

Ultrafast Optics and Spectroscopy
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Lecture - 24
Measurement of Ultrafast Pulse

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Module 7: Measurement

Intensity Autocorrelation

Time sensitive detector measures the time-integrated SHG intensity

$$V_{SHG}^{signal} \propto \int_{-\infty}^{+\infty} I_{SHG}(t) dt = \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} |a(t)|^4 dt + \int_{-\infty}^{+\infty} 4 |a(t)|^2 |a(t-\tau)|^2 dt + \int_{-\infty}^{+\infty} a^2(t) a^{*2}(t-\tau) e^{i2\omega\tau} dt \right]$$

$\int_{-\infty}^{+\infty} |a(t)|^4 dt = \int_{-\infty}^{+\infty} I_{SHG}(t) dt$

Constant SHG Energy from Individual Pulses

$\int_{-\infty}^{+\infty} 4 |a(t)|^2 |a(t-\tau)|^2 dt = \int_{-\infty}^{+\infty} I_{SHG}(t) I_{SHG}(t-\tau) dt$

Intensity Autocorrelation $T^{(2)}(\tau)$

Third and fourth terms are interferometric terms *time average*

Now, if you look at this different terms, the first term this term that is a constant SHGs energies from individual pulses this is there is no square here. So, this is the constant SHG energy from individual pulses, which means if the pulses are not overlapped. Let us say these two pulses are coming out of the Michelson interferometer, they are delayed by tau. If they are not overlapped with each other still we get this is the detector, this is the SHG pulses, and detector is seeing these two pulses it will give me a constant background signal first.

So, we plot as a function of tau[τ] and this is the total V signal of SHG, then at a very longer distance sorry as at a very longer delay on both side positive delay or negative delay we get a constant signal. A constant signal is coming from individual pulses, each pulse will produce always produce SHG because this is a collinear configuration.

The second term, if you look at this term this is called intensity autocorrelation term this term. This intensity autocorrelation term is represented by second order autocorrelation

tau and these autocorrelation terms exist as long as the pulses are overlapped. The overlap could be this much, or overlap could be this much, or overlap could be of this much.

So, this is the overlap regime, this is also an overlap regime, or this could be a little bit of overlap. So, this term exists as long as we have the overlap between two pulses and if we have overlap between two pulses we will create extra SHG energy and that is why at 0 delay we will see that we will have maximum and then slowly they are going down to the constant background signal. This regime what we see the enhancement of the SHG signal is because of the overlap between two pulses. Because of this extra term this is called intensity autocorrelation and that exists only when two pulses are overlapped.

Other terms; third term and fourth terms this one and this one if you carefully looked at it they have an oscillatory term. This complex term representing oscillatory term and real component of that is going to be $\cos 2\omega_0 \tau$ and this real component is going to be $\cos \omega_0 \tau$; whenever there is an oscillatory term, if we time average it if we time average it then they would become 0.

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Intensity Autocorrelation

Time sensitive detector measures the time-integrated SHG intensity

$$I_{SHG}^{signal} \propto \int_{-\infty}^{+\infty} I_{SHG}(t) dt = \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} |a(t)|^4 dt + \int_{-\infty}^{+\infty} 4 |a(t)|^2 |a(t-\tau)|^2 dt + \int_{-\infty}^{+\infty} a^2(t) a^{*2}(t-\tau) e^{i2\omega_0 \tau} dt \right]$$

~~ZERO~~ ~~ZERO~~

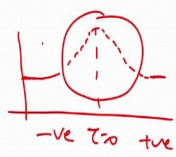
$+ \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} (|a(t)|^2 + |a(t-\tau)|^2) |a(t) a^*(t-\tau) e^{i\omega_0 \tau} dt + c.c. \right]$

$\int_{-\infty}^{+\infty} |a(t)|^4 dt = \int_{-\infty}^{+\infty} I_{SHG}^2(t) dt$ Constant SHG Energy from Individual Pulses

