Ultrafast Optics and Spectroscopy
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Lecture – 17
Nonlinear and Dispersion Effects (Continued.)

Welcome back, we are continuing module 4, Dispersion Effect. We are trying to understand the relationship between the chirp pulse and the transform limited pulse. The chirp pulse introduced due to second order spectral phase or the group velocity dispersion in the medium and previously, we have already seen that relationship, but we have not seen the full derivation now we are going to go over the full derivation.

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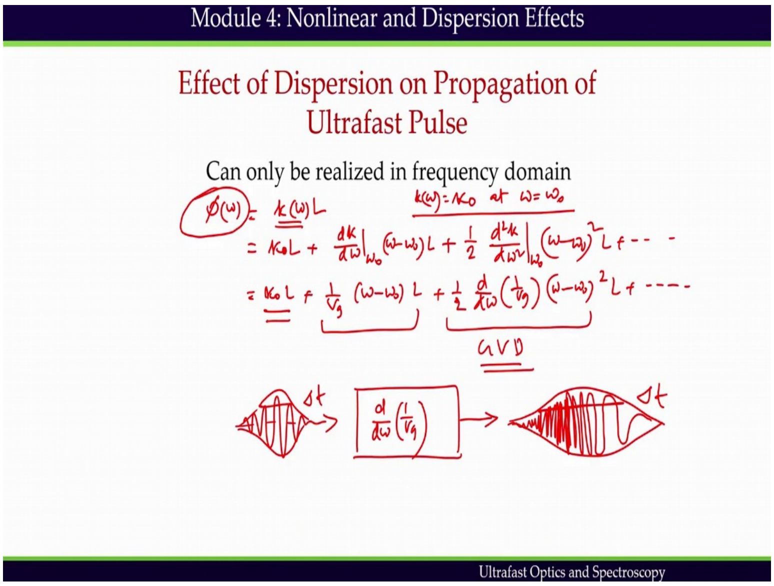

So, what we have seen is that this different component of the spectra phase is going to introduce different effects in the pulse and we are only interested in GVD group velocity dispersion effect.

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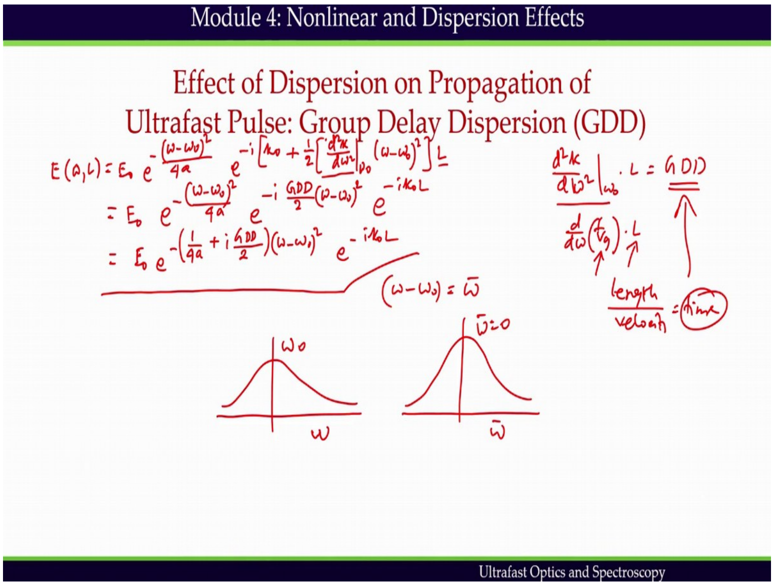

So, what will do will find out we will assume that due to propagation of the pulse through the medium only additional phase which is introduced is due to group velocity dispersion that is second order dispersion term. Then, electric field of the pulse emerging from the medium can be written as E that is E omega comma L equals E naught e to the power minus omega minus omega naught square divided by 4a multiplied by e to the power minus i, now, this is the total phase I am going to write down.(Please look at the slides for mathematical expression)

So, this k naught contribution coming from the vacuum contribution that would be always there plus there is additional phase I am going to introduce in the spectral domain, in the frequency domain and that phase is nothing, but group velocity dispersion d 2 k d omega 2 at omega naught; at omega naught multiplied by omega minus omega naught whole square multiplied by L. So, this is the new spectral phase we introduced.

