Ultrafast Optics and Spectroscopy
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Lecture - 15
Dispersion Effects

Welcome to module 4 of this course Ultrafast Optics and Spectroscopy. In this module we will study Dispersion Effects which is experienced by ultrafast pulse, while propagating through the dielectric medium.

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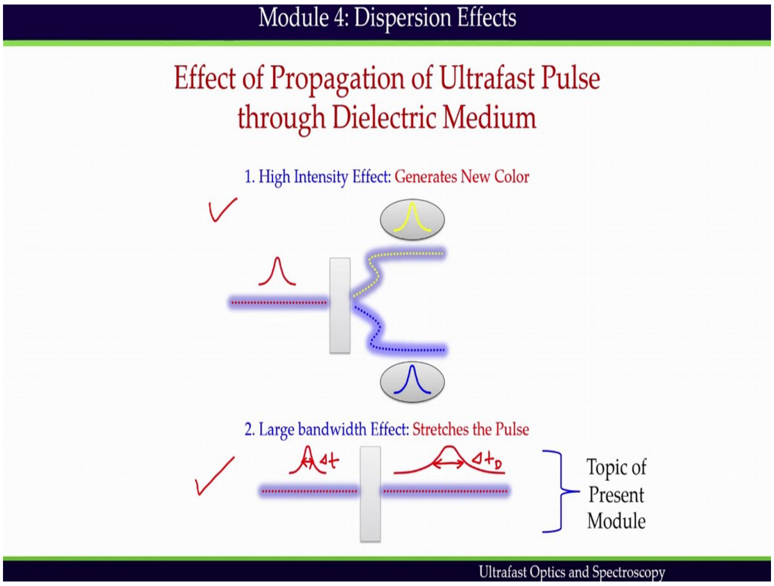

We have already mentioned that when ultrafast pulse propagates through a medium it experiences two effects. The first one we have already studied high intensity effect, which is collectively called non-linear optical effects and a second effect experienced by the pulse while propagating in dielectric medium is large bandwidth effect which is call dispersion effect.

In this module we shall learn dispersion effects experienced by an ultrafast pulse, when it propagating through a dielectric medium. Material dispersion always stretches a pulse in time and that is why a short pulse here the pulse duration is expressed by delta t before propagating through the medium and this delta t time duration of a pulse is defined as the full width half max of the intensity profile which will become larger as it propagates through the medium. This is delta t after dispersion and that is unavoidable for any dielectric medium.

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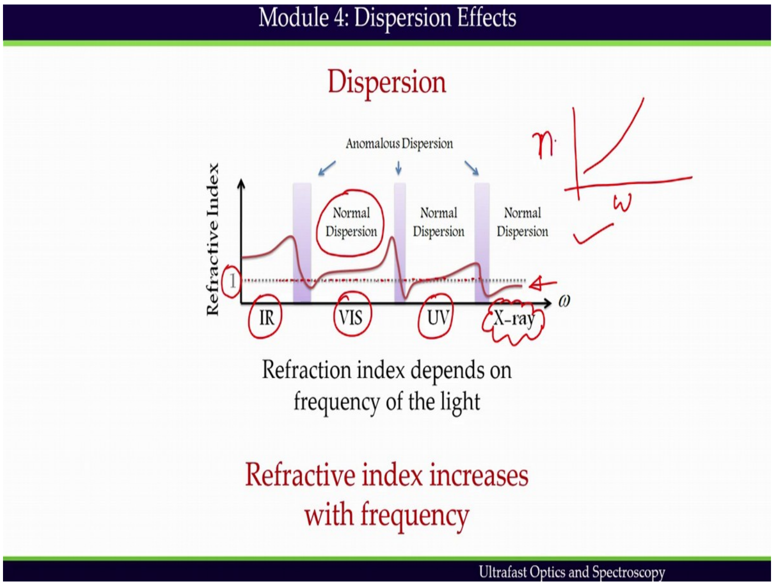

So, this is the topic of the present module, but what is dispersion by the way? Dispersion is a variation of refractive index as a function of frequency which is shown here in this figure. We see that in the visible regime and UV regime or IR regime where most of the spectroscopies are performed refractive index of the medium is much higher than 1 refractive index of the medium in IR and visible or UV regime they are greater than 1, but in the X-ray regime the medium dispersion becomes less than 1.

