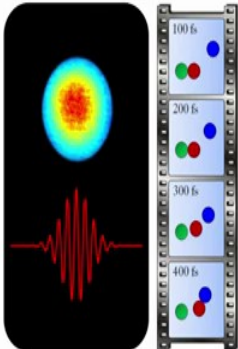


Ultrafast Optics and Spectroscopy
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Lecture – 18
Transverse Electromagnetic Mode

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Module 5: Transverse Electromagnetic Mode (TEM) of Ultrafast Pulse



Ultrafast Optics and Spectroscopy

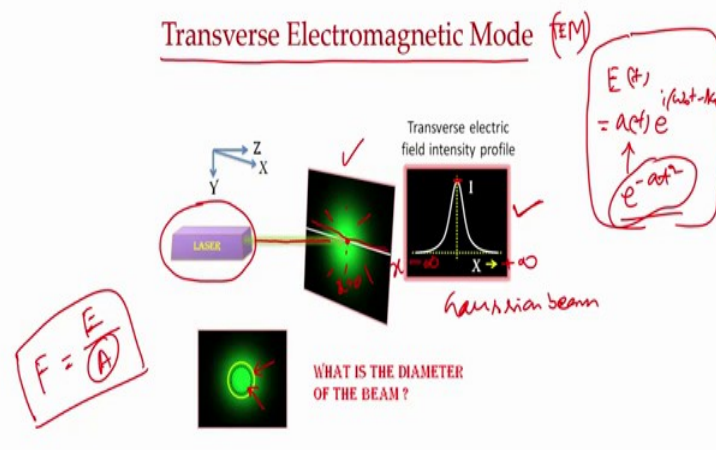
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Welcome to module 5 of the course Ultrafast Optics and Spectroscopy. In this module we will study Transverse Electromagnetic Mode of Ultrafast laser.

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Module 5: TEM

Transverse Electromagnetic Mode (TEM)



Transverse electric field intensity profile

Gaussian beam

WHAT IS THE DIAMETER OF THE BEAM?

$E(r) = A(r) e^{i(\omega t - k_z z)}$

$F = \frac{E}{A}$

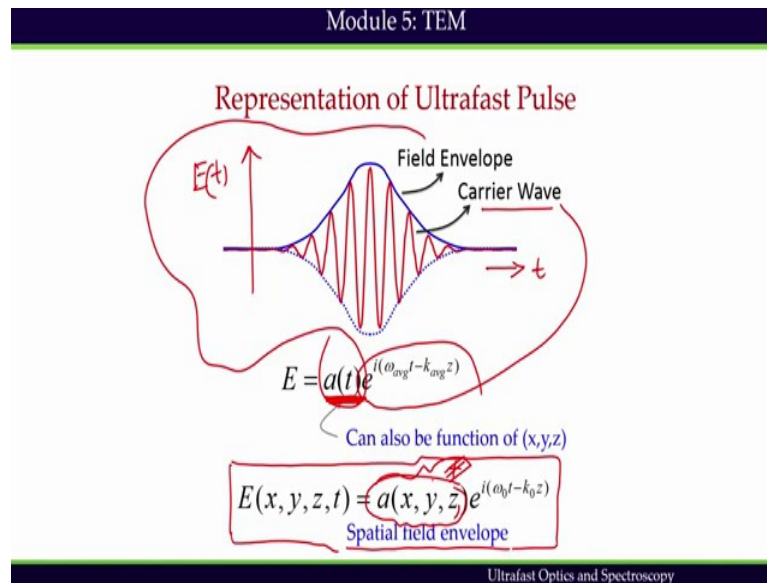
Often in ultra fast laser spectroscopy lab, we represent propagation of laser beam with a ray. Rays are idealized or oversimplified model of the laser beam let us say I have a source and we show that there is a line we show that laser is propagating along this direction. But in reality a laser emits a beam of finite width or diameter. If a piece of paper is placed on the laser beam just like the one here, we get a cross sectional area illuminated by the laser beam what we observe is called transverse electromagnetic mode.

This is also in short is also called TEM. When transverse electric field intensity of the laser beam can be expressed by a Gaussian function, which means if I take a cross section of what I am observing and plot this cross section as a function of x this is to be x equals 0 then the intensity profile we find the maximum of the intensity occurs at the center, here and the wings are less intense these are less intense regime.

