Ultrafast Optics and Spectroscopy
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Lecture - 03

Welcome to the module 2 of the course Ultrafast Optics and Spectroscopy. In this module we will go over the mathematical representation of ultrafast pulse.

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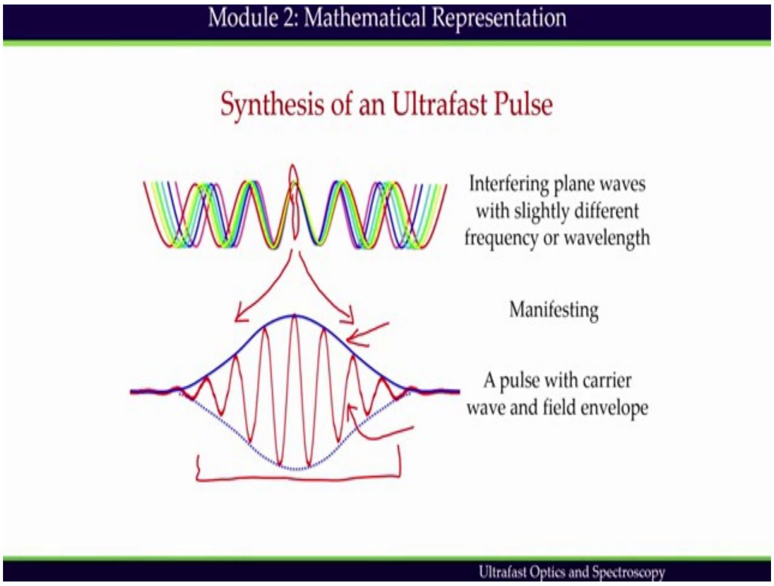

We have already pointed out in our last module that a very short flash of light is technically called an ultrafast pulse in which electromagnetic energy is localized in a very short time. Synthesis of such light pulses requires a large number of plane waves or color components or wavelength components. So, in this figure, if you see this figure we have represented a number of waves with different colors – red, yellow, green, blue. They have slightly different frequencies and they are interfering. They are interfering and having constructive interference in this regime. This is localizing electromagnetic energy in time.

So, the interference of different plane waves results in localization of electromagnetic energy in time which is nothing, but a pulse. Pulse has two different components; one, this is called envelop function and another one is carrier wave. The exact temporal duration of a pulse depends on these frequency components that constitute the spectrum of the pulse and the phase relationship between the frequency components. Before we begin discussion of the synthesis, manipulation, control, measurement and application of ultrafast pulses we must first understand some important properties of the ultrafast pulse.

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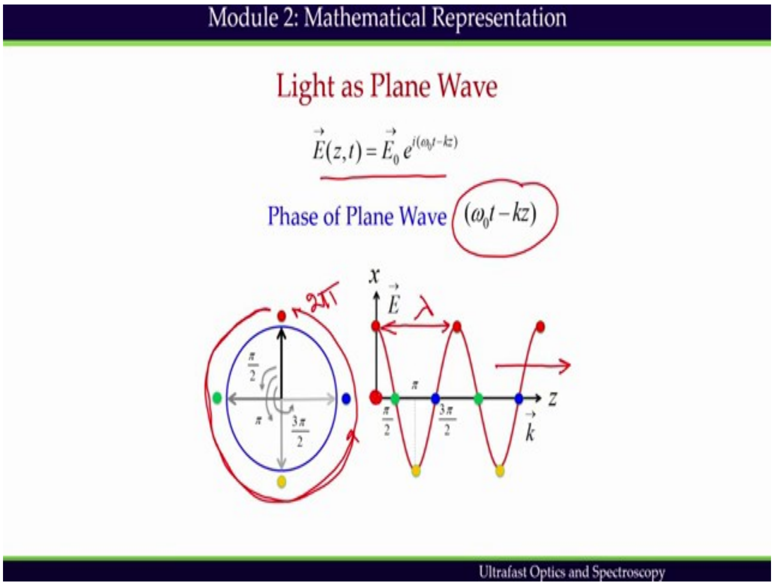

As an ultrafast pulse is a sophisticated flash of light, we will start the discussion with a short introduction to light. The true nature of light is unknown. Light is said to be dual in nature. Certain phenomena such as interference exhibit the wave nature of the light and in that case light has both electric and magnetic fields. The other phenomena associated with light such as photoelectric effect, display, the particle aspect of light. Light particles are called photons. This dual characteristic of light successfully explains all experimental observables.

However, true nature of light is still unknown. A large portion of this course will deal with only wave character of light and this is why we shall focus on the wave character of light.