$\int_{-\infty}^{+\infty} 4 |a(t)|^2 |a(t-\tau)|^2 dt = \int_{-\infty}^{+\infty} I_{SHG}(t) I_{SHG}(t-\tau) dt$ Intensity Autocorrelation

Third and fourth terms are interferometric terms: Time-averaging makes them ZERO

average $\cos \theta = 0$



And which means that I can suppress this third and fourth terms by time averaging, because $\cos \theta$ average of $\cos \theta$ is 0 and we can time average it. The moment we time average it we get these terms 0 and we get in the collinear configuration we have only these two terms. And how does it manifest this term is SHG signal does not matter what is the delay. So, at longer delay in both positive and negative delays, we

get a constant background signal. Then there will be an enhancement due to this extra term and this enhancement occurs only when the pulses are overlapped.

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Intensity Autocorrelation

Time sensitive detector measures the
time-integrated SHG intensity

Collinear + Time-averaged + Use filter to filter out incident beam

$$\begin{aligned}
 V_{SHG}^{signal} &\sim \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} |a(t)|^4 dt + \int_{-\infty}^{+\infty} 4 |a(t)|^2 |a(t-\tau)|^2 dt \right] \\
 &\sim \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} I_{SHG}^2(t) dt + 4 \int_{-\infty}^{+\infty} I_{SHG}(t) I_{SHG}(t-\tau) dt \right] \\
 &\sim \frac{1}{16} \left[2 \int_{-\infty}^{+\infty} I_{SHG}^2(t) dt + 4 \Gamma^{(2)}(\tau) \right]
 \end{aligned}$$

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Thus, under correct experimental condition which means we have used collinear configuration, time averaging we have used to suppress other fourth and third terms, and we use filter to filter out incident beam. Under this condition we have only these two terms.

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Collinear Intensity Autocorrelation

Collinear + Time-averaged + Use
filter to filter out incident beam

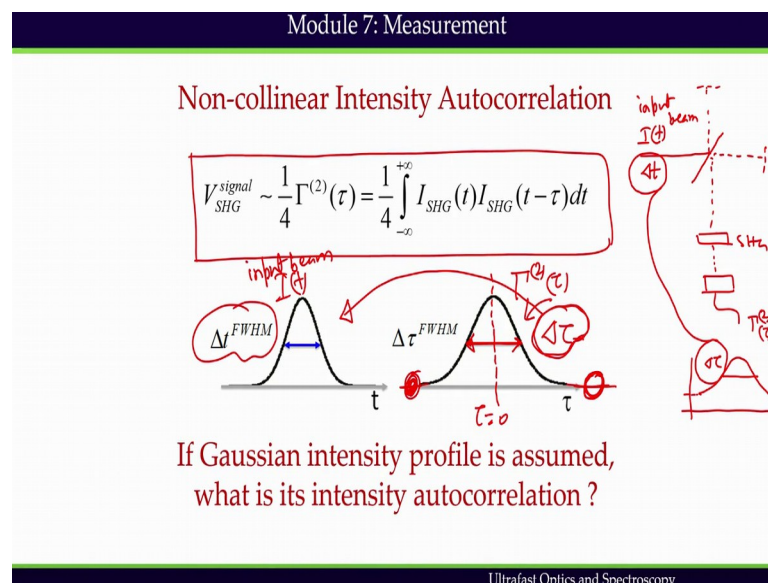
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Now, this constant background can be eliminated if we think of non collinear configuration. In non-collinear configuration we have rearranged the interferometer a bit, here we are now using mirror pair. So, let us look at this input beam is split into two by 50-50 beams meter, they go this way the first one goes this way and the second one goes this way, then this is reflected back and this beams also is reflected back.

So, these two beams now time delayed with respect to the delayed stage and they are focused on SHG signal. And we know that conservation of momentum and energy two beams going like this way this is k_1 , this is k_2 non-collinear configuration we create some frequency generation which we are calling it in here second harmony generation. And this is going to be the direction of the sum frequency generation and that is exactly what happens here.