Now, if this kind of intensity profile can be expressed by Gaussian function, the beam is called Gaussian beam. Now we have to distinguish here if this beam is Gaussian beam this is different from the Gaussian envelop which you talked about, electric field in time (Refer Time: 02:58) was expressed by $e^{-\frac{1}{2}(\omega t - kz)^2}$. Sometimes we said that electric field envelope can be expressed as $e^{-\frac{1}{2}at^2}$, this is time domain description of the pulse and we said that field envelop can be expressed or represented by a Gaussian function in time, but this Gaussian function is in space is a spatial profile. (Please look at the slides for mathematical expressions)

But we have to remember that even if it is represented by a Gaussian beam Gaussian function, we know that the Gaussian beam spreads from minus infinity to plus infinity then how do we define diameter of a Gaussian beam? Should we consider this circle yellow circle or should we consider this black circle? In many laser based experiments finding out fluence which is defined as energy power cross sectional area fluence is defined by energy bar cross sectional area and which depends on the size of the laser beam that is cross sectional area becomes very important that is why it is important to define the diameter at the beam.

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We have already seen that a pulse can be decomposed into carrier wave and the field envelop as shown here. So, this is your time axis and this is your field temporal field and a pulse has carrier wave that is represented by this complex component and it has a field envelop as well which is expressed by this $a(t)$. So, far this field envelop is expressed as a function of time, but we have to remember that this field envelop can also be a function of spatial coordinate this is called spatial field envelop.

If we express this envelop function there should be another t here. So, it depends on time as well as space. So, the final expression for more appropriate electric field of a pulse is this one where the envelope function is a function of space as well as time. But to simplify the problem for the present context we will consider will turn off this time dependents and we will just check this spatial dependent field envelop.

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Module 5: TEM

Paraxial Wave Equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E(x, y, z, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E(x, y, z, t) \quad \leftarrow$$

Wave equation in vacuum

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left[a(x, y, z) e^{i(\omega_0 t - k_0 z)} \right] = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[a(x, y, z) e^{i(\omega_0 t - k_0 z)} \right]$$

Here,

$$\left\{ \begin{aligned} \frac{\partial^2}{\partial z^2} \left[a(x, y, z) e^{i(\omega_0 t - k_0 z)} \right] &= \frac{\partial}{\partial z} \left[\frac{\partial a}{\partial z} e^{i(\omega_0 t - k_0 z)} - i k_0 a e^{i(\omega_0 t - k_0 z)} \right] \\ &= \frac{\partial^2 a}{\partial z^2} e^{i(\omega_0 t - k_0 z)} - i k_0 \frac{\partial a}{\partial z} e^{i(\omega_0 t - k_0 z)} - i k_0 \frac{\partial a}{\partial z} e^{i(\omega_0 t - k_0 z)} - k_0^2 a e^{i(\omega_0 t - k_0 z)} \end{aligned} \right.$$

and, $\frac{\partial^2}{\partial t^2} \left[a(x, y, z) e^{i(\omega_0 t - k_0 z)} \right] = -\omega_0^2 a e^{i(\omega_0 t - k_0 z)}$

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If we take this equation and plug that in this equation, this is the wave equation in vacuum then we find this expression. And in order to solve this equation we have to get the second derivative with respect to space if we get the second derivatives and with respect to time also second derivative is very simple math one can go over it quickly. Finally, we get this expression.

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Module 5: TEM

Representation of Ultrafast Pulse under Paraxial Approximation

$$\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + \frac{\partial^2 a}{\partial z^2} \right) e^{i(\omega_0 t - k_0 z)} - 2i k_0 \frac{\partial a}{\partial z} e^{i(\omega_0 t - k_0 z)} - k_0^2 a e^{i(\omega_0 t - k_0 z)} = -\frac{\omega_0^2}{c^2} a e^{i(\omega_0 t - k_0 z)} \quad \checkmark$$

$$\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + \frac{\partial^2 a}{\partial z^2} \right) e^{i(\omega_0 t - k_0 z)} - 2i k_0 \frac{\partial a}{\partial z} e^{i(\omega_0 t - k_0 z)} = 0$$

because, $\frac{\omega_0^2}{c^2} = k_0^2$

$$\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + \frac{\partial^2 a}{\partial z^2} \right) - 2i k_0 \frac{\partial a}{\partial z} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - 2i k_0 \frac{\partial}{\partial z} \right) a(x, y, z) = 0$$