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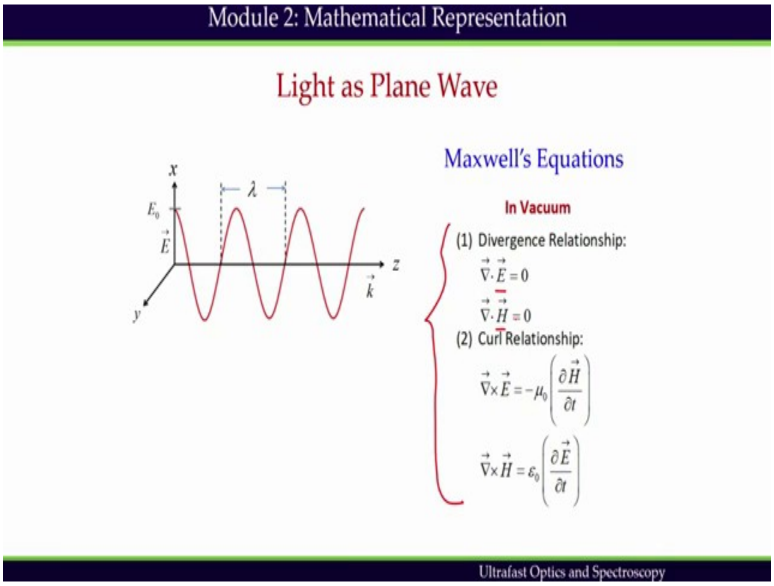

The wave character of light is fully described by Maxwell's equations which are given here. These equations are called Maxwell's equations which are nothing, but vector differential operations on the electric and magnetic fields; this is electric field, this is magnetic field.

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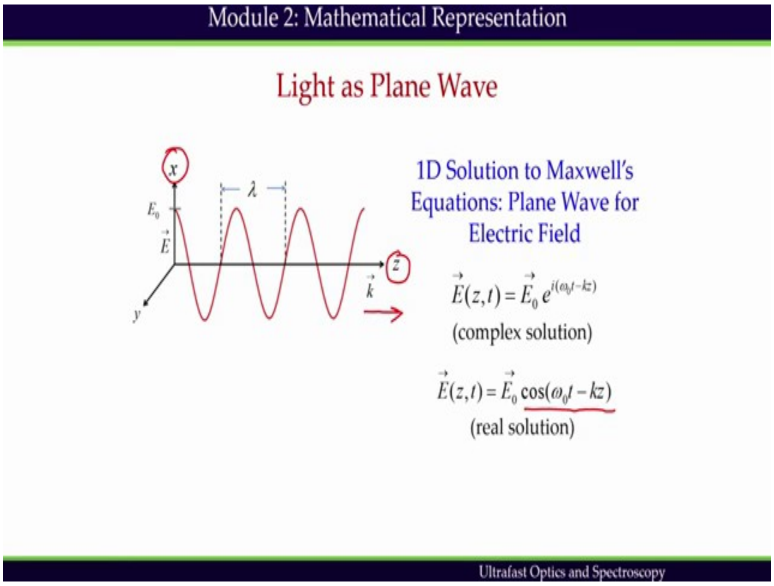

In vacuum one-dimensional solution to these equations renders plane wave both for electric and magnetic fields. As magnetic field strength is very weak compared to the electric field strength we will neglect magnetic field here, we will only focus on the electric field which is shown here the complex representation of the electric field looks like this E z, t that is one-dimensional propagating along z direction equals E naught which is absolute magnitude of the electric field omega naught is angular frequency, and k is the magnitude of the wave vector. Wave vector representing the propagation vector of the wave; propagation vector of the wave is along z direction. (Please look at the slides for mathematical expressions)

Now, what does it mean by this electric field and the propagation vector? Let us say light wave is propagating along this direction and if we have a free electron here then what will happen, this electron will start vibrating along this direction which would be the perpendicular direction perpendicular to this propagation direction, and this will vibrate along this direction because electric field is acting along this direction; an electric field is nothing, but the force per unit charge.

We must note here that the actual electric field is the real part of the solution and real part of the solution is given by the cos omega naught t minus kz which is nothing, but an amplitude modulated cosine wave. Its electric field oscillates on the plane x, z this is the plane x and z. On this x, z plane the field is oscillating and the propagation direction is along this z direction. (Please look at the slides for mathematical expressions)

The term omega naught t minus kz, this term in this equation features the phase of the plane waves which represents an angle to manifest certain linear advancement of the propagating wave. This depicts a close relationship between rotational motion and linear motion. When a wave propagates along z direction and advancement of the wave along the z direction with respect to a certain frame or a point, let us say with respect to this point wave is propagating along; wave is propagating along z direction. So, with respect to this point the advancement can be represented by an angle in the rotating dial in their angle in this rotating dial.