And there is a consequence for it we will go over the consequences later, but one thing we should note here that in the normal dispersion regime refractive index always increases as a function of frequency.

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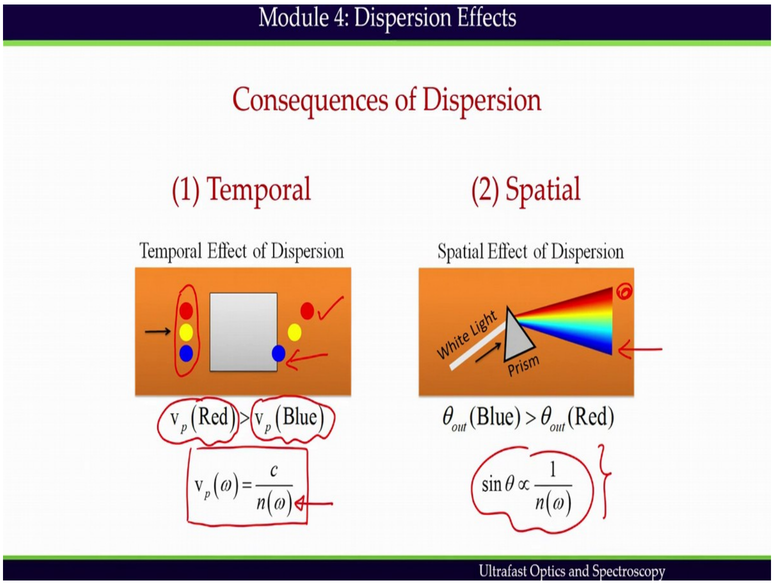

This has two important effects in light propagation. The first effect is temporal effect and the second effect is spatial effect. In temporal effect we have already come to know that phase velocity depends on refractive index Vp. It depends on the refractive index and refractive index depends on frequency that is why phase velocity of red light low frequency component would be would be much higher than phase velocity of the blue light that is why if I have a combination of red, blue and yellow lights, they are propagating in vacuum. And once it is propagating through the medium a dispersive medium, then we see that red light propagates at faster velocity than the blue light.

So, this is the temporal effect we see which means that if I take this analogy and then try to understand what might happened to the ultrafast pulse, one can think of like this way. I have red and blue components in the vacuum they are propagating, but the moment it is propagating through the medium what will happen?

This red component would travel at a faster velocity than blue component and slowly I am stretching the pulse and this effect is unavoidable effect for any medium we will calculate how much stretching will experience very soon. Another effect dispersion effect is spatial effect according to Snell's law when light propagates across an interface; angle of refraction is inversely proportional to the refractive index of the material or medium. And this inverse relationship suggests that if light propagates from low refractive index material to high index material, higher frequency light bends more as illustrated here.

So, blue light will bend more than red light and we will find out what are the effects one can anticipate for the ultrafast pulse when ultrafast pulse is propagating through the medium.

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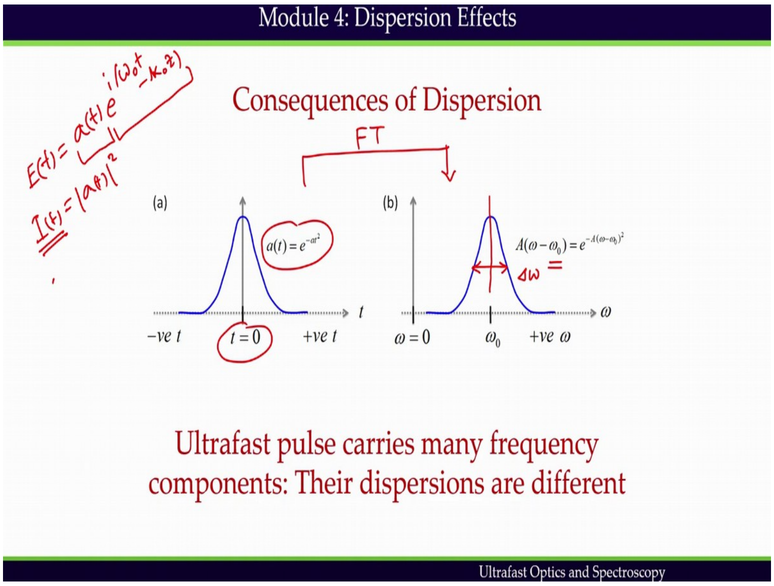

