Ultrafast Optics and Spectroscopy

Dr. Atanu Bhattacharya
Department of Inorganic and Physical Chemistry
Indian Institute of Science, Bengaluru

Lecture - 16
Nonlinear and Dispersion Effects (Continued.)

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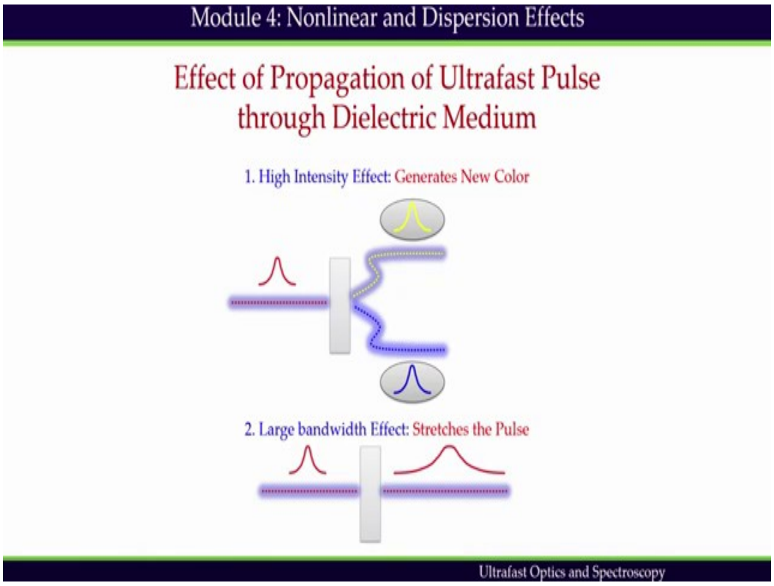

Welcome back to module 4. In this module, we will be continue more discussion on Nonlinear Effects and Dispersion Effects, which originates due to propagation of ultrafast pulse in dielectric medium. We have seen already that when an ultrafast pulse propagates through dielectric medium, it experiences both nonlinear and dispersion effects.

We have also seen that nonlinear effects can be realized in the time domain, but dispersion effects cannot be realized in the time domain. Dispersion effects can only be realized in frequency domain. We will go over the details of a few nonlinear and dispersion effects which originates from propagation of an ultrafast pulse in a dielectric medium.

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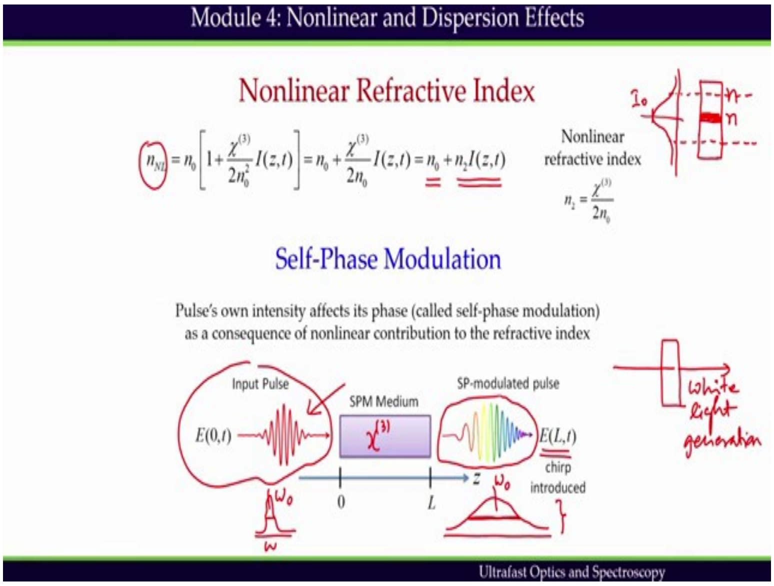

We will begin with full derivation of self phase modulation. Self-phase modulation has been introduced in our earlier lecture. This self-phase modulation originates from the intensity dependent refractive index which we have seen in the nonlinear effects already. The intensity dependent refractive index nonlinear refractive index is nothing but vacuum refractive index n naught plus this intensity dependent refractive index.

What it suggest is that, if I have a medium and an ultrafast pulse propagating will assume that the Gaussian profile it has, it means this is a special profile is Gaussian which is maximum this is I naught maximum and when its propagates through the medium what happens, the medium experiences intensity high intensity. The peak power will be very high, that is why intensity would be very high and due to this intensity dependent refractive index.

So, what does it mean it means that if we look at the pulse which is propagating through the medium if I take this dotted line and which is representing the lesser diameter of the laser beam then it is quite clear that the refractive index here at the center would be different from refractive index at the wings. This is the refractive index two different refractive index we see and this is refractive index induced intensity induced refractive index which we see in the medium is because of the third order nonlinear effect.