For an example, 2 pi phase in rotating frame; 2 pi phase in rotating frame is represented by this 2 pi angle. This is 2 pi angle, represents an advancement of the way by lambda distance in the linear frame. This is the lambda distance; phase is an important physical quantity of a plane wave. The role of phase on the synthesis behavior of a pulse would be more evident very soon. (Please look at the slides for mathematical expressions)

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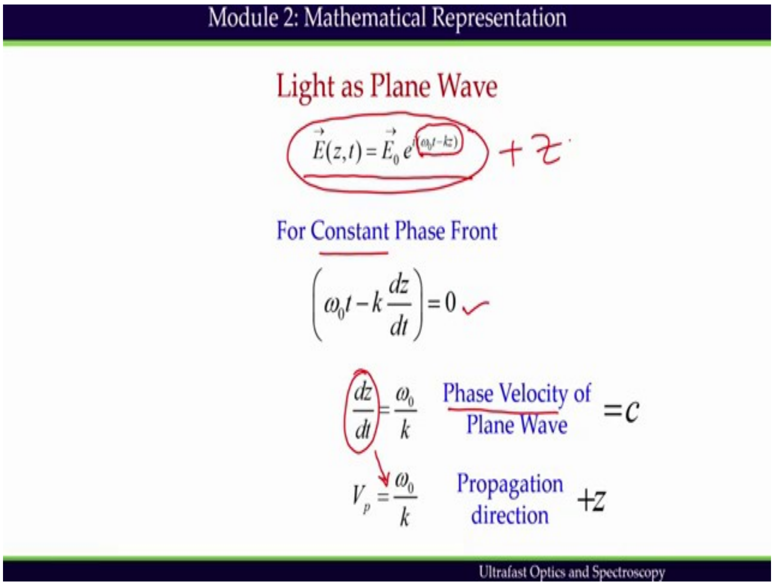

For a constant phase front, the first derivative of the phase with respect to time becomes 0. So, if we take the first derivative with respect to t; the first derivative of this phase with respect to t for the constant phase front it becomes 0. Here dz dt, this dz dt is called phase velocity which is c speed of light in vacuum. Thus phase velocity of a plane wave is defined as the velocity with which the constant phase front of a plane wave travels. (Please look at the slides for mathematical expressions)

This equation represents a positive value. Look at this positive value here which suggests that the direction of the velocity is along positive z direction that is why the equation given here the complex representation of the plane wave E z, t equals E naught e to the power to the power i omega naught t minus kz. This equation representing a plane wave propagating along positive z direction. (Please look at the slides for mathematical expressions)

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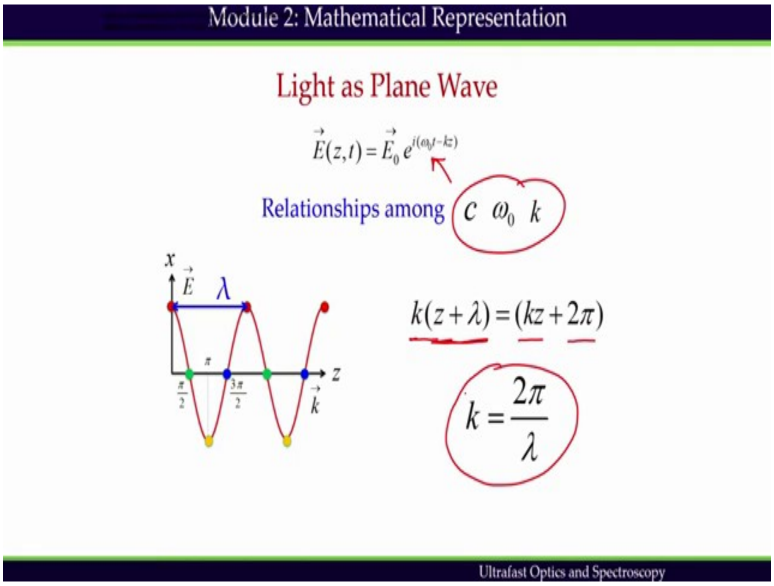

It is important to understand the relationship among c, omega naught and k in this equation. c is speed of light in vacuum, omega naught is angular frequency and k is the magnitude of the wave vector.

We have already pointed out that the 2 pi phase in rotating frame represents an advancement of the wave by lambda distance in the linear frame. This can be mathematically represented by k multiplied by z plus lambda; this is the lambda phase and advancement equal is equal to kz plus 2 pi or k equals 2 pi by lambda. (Please look at the slides for mathematical expressions)

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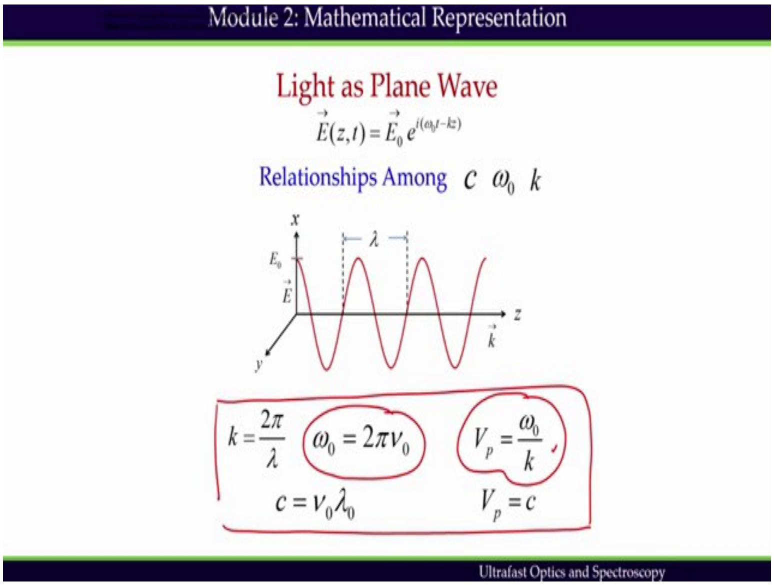