Now, if we simplify this then what we get here is that this can be simplified as following we can write down as E naught e to the power minus omega minus omega naught square by 4a, then e to the power minus i I will write down GDD, what does it mean I will write down very soon, omega minus omega naught whole square. GDD is nothing, but this d 2 k d omega 2 at omega naught multiplied by L is GDD, Group Delay Dispersion. .(Please look at the slides for mathematical expression)

What does it mean? It means that this second derivative is nothing, but d d omega of 1 by V g multiplied by L. So, V g is certain kind of velocity and this L is length. So, length divided by velocity is nothing, but time. So, GDD is representing a group delay dispersion, certain kind of time it is expressing. So, what I have done here this multiplied by this term I have written as GDD divided by 2 this half is coming here and then omega minus omega naught square remaining as the one which we have in the previous expression and then I am going to write down minus i k naught L that is coming from this first term.

So, if we have this then I can rewrite this one as E naught e to the power minus 1 by 4a plus I GDD divided by 2 then multiplied by omega minus omega naught square multiplied by e to the power minus i k naught this is another simplification we have made for this expression. And we will defined omega minus omega naught that we have done previously also as omega bar this is quite a general derivation we are following. We do it because that we simplify our derivation and when we say omega what is the difference between omega and omega bar? .(Please look at the slides for mathematical expression)

If we express those the frequency domain field then it is centered at omega naught, but if we express the same field in the omega bar of domain then it is centered at as omega bar equals 0 that is the only difference we have. It is just a change of the convention that is all. It does not change any the does the shape of the spectrum. So, with this expression what we need to do is that now we know this frequency domain field and we are interested in time domain field because we want to get the intensity profile and from intensity profile we would like to know the full width half max.

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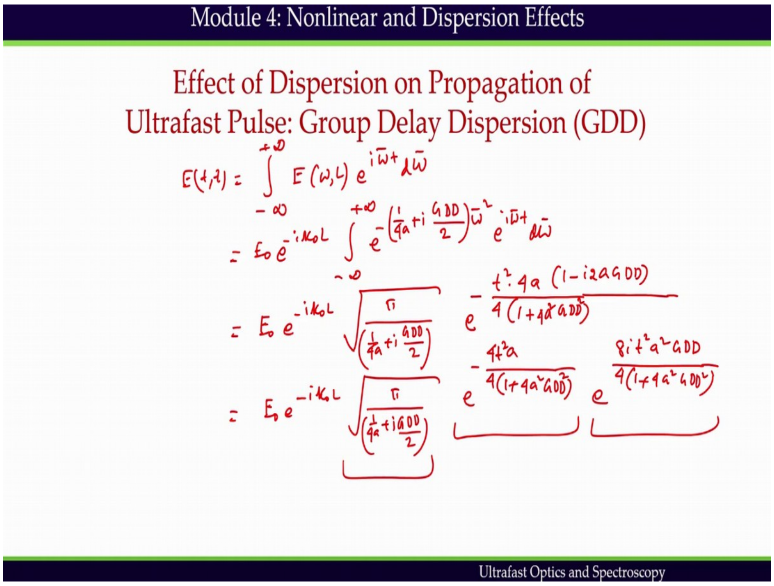

So, what we will do? We will first do the Fourier transform of that time frequency domain field to get the time domain field E t, z and Fourier transform inverse Fourier transform and that is going to be omega L e to the power i omega bar t d omega bar. So, that simplifies that this is the reason why we have taken this omega bar because if we change the variable it does not change the shape. .(Please look at the slides for mathematical expression)

So, I am changing variable is helpful for getting this integration done. So, finally, what we get is that E naught then e to the power minus i k naught L that comes out of the integral then minus infinity to plus infinity into e to the power minus 1 by 4a i GDD divided by 2 then omega bar square then e to the power i omega bar t then d omega bar. This is the expression we get. .(Please look at the slides for mathematical expression)

And if we use the standard identity then we can write down this standard Gaussian integral as e to the power minus i k naught L then square root of pi by 1 by 4a plus i GDD divided by 2 GDD divided by 2 multiplied by e to the power; e to the power minus t square multiplied by 4a divided by 4 multiplied by 1 plus i 2a GDD, this is the expression we get. And we can again simplify by multiplying 1 minus i 2a GDD and here also multiplied 1 minus i 2a GDD we multiply this. .(Please look at the slides for mathematical expression)