An ultrafast pulse carries a number of frequency components that we have already seen we have represented ultrafast pulse and the representation was following we can remind ourself. We have represented the electric field in time domain as a t multiplied by e to the power i omega naught t minus k naught z. This part is career web part oscillatory part and this part is field envelope. (Please look at the slides for mathematical expressions)

And then we get the intensity which is nothing, but a t square modulus and when express intensity of an ultrafast pulse assuming a Gaussian pulse like this ready centred at t equals 0. The moment I do Fourier transform of this time domain pulse, I get a frequency domain description of the field and which suggests that, the frequency domain field is centred at omega naught equals at the centred omega naught.

So, frequency domain field is centred at omega naught and it has a bandwidth delta omega. So, when an ultrafast pulse propagates through a dispersive material, it will experience both temporal and spatial effects due to dispersion in every ultrafast optical spectroscopy lab different optics such as lens, polarizer, non-linear crystals are used and when ultrafast pulse propagates through these optics it experiences dispersion. Effect of dispersion on propagation of ultrafast pulse is unavoidable because every medium exhibits some extent of dispersion.

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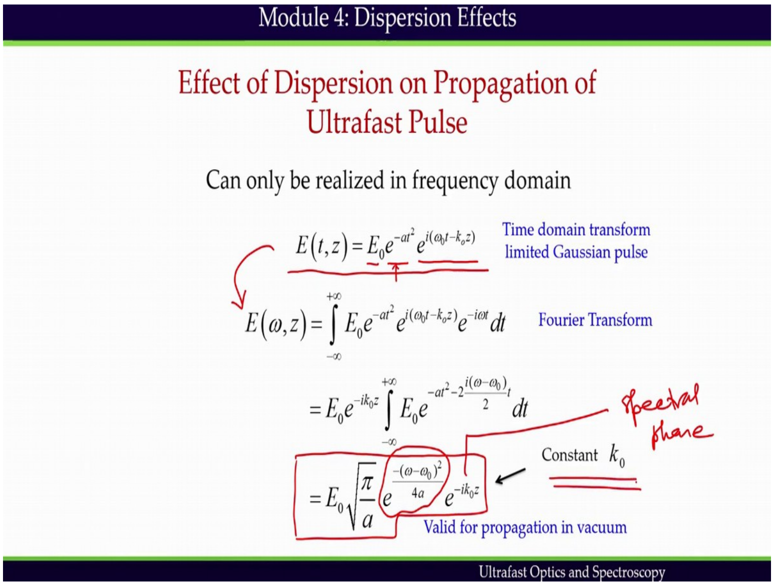

Effect of dispersion cannot be realised with time domain description of the pulse, it can only be realised in frequency domain description of the pulse and that is why to understand the dispersion effects, we will express time domain pulse here we have taken a Gaussian pulse in frequency domain by Fourier transform. The transform limited Gaussian pulse is expressed by E naught e to the power minus at square that is the field envelope Gaussian field envelope and this is the carrier wave. Once we get the Fourier transform done then we get this equation, this expression for field in the frequency domain.

This expression shows that in the frequency domain we get another Gaussian envelope and this is spectral phase. A phase factor which comes in frequency domain that is why it is called spectral phase. Now this equation is valid when a pulse propagates in vacuum that is why k naught is constant it is not varying, it is not changing.

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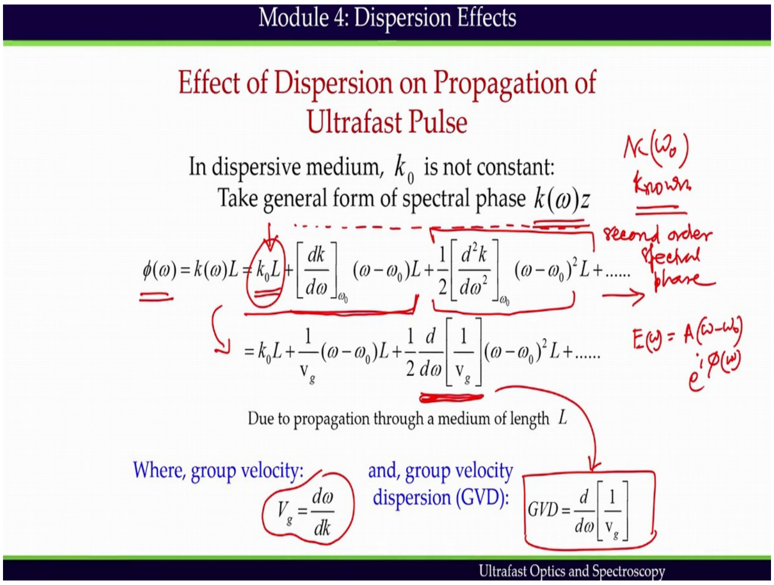