Now intensity dependent refractive index imposes an additional phase shift on the pulse envelope during propagation. We have seen that refractive index can depend on intensity following the equation given here, one of the consequences of this intensity dependent refractive index is self-phase modulation which you have seen and also we have seen that self focusing was also one of the consequences due to this intensity dependent refractive index.

Now self-phase modulation what happens, I have a pulse which is let us say propagating through the medium this is the input beam input pulse which is propagating through the medium third order nonlinear effect, this is chi(χ)3 nonlinear medium and due to this propagation we get an another pulse and what we see is that continuum generation occurs here. (Please look at the slides for mathematical equations)

So, if the pulse this is time domain representation what if I represent in frequency domain, I see frequency domain representation like this. This is omega the same wavelength as omega naught and due to this propagation, we see large bandwidth. New frequency components are generated on both side of this center wavelength and we see that the large bandwidth is created from here.

So, ideally one can say that this pulse would be much shorter than this pulse, because it has enough bandwidth, but it does not happen. In general when we do this self-phase modulation due to self-phase modulation we create white light, this is white light generation, we will go over the derivation, this is white light generation because big continuum we create that is why we call white light generation, many frequency components we in create here.

Now, ideally one can say that due to this large bandwidth one can say that the pulse duration will be much shorter than the input beam but generally that does not happen because we know that dispersion will actually elongate the pulse. So, although we have enough bandwidth to create a short pulse but another condition to create short pulse is the mode locking and we have to bring all the phases all the frequency component in phase and that does not occur due to dispersion.

So, the mediums dispersion will not allow us to have much shorter pulse. It will broaden the pulse; it will elongate the pulse in time. At the same time it will create a new frequency components. That is why it is called white light generation. White light means it is including all the visible light frequency components. So, it creates a continuum. We will take a look at how we create this continuum in the self-phase modulation process.

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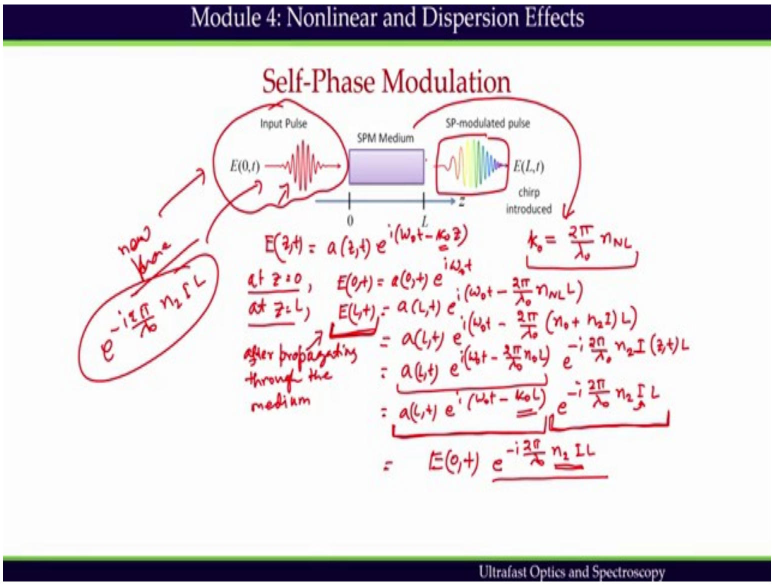

To visualize the effect of the self phase shift on the pulse while propagating through the dielectric medium, let us express the electric field of an incident pulse propagating in z direction just like the one incident pulse this incident pulse we can write down E z comma t equals a z comma t e to the power i omega naught t minus k naught z. So, this is the input beam which is propagating which will propagate through the medium. (Please look at the slides for mathematical equations)

Now, we will assume that we have a nonlinear medium of thickness L as shown here upon entering the nonlinear optical medium k must be expressed in terms of nonlinear refractive index. That is the general k should be reflected. So, in the medium I will have k, in the vacuum I will have k naught that is the vacuum wave vector, but the moment it enters the medium I will not be able to use this k. I have to use I will not be able to use k naught. I have to use the intensity dependent k which is nothing, but 2 pi by lambda naught. This k naught is going to be 2 pi lambda naught then n nonlinear which is nothing but a nonlinear refractive index. (Please look at the slides for mathematical equations)