On the other hand, omega naught is the angular frequency of the plane wave which is related to the optical frequency nu naught. The constant phase front of the wave propagates with a velocity called phase velocity which is given by omega naught by k.

We get this phase velocity to remind you from the first derivative we take. The first derivative of the phase and then we make it 0 which means phase velocity represents the velocity of the constant phase front. Thus a plane wave holds all these relationships. It is a good idea to remember them. So, this is all about the plane wave.

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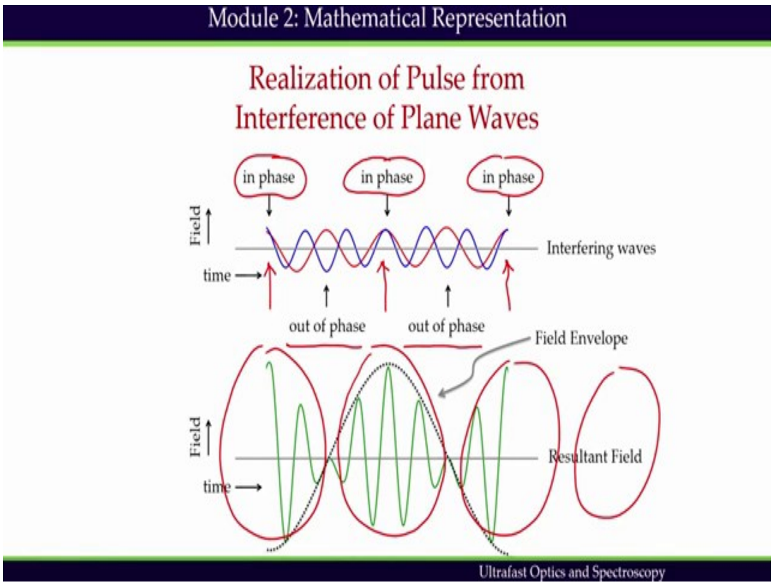

Question is, how do we realize a pulse from the concept of plane wave? The idea of pulse comes from the theory of optical interference or plane waves. Here we have shown two interfering waves with slightly different frequencies. The resultant field represents the pulse. So, the blue and red waves we have represented to show the slightly different frequencies or wave length for the interfering waves and when they interfere in time there are regions where these two waves would be in phase. So, these are the regions where they are in phase and there are regions where they are out of phase.

When two waves are interfering in phase they are called constructive interference and when they are in they are out of phase that is called destructive interference due to this in constructive and destructive interferences we get the localization of electromagnetic energy which is nothing, but a pulse. A pulse here, again a pulse here, again a pulse here and so on.

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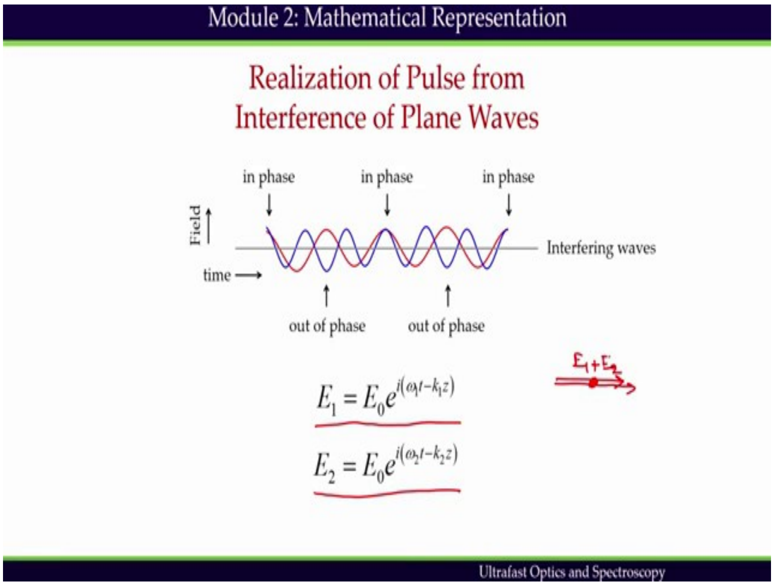

The theory of optical interference states that at a point total electric field can be calculated based on linear superposition of electro magnetic plane waves because electric fields are additive at a particular point. So, if I have two electric fields propagating along the same direction E 1 and E 2, at this point total field is going to be linear combination of these two linear superposition of these two electromagnetic plane waves.