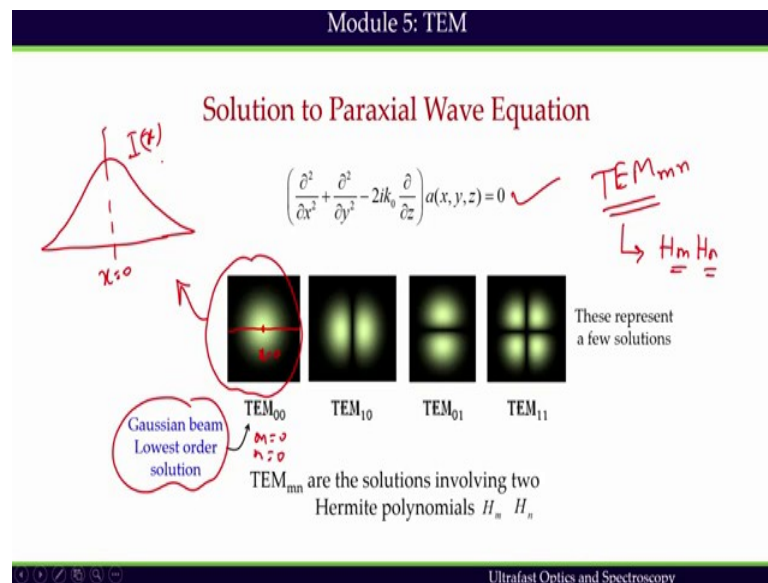
Under paraxial approximation (field envelope gradually varies along z-direction): $\frac{\partial^2 a}{\partial z^2} \ll \frac{\partial a}{\partial z}$

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And we know that in vacuum k is related to ω by this expression. So, we can plug that in and we get this expression. Now we shall make paraxial approximation which

says that field envelope gradually varies along the z axis. Therefore, we can write down the second derivative of a is much less than the fast derivative of a and that is why the second derivative term can be considered to be 0. So, we reorganize the equation and we finally, get an equation which is called paraxial wave equation.

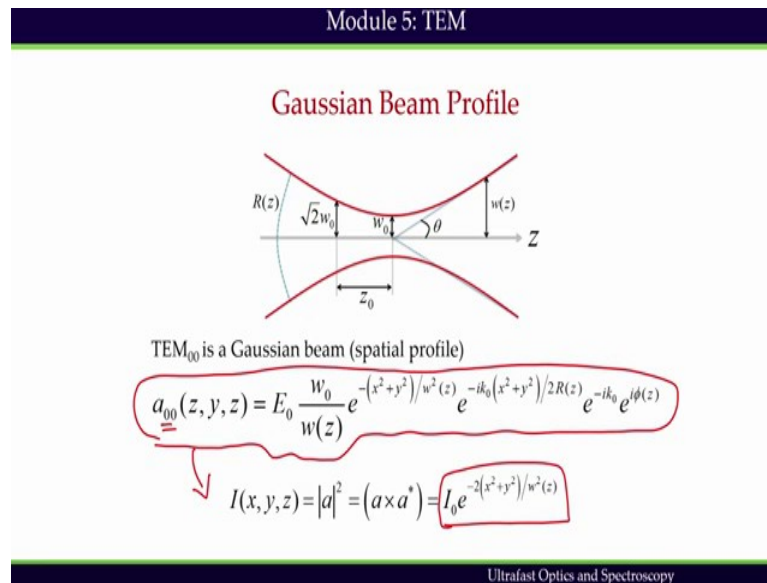
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The paraxial approximation assumes that all light waves are traveling nearly parallel to the z axis, laser beams are coherent and directional and thus laser beams typically obeys this approximation. We can solve the paraxial wave equation and we can get infinite set of solutions each one represents transverse electromagnetic mode.

