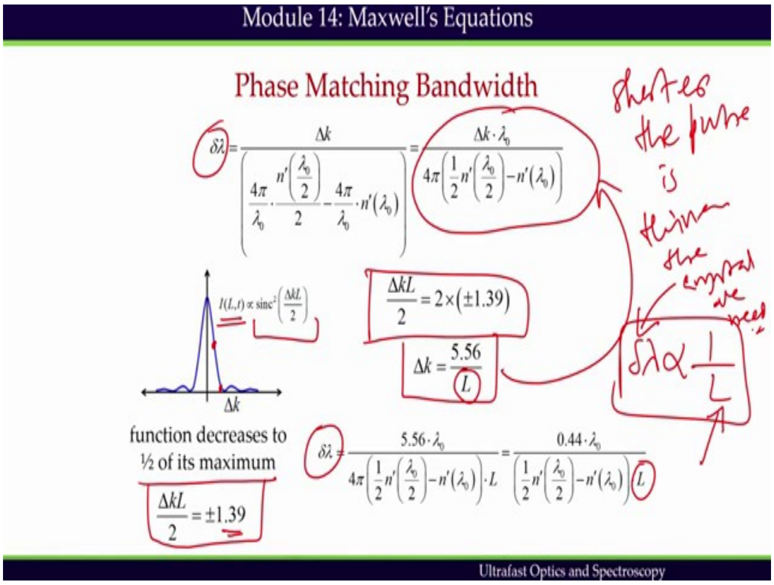

So, finally, what we get is this expression and further we can reduce the expression in terms of delta lambda. What we get delta lambda equals this delta k by this and from intensity, we know that the intensity will depend on the cardinal sine function. So, from that expression we know that the function this kind of cardinal sine function will decrease to half of its maximum intensity, when I have this value delta k L by 2 equals 1.39.

So, I will consider that the intensity will go down to 0 when I have twice this value. So, which means I am not here at this point, I am actually here in this point. So, I can write down this expression and finally, delta k has an relationship with the thickness. So, if we plug that in here, we get delta lambda inversely proportional to L delta lambda inversely proportional to L.

What is important here to note here is that, if we have a non-linear medium and if we have a very short pulse, short pulse will have a large delta lambda large delta lambda thin crystal we need. So, this is suggesting that although in a medium I may achieve phase matching for the centre frequency component, but in order to gather other frequency component I need to use a thin crystal. The shorter the pulse is thinner the crystal we need. So, this is the take home message we have.

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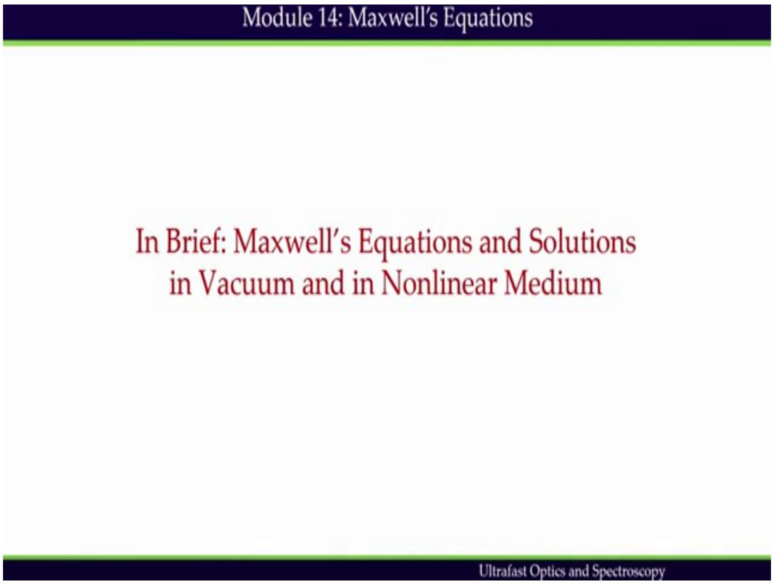

With this we have come to the end of this module, in this module we have gone over more details of the Maxwell's equations its implementation in understanding plane wave propagation in the vacuum then ultrafast pulse propagation in the medium and how we can use Maxwell's equations to understand different phenomena in ultrafast optics. We will meet again for the next module.