In a dispersive medium, however, magnitude of wave vector must take more general form k omega. Consequently spectral phase of the pulse is controlled by k omega z this term. So, instead of k naught z we have to consider k omega z k is the function of omega and the moment we do that, we can express it in Taylor series expansion because the k omega naught value is known which is the vacuum value and we have to expand this function around omega that is why we can use this Taylor series expansion.

The moment we look at the Taylor series expansion we get the spectral phase description, which includes vacuum contribution k naught L that is the L distance. So, instead of z we are considering L and the first term which exist for plane wave as well is the natural phase advance due to propagation through L distance k naught L.

So, if you look at the expression for frequency domain field it looks like this multiplied by e to the power i phi omega. Now second term which we get is related to group velocity, the velocity of the envelope function. We remind our self in a pulse I have two components envelope and the carrier wave. Envelope is this one and the carrier wave which is the oscillatory part and in vacuum they travel when their phases are lot like this way. And we said that there are two different velocities phase velocity the velocity of the carrier wave and group velocity of the envelope.

So, group velocity is expressed by this d omega d k which exist in pulse only and the third term here is related to variation of group velocity as the function of frequency. So, just little modification of this equation simple modification by introducing group velocity we get this term in the second order spectral phase. So, this is your second order spectral phase. Second order spectral phase. In the second order spectral phase we get this group velocity dispersion this is the definition we are giving based on Taylor series expansion. (Please look at the slides for mathematical expressions)

Now when a pulse propagates through a dispersive medium, additional phase is added to the propagating pulse. Because in addition to k naught L we get start getting all these terms and I can have even higher order terms as well depending on the level of dispersion we have. Level of dispersion an ultrafast pulse experiences while propagating through the medium decides how many terms in this equation we should consider.

This undoubtedly depends on the experimental conditions. However, to understand the influence of each term on the propagation of an ultra fast pulse one can think of adding systematically each term and subsequently check the effects. If only the first term in the equation is introduced; that means, this one the pulse does not experience any dispersion at all, it is just like vacuum propagation. This is true for the vacuum propagation. In order to see the dispersion effect we should introduce at least first two terms; that means, these two terms we have to introduce to get the dispersion effect.

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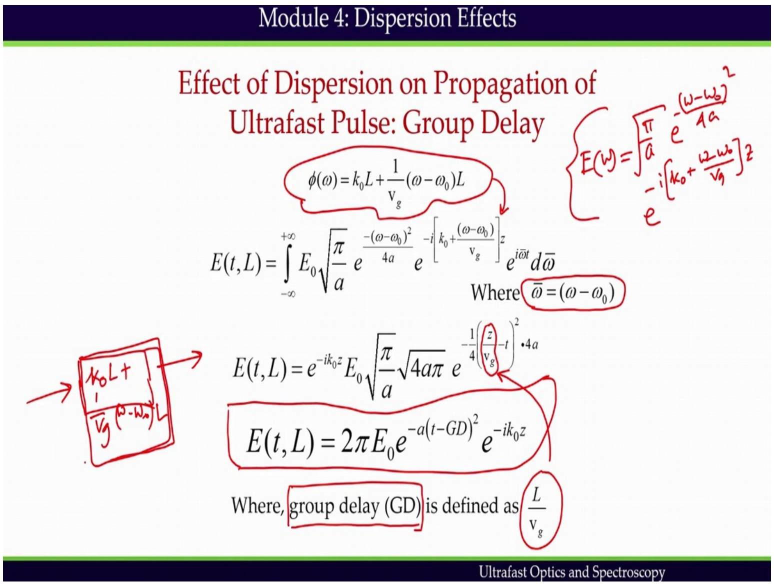

