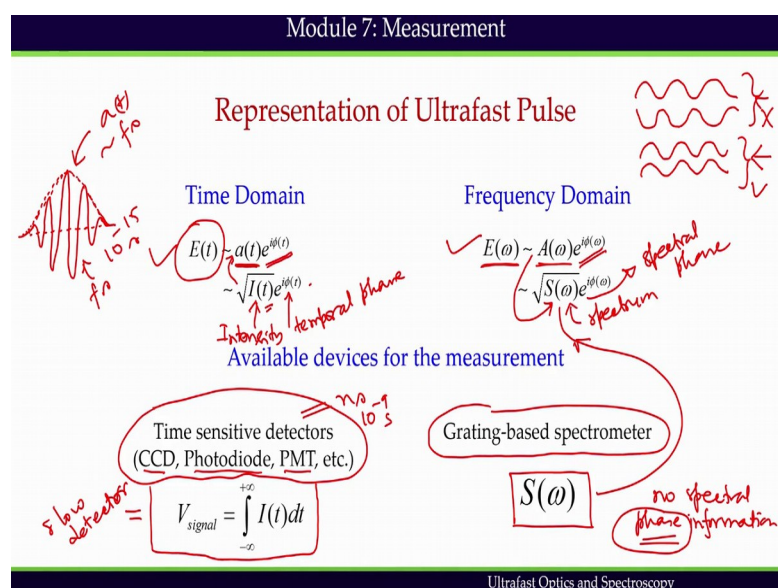


Ultrafast Optics and Spectroscopy
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Lecture - 23
Measurement of Ultrafast Pulse

Welcome to module 7 of the course Ultrafast Optics and Spectroscopy. In this module we will study how to measure Ultrafast Pulse.

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And ultrafast pulse is represented both in time and frequency domains which we have already seen in this course. In the time domain we express electric field as a product of envelop function field envelop and carrier wave $e^{i\phi(t)}$.

And we have seen that this $a(t)$ can be represented as square root of $I(t)$. On the other hand in frequency domain a pulse is represented as field envelop in the frequency domain and its spectral phase. And this field envelop is related to the spectrum which we record following this equation $a(\omega) = \sqrt{S(\omega)}$. Now, if we want to measure a pulse we have to measure $I(t)$ temporal phase.

So, this is intensity this is temporal phase and we have to measure spectrum or a spectrum and spectral phase. So, these are the physical quantities we need to measure in order to represent a pulse, but if we look at the available devices for the measurement

which we have. They are two different kinds one is time sensitive detectors, including CCD camera, photodiodes, PMT photomultiplier tube. On the other hand in the frequency domain we have grating based spectrometer.

So, this is the two devices we have for the measurement. Now, unfortunately available time sensitive detectors are way too slow to measure ultrafast pulse intensity profile and field oscillation as well. So, if we look at a pulse lets say 10 femto second pulse or 100 femto second pulse. I have an envelope function I am drawing here a transformed limited pulse. So, if we think of the rise time and fall time of a t .

This is of the order of let say femtosecond and also if you look at carrier waves the characteristic time of carrier wave is also femtosecond this is we have already seen. But all the time sensitive detectors having response time of the order of nanosecond, 10 to the power minus 9 second this one 10 to the power minus 15 second.

So, clearly they are too slow to measure either the intensity profile directly or the temporal phase in the time domain. So, if we try to measure anything faster with the slow detector we end up with getting this is nothing, but total energy content in the pulse. So, this integral what we measure with the slow detector on the other hand spectrometer can only measure spectrum.