So, this one is going to be 1 minus of 1 this is going to be 1 plus 4a square this is going to be 4a square GDD square and this is gone and that is the way we get back this expression. And finally, what we get is that we can separate this complex part and the real part and we can get this E naught e 2 to the power minus i k naught L square root of pi by 1 by 4a plus i GDD divided by 2 multiplied by e to the power minus 4t square a divided by 4 1 plus 4a square GDD square multiplied by e to the power then 8 pi t square a square GDD divided by 4 multiplied by 1 plus 4a square GDD square. So, this is the expression we get and we are interested. So, basically what we get this is the real part, this is the imaginary part and this part also imaginary amplitude. .(Please look at the slides for mathematical expression)

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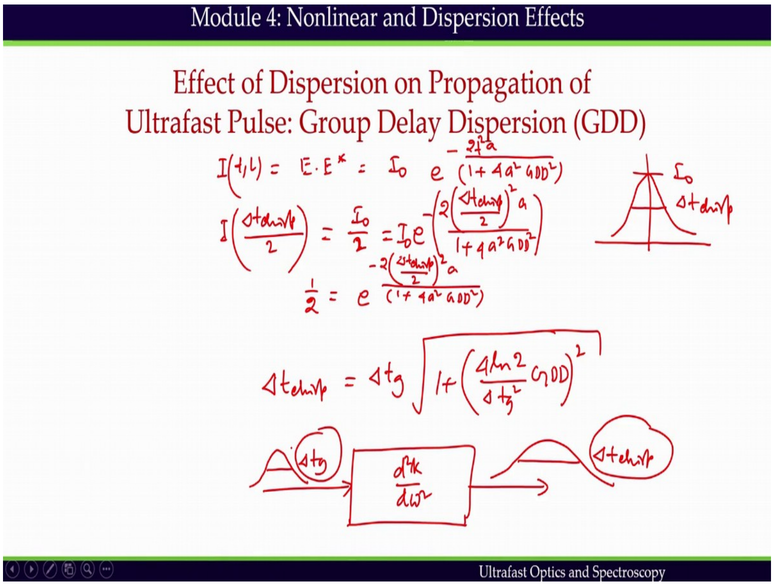

And if we take the intensity the moment we take the intensity then what we get is that I t, L we need to know intensity because pulse duration the way we express pulse duration is the full width half max of the intensity profile. So, intensity we need to know and what we get is that finally, it is nothing, but I naught I am just taking the amplitude the total amplitude is to be I naught e to the power the expression finally, get e to the power minus 2 then t square then a divided by 1 plus 4a square GDD square this is the expression what we get finally. .(Please look at the slides for mathematical expression)

And we know that the way we get the this is the time domain intensity profile of the group delayed pulse originated due to the dispersion effect and this maximum intensity is

I naught and we know that the full width half max delta t chirped this is I will call it chirped because this is a chirped pulse already we have reduced this chirped due to the dispersion second order dispersion effect and with this delta t we know that by definition I delta t chirped divided by 2 at this I get the half of this intensity which is nothing, but I naught by 2 which is nothing, but e to the power minus 2. Then instead of t we have to insert delta t chirped divided by 2 whole square a divided by 1 plus 4a square GDD square GDD square. .(Please look at the slides for mathematical expression)

So, this is the definition of full width half max. We have been giving this definition for a long time and what we can write down is that which means that it is nothing, but this is I naught. So, it is nothing, but half equals e to the power minus 2 delta t chirped divided by 2 whole square a divided by 1 plus 4a square GDD square. So, if we simplify it then we can write down finally, that delta t chirp is going to be delta t Gaussian beam delta t g square root of 1 plus 4 ln 2 by delta t g square GDD this is whole square. So, this is the expression we get. (Please look at the slides for mathematical expression)

So, what we write down here is that what we write down here is that if we have a medium and this medium is introducing this second order spectral phase that is d 2 k d omega 2 and if I plot the intensity profile of the transformative pulse which is delta t g Gaussian pulse we have considered, then I get an elongated pulse and this elongated pulse is delta t chirp and what we have got is that a relationship between delta t chirp and delta t g here in this expression. (Please look at the slides for mathematical expression)