So, that is the way we have to express in the medium. So, if we if we express that in the medium, then at z equals 0 at z equals 0 this distance at z equals 0, field can be expressed as 0 t which is nothing but a 0 t into e to the power i omega naught t at z equals 0 but at z equals L distance what we get after propagating through the medium of thickness L, the field can be expressed as E L t which is nothing but a L t e to the power i, then omega naught t minus as I mentioned before that we have to use this k which depends on nonlinear refractive index which is nothing but 2 pi by lambda naught nonlinear refractive index multiplied by L distance. (Please look at the slides for mathematical equations)

Now, combining these two equations what I get, we get that this a L t e to the power i omega naught t omega naught t then minus 2 pi lambda naught. Now nonlinear refractive index is nothing but n naught plus n 2 I, that we have seen previously multiplied by L. So, which can be written as L t e to the power i omega naught t minus 2 pi by lambda naught n naught L multiplied by e to the power minus i 2 pi by lambda naught n 2 I which is intensity depends on z and t and multiplied by L. (Please look at the slides for mathematical equations)

So, what we see here is that this part this part can be written as a L t e to the power i omega naught t minus this is k naught L if I did not have the medium. That is k naught L multiplied by e to the power minus i 2 pi by lambda naught n 2 I L. So, what we see here is that this field after propagating this field is after propagating through the medium this field we get after propagating through the medium. (Please look at the slides for mathematical equations)

Now this field is nothing but the vacuum field, this is the field which we should have expected if we did not have this medium. Now this is the vacuum field which is augmented by another phase factor this whatever we get in the complex notation e to the power this exponential complex notation we call it phase.

So, additional phase is introduced and that is why we are saying that this additional phase coming due to its own intensity that is that is why it is called self-phase modulation. So, you are modulating you are augmenting its own intensity profile by its; it by additional phase factor additional phase factor is coming due to the intensity dependent intensity dependent refractive index. So, we can write down that this field is nothing but E 0 t field which is equivalent to E 0 2 field and what propagated due to L distance multiplied by minus i 2 pi by lambda naught n 2 I L. (Please look at the slides for mathematical equations)

This is the mathematical expression for self phase modulated pulse after propagating L distance in third order nonlinear medium. We note that time dependence of the additional temporal phase, this is also complex temporal phase. We call it complex envelop phase. Whatever comes in the complex notation that is the envelop phase according to the definition which you given which we have given in module two so, this additional temporal phase comes due to self-phase modulation and the induced phase change induce phase change in self-phase modulation therefore, arises only from the intensity dependent refractive index of the material.

So, this intensity dependent this part is actually controlling the additional phase which is introduced to the pulse. So, the pulse this pulse would have been similar or the same after propagating for the L distance, but now I have introduced a new phase to it, a new phase has been introduced and that phase is nothing but e to the power minus i 2 pi by lambda naught n 2 I L, this new phase has been introduced to this pulse and effectively the pulse is now chirp. We will prove how we are getting the chirp pulse very soon. <u>(Please look at the slides for mathematical equations)</u>

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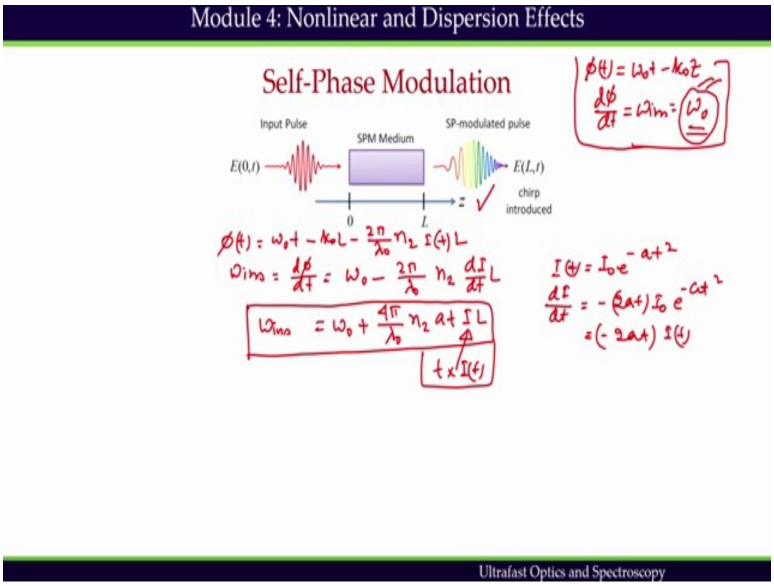

Now, we can calculate the instantaneous frequency. Instantaneous frequency we have defined instantaneous frequency in a module two and the by definition instantaneous frequency is nothing, but the time derivative of the total phase. Total phase phi t temporal phase is given as omega naught t minus k naught L minus 2 pi by lambda naught, n 2 I which is function of z t L, we will; for the time being we will call it, because we are dealing with time domain in the perspective of this problem so, we will call it I t time dependent intensity profile multiplied by L. (Please look at the slides for mathematical equations)