So, the first task would be we will introduce this two terms the first two terms and then we will plug that in the frequency domain field which was obtained previously pi by a e to the power minus omega minus omega naught square divided by 4 a multiplied by e to the power minus i. (Please look at the slides for mathematical expressions)

Then instead of k naught now I have to write down k naught plus omega minus omega naught divided by vg z. So, this was the expression for the field in frequency domain if we consider the first two terms and then all we need to do is we have to convert it to time domain by inverse Fourier transform. (Please look at the slides for mathematical expressions)

In this we will consider a new variable omega bar. So, this integration will be performed with omega bar as variable and omega bar is defined as omega minus omega naught that is just simplifies the problem and then we get this expression in time domain. And now if we define another term group delay which is nothing, but L by vg. This L by vg or z by v g we have here we are giving another definition to this term. Here GD group delay is defined as L by vg this equation represents the outgoing pulse in time domain after experiencing first two terms due to dispersion and L by vg distance divided by velocity is nothing, but time that is why it is called group delay what does it mean by this delay? (Please look at the slides for mathematical expressions)

I will just talk about it, but this is the way definition is coming and final expression for the outgoing pulse after experiencing them. So, I have an medium a pulse is propagating through the medium and we have assumed that medium is giving me only this dispersion effects. So, it is called first order diffraction effects is included.

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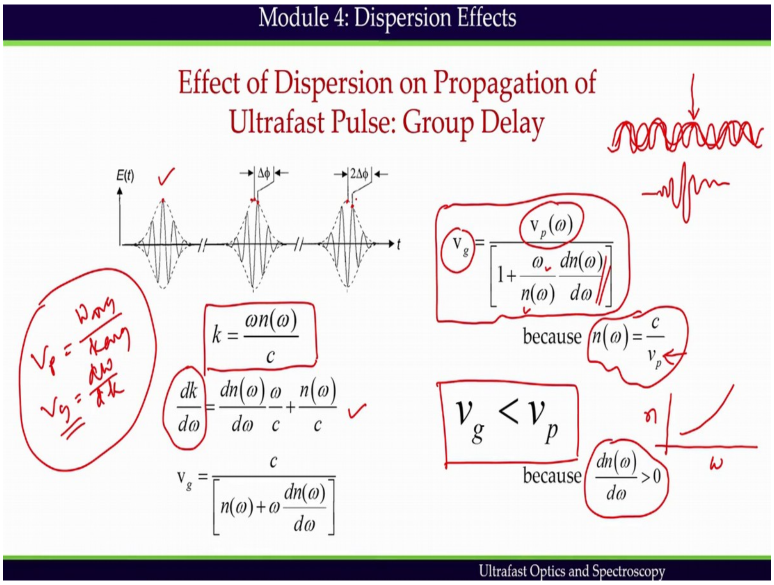

What does it mean by this delay? We have already seen that optical pulse is synthesized by interference of electromagnetic waves with slightly different frequency propagating along the same direction and due to that interference. So, I have a number of frequency components this is one, second frequency components, then third frequency component and due to the interference we see that at this point of time we get constructive interference and that is why pulse looks like this.

This is the resultant electric field how is changing. And we have also seen that a pulse has two velocities phase velocity and group velocity. Phase velocity is defined by v p phase velocity is the velocity of the carrier wave which is defined by omega average by k average and group velocity is defined by d omega d k that these are the definitions we have already found in module 2. Now in any dielectric medium dispersion relationship can be written as k equals omega n by c this also we have found previously. (Please look at the slides for mathematical expressions)

If we take the first derivative of k with respect to omega to obtain group velocity, then we get this expression simple math and then if we insert refractive index definition that is the speed of light in vacuum and the speed in the medium. Then finally, we get an expression which connects v g and v p phase velocity and group velocity and we are trying to find out what is the relationship between phase velocity and group velocity in a medium. Because dn d omega there is a slope, this dn d omega and we have already shown that in non absorbing regime refractive index will always increase with respect to omega, which means that the slope would be always positive. (Please look at the slides for mathematical expressions)