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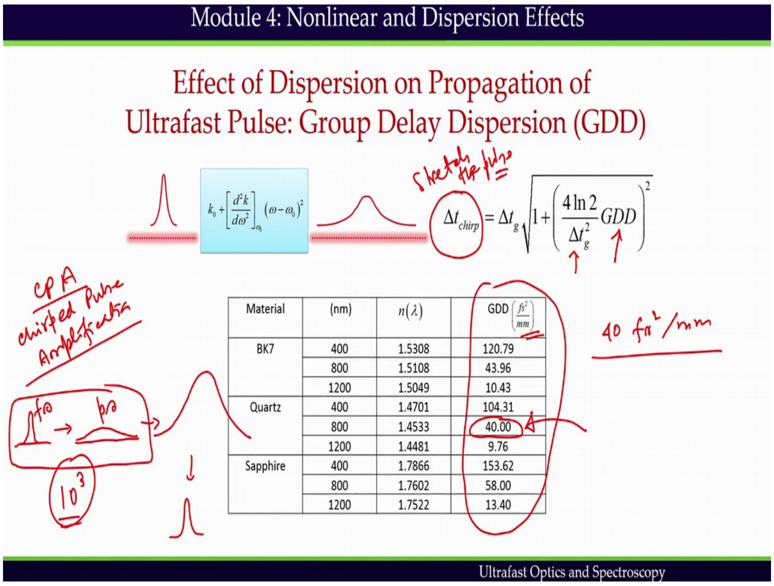

So, what we see here is that in this expression all these terms GDD this is the value of GDD in femtosecond square per mm, this is the unit of GDD. Now, per mm we are introducing this amount of GDD for different material. For example, quartz 800 nanometer it can introduce 40 centimeter 40 femtosecond square 40 femtosecond square per mm GDD I will introduce. So, all the terms are positive and that is why this will always stretch it will stretch the pulse always stretch the pulse that is the that is this is unavoidable. Any time, any ultra fast pass propagating through the medium it will stretch the pulse.

Now, question is if we try to look at the one simple example nearly all high power ultra fast laser systems make use of a technique called chirped pulse amplification, that is the idea is called chirped pulse amplification CPA. We will introduce this chirped pulse amplification idea in the construction of the ultrafast laser module and the CPA this chirped pulse amplification. What does it mean? It means that I have a short pulse very short pulse I am just showing the frequency intensity profile.

A short pulse is stretched like this intensity has to be conserved the sorry total energy has to be conserved that is why intensity massive intensity has dropped down I have stressed it. Then, I should amplify the pulse and then I can compress the pulse and this is the way amplification should be done. CPA scheme can increase the energy of a short pulse while avoiding very high peak power in the laser amplification process. This is done by lengthening the duration of the pulse before amplification. Before injection into the amplifier short pulse is stretched in time by introducing a frequency chirp into the pulse which increases the duration of the pulse by a factor of 10 to the power 3 to 10 to the power 4.

So, this stretching step like if we have femtosecond pulse we make it picosecond pulse. So, 10 to the power 3 times longer pulse we create first and then we amplify, that is the basic idea of chirped pulse amplification. This chirped pulse amplification is very important to create high power laser because if we directly introduce a short pulse into the amplifier due to peak power it will damage the optical mediums. That is why we have to reduce the peak power and in order to reduce the peak power we have to elongate the pulse or stretch the pulse and there are or in other words we have to introduce the chirp.

So, we deliberately introduced this chirp to elongate a pulse and then amplify and in this scheme CPA scheme; CPA scheme is very very commonly use scheme to do synthesized high power laser system a laser pulses. Now, chirp pulse can be obtained, we know that it can be obtained simplify propagating a short pulse through optical medium. So, one can suggest that we want to stretch the pulse, we can just propagate the pulse short pulse through the media and such as let us say quash medium.

Now, the problem is that that suggestion can be a good suggestion, but the problem is that material dispersion does not introduce a big chirp. We need we need to have big chirp, so that we can increase the pulse duration by a factor of 10 to the power 3. So, what we will do; we will do a simple calculation.