For the given problem which deals with only two plane waves of slightly different frequencies having the same maximum field amplitudes and traveling along the same direction that is positive z direction, these two fields E 1 and E 2 can be represented by in complex notation as E 1 equals E naught e to the power i omega 1 t minus k 1 z and E 2 equals E naught e to the power i omega 2 t minus k 2 z. Here we note that we have taken two different magnitudes of k vectors because their frequencies are different and we have seen that k is related to frequency. However, their directions can be same. (Please look at the slides for mathematical expressions)

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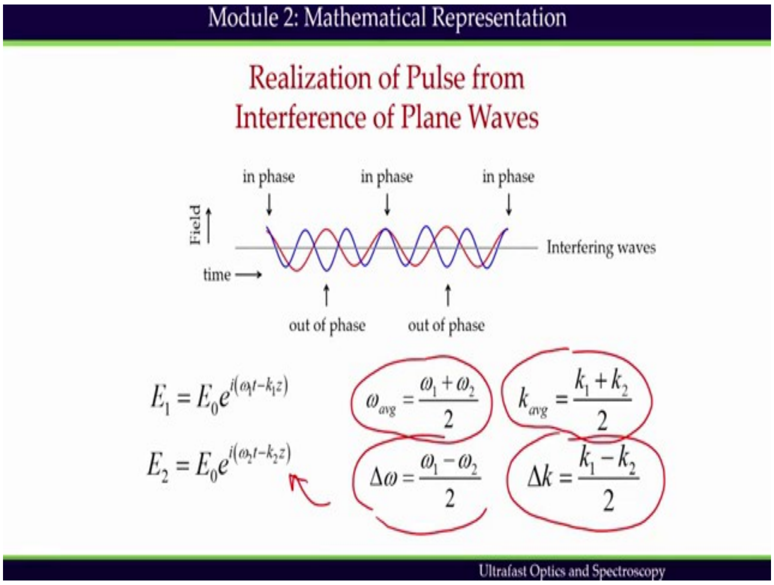

Now, let us define omega average by omega 1 plus omega 2 by 2, k average as k 1 plus k 2 by 2 and delta omega as omega 1 minus omega 2 by 2 and delta k as k 1 minus k 2 by 2.

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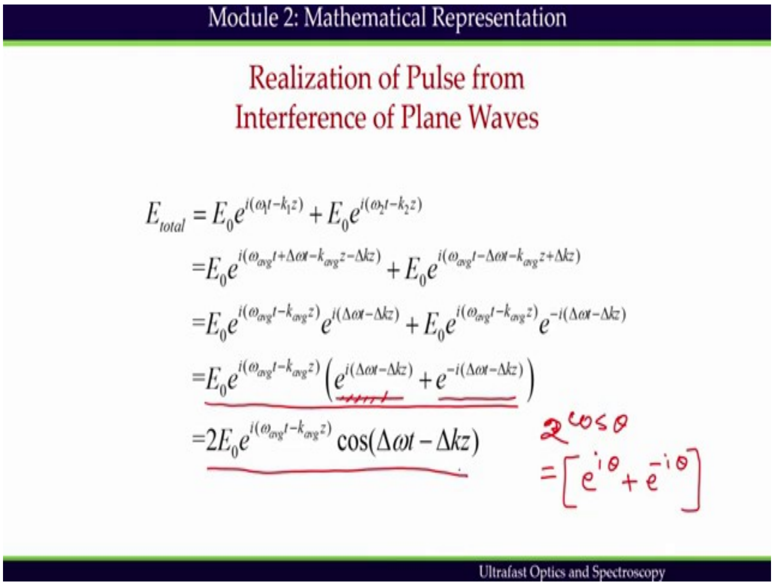

If we insert them in these equations and, then we calculate total field due to the interference of these two electromagnetic waves propagating along the same direction, we find after doing simple math we get this. And, if we employ this part is the complex of this part is complex conjugate of this part. This is why we can write down a small like cos theta 2 cos theta equals e to the power i theta plus e to the power minus i theta. With this we get this final equation. (Please look at the slides for mathematical expressions)

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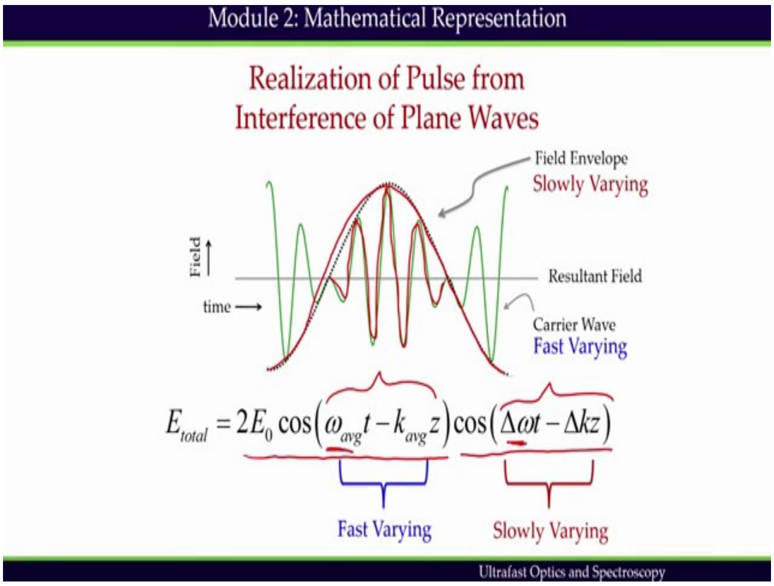

