Ultrafast Optics and Spectroscopy
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Lecture - 19
Transverse Electromagnetic Mode (Continued)

Welcome back to module 5. We have discussed Transverse Electromagnetic Mode. And, transverse electromagnetic mode one can visualize this mode if I have a laser beam propagating along this direction and if I place a piece of paper, then I will be able to see on the piece of paper I will be able to see the transfers electromagnetic mode.

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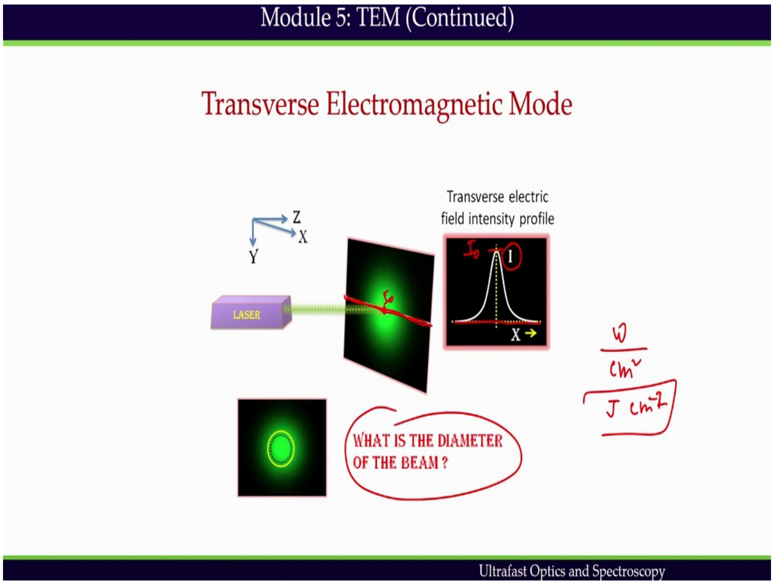

And, we have already pointed out that rays are idealized or oversimplified model of the laser beam. In reality a laser emits a beam of finite width or diameter. If a piece of paper is placed on the laser beam we get the cross sectional area illuminated by the laser beam as shown here. What you observe is called transverse electromagnetic mode of the laser.

When transverse electromagnetic mode or transverse electric field in density of the laser beam can be expressed by Gaussian function, the one we have shown here. So, let us say this is the laser beam which we are seeing and if we take the cross section of the of this region and if we plot this cross section this X versus this intensity versus I then the maximum is I naught maximum is at this point which is I naught and it will be decay following a Gaussian decay.

As the energy of a Gaussian beam spreads from minus infinity to plus infinity, theoretically it is a question, how do we define the beam diameter? If the energy a sprits from minus infinity to plus infinity, how do we define the beam diameter? In many laser based laser based experiments determining fluence which is nothing, but the energy by cross-sectional area that is we express Joule per centimeter square m.Joule per square centimeter something like that.

In many laser based experiments determining fluence which is defined as energy per cress sectional area and which depends on the size of the laser beam becomes important and that is why we need to know how to define the diameter.

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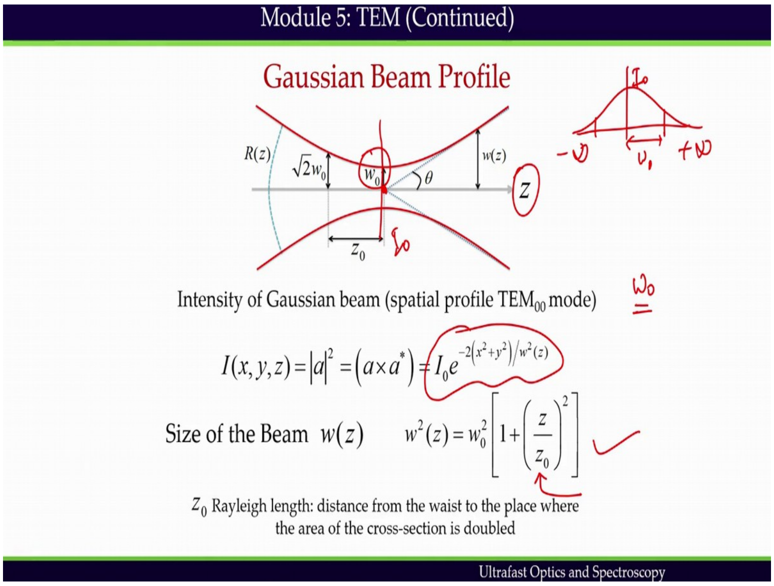

We have seen that the intensity of TEM 00 solution of the paraxial wave equation which features Gaussian beam can be written as I is nothing, but a special Gaussian profile e to the power minus 2 x square plus y square divided by w square. The beam waist this w naught we have already seen this one is the beam waist which is the narrowest size of the beam. Size of the beam is defined as the radius; it is not the diameter, is the radius. Gaussian beam spot size is represented by this radius.