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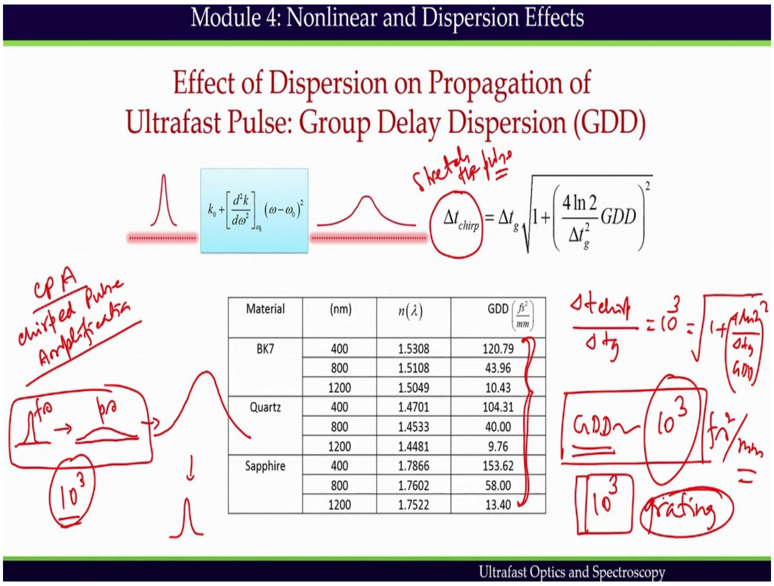

Let us say if I would want to get one 10 to the power so, basically this ratio delta t chirp divided by delta t g this ratio has to be 10 to the power 3 that is the expectation we have. Now, in order to have this 10 to the power 3 ratio what kind of GDD I need that is the question we have and if the GDD is comparable to the values given in this table, then we can select one of the material to increase the pulse duration.

So, let us check if 10 to the power 3 is the ratio which we are looking at then is going to be equivalent to 1 plus 4 ln 2 by delta t g GDD square. So, what we have is that approximately we need to have 10 to the power 3 multiplied by 2 equals approximately we need, this GDD is going to be also 10 to the power 3 times. <u>(Please look at the slides for mathematical expression)</u>

So, in order to achieve a 10 to the power 3 factor stretching due to dispersion I need equivalent amount of GDD and we see that material dispersion does not give me such a high GDD. I need 10 to the power 3 femtoseconds square per mm kind of GDD from a medium. No material can actually produce this amount of GDD to achieve the desired pulse stretching in CPA scheme. We have to use a grating because only grating can provide this amount of GDD.

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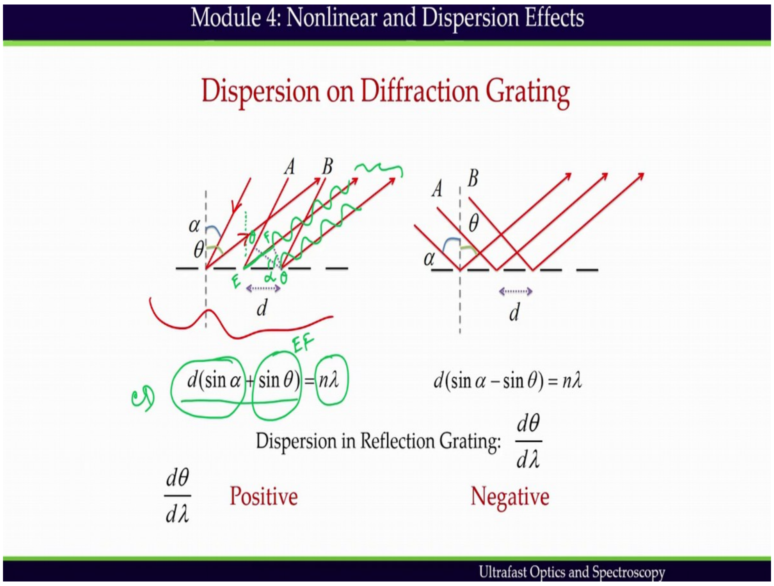

So, we will look at what is going on with grating. A diffraction grating is an optical element that possesses an arrangement of a large number of parallel, equidistant and closely spaced slits of same width which are placed side by side as shown here in this let us say figure. And, slit sizes are comparable to the wavelength of the light. Diffraction gratings are manufactured by ruling liens on a well polished thin uniform sheet of glass or reflecting metal surface.

The number of grooves which are drawn may vary from 15,000 to 30,000 per inch or even more. The grooves scatter light and acts like opaque substance while un-disturbed polished part behave like slit. Similar to a prism a diffraction grating disperses light composed of different wavelengths. As depicted here in this figure alpha is the incident angle and theta is the diffraction angle.