Now if we take the first derivative we get instantaneous omega which is nothing but the first derivative of this total temporal phase which we get minus 2 pi by lambda naught n 2 d I d t multiplied by L. Now, if we assume that I has intensity profile this temporal intensity profile is represented by a Gaussian beam Gaussian pulse I t equals I naught e to the power minus a t square. (Please look at the slides for mathematical equations)

If we consider that then the first derivative of this function is going to be minus 2 a t multiplied by I naught e to the power minus at square which is nothing but minus 2 at multiplied by I t there is the intensity profile. So, thus in instantaneous frequency for shelf phase modulated Gaussian pulse can be written as omega naught plus 4 pi by lambda naught n 2 a t I L. So, this is your instantaneous frequency. (Please look at the slides for mathematical equations)

Now if we recall that if we did not have the medium, then we could have written the temporal phase as omega naught t minus k naught z. In that case the first derivative with respect to t which is instantaneous omega would be omega naught and; that means, it is a transform limited pulse because there is no chirp introduced. Chirp what does it mean it is chirp we when you introduce a chirp to the pulse, we sweep the frequency instantaneous frequency as a function of time that we are not doing here. So, if we had a transformative pulse propagating in vacuum then we should have got this, and it means that we did not have introduced any chirp.

But our medium is self phase modulated medium and that is why instantaneous frequency will vary and it will vary as a function of t, it is a complex very complicated function it is t multiplied by I t that is the temporal behavior we have introduced and that is why it is clearly that instantaneous omega instantaneous frequency depends on time, which means that we have introduced chirp in the shelf phase modulated pulse. (Please look at the slides for mathematical equations)

Now we shall reexamine the self phase modulated phenomenon in frequency domain. Why? Because we want to prove that during this self-phase modulation, we are not only introducing chirp in the pulse, it is not a linear chirp. If it was a linear chirp, we could have seen some kind of function of t but it is a very complicated function; it is a product of t and I, t Gaussian function and t and so, it is a very complicated chirp we introduced in the self-phase modulation process.

What we want to know is that, if we introduce this chirp and finally, we have the time domain expression for the self phase modulated pulse, can we get the spectrum out of it; and yes, answer is yes, we will be able to get the spectrum can be calculated from this Fourier transform.

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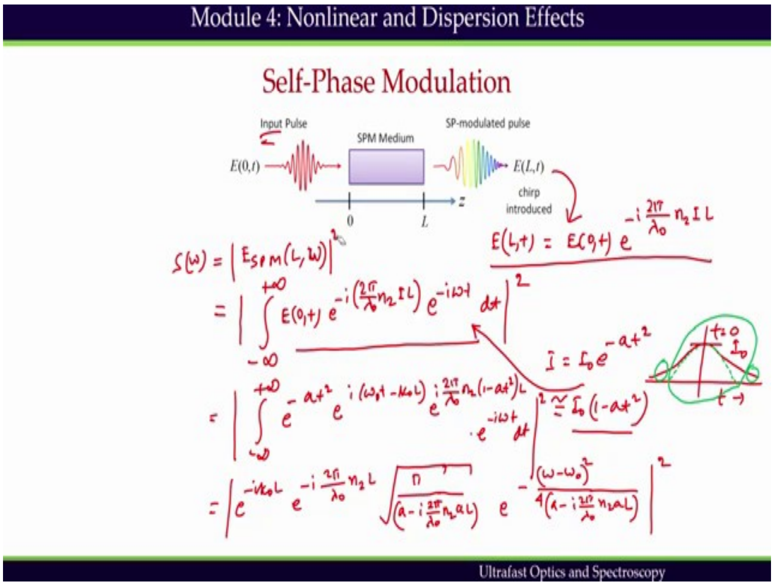

So,we know that this E this E L t, this field was expressed as E 0 t. This is the field which we could expect without the medium but now we are introducing new phase to the pulse due to self-phase modulation and that is lambda naught then n 2 I then L. So, this is this is the final expression of the self-phase modulated pulse. All we need to do if we want to find out the frequency then, we have to get the Fourier transform and then we have to take the square modulus of this. (Please look at the slides for mathematical equations)

So, s omega s omega that is the power spectrum is nothing but self phase modulated field in the frequency domain square modulus of that and that is nothing but Fourier transform of the time domain field which is nothing but E 0 t e to the power minus i 2 pi by lambda naught n 2 I L e to the power minus i omega t d t; that is the Fourier transform square modulus. To obtain so, now, now in order to get this Fourier transform probably we have to use numerical method. (Please look at the slides for mathematical equations)

But we can take one very simple approach and what we can do; we can expand i in exponential series expansion to the first order only. So, what we can say that this I we have said that it follows a Gaussian behavior which is nothing but this one which looks like this and this is t equals 0. It is centered at t equals 0 and this is the maximum intensity I naught. So, this is the when temporal behavior of the of the intensity profile.