So, input beam can be blocked; two input beams can be blocked and only SHG signal can be extracted, which means that previously the equation we had was $V_{\text{signal}} = \frac{1}{16} \int_{-\infty}^{+\infty} I_{\text{SHG}}(t) I_{\text{SHG}}(t - \tau) dt$ this intensity autocorrelation term. Now this term can be non suppressed because you can block them they are along two different direction and constant background is eliminated. We have no background then the moment they become overlapped they will start to increasing the SHG intensity. So, we get the signal when two pulses are temporally and spatially overlapped.[Please look at the slides for mathematical expression]

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Thus the output signal in the background free non collinear geometry becomes only this correlation term. The intensity auto correlation function assumes it is maximum value at τ equals 0 this is the intensity auto correlation and this is your τ equals 0 we get the maximum at τ equals 0. The pulse exhibits no overlap for the delays much greater than

the pulse with and correlation function become 0 and that is why in this regime at a longer delay we do not see any correlation.

The meaning of correlation is the memory which means that the pulse in this regime, they cannot remember how they are interacting, here also the pulse cannot remember how they are interacting, but here they can remember how they are interacting. The width of the correlation profile we get this correlation profile intensity auto correlation profile which is expressed by $\Delta\tau$ and we are considering again full width half max of this correlation profile and this $\Delta\tau$ is, this is the full width half max of the correlation profile. This is the τ^2 profile, this Δt is related to the intensity correlation intensity full width half max of pulse Δt this is I_t , input beam intensity.

So, what we are trying to emphasize here, we used certain kind of interferometer and I had input beam intensity I_t . Every intensity profile has a width that is represented by Δt or pulse width of the input beam this is input beam, then it goes through the interferometer and then comes out of the interferometer. I have SHG and then I record the signal of SHG, in the end I get correlation function the correlation profile.

This correlation profile looks like this depending on whether collinear configuration or non collinear configuration, I may have the width $\Delta\tau$ and also the background I may have I may not have depending on what kind of configuration we have used. Now, what we are trying to say here is this $\Delta\tau$ is related to the input intensity profile Δt .

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Intensity Autocorrelation of a Gaussian Pulse

Gaussian intensity profile $I(t) = I_0 e^{-\frac{4 \ln 2 t^2}{\Delta t^2}}$ ✓

$$\begin{aligned} \Gamma^{(2)}(\tau) &= \int_{-\infty}^{+\infty} I(t)I(t-\tau)dt \\ &= \int_{-\infty}^{+\infty} I_0 e^{-\frac{4 \ln 2 t^2}{\Delta t^2}} I_0 e^{-\frac{4 \ln 2 (t-\tau)^2}{\Delta t^2}} dt \\ \Gamma^{(2)}(\tau) &= I_0^2 \int_{-\infty}^{+\infty} e^{-\frac{4 \ln 2 (t^2 + t^2 - 2t\tau + \tau^2)}{\Delta t^2}} dt \\ &= I_0^2 e^{-\frac{4 \ln 2 \tau^2}{\Delta t^2}} \int_{-\infty}^{+\infty} e^{-\frac{8 \ln 2 t^2}{\Delta t^2} + \frac{8 \ln 2 t\tau}{\Delta t^2}} dt \end{aligned}$$

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And we will take an example of Gaussian beam to find out how they are correlated. Because then we will be able to use the measurement and predict what was the intensity full width half max of the input beam. So, we take a Gaussian intensity profile which is now very familiar to us, this delta t is representing intensity full width half max we have to emphasize here.

Then the term which we are interested in we will calculate this is the intensity autocorrelation term which is nothing, but the product of these two Gaussian beams with a delay that is the tau delay we have introduced, this is the tau delay we have introduced and after doing little bit of math very simple math.