Now, ray A we are considering two different rays A and B. A has to travel an extra distance in comparison; in comparison with ray B in order to in order for constructive interference to occur. So, what is going on this two beam can interact constructively if I can have constructive interference only when only when this extra distance travelled by A this is the extra distance travel by A has to be; has to be n lambda to in order to have that constructive interference. And, how do we get that length? We have to get that, this is the extra length which will be traveled by the beam A.

So, we have to get that length we can get this length by employing trigonometry. We know that this is alpha, this angle is alpha; if this angle is alpha, then this angle is going to be 90 degree minus alpha. So, this angle is going to be now alpha. Similarly, if this angle is beta, if this angle is theta, then this angle is going to be 90 minus theta and because it is 90 minus theta, this angle is going to be theta. So, if these two angles are known then we will be able to find out what would be the extra distance because this is this distance if I consider this is let us say C, D distance CD; the CD distance is nothing, but d sin alpha this is CD distance and this EF distance is nothing, but d sin theta, this is EF distance. (Please look at the slide)

So, the total distance extra distance traveled by A is going to be d sin alpha plus sin theta and that has to be equal to the n lambda, in order to have the constructive interference between two diffracted beam A and B.

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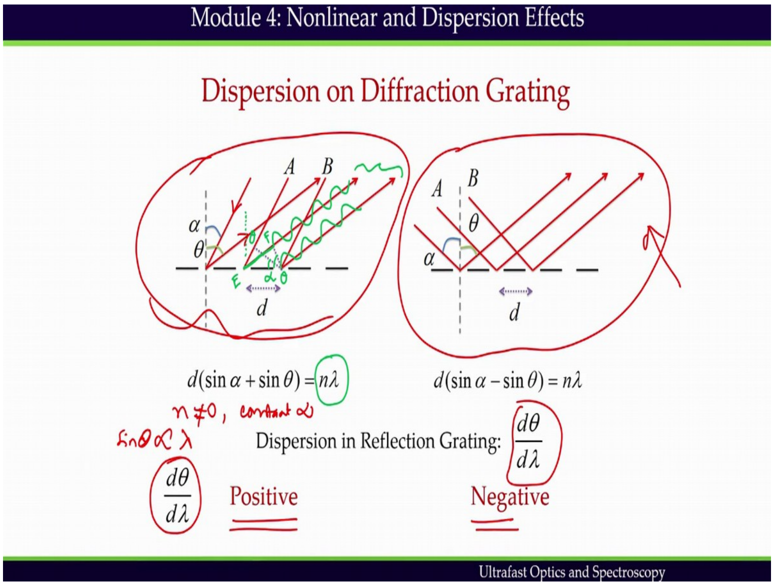

And, for n naught equals 0 is it means that if we look at the higher order term n naught equals 0 for a constant alpha what we get? We get that sin theta is going to be proportional to lambda or in other words d theta d lambda is going to be positive. Similarly, we can have this expression what is the difference between this geometry and this geometry? In this geometry both rays A and B are on the same side of the surface normal, but in this configuration both rays are on the opposite side of the surface normal.

If they are an opposite side of the surface normal we can prove that this d theta d lambda is going to be negative.

So, depending on which side they are it can be positive and negative. And, this d theta d lambda is nothing, but dispersion which means the angle will change depending on the wavelength angle of this angle of the diffraction. So, this theta will depend on the lambda.

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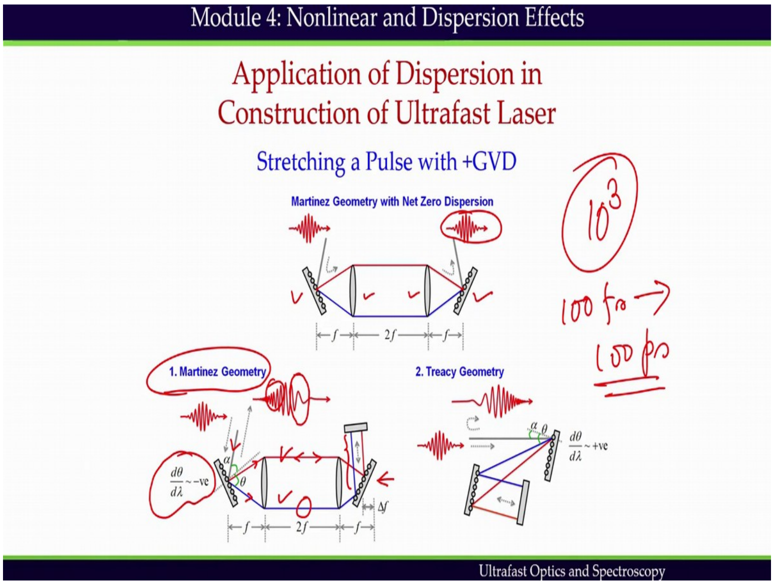