What we are going to do is that to the first order we are going to expand this equation expand this function and if we expand this function approximately, we can write down that a t square I naught 1 minus a t square. We are doing this simplification over simplification just to get an idea or prove analytically that we are going to expand the spectrum we are going to generate new frequency component when the input pulse propagating through the self phase modulated medium. (Please look at the slides for mathematical equations)

So, and in this approximation if we do this approximation what will happen, under this approximation we are not to off. We are actually if we plot this 1 minus a t square then it is going to be something like this is something like this a t square. So, so, what we are missing is that the wing part is missing, but mostly it is describing the variation the Gaussian variation near t equals 0.

So, we are good to go with this with this function and if we do that, then the if we plug that in; if we plug this in here then what we get, we can write down that this is nothing but minus infinity to plus infinity minus infinity to plus infinity then e to the power; e to the power minus a t square e to the power i omega naught t minus k naught L. (Please look at the slides for mathematical equations)

This is the vacuum contribution this is the vacuum contribution and then additional phase which has been introduced that is 2 pi by lambda naught n 2 I; I will write down 1 minus a t square and then 1 minus a t square multiplied by L multiplied by e to the power minus I omega t d t then square modulus and this expression can be simplified and one can write down the simplified expression as following. (Please look at the slides for mathematical equations)

One can write down the simplified expression as e to the power minus i k naught L multiplied by e to the power minus i 2 pi by lambda naught n 2 L then square root of pi by a minus i 2 pi by lambda naught n 2 a L multiplied by e to the power minus omega minus omega naught square divided by 4 a minus i 2 pi by lambda naught n 2 a L; n 2 a L this is going to be square modulus. (Please look at the slides for mathematical equations)

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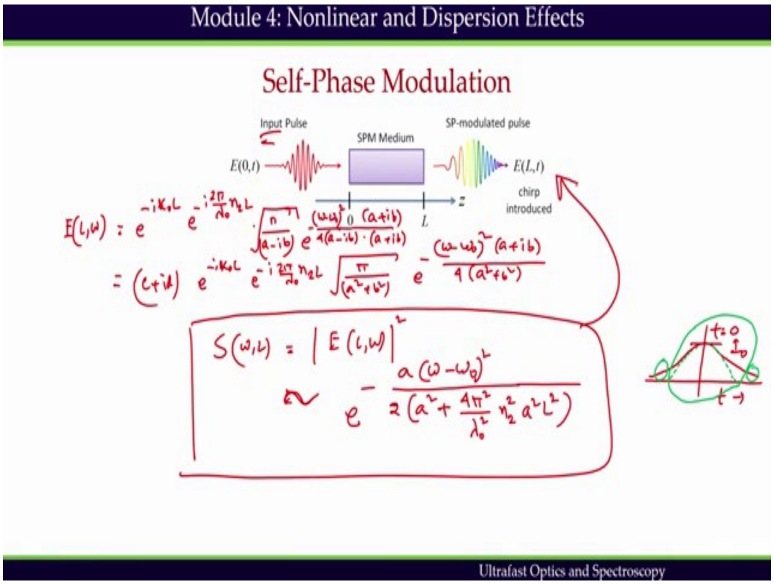

Now what we will do here; we will represent this to be b. So, we can write down ok, we erase this part and we can we can say that this 2 pi by lambda naught n 2 a L, this part is nothing but b. So, if we express that then what we get is that this field e L omega, this field can be written as e to the power minus i k naught L e to the power minus i 2 pi by lambda naught n 2 L multiplied by square root of pi by a minus i b.

That we can write down multiplied by e to the power minus omega minus omega naught square divided by 4 into a minus i b, we can write that out and finally, we can multiply with a plus i b. So, i can multiply a plus i b. So, we can write down here also a plus i b s. Similarly, I can do the same thing here within this square root and finally, I can get this expression. (Please look at the slides for mathematical equations)

The expression which I will get is that I will have one more expression for square root of a plus i b then e to the power minus i k naught L e to the power minus i 2 pi by lambda naught n 2 L square root of pi by a square plus b square e to the power minus omega minus omega naught square multiplied by a plus i b by 4 a square plus b square. (Please look at the slides for mathematical equations)

So, we get this expression and remember we are interested only in the square modulus term of this, because we are interested on the spectrum only and if we want to if we want to get the square modulus of it we can easily say that instead of square root of a plus i b we can write down this we can we can convert this complex number a plus i b to c plus i d.