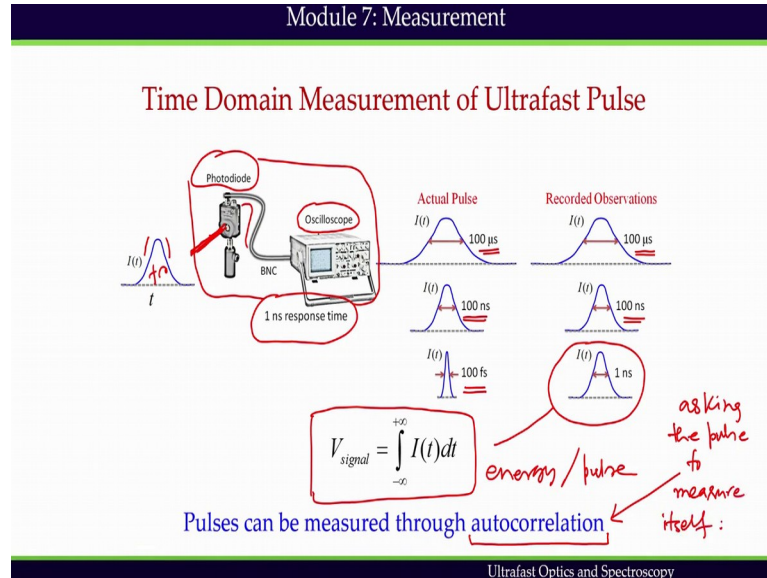
So, good thing about spectrometer it can directly measure $S(\omega)$ which is present in the description of the frequency domain description of the pulse, but it does not give any spectral phase information. And we have seen that the phase information is very important spectrum gives how many frequency components we have and phase gives how their phase are correlated.

So, let say I have two different waves having phase relative phase is like this and another one like. So, these two if we look at they are undergoing destructive interference on the other hand if they are like this they are undergoing constructive interference and we create the pulse. So, this is contributing to the pulse and this is not contributing to the pulse that is why having how many frequency components.

We have knowing how many frequency components we have in the pulse is not enough information we need to know how the phase or the relative phase of different colour

components. In spite of these problems in this module we will try to understand how to measure a pulse with these devices.

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So, the first thing we will discuss is time domain measurement of ultrafast pulse. A time sensitive detector such as photodiode emits electrons in response to photon. So, if we direct the beam laser beam to the photodiode this will give me electrical response and that electrical response can be plotted in an oscilloscope. Time sensitive detectors include photodiodes, photomultiplier tubes.

But available time sensitive detectors have very slow rise and fall times of the order of nanosecond response time. So, if the whole device the detector including photodiode and the oscilloscope they have nano second response time. On the other hand if we think about the femtosecond pulse it is rise time and fall time is of the rough femtosecond and that is why if we direct femtosecond laser beam to the photodiode to this configuration. Then what will happen? Let us say that we are directing a beam of 100 microsecond pulse to the photodiode.

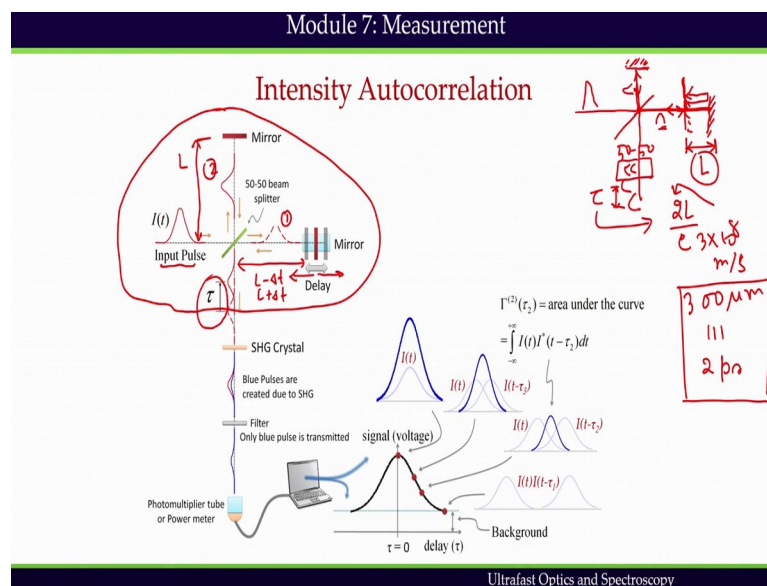
Then we will be able to immediately record the intensity profile of 100 microsecond. Because response time of the detector is nanosecond is much faster than the pulse duration, but if we take 100 nano second pulse then oscilloscope in the oscilloscope we will be able to see 100 nano second pulse as well. But if we take 100 femto second pulse

and direct the beam to the photodiode, then what will be our observation our observation would be a nano second pulse not a femtosecond pulse.

And this is related to the total energy of the pulse which is given by this integration minus infinity to plus infinity $I(t) dt$. This is related to energy per pulse thus detector output voltage is proportional to the ultrafast pulse energy. The slow response time of available time sensitive detectors does not permit us to make time domain intensity profile measurement of the ultrafast directly.