So, with this idea one can stretch a pulse and simply by we will discuss this stretching part in the when we will discuss the construction of ultrafast laser system, but we can quickly take a look at it. In the stretching part, one can actually think of having two gratings facing each other and two lenses. And gratings are placed at the focal point of these lenses. So, what will happen if the pulse is entering this grating pair then because they are at the this grating positions are at the focal point of these two lenses they will not experience different color components will not experience different path length and that is why we will get back the transform limited pulse.

But, if we slightly pushed this grating here in the Martinez configuration; we will look at Martinez configuration first. If we push it towards the lens then what will happen is clearly shown that the red part is going to be traveling shorter distance when is coming out and that is why the low frequency component is in the front part of the pulse and blue component which is the higher frequency component it is traveling longer distance. That is why it is little bit delayed with respect to the red component.

And why we know? We know that this d theta d lambda is going to be negative is going to be negative because the input beam and diffracted beam are there on the opposite side of the surface normal and that is why it is going to be negative. And that is the reason why this red part should be here and blue part should be here. So, this is the; this is the way one can stretch a pulse and the stretching when we are doing the stretching with this kind of gritting configuration one can achieve 10 to the power 3 factor of stretching in the pulse which means that if I have 100 femtosecond pulse which can be stretched to 100 picosecond pulse with the help of this kind of configuration.

No material can induce this kind of 10 to the power 3 order of the 10 to the 3 factor of stretching due to the dispersion effect. We have seen that the quartz is going to be almost 40 as femtosecond square per mm, that is very very small.

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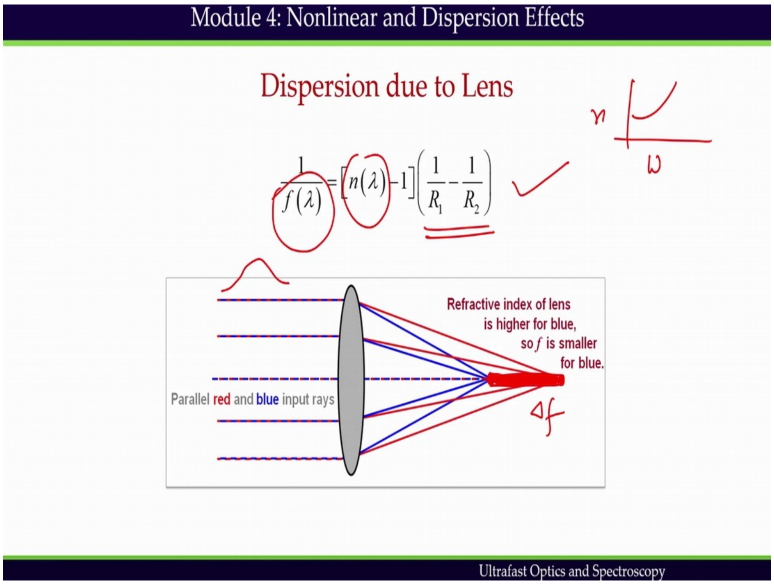

Now, so far what we have discussed is the material dispersion and how it is affecting the pulse duration. Now, we will talk about the lens how lens can actually affect the pulse duration. What we see here is that different application of ultra fast laser pulses involve focusing laser pulses and focusing on ultra fast laser pulses influences the spatial and temporal characteristics of the laser pulses in the focal region. In the thin lens any incident ray traveling parallel to the principle axis. Principle axis will refract through the lens and travels through the focal point on the opposite side of the lens.

Well known lens maker formula is given by this equation which has which clearly shows that for the constant curvature; curvature is constant for a particular lens. For a particular lens, focal length will be inversely proportional to this n lambda and we know that n refractive index will increase with respect to omega or decrease with respect to lambda.

So, what is going on? The higher frequency components will have shorter focal length and that is why we have shown here the blue part is focused quickly than the red part. So, when a pulse is propagating through the transitive lens then what we see is that the result is following the ultra fast pulse is broadband source with different frequency components. Lens maker's formula such as that different frequency components are focused at different points on the focal plane. Thus due to dispersion the focal length of a lens varies with the wavelength. This variation is called chromatic aberration. This results in an expansion of the focal region along the optical axis.