Any complex number can be converted to another complex number and if we do that then all we are interested in is s omega which is nothing, but the square modulus of the L omega and e L omega and that is that can be expressed as e to the power minus a multiplied by omega minus omega naught square divided by divided by 2 a square plus 4 pi square. I am inserting the value of b lambda naught square n 2 square a square L square. This is what we get. So, this is the final expression for the spectrum which we are expecting associated with this self-phase modulated pulse(Please look at the slides for mathematical equations).

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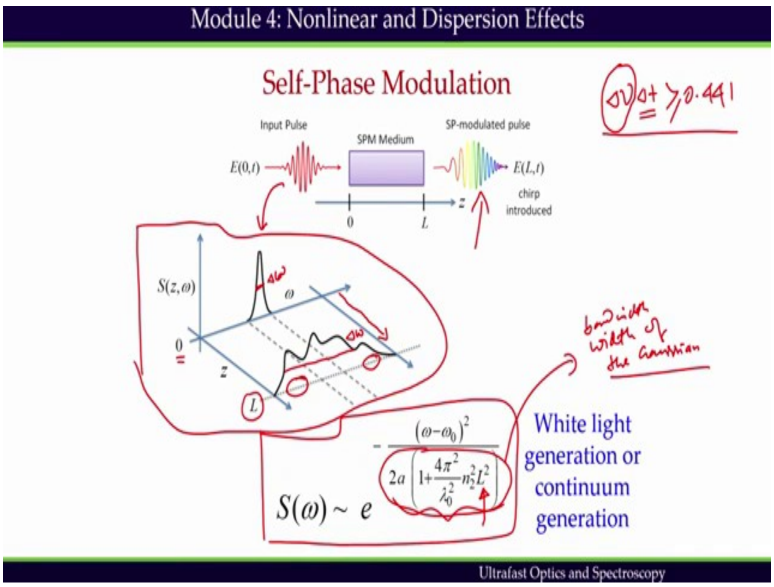

So, here we are interested in the shape of the power spectrum we can consider only the real part and the normalized maximum amplitude and we see that the power spectrum has an expression like this, which suggests that this denominator in this expression in a Gaussian function defines the width of the Gaussian. So, basically this is defining the bandwidth of the Gaussian and which suggests that this bandwidth is going to depend on L, the distance and that is exactly we have depicted here.

Let us say we started with this is the input beam spectrum and we started and this is the 0 position, we started with this spectrum the bandwidth was let us say delta omega. Now as it propagates through the medium after propagation of L distance what we see is that new frequency components has been generated and these frequency components are generated due to self-phase modulation process.

So, it is evident then new frequency components are generated due to self-phase modulation. Extra frequency components brought in the spectrum on both sides of the center wavelength. This is called white light generation. We have already mentioned that when ultra-fast pulse propagates through a medium it experiences dispersion effect. Material dispersion always stretches a pulse in time. So, white light pulse must be a broad pulse although it contents large bandwidth suitable for a short pulse.

So, just looking at the power spectrum evolution, one can say that because we have large bandwidth now, we have very short pulse here but that does not occur and that does not occur just because dispersion will start playing role immediately. So, the moment we create new frequency components in the medium that will face the dispersion and what does it mean by dispersion red light or the high sorry low frequency components will travel at a faster velocity than the high frequency components and that will stretch the pulse in time.

So, having what it suggest, it suggest that having the enough frequency components does not mean that we should have short pulse and that is the reason time bandwidth product is always written like this way; delta nu delta t is greater than equals 0.441 for a Gaussian pulse. Why? It suggest that for a given frequency component for a given bandwidth the let us say I have a certain bandwidth I can have a shortest duration pulse delta t which is defined by this equation but rest of the pulses are going to be always broader or the longer than the short shortest duration pulse.

The shortest duration pulse for a given bandwidth is called transformative pulse, rest of them are chirp pulse. What kind of chirp we have that depends on the temporal behavior of the instantaneous frequency. So, this is all about self-phase modulation. We discussed this self-phase modulation previously, but this time we are giving more details and there will be derivation of this power spectrum.