But, how do I define this radius that is the question because Gaussian beam if you look at we take this cross sectional area then it is the profile would be extending from minus infinity to plus infinity middle one is I naught. So, at this point intensity is I naught, but intensity is decaying. So, question is what point in space will define that this is my radius, that is the question we are going to address in this lecture.

Size of the beam at any point as a function of z is given by this equation which means the size of the beam will depend on the where we are measuring in the Z-axis and here in this expression we have seen previously that z naught is nothing, but Rayleigh length ah. Rayleigh length is the distance along the propagation direction of a beam from the waist to the place where the area of the cross section is doubled. So, this is something which we have already gone over, we have understood this general feature of the Gaussian beam already.

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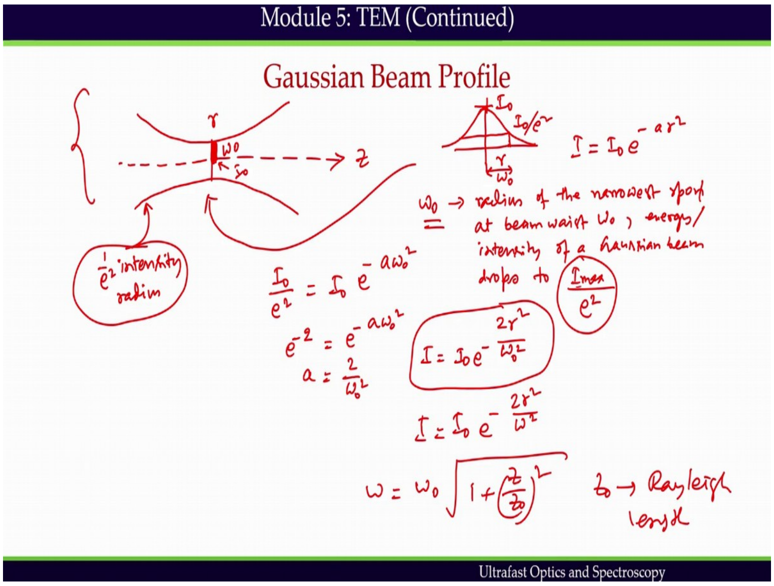

So, now, we will take a closer look at the beam waist and its meaning. What we have drawn here is that this is a profile we have drawn, the meaning of this profile I have not been discussed yet. We said that at this point this is r and we can express this cross-sectional, area this intensity variation in this cross sectional area as a function of r or x 1 can say x also and this cross-sectional area is following in Gaussian intensity profile spatial profile. This is the I naught is the maximum intensity. This is intensity at this point is I naught.

So, one can express this profile cross-sectional profile as I naught e to the power minus a r square and we said that there is a definition of beam waist which is nothing, but this radius this is omega naught and omega naught is nothing, but the radius of the narrowest spot in the laser beam. But, at the same time one can define it is at beam waist w naught energy or intensity of a Gaussian beam drops to I max by e square.

So, 1 by e square times I maximum value when I get that is the point I consider it to be omega naught. That is why, although we have been define this profile, what does it mean by this line. This line is nothing, but 1 by e square intensity radius profile. So, I can write down that I naught by e square this happens when? When I have e to the power minus a omega naught square r equals omega naught.

So, I can write down e to the power minus 2 equals e to the power minus a omega naught square. So, a equals 2 by omega naught square and according to this definition I can write down I as omega sorry I naught e to the power minus 2 r square divided by omega naught square. So, this is the intensity profile expressed in terms of omega naught and what is omega naught? It is the radius of the narrowest spot which is defined by the point where the intensity drops to the intensity drops to 1 by e square times of times e I max. (Please look at the slides for mathematical expressions)

So, let us say this is the point where intensity is I naught by e square. This point is called omega naught, that is the way we give the definition and in general as omega will change in general we can also write down 2r square by omega square where omega is the size of the beam which will depend on beam waist, narrowest size multiplied by 1 plus z by z naught square z naught is the Rayleigh length. So, this is the definition. So, whenever we show the Gaussian profile like this we this profile indicates that we are drawing 1 by e square intensity radius profile.