Now, if we take the real part of the total electric field we get two cosine functions. The final expression becomes 2 E naught cos omega average t minus k average z cos delta omega t minus delta kz. The first term associated with this omega average is a fast varying component. And, the second term associated with this delta omega is a slowly varying component of the resultant electric field because the difference between two frequencies is very small and that is why delta omega is very small number, but omega average is very high number, big number. (Please look at the slides for mathematical expressions)

These two components are shown in this figure; the first varying component is represented by this oscillation and the slowly varying component is represented by this oscillation. Therefore, interference of two plane waves of slightly different frequencies propagating along the same direction yields a pulse which is nothing, but a product of rapidly varying and slowly varying cosine waves. These points to two important features: a plane wave with single frequency component can never produce a pulse and to realize a pulse we need more than one frequency component.

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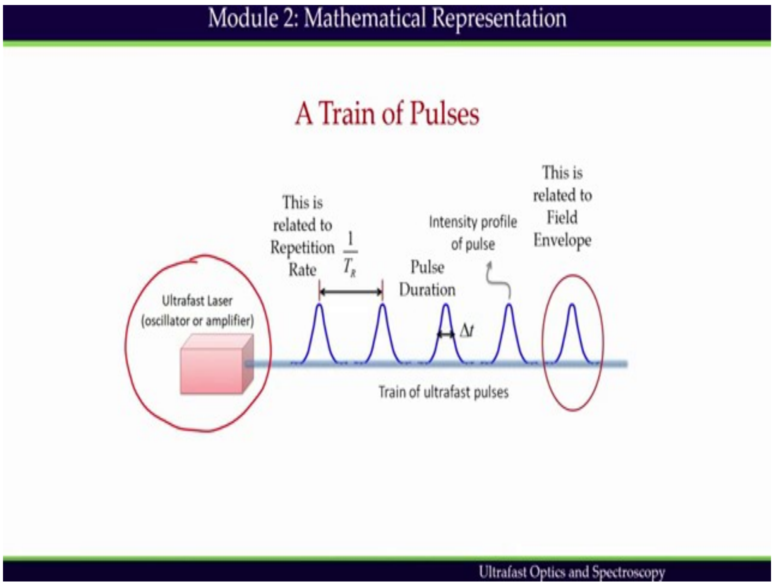

However, we must remember that from any ultrafast source it could be oscillator, it could be amplifier we produce a train of pulses just the way we have shown here, a number of pulses coming at a particular repetition rate. This train of pulses will always have slowly varying and fast varying oscillations which are represented by this equation. This is the fast varying component; this is the slowly varying component.

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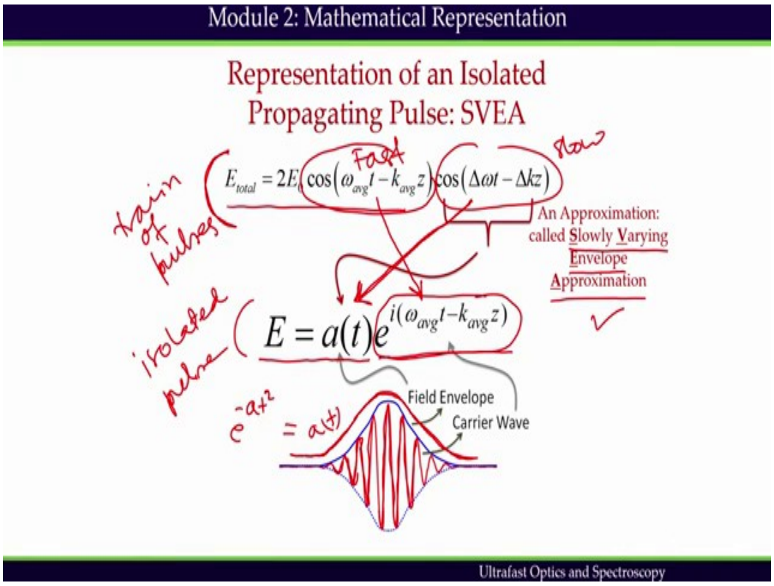

But, instead of using this complicated mathematical form obtained for an ultrafast pulse train produced due to interference of multiple frequency or color components and isolated propagating pulse can be approximately represented as E equals a t multiplied by e to the power i omega average t minus k average z. We note here that we have kept the fast varying component here, but this slowly varying component is approximated to an envelope function and this approximation is called slowly varying envelop approximation.