So, then question is how do we measure the profile? We measure the profile by a technique called autocorrelation with this device autocorrelation which means you are asking the pulse to measure itself that is called autocorrelation.

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A simple setup which can do autocorrelation measurement is Michelson interferometer. This is a configuration of Michelson interferometer where we have two mirrors and one beam splitter.

So, input beam is split into two identical pulses this input pulse is split into two identical pulses 1 and 2 with the help of 50-50 beam splitter. And then we place this mirror on a motorized stage that is why mirror can be taken backward or forward. And depending on where they are depending on if this length is L and if this length is L minus δL or L

plus ΔL . We introduced the delay between two pulses and this delay is represented by τ . A 50-50 beam splitter will always create two identical pulses.

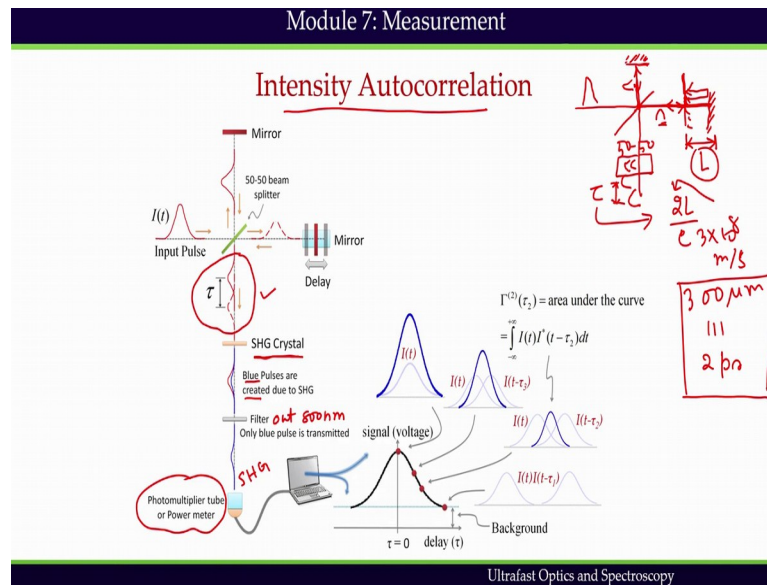
So, this is 50-50 beam splitter then first 50 percent will be transmitted 50 percent will be reflected. That is why a pulse I get to baby pulses then I have mirror here and if the mirrors are placed at equal distance then they will again come back to this point after the reflection they will come back to this 50-50 beam splitter. At the same time and again 50 percent of that baby beams will be so, this beam will be transmitted and this beam also will be reflected.

So, this output beam is this two now if we step it back then this is a longer path travelled by this arm or this pulse and that is why after the beam splitter this transmitted beam and the reflected beam will have a delay this is called τ . Now, if we move the mirror backward by a distance L , let say I have moved the mirror distance L with respect to the equal distance position so with respect to this point only.

At this point this arm distance and this arm and this arm they are of equal distance now with respect to this distance I have step back this mirror by a distance L . If you do that then the delay can be represented as $2L$ by C distance by C velocity a speed giving the time. This factor 2 is coming because light must travel at the extra distance to the mirror and come back from the mirror. So, it will go to the mirror and then come back. So, total distance travelled is $2L$.

So, we are considering the velocity a speed of light as three into 10 to the power eight meter per second one can remember this that 300 micron displacement yields a delay of 2 picosecond from this expression. So, 300 micron displacement will give you 2 picosecond the delay between 2 pulses coming out of the Michelson interferometer. So, with this information we can go back to the Michelson interferometer and the and we see that in the Michelson interferometer.

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We have now 2 pulses coming out of the interferometer they have τ delay and we have inserted now 1 SHG crystal, second harmonic generation crystal. Because these two beams are coming collinear each beam will produce second harmonics as well as there will be coherent generation of the second harmonic due to action joint action of two beams as well.

So, in the intensity autocorrelation measurement what we measure we filter out 800 nano meter that is the fundamental beam and with the help of photomultiplier tube or a power meter. We just check total energy or total second harmonic generation energy because we are looking at second harmonic generation it is called SHG intensity autocorrelation.