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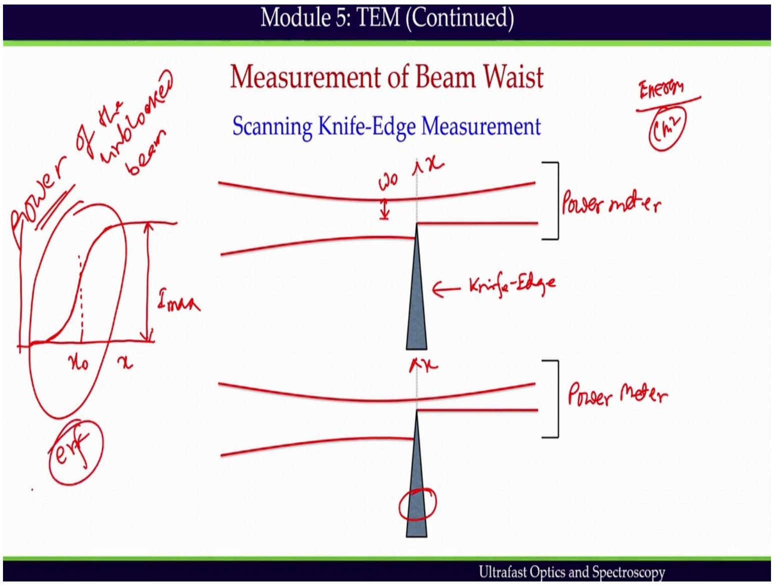

For many laser applications including ultrafast spectroscopy measuring beam with becomes important for the lesser output characterization because generally we express the fluence as energy per cross-sectional area and this cross-sectional area depends on how we are defining the width. I said this is my width and I can express this width as the beam waist, the w naught. So, question is how do I get the w value at different point of the laser beam?

A technique called the knife edge method is widely used for measuring the laser beam width. Its main advantage is at the simplicity and possibility to use for wide range of radiation wave lengths and powers. The measurement principle is depicted here, the beam is progressively blocked by a knife edge. We take a knife edge, this is called knife edge. We can use a razor blade for the knife edge and we progressively block the beam, we scan it along the x-axis and this is the power meter. We measure the power as we scan the as we block the beam by a knife edge and the transmitted power is measured as a function of position across the beam as a function of x as shown here.

Therefore, in the position where the beam is not blocked by the knife edge the maximum power is measured on the other hand the major power is minimum typically zero when the blade covers the whole beam. In this experimental setup the knife edge measurement setup we use a translational stage. So, this knife edge is translated along the x-axis and we at different position we measure the power which is not blocked.

Now, this is the way we get the knife edge measurement and if we measure it, then experimentally what we see? We see a typical profile something like this. This point is x naught, this is the x and we start with 0 because we have blocked it or vice versa. If we do not block it we start it maximum, then slowly this is the I max, slowly it drops down and then goes to zero.

And, the typical nature of this is the power meter we are plotting the power unblock power of the unblocked beam. So, if we plot them then we get this profile. This profile is equivalent to an error function and how it is related to error function that is exactly what we are going to discuss next.

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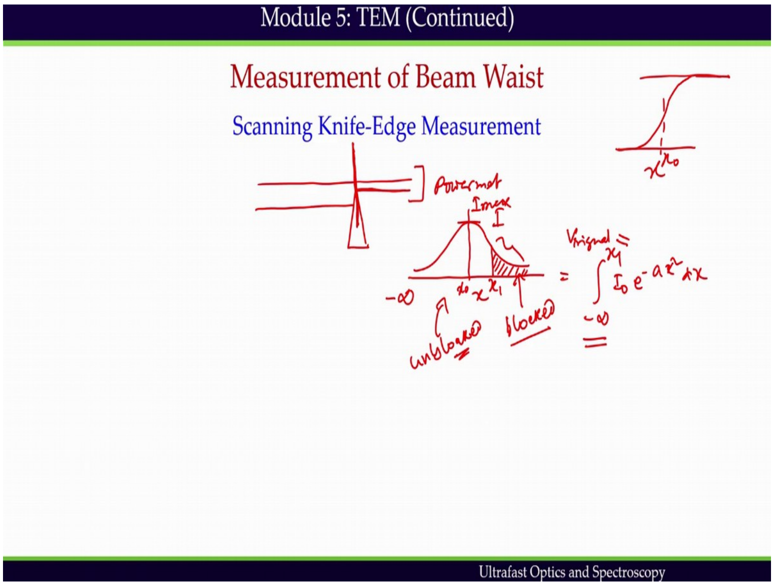