Because slope is positive, omega is positive, n is positive in this equation, omega is positive n is positive v g would be always less than v p. Therefore, in non absorbing medium relative phase of envelope and carrier wave changes due to phase and group velocity mismatch and that is demonstrated in this figure. If you look at the tip of the envelope and tip of the carrier wave, then we find that in the first pulse they are at the same time, but in the second pulse at it is as it propagates through the medium, the tip of the envelope and tip of the carrier wave they are separated in time, it shows a mismatch again it is further separated as it propagates through the medium. So, v g is always less than v p in the medium and that is representing the group delay which we discussed in the previous slide. (Please look at the slides for mathematical expressions)

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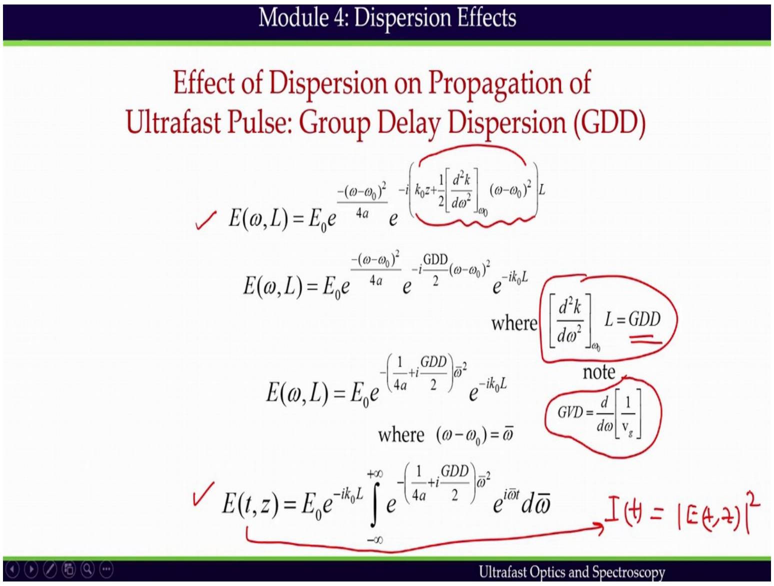

Now, if we assume that due to propagation of pulse through a medium only additional phase which is introduced is due to group velocity dispersion, previously we have included group velocity only now you are including second order spectral phase, similarly we are representing electric field in the frequency domain and we have taken second order spectral phase and we are now defining another term GDD; GDD is nothing, but this term which we obtain from this equation is just a definition group delay dispersion and here we remind ourself that, GVD is defined by this definition again comes from the Taylor series expansion.

So, with this GDD definition we solve and then we get the time domain field by inverse Fourier transform and taking the full width half max. So, once we get the field in time domain, we can get intensity in the time domain from field to intensity we know how to do that.

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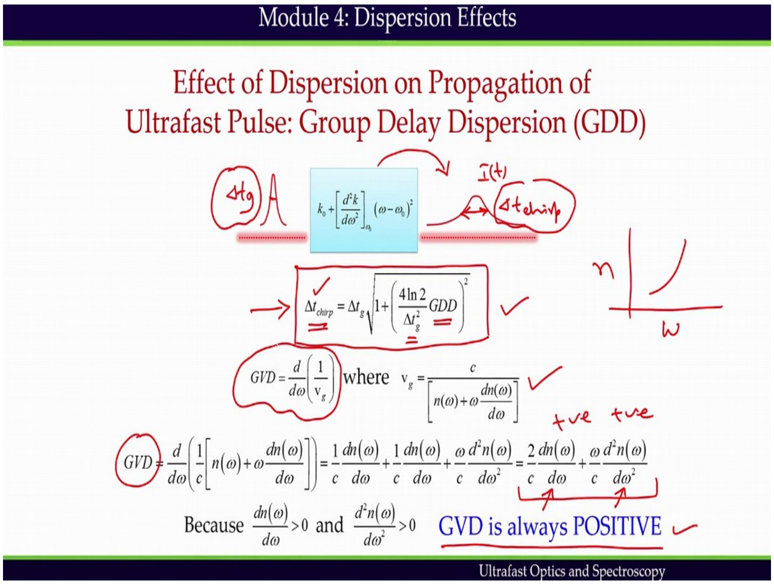