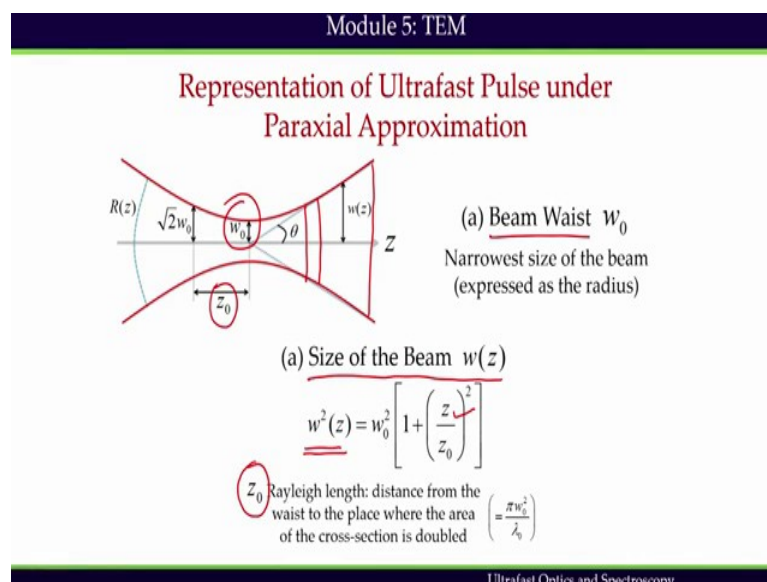
So, if you solve this equation you get TEM Transverse Electromagnetic Mode different transverse electromagnetic mode. This TEM is expressed in terms of two Hermite polynomials and this is expressed by 2 Hermite polynomials of m and n orders. The lowest order transverse electromagnetic mode we get when m equals 0 and n equals 0 and that is why this lowest order solution is called transverse electromagnetic mode 0 0 TEM 0 0 mode. This lowest order solution represents a Gaussian beam which means that if I take a cross section like this way make it as a center x equals 0 and if I plot the intensity variation as a function of time as a function of x then I get a Gaussian function. (Please look at the slides for mathematical expressions)

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The TEM 0 0 solution of the paraxial wave equation which features Gaussian beam can be written as this. So, this is the final solution for the lowest order solution a 0 0. This represents this is the this represents a Gaussian beam spatial beam and intensity of this is nothing, but square modulus of the solution, the field and we get this expression for the intensity.

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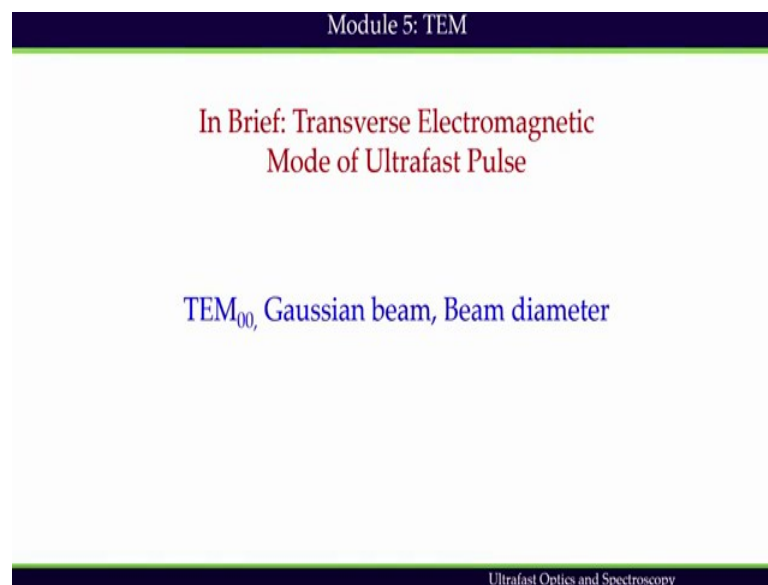


A Gaussian beam propagating along z direction is depicted here featuring some of the important parameters. The first parameter you should note is the beam waist, omega w

naught which is the narrowest size of the beam. Size of the beam is defined as the radius it is actually expressed in terms of radius Gaussian beams pore size is represented by this radius. Then we look at another parameter size of the beam, size of the beam is changing as we move along the z direction. So, it is expressed in terms of as a function of Z with the help of this expression here z naught is called Rayleigh length Rayleigh length is the distance along the propagation direction of the beam from the waist to the place where the area of the cross section is doubled.

So, this length is called Rayleigh length z naught. So, these are the characteristics of TEM 0 0 mode mostly most of the laser system which are built in the ultra fast lab they are off TEM 0 0 quality that is why you should know this characteristics of TEM 0 0 mode.

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With this we have come to the end of this talk in this module, we have discussed TEM 0 0 mode what is the source of TEM 0 0 mode how do we get that mode and what are the characteristic features of that mode. We will meet again in the next module.