We obtain a curve similar to the one depicted in this figure. This kind of curve we obtain in this measurement as a function of x and this is x naught. And, as many laser applications we use Gaussian beam TEM 00 more we use the power detected by the scanning edge measurement can be easily estimated from the area under a portion of the Gaussian function.

So, basically what is going on I have a beam propagating along this and a knife edge is cutting the beam and allowing the beam this much beam at a particular position at a particular position and I am checking with the power meter. So, this cross-sectional area can be expressed as a Gaussian beam, special profile that we have already seen and maximum is max I.

So, when we are blocking the beam we are let us say blocking up to this part this shadow regime is blocked and if the shadow regime is blocked, this middle is x naught. If the shadow regime is blocked then what we are doing power meter is finding this is let us say x 1 position. So, what power meter is doing V signal is nothing, but minus infinity to x 1 integration of I naught e to the power minus a x square dx.

So, this measurement estimates the area under the portion of the Gaussian beam which is not blocked yet. So, this shadow part is blocked and this part is unblocked and with the power meter we are trying to find out the total energy and that is why instead of taking the limit from minus infinity to plus infinity we are taking limit from minus infinity to the x 1 point.

Let us assume that a knife edge blocks everything at x equals x 1. So, this part is blocked let us say. If this part is blocked then the power meter records energy of the beam which is unblocked. This is nothing, but the area of the Gaussian from minus infinity to x 1 because the power detected by the power meter is directly proportional to the area under the Gaussian from minus infinity to x 1.

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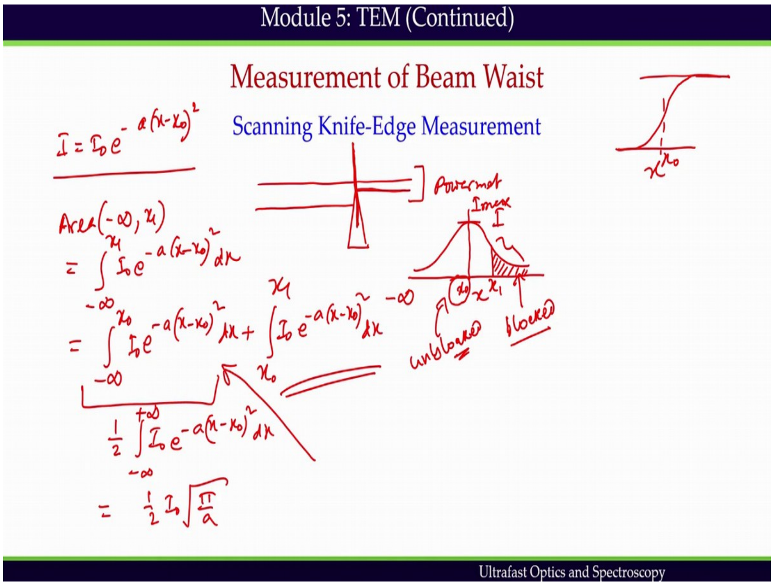

So, instead of this general form I will use this general form which is suitable for the present work is I equals I naught e to the power minus a x minus x naught square. So, this is representing the Gaussian profile which is centered at x naught. The area under the unblock portion is now given by the area under the unblock portion which is minus infinity to x 1 this is given by minus infinity to x 1 I naught e to the power minus a x minus x naught square dx, that is the area. (Please look at the slides for mathematical expressions)

I split this into two regimes minus infinity to x naught I naught e to the power minus a x minus x naught square dx another regime is x naught to x 1 I naught e to the power minus a x minus x naught square dx. So, this is a two regimes I have separated; I have separated this two regimes because I can make use of this regime, minus infinity to x naught that is minus infinity to the maximum. (Please look at the slides for mathematical expressions)

It is nothing, but because it is a symmetric function it is nothing, but this part is nothing, but half of minus infinity to plus infinity total area it is nothing, but e to the power minus a x minus x x naught squared dx this part which is half I naught square root of pi by a standard integral we are using. (Please look at the slides for mathematical expressions)

So, it is a so, the first part this integral is giving me half of the total area under the curve, but next task would be to find out this area. So, question is how do we find out this area? You know to find out this area we have to introduce error function.