This representation of an isolated propagating pulse is valid only when the period of the field envelop a t. So, this is the field envelope a t, this is represented by a t, it looks like a Gaussian function. So, one can think of a t equals e to the power minus a t square a Gaussian function that is possible. So, this SVEA, Slowly Varying Envelope Approximation, is valid only when the period of the field envelop that is the characteristic time over which the variation of a t occurs and the period of carrier wave this part is representing the carrier wave part which is represented by this oscillation.

When these two periods are significantly different, then we can use this slowly varying envelope approximation. In general for pulses longer than 50 femtosecond this approximation is valid, but pulse is less than 10 femtosecond this approximation may not be valid. So, the whole idea is that if the characteristic time scale of this envelope function is much longer than the characteristic time scale of this oscillation then we can use slowly varying envelope approximation. And, we can approximate this slowly varying component to an envelope function to get the isolated a second pulse isolated ultrafast pulse.

We have to remember that this kind of equation representing train of pulses and this kind of equation is representing isolated pulse, but both of them are propagating.

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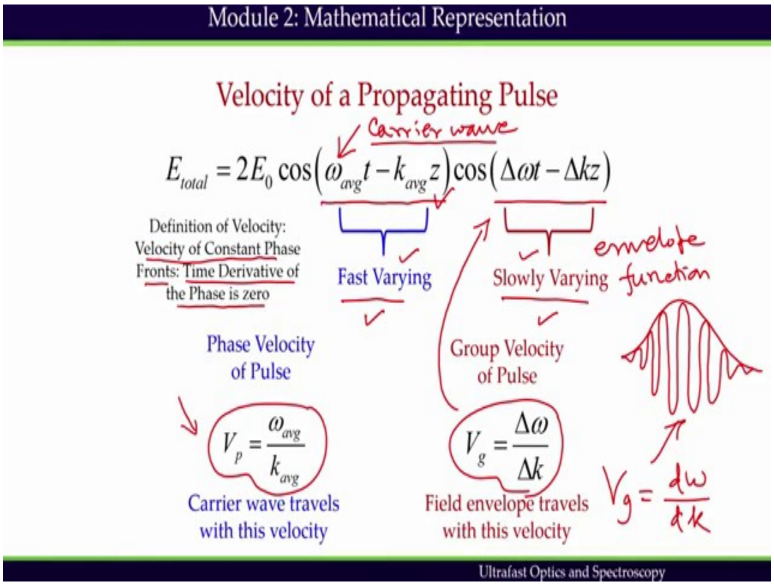

A pulse is also a propagating wave no matter whether we are representing in its isolated form or in a strain or pulses form, but a pulse is a propagating wave. So, it should travel with some velocity. What is the velocity of a pulse? To find an answer we will re-examine the equation obtained from interference of 2 plane waves of dissimilar frequency components propagating along the same direction. The first component which is omega average t minus k average z is the fast varying component and the second component delta omega t minus delta kz a slowly varying component.

The frequency of carrier waves which is given by omega average, this is associated with carrier wave so, I have a pulse, envelope and then we have this carrier wave oscillation. The velocity of a pulse is defined based on these two components, associated with the fast varying and slowly varying component. Mostly this has been approximated to an envelope function and this is representing carrier wave and we will define the velocity with respect to these two components.

Already we have seen that the definition of velocity is given by the velocity of constant phase fronts which means I have to take the time derivative of the phase and make it 0, and we get the corresponding velocity. We have two different phases associated with fast varying and slowly varying that is why we get two different velocities of a pulse. The carrier wave fast varying component of a pulse travels with phase velocity.

So, the phase velocity is associated with this carrier wave which is defined by omega average by k average. We get this expression by taking time average a sorry by taking the time derivative of this function. Thus average or resultant frequency component of a pulse travels at phase velocity. The field envelope of a pulse on the other hand travels with group velocity which is defined by Vg, and Vg equals delta omega by delta k. Again we get this expression by taking time derivative of this phase and make it 0.

And, if we consider that we have many frequency components and considering very small difference in frequency we can also write down Vg equals d omega d k. So, what does it mean by this phase velocity and group velocity of a pulse? I will represent it with my hands. This curvature representing the envelope function and my fingers are representing carrier wave. They remain locked in vacuum but, we have to remember that this envelope function can travel with group velocity and carrier wave can travel at phase velocity and in vacuum they are same. So, they travel like this. But, in a dispersive medium they can be different and when they are traveling one of them may have different velocity than the other.

So, the basic idea of group velocity and phase velocity comes from the fact that group velocity is always going to be the velocity of the envelope function and phase velocity is going to be the velocity of the carrier wave.