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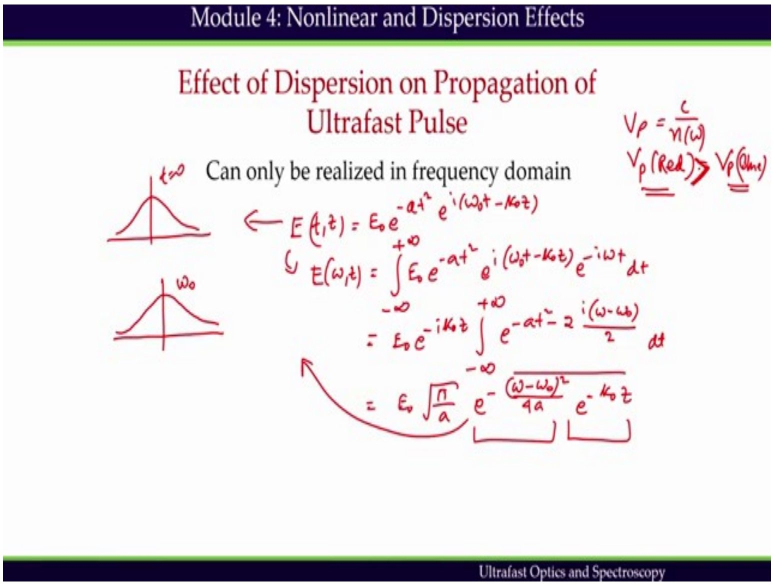

We have seen that ultra-fast pulse carries many frequency components. So, it propagates through a dispersive material and when it propagates through the dispersive material it experiences dispersion. In every ultra-fast optical spectroscopy lab different optics such as lens, polarizer, nonlinear crystals etcetera are used and when ultra-fast pulse propagates through these optics it experiences dispersion. Effect of dispersion on propagation of ultra-fast pulse is unavoidable because every medium exhibits some extent of dispersion and we know that phase velocity that Vp phase velocity, Vp depends on the refractive index c by n omega. It inversely depends on refractive index like this.

So, that is why the velocity of the red component which is low frequency components which would be much larger than velocity of the blue component, the high frequency component. So, this two different frequency component will have different velocities and that is why pulse will elongate in time and that is the effect we have already seen. Effect of dispersion cannot be realized with time domain description of the pulse it can only be realized with frequency domain description of the pulse. Therefore, to obtain a mathematical formulation of propagation of ultra-fast pulse in a dispersive medium, we have to treat ultra-fast pulse in frequency domain.

We may begin with transform limited pulse in Gaussian pulse which can be written as E t z equals E naught e to the power minus a t square e to the power i omega naught t minus k naught z. That is the transformative pulse. All we need to do is you have to convert to frequency because in order to understand dispersion effect we have to go to the frequency domain; why, I will explain immediately very soon. (Please look at the slides for mathematical equations)

E omega z that is the Fourier transform of this function which is nothing but minus infinity to plus infinity E naught e to the power minus a t square e to the power i omega naught t minus k naught z e to the power minus i omega t d t. That is the Fourier transform and if we do the Fourier transform then we can write down E naught e to the power minus i k naught z that comes out of the integral because it is does not depend on time minus infinity to plus infinity then e to the power minus at square minus 2 multiplied by i omega minus omega naught divided by 2 then d t and we know this integration we have done this integration before which is nothing, but E naught square root of pi by a e to the power minus omega minus omega naught square divided by 4 a multiplied by e to the power minus k naught z. (Please look at the slides for mathematical equations)

This is is time domain field profile which is centered at t equals 0 that we have seen previously and this is represented by a Gaussian which is centered at omega naught. But it has additional phase factor this k naught is constant for vacuum but it is not constant for a medium and that is why what we did is that we have expressed in the dispersive medium the magnitude of the wave vector must take more general form that is k omega.

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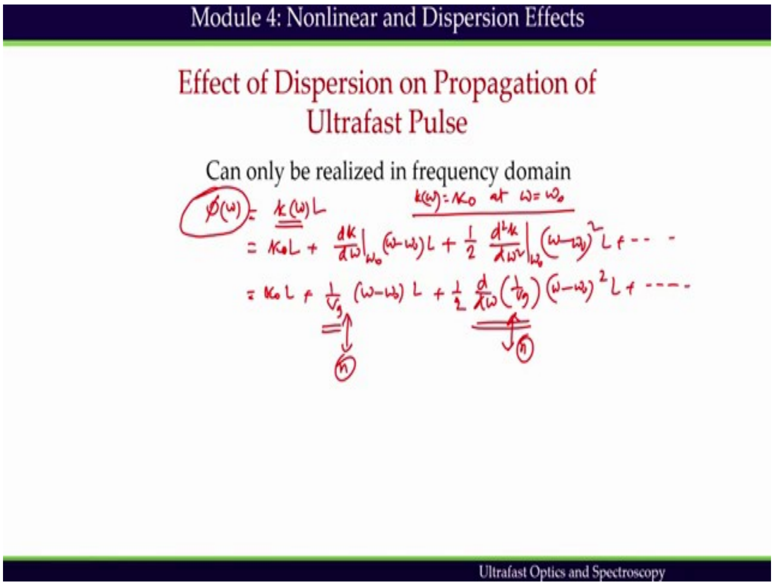