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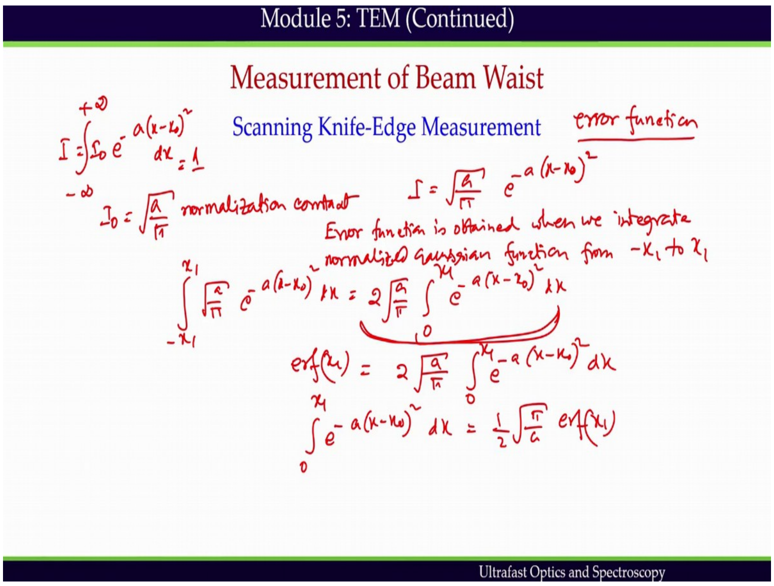


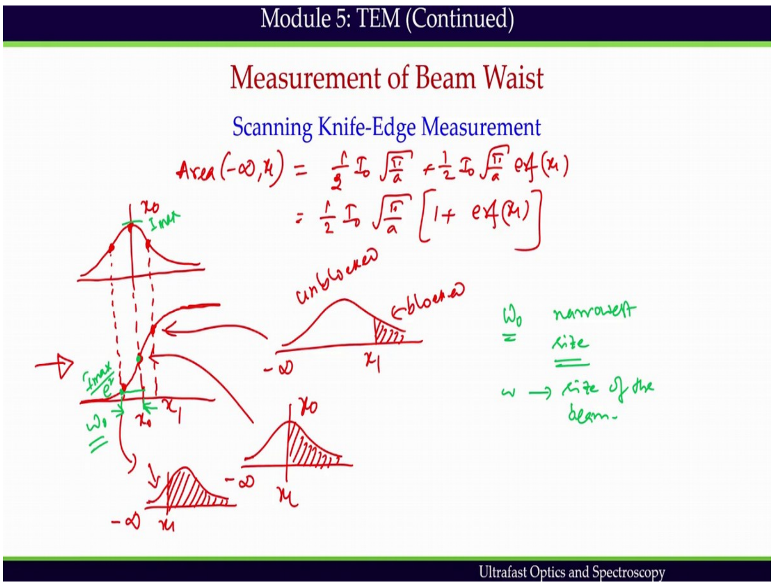
To get error function we have to consider normalize Gaussian function because error function is related to the normalized Gaussian function. So, I will first normalize our

Gaussian function e to the power minus a x minus x naught square, this can be normalized. If I normalize it, then I will integrate it minus infinity to plus infinity and area under the curve integration is going to be 1.

So, this gives me directly I naught equals square root of a by pi, this is normalization constant; this is the normalization constant ok. So, a normalize Gaussian function of the form which we have taken can be written as a by pi e to the power minus a x minus x naught whole square. This error function is obtained when we integrate normalized Gaussian function from minus x 1 to plus x 1.

So, I have to now integrate this normalize Gaussian function minus x 1 to x 1 square root of a by pi e to the power minus a x minus x naught whole square dx which is nothing, but 2 multiplied by 0 to x 1 because it is a symmetric function. So, twice the half of this function which is it is nothing, but a x minus x naught whole square dx or we can write down as square root of e by pi minus 0 to x 1. So, this function is called error function. (Please look at the slides for mathematical expressions)

So, this error function of x 1 limit is going to be up to x 1 is nothing, but 2 square root of a by pi 0 to x 1 e to the power minus a x minus x naught whole square dx, this is error function. And, if this is error function then I can write down this integral 0 to x 1 e to the power minus a x minus x naught whole square dx is nothing, but half square root of pi by a error function of x 1. (Refer Slide Time: 24:07) (Please look at the slides for mathematical expressions)



So, I can now given this fact and how does it look like? I will just show you how does it look like. So, finally, we get the area which we are interested in minus infinity to x 1 is nothing, but half I naught square root of pi by a plus half I naught square root of pi by a error function of x 1 which is nothing, but half I naught pi by a 1 plus error function of x 1.