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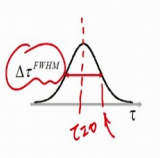
Intensity Autocorrelation of a Gaussian Pulse

$$\Gamma^{(2)}(\tau) = I_0^2 e^{-\frac{4 \ln 2 \tau^2}{\Delta t^2}} \sqrt{\frac{\pi \Delta t^2}{8 \ln 2}} e^{\frac{(4 \ln 2 \tau^2)^2}{\Delta t^4} \frac{\Delta t^2}{8 \ln 2}} \checkmark$$

$$= I_0^2 e^{-\frac{4 \ln 2 \tau^2}{\Delta t^2}} \sqrt{\frac{\pi \Delta t^2}{8 \ln 2}} = I_0^2 \sqrt{\frac{\pi \Delta t^2}{8 \ln 2}} e^{-\frac{2 \ln 2 \tau^2}{\Delta t^2}}$$

using $\int_{-\infty}^{\infty} e^{-Ax^2 - 2Bx} dx = \sqrt{\frac{\pi}{A}} e^{\frac{B^2}{A}}$

By the definition of FWHM of the function $\Gamma^{(2)}(\tau)$



$$\frac{\Gamma_0^{(2)}}{2} = I_0^2 \sqrt{\frac{\pi \Delta t^2}{8 \ln 2}} e^{-\frac{2 \ln 2 \left(\frac{\Delta \tau}{2}\right)^2}{\Delta t^2}}$$

$$\frac{\Gamma_0^{(2)}}{2} = \Gamma_0^{(2)} e^{-\frac{2 \ln 2 \left(\frac{\Delta \tau}{2}\right)^2}{\Delta t^2}} \quad \text{as, } \Gamma_0^{(2)} = I_0^2 \sqrt{\frac{\pi \Delta t^2}{8 \ln 2}}$$

$$\frac{1}{2} = e^{-\frac{2 \ln 2 \left(\frac{\Delta \tau}{2}\right)^2}{\Delta t^2}}$$

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We get this expression for the autocorrelation profile and we will use the standard integral to solve this and finally, once we get this autocorrelation this autocorrelation again representing a Gaussian function. So, if I start with a Gaussian input beam it is intensity autocorrelation profile is also representing a Gaussian beam, but the as a function of tau now is a time delay and this is your tau equals 0. And this function has width which is represented by delta tau which is at the full width half max. So, by definition of full half max we will be able to write down this that is the intensity becomes half of it is maximum intensity at this delay.

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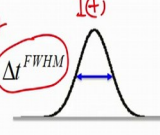
Intensity Autocorrelation of a Gaussian Pulse