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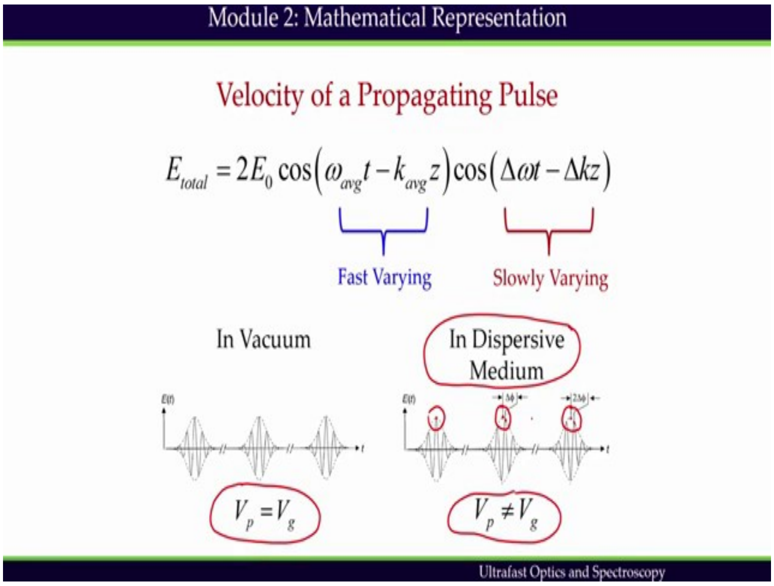

In vacuum Vp equals Vg, this is true only in vacuum which means that this is locked when they are traveling. But, in medium particularly in dispersive medium they are not equal and what happens this envelope function travels at a different velocity and the carrier wave travels at a different velocity and that is why it looks like this, when it is travelling it is changing like this and that is represented here.

If you think about the tip of the envelope function and the tip of the carrier wave they are different at different time. Here, there are the same position then there is a slightly difference, then more difference is evident here.

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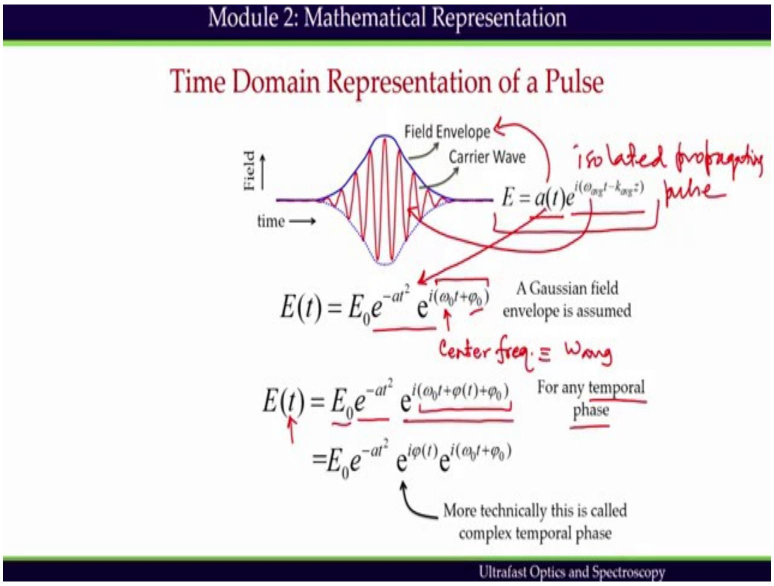

So, thus far we have seen that a pulse has a slowly varying component called filled envelope and a fast varying component called carrier wave. This is called time domain representation of a pulse. Here we note that only time domain representation does not feature all characteristics of a pulse. A pulse carries a number of frequency components. Time domain representation only features resultant frequency, information in particular phase or spectral phase or the phase of all spectral components is missing in the time domain representation.

This is why for unique representation of a pulse both time and frequency domain representations are important. We will represent frequency domain representation of a pulse very soon. However, before we go over the frequency domain representation will closely look at an important feature of time domain representation. Under SVEA that is slowly varying envelope approximation if a Gaussian filled envelope is assumed then we can write a pulse as a t. So, that was the general expression of an isolated isolated propagating pulse where a t is the envelope function and this complex part is representing the carrier wave. So, a t is representing the field envelope and this part is representing the carrier wave. <u>(Please look at the slides for mathematical expressions)</u>

And, if we assume that a t can be represented by a Gaussian function e to the power minus a t square where omega naught is the center frequency of the pulse, this is called center frequency center frequency of the pulse. And, phi naught which is this, omega naught is center frequency which is nothing, but omega average which we have seen before. And, phi naught; we have introduced phi naught here, this phi naught is some kind of initial constant absolute temporal phase which we have included. But, we have to remember that total temporal phase may not be always linear with respect to time just like here.

Here this temporal phase may not be always linear with respect to time. It can be a complicated function of time and that is why the more general form of a Gaussian isolated propagating pulse we can write down as E naught absolute magnitude of the electric field then Gaussian function which is the envelope function and the total temporal phase. This is called temporal phase because this phase appears in the time domain representation of the pulse; phi t can be a complex number of sorry complex function of time.