And then once we get the intensity we can take the definition of full width half max of the intensity to get the width because we are finally, interested in the width and we get an equation a very simple equation this one what does it mean? Here delta t chirp we have to remember that anytime for a given spectral bandwidth if number of frequency components are fixed, then anytime we are trying to broaden a pulse, it means we are introducing a chirp. (Please look at the slides for mathematical expressions)

So, dispersion will always broaden a pulse that is why we are introducing chirp. So, we are defining this pulse to be chirped pulse and that delta t is the intensity full width half max of this pulse. On the other hand delta t g is before dispersion this one again this is also intensity full width half max and this equation shows the this equation shows how this pulse duration and this pulse duration are related. It suggest that as we increased GDD will have broader pulse.

As already stated earlier the dispersive pulse broadening is unavoidable in any optical medium because all optical material exhibits positive GVD and how do we know is positive GDD? We know from this equation we know the definition of GVD this definition is coming from Taylor series expansion where vg is expressed like this and then we can calculate GVD we get this expression, in this expression again we have first derivative and second derivative. We have to remember that refractive index increases as a function of frequency in non absorbing medium non absorbing regime that is why these terms are all positive that makes GVD always positive.

So, you will get a positive GVD from every material and due to this positive GVD, the pulse will always broaden in time it will be stretched in time. So, it is desirable that we find out an optical material which features negative GVD as well because we have to compensate the pulse. For some reason if the pulse is propagating through a dispersive medium it will always broaden and chirp will be introduced.

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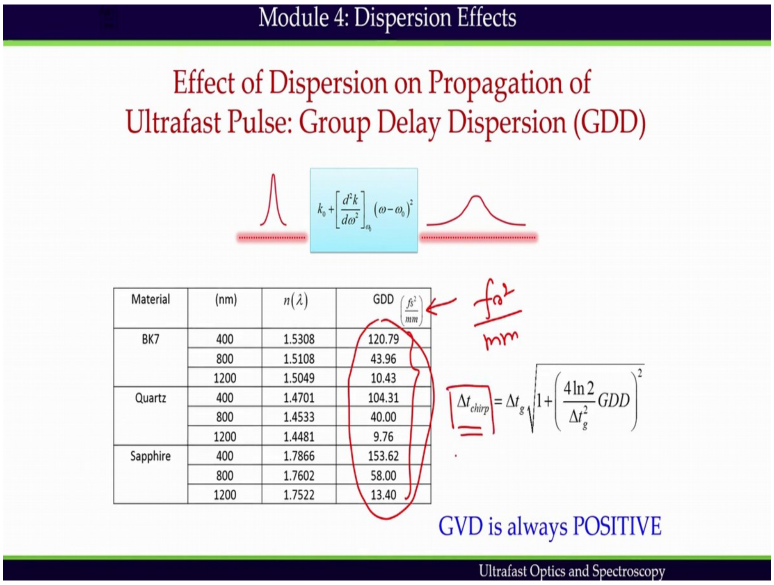

So, I need another medium which will introduce negative GVD to get my broaden pulse back to the compressed pulse. This table shows GDD which is always positive and I suggest that the unit of this GDD is suggesting that femtosecond square divided by mm. So, per millimetre propagation of the pulse it will introduce this amount of GDD and if we plug that in we will be able to find out what will be the final pulse duration after the propagation.

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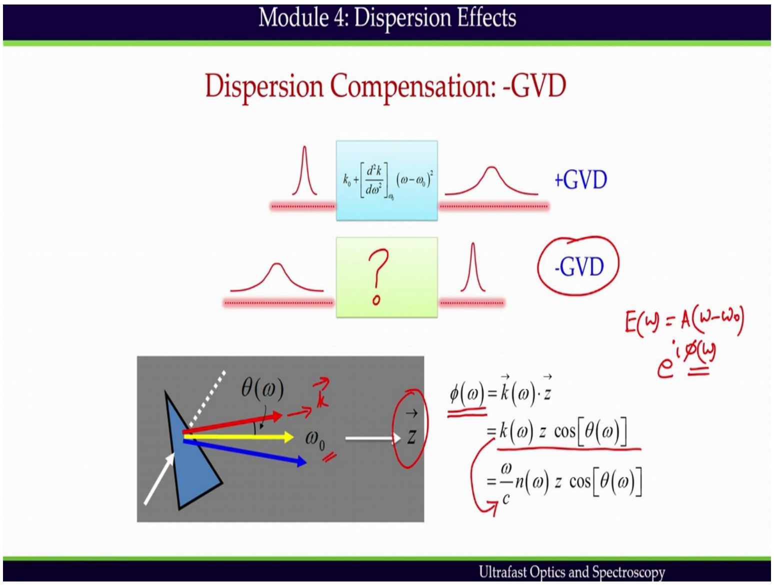