$$\ln 2 = \frac{2 \ln 2 \left(\frac{\Delta \tau}{2}\right)^2}{\Delta t^2}$$

$$\Delta t^2 = 2 \left(\frac{\Delta \tau}{2}\right)^2$$


$$\Delta \tau = \sqrt{2} \times \Delta t = 1.41 \times \Delta t$$

Input beam $I(t)$



Δt^{FWHM}

$\Gamma^{(2)}(\tau)$



$\Delta \tau^{FWHM}$

\downarrow

$\frac{\Delta \tau}{1.41}$

$= \boxed{\Delta t}$

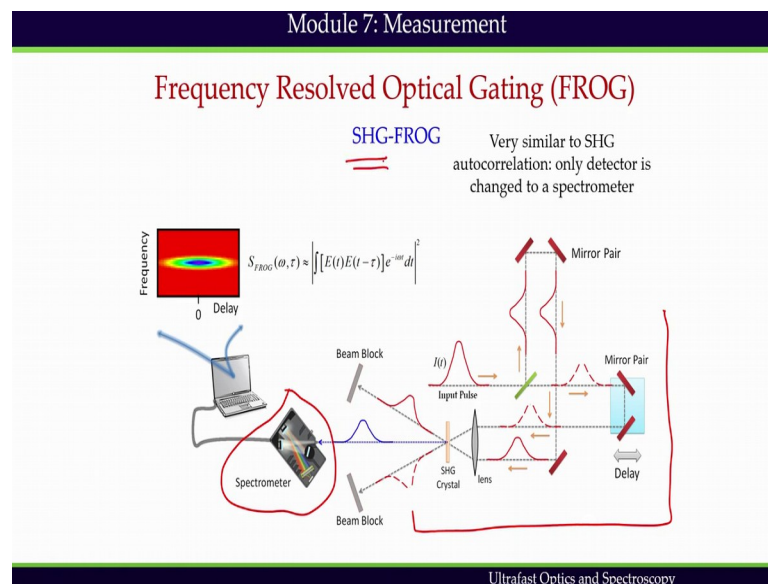
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And if we use that then finally, we get an expression which correlates delta t and delta tau, delta t is the intensity full width half max. So, this is your I t profile of the input beam. And this is the corresponding intensity autocorrelation profile and this tau and delta t is now related for a Gaussian pulse it is by the factor of 1.41.

So, after the measurement when we get this delta tau, if we assume Gaussian intensity profile we can simply divide the intensity autocorrelation with this delta tau which is measured quantity from experiment which can be divided by 1.41 to get this delta t, delta t is the full width half max of the of the intensity profile. So, by this measurement one can quickly get the full width half max of the intensity profile or the time duration of the pulse.

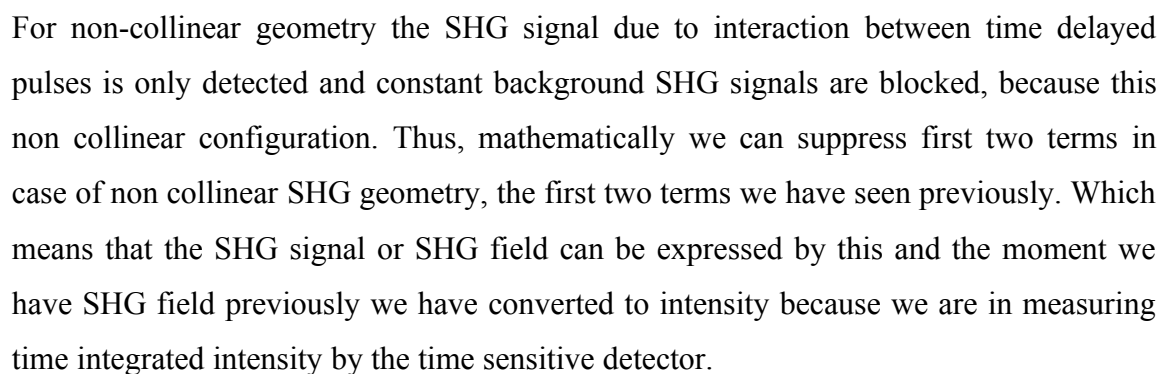
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So, far we have considered ultrafast pulse measurement techniques that are operated purely in the time domain and that is called autocorrelation measurement. Autocorrelation measurements are undertaken often in ultrafast lab; however, we should remember that all the measurements discussed above do not characterize the pulse completely.

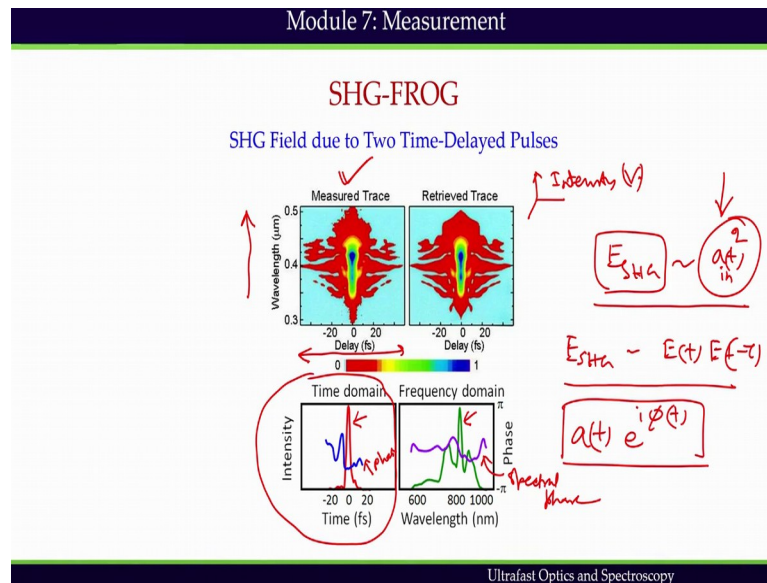
The approach that characterizes a pulse completely involves a hybrid domain methodology combining time and frequency domains the technique is called frequency resolved optical gating. In the FROG measurement what we do the only modification we do this is the exactly non collinear configuration autocorrelation measurement. Instead of