So, basically my energy is now is distributed in this focal region. It is not focused at a particular point, but over a region in the on the optical axis. This distributes the focused energy over a large region than desired. So, what will do next our task is to find out what would be the delta f associated with it during this; during this process.

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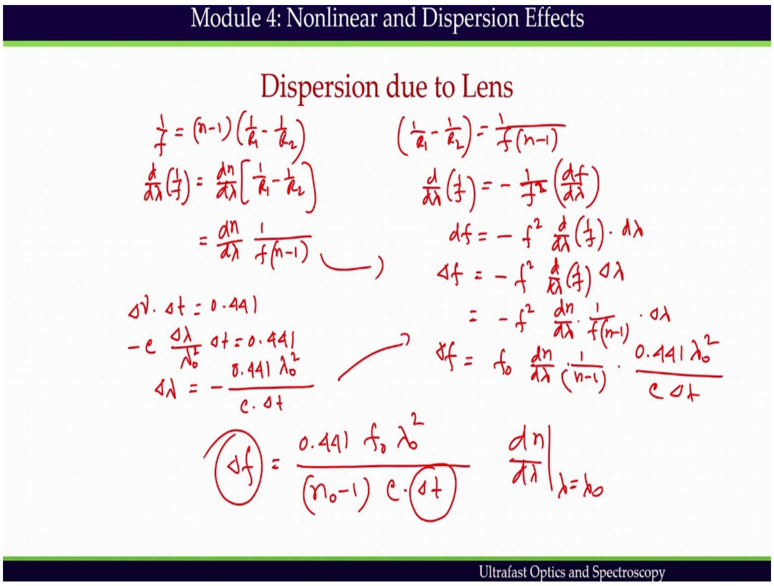

Now, this is exactly what we are trying to find out. So, what we will do is that we know that lens makers formula is nothing, but n minus 1 1 by R 1 plus 1 by R 2. And if we take the derivative with respect to d lambda then what we get is that dn d lambda then 1 by R 1 minus 1 by R 2. And we also know that 1 by R 1 plus 1 by R 2 this is this sorry, this is minus may 1 by R 1 minus 1 by R 2 this can be written as 1 by f multiplied by n minus 1. So, I can replace this here dn d lambda multiplied by 1 by f n minus 1 I can write down this one. (Please look at the slides for mathematical expression)

Now, we can this part left hand side can be written as d d lambda of 1 by f this can be rewritten as 1 by f square df d lambda this is simple derivative of this. So, I can write down d f is nothing, but d f is nothing, but minus f square d d lambda of 1 by f multiplied by d lambda. And we can express as if this can this can be rewritten as d d lambda of 1 by f delta lambda if we consider delta with. So, I can plug that in here and I can get this minus f square this derivative is nothing, but dn d lambda multiplied by 1 by f n minus 1 multiplied by delta lambda.

Now, we know that delta nu multiplied by delta t equals 0.441 that is the time bandwidth product for a Gaussian profile. So, this can be rewritten as c delta lambda by lambda naught square multiplied by delta t equals 0.441. This is we have seen this kind of derivation previously. So, delta lambda we are expressing the time bandwidth product in terms of delta lambda delta t which is nothing, but 0.441 multiplied by delta lambda square divided sorry, lambda naught square divided by c multiplied by delta t which can be plugged in here. (Please look at the slides for mathematical expression)

So, finally, delta f is nothing, but I have minus this minus and this minus both are cancelling out. So, I get f naught a sorry f naught multiplied by dn d lambda multiplied by 1 by n minus 1 multiplied by 0.441 lambda naught square divided by c delta t. So, final expression of delta f the spread of the focal point is going to be 0.441 f naught lambda naught square divided by n naught minus 1 c delta t dn d lambda at lambda equals lambda naught center wavelength. So, this is suggesting that delta f the spread of focal point focal length will be inversely depend on delta t. So, shorter pulse will have much longer spread of the energy over the focal point on the axis optic axis optical axis.

So, with this we have come to the end of this module. In this module, we have one more time gone over few important non-linear effects and dispersion effects which are experienced by the ultrafast pulse when the pulse is propagating through the dielectric medium. We have discussed lens effect and group velocity dispersion one more time we have discussed and we have also discussed the continuum generation. We will meet again for the next module.