So, I have a Gaussian special profile like this which is centered at x naught and I am now plotting up as a function of x 1; x 1 can be anything. So, this if you plot x x 1 error function should behave like this. So, this is x naught. So, at this point, how do I get this point? At this point the value of error function indicates this integration up to x 1. So, integration of this.

So, basically this shaded regime is the blocked beam; only beam which is unblocked is this part and that is why you have taken integration from minus infinity to x 1 and I got this point. I got this point by taking integration from minus infinity to plus infinity sorry minus infinity to x naught. So, this regime is blocked.

On the other hand, I get another point here. How do I get this point? I get this point by taking integration from minus infinity to x 1 this is x 1; this is x 1. So, this regime is blocked and this regime is unblocked and this is the way we get this error function and question is how do we define? So, basically in the experiment we get this error function behavior, this is the measurement we see and once we get the measurement done we can find out from x naught to x naught is where the maximum intensity I should have I should the maximum of the I max should appear this is where I max should appear.

And, so all we need to do is that we have to get from this I max we have to get e square by I max point. Let us say e square by I max point is here, at this point I have I max divided by e square. So, in the end I will be able to define this as omega naught from the experiment. This omega naught is nothing, but the narrowest size and if it is not omega naught, then it is omega this is the size of the beam.

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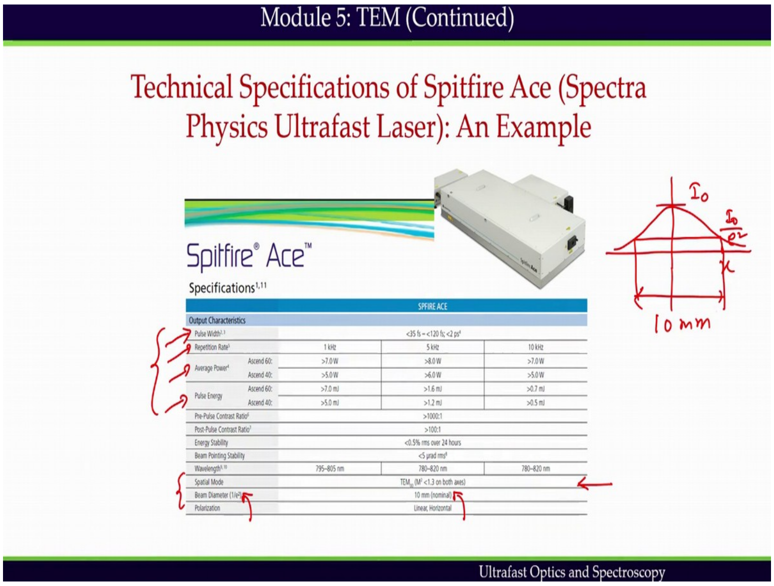

So, with this we will take a look at the specification. In module 2 we have looked at different specification of the laser system manufactured by different vendors; we have gone over the meaning of pulse with reputation rate, average power, pulse energy already we are familiar with this features of the laser system ultra fast laser system.

Now, based on the present module we can also understand two features to additional features of the femtosecond laser beam. This feature suggesting that what kind of mode quality we have that is TEM 00 mode which is frequently used in the laser system, but in the in addition to that we are defining beam diameter as 1 by e square.

So, when you say that beam diameter is 10 mm it means that it is the 1 by e square intensity radius we are talking about. So, basically we have an intensity profile at a particular point that is the x cross-sectional profile and this is the maximum I naught. We are going to take I naught by e square and this diameter this is the radius and this is the diameter. So, we are talking about when we take when say 10 mm 1 by e square beam diameter is suggest this.

So, with this we have come to the end of this module. Here we have discussed little more details of the TEM 00 mode, how to measure them diameter of an ultrafast laser beam and how to express it and we see that error function is the response which we see in the knife edge measurement. We will meet again for the next module.