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Module 7: Measurement

Intensity Autocorrelation

SHG Field due to Two Time-Delayed Pulses

$$E_{SHG}(t) \sim (E_{out}(t))^2$$

$$\sim \left[\frac{1}{2} a(t) e^{i\omega_0 t} + \frac{1}{2} a(t-\tau) e^{i\omega_0(t-\tau)} \right]^2$$

$$\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{i2\omega_0(t-\tau)} + 2a(t)a(t-\tau) e^{i2\omega_0 t} e^{i2\omega_0(t-\tau)} \right]$$

$$\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{-i2\omega_0 \tau} e^{i2\omega_0 t} + 2a(t)a(t-\tau) e^{i2\omega_0 t} e^{-i\omega_0 \tau} \right]$$

$$\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{-i2\omega_0 \tau} + 2a(t)a(t-\tau) e^{-i\omega_0 \tau} \right] e^{i2\omega_0 t}$$

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So, if we look at one more time the expressions which will be responsible to create second harmonic beam. Here we have 50-50 beam splitter the input beam is expressed as $a(t)e^{i\omega_0 t}$ that is $E(t)$ that is the input beam I have expressed. 50-50 beam splitter will split the intensity by it will split the intensity into two equal intensity pulses.

So, we know that E is square root of I and that is why the reflected beam and transmitted beam can be represented as $1/\sqrt{2}$ by square root 2 $a(t)e^{i\omega_0 t}$ and this one this is transmitted and reflected beam would be all again represented square root $a(t)e^{i\omega_0 t}$. (Please look at the slides for mathematical expressions)

Then due to mirror I have a mirror here; I have mirror here. This beam will come back and again be transmitted through the 50-50 beam splitter. So, that transmitted beam would be again $1/\sqrt{2}$ by square root of that beam that is the field. So, one field is this 1 and then again thinks about this beam this will be reflected back and reflected by this 50-50 beam splitter. I get again $1/\sqrt{2}$ into to the beam $e^{i\omega_0 t}$.

So, this is the total field coming out of the interferometer which is this is the pulse E_1 and this is E_2 and total field is going to be $E_1 + E_2$ which is coming out of the interferometer. Now, if we introduce a delay with respect to the other pulse instead of t

of the second pulse we will write down E_2 as this is nothing, but 1 by 2 a t minus τ that much delay I have introduced multiplied by e to the power $i\omega_0(t - \tau)$. (Please look at the slides for mathematical expressions)

So, this is the field which is delayed by τ time and that is exactly what we have written. E_{out} is the field coming out of the interferometer at this point. In the coming out of the Michelson interferometer E_{out} is represented by half because square root square root if you multiply 1 by square root 2 .

And 1 by square root of 2 we get half factor then a t into 2 to the power $i\omega_0 t$ that is the first pulse. And the second pulse is delayed with respect to the first pulse and that is half a t minus τ into e to the power $i\omega_0(t - \tau)$.

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Module 7: Measurement

Intensity Autocorrelation

SHG Field due to Two Time-Delayed Pulses

$$\begin{aligned}
 E_{SHG}(t) &\sim \left(E_{out}(t) \right)^2 \\
 &\sim \left[\frac{1}{2} a(t) e^{i\omega_0 t} + \frac{1}{2} a(t-\tau) e^{i\omega_0(t-\tau)} \right]^2 \\
 &\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{i2\omega_0(t-\tau)} + 2a(t)a(t-\tau) e^{i2\omega_0 t} e^{i2\omega_0(t-\tau)} \right] \\
 &\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{-i2\omega_0 \tau} e^{i2\omega_0 t} + 2a(t)a(t-\tau) e^{i2\omega_0 t} e^{-i\omega_0 \tau} \right] \\
 &\sim \frac{1}{4} \left[a^2(t) e^{i2\omega_0 t} + a^2(t-\tau) e^{-i2\omega_0 \tau} + 2a(t)a(t-\tau) e^{-i\omega_0 \tau} \right] e^{i2\omega_0 t}
 \end{aligned}$$

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Then after this, what we have? We have a SHG crystal placed and SHG occurs. Because of the second order polarization which means the term which will be responsible due to SHG field, then SHG field is going to be nothing but E_{out} square and that is why we have to take the square of this whole term. If we take the square and if we look if we do little bit of rearrangement of the terms we get this is very easy to get and then again reorganizing it.