So, that phi(φ) omega that is the that is related to the spectral phase phi omega should be written as, k omega multiplied by L. L distances if it is travelling in a dispersive medium and this k omega can be is a frequency dependent unknown function which can be expressed and its value at k naught omega naught is known because it is this k omega equals k naught at omega equals omega naught. (Please look at the slides for mathematical equations)

So, this is this is known and its first derivatives are known also, I will show you how it is known the first derivatives are related to its group velocity, group velocity dispersion and group velocity, group velocity dispersions are related to their refractive indices. So, here is the point; point is that if we express the ultra-fast pulse in time domain, then there is no way you can express k in terms of a refractive index.

But if you express k in terms of frequency or if you express k in frequency domain then there are physical quantities which can be correlated to this k in the Taylor series expansion. So, the Taylor series expansion will write down this k naught that is the initial value plus the first derivative with respect to omega at omega naught, that is omega minus omega naught multiplied by L plus the second derivative, which is d 2 k d omega 2 at omega naught omega minus omega naught square L plus blah blah blah. All those terms can be introduced. (Please look at the slides for mathematical equations)

And we know that by definition that is nothing but k naught plus 1 by Vg group velocity multiplied by omega minus omega naught into L plus half of this is nothing, but group velocity dispersion 1 by Vg, that we have seen omega minus omega naught square L plus blah blah blah. All these terms are known Vg this in the previous lecture we have seen that they are all related to refractive index they are all related to refractive index and refractive index is something which can be experimentally determined that is why this spectral phase this omega the omega of a phi omega is not exactly spectral phase because spectral phase has to be related to the spectrum but it is related to the frequency dependent phase factor which comes in the frequency domain representation of the electric field.

So, what we see is that in the time domain there is no scope for us to represent k in terms of different physical quantities, which will account for the dispersion effect. But in frequency domain there is a possibility to express k in terms of physical quantities which is directly connected to the dispersion. That is material refractive index n and that is the reason why we have to express the pulse in the frequency domain.

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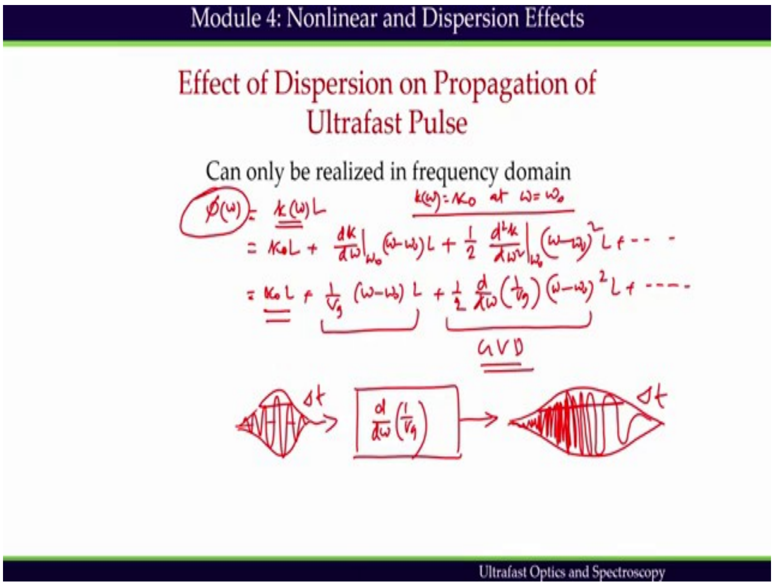

Now, this can be this is so, the first term here we know that this is the vacuum contribution, then second term here it we know that this is coming due to group velocity. This group velocity will introduce group delay which is which introduced in the pulse because of the dispersion effect and that creates the carry envelop phase and the third term is the second order spectral phase term; which is responsible for GVD group velocity dispersion we which we have already seen. What is our task next; we are going to do the derivation for the pulse which is experiencing this GVD.

So, I have a pulse transform limited pulse propagating through the medium like this way I have a pulse propagating through the medium and this medium has this kind of a dispersion effect second order spectral phase we will have to introduce and that is group velocity dispersion and due to this dispersion we would like to see what is going on in the pulse.

Definitely we are going to introduce a chirp and a chirp pulse is nothing but something like this. So, frequency is changing over the over the pulse as a function of time. So, that is called chirp which introduced a chirp. We will find out this with delta t and this with delta t and we will find out what is the relationship among them. We will stop here; we will continue this module in the next lecture.