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Now we are not using time sensitive detector we are using spectrometer that is why we will not convert into intensity we will convert to frequency domain first. So, this is frequency domain SHG signal which is nothing, but Fourier transform and then we are measuring the spectrum which is nothing, but square modulus of the frequency domain field which is expressed by this equation.

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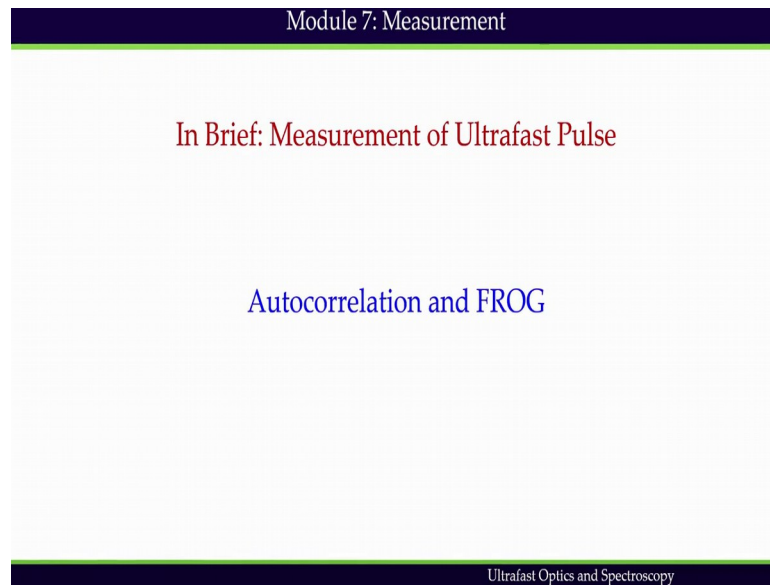


So, one example of SHG-FROG phase is shown here this is two dimensional plot because along this direction we are plotting the frequency wave length along these direction we are plotting delay. At different we are checking the spectrum and the vertical axis is representing the intensity, which means the V's voltage coming out of the spectrometer. It can be easily shown that SHG FROG trace is symmetric function of tau after getting the FROG trace question is how to retrieve the pulse?

So, we first measure a FROG trace and then we make a theoretical gaze for E SHG; what would be the field of E SHG and E SHG is related to it is input field by this. So, we will first make a gaze of the pulse, then we will try to theoretically simulate the experimentally measured spectrogram and we will try to feed full filling the non-linear equation which is nothing, but E t multiplied by E t minus tau.

And if we do not get the fitting then we change the parameter in the input beam and iterate the fitting algorithm until we get the best fitting. This is the way we can retreat the pulse shape completely because we made an assumption and we are getting the best solution for that an assumption included field envelop as well as spectral phase both together. And that is why final in the time domain we get this profile intensity profile as well as phase information. Here we get spectrum as well as spectral phase information. So, FROG measurement is more advanced technique to measure the pulse completely.

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With this we have come to the end of this module. In this module we have studied different aspects of pulse measurement; autocorrelation is a simplest idea more advanced idea is FROG based measurement. We will meet again for the next module.