Now, as I have told you that dispersive pulse broadening is unavoidable and in experiments there are many optics there are many transmission optics which we use that is why pulse will always broaden and we need a material which will recompressed a pulse introducing negative GVD. Unfortunately there is no material directly can introduce my negative GVD prism and gratings are two optical elements which are capable to introduce negative GVD by means of angular dispersion.

So, here we have to use angular dispersion effect to introduce GVD. To understand this angular dispersion and it is effect on in producing negative GVD, let us assume the centre frequency component of the pulse travels along z direction here and k is the direction of any other frequency components. So, this is k and centre frequency is propagating omega naught propagating along z direction.

A prism we know that light will bend due to angular dispersion and one can write down the spectral phase if we recall it omega E omega that is the field in frequency domain is expressed as A omega minus omega naught that is the envelope function and there is a spectral component. The spectral phase is expressed as k is a dot product of k and z which gives me k z omega k z cos theta and k can be expressed in terms of omega by c n that is dispersion relationship. So, finally, we get this spectral phase. (Please look at the slides for mathematical expressions)

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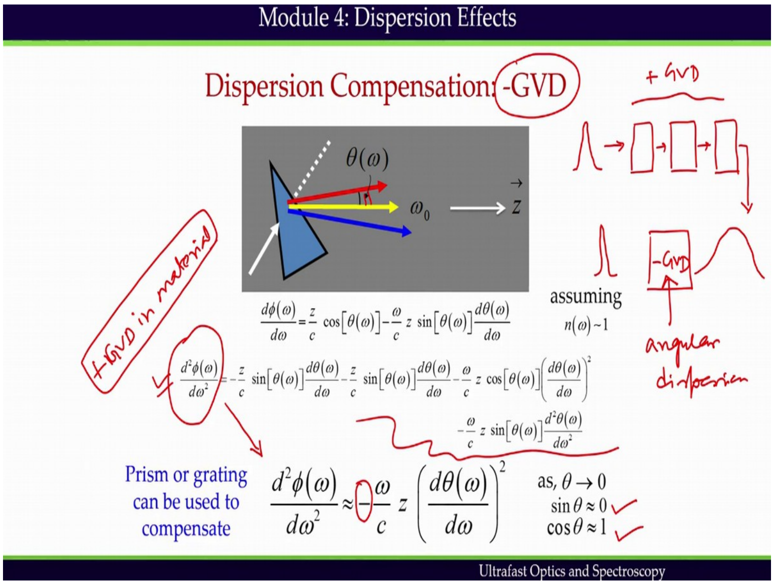

And we are interested in second order spectral phase because that is introducing positive GVD in material. So, we want to find out can we get negative GVD from the second order spectral phase. So, when you take the second derivative with respect to omega, we get this expression and if we assume that theta angular dispersion is very small, this theta with respect to omega naught centre frequency is very small, then sin theta can be approximately considered to be 0 and cos theta can approximately be considered as 1 and we can reduce this second order spectral phase, what we find is that a negative signature negative sign. Presence of this negative sign shows that negative GVD is possible to achieve with the help of prism or grating. (Please look at the slides for mathematical expressions)

So, in the end I let us say I have a short pulse I have many mediums through which light is propagating the pulse is propagating and due to this propagation in the end I get a broad pulse a chirped pulse as well. It has to be chirped otherwise we could not broad the pulse. Now with the help of this negative GVD all material will produce positive GVD, these are actually positive GVD with the help of negative GVD which is achieved by not by any material, but angular dispersion we may compress the pulse again.

There is enormous application of this negative GVD in the construction of ultrafast pulses we will study that very soon. So, with this we have come to the end of this module, in this module we have studied group velocity dispersion meaning of positive GVD, meaning of negative GVD, how to achieve negative GVD we will meet again in the next module.