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Module 7: Measurement

Intensity Autocorrelation

SHG Intensity due to Two Time-Delayed Pulses

$$\begin{aligned}
 I_{\text{SHG}}(t) &= |E_{\text{SHG}}(t)|^2 \\
 &\sim \frac{1}{16} \left[a^2(t) e^{i2\phi t} + a^2(t-\tau) e^{-i2\phi\tau} + 2a(t)a(t-\tau) e^{-i\phi\tau} \right] e^{i2\phi t} \\
 &\sim \frac{1}{16} \left[|a(t)|^4 + |a(t-\tau)|^4 + 4|a(t)|^2 |a(t-\tau)|^2 + a^2(t)a^{*2}(t-\tau) e^{i2\phi\tau} + 2\left(|a(t)|^2 + |a(t-\tau)|^2\right) a(t)a^*(t-\tau) e^{i\phi\tau} + c.c. \right]
 \end{aligned}$$

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And finally, if we have SHG field then we can get intensity of the SHG and that is square modulus of the SHG field and SHG field expression we have got in the previous slide. So, we have to take square modulus of that which is nothing, but this multiplied by its complex conjugate which is this one. And we get an expression for the intensity of the SHG field.

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Module 7: Measurement

Intensity Autocorrelation

Time sensitive detector measures the time-integrated SHG intensity

$\epsilon_{\text{SHG}} = \frac{2}{\omega_{\text{SHG}}}$
 $I_{\text{SHG}} = |E_{\text{SHG}}|^2 = |A(t)|^2$

$$\begin{aligned}
 V_{\text{SHG}}^{\text{signal}} &\propto \int_{-\infty}^{+\infty} I_{\text{SHG}}(t) dt = \\
 &\frac{1}{16} \left[\int_{-\infty}^{+\infty} |a(t)|^4 dt + \int_{-\infty}^{+\infty} |a(t-\tau)|^4 dt + \int_{-\infty}^{+\infty} 4|a(t)|^2 |a(t-\tau)|^2 dt + \int_{-\infty}^{+\infty} a^2(t)a^{*2}(t-\tau) e^{i2\phi\tau} dt \right] \\
 &\quad + \frac{1}{16} \left[\int_{-\infty}^{+\infty} 2\left(|a(t)|^2 + |a(t-\tau)|^2\right) a(t)a^*(t-\tau) e^{i\phi\tau} dt + c.c. \right] \\
 V_{\text{SHG}}^{\text{signal}} &\propto \int_{-\infty}^{+\infty} I_{\text{SHG}}(t) dt = \\
 &\frac{1}{16} \left[2 \int_{-\infty}^{+\infty} |a(t)|^4 dt + \int_{-\infty}^{+\infty} 4|a(t)|^2 |a(t-\tau)|^2 dt + \int_{-\infty}^{+\infty} a^2(t)a^{*2}(t-\tau) e^{i2\phi\tau} dt \right] \\
 &\quad + \frac{1}{16} \left[\int_{-\infty}^{+\infty} 2\left(|a(t)|^2 + |a(t-\tau)|^2\right) a(t)a^*(t-\tau) e^{i\phi\tau} dt + c.c. \right]
 \end{aligned}$$

because $\int_{-\infty}^{+\infty} |a(t)|^4 dt = \int_{-\infty}^{+\infty} |a(t-\tau)|^4 dt$

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Actual measurement as I have told previously in the actual measurement we are we are not checking the intensity profile. But we are checking the total energy dump to the

detector and that is given by the signal that is related to total energy per pulse when use power meter and check how much energy we have per pulse that exactly what we monitor it is an integration of the intensity profile.

So, in the end what we measure is proportional to this re signal which is nothing, but this expression and in this expression it is very easy to do this math I mean one can one can carefully looked at the different terms and then we will be able to get this expression. Now, if you look at this first two terms what does it mean? It means that they are the intensity of the SHG beam created by individual pulses a t is the field envelop of the input beam.

We have to first square it to get SHG field. And then I SHG intensity of the SHG is going to be a t square modulus which is nothing, but a t to the power 4 that is why this is I SHG; SHG created by individual pulses. Each pulse one of the pulses is not time delayed with respect to the other one is time delayed. So, each pulse is producing SHG and their intensities should be equal.

Whether a pulse is time delayed or not in the end their individual SHG signal should be the same that is why we have this they are equal and in that case we can write down with the two factor and rest of the terms remains here. We will stop here and we will meet again for the same module.