Symmetry and Structure in the Solid State

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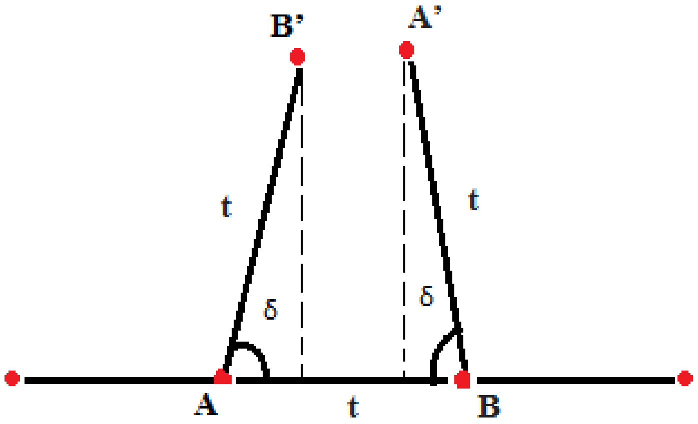
Lecture – 04
Properties of Crystal

So, if we now calculate the distance between A' and B', the A' B' will be a multiple of t, because by definition of a periodic lattice. It has to be a multiple of t, it could be one times t, two times t, three times t, but that value of m is an integer. It is only an integral multiple of t. In that situation, we define a two dimensional lattice. Therefore, the definition of a two-dimensional lattice comes into the picture here. And the presence of the two-dimensional lattice puts a restriction on how we can get the value of delta are organized. Let us examine it now from a simple arithmetic point of view or a trigonometric point of view



B'A'=mt

AB'=AB=t and ∠BAB'= $2\pi/n = \delta$

A'B=AB=t and ∠ABA'= δ

B'A'=AB-AB'cosδ-A'Bcosδ

B'A'= t(1-2cosδ)

mt = t(1-2cosδ)

cosδ= (1-m)/2

So, let us take this distance A' B, A' B has by definition is A B, because now we said that this is the n-1 th lattice point. So, one point before the nth point, which happens to be B. And therefore, we have A B', and therefore it is equal to AB and that is t. And the angle which is BAB' So, this should be this angle which is referred to the angle ABA', which I should be referring to here. The angle ABA' will be equal to 2 π divided by n, because n is the fold of rotation. So, if n is equal to 1, it is 360 degrees; if n is equal to 2, it is 180 degrees and so on.

So, BAB' is 2 π divided by n, so also ABA'. And both of them define our delta value. Now, the question is that these two values remain t, and B prime A prime defines mt, but what we can do by geometry is we can write B' A' as equal to AB, which is the translational periodicity minus AB' cosine δ, AB' cosine δ, and B A' cosine δ.

So, this expression for B' A' will ensure that we have now B' A', which now can be written in terms of t. So, A B is t, the A' B' is t, A' B is t, and therefore this is with two times 1 minus 2 cosine δ. So, 1 minus 2 cosine delta times t defines B' A'. And by the laws of periodicity, this should be a multiple of t, which happens to be the translational periodicity.

Therefore, we can write an expression mt is equal to t times 1 minus 2 cosine delta. So, this would give us the value of cosine delta as 1 minus m divided by 2, we all know the property of a cosine function. And the cosine function will now be an oscillatory function. And the values corresponding to 1 minus m divided by 2, therefore is a restricted value. The restrictions which come on m will be between -2 and +2, so m cannot take any other value other than these -2, -1, 0, 1, and 2 representing different types of rotation axis. For example, in this table we have given the value of 1 minus m is equal to minus of 2, which corresponds to a cosine delta of -1. And cosine delta of minus 1 is the value part, which is 180 degrees.

And therefore, it represents a 2-fold axis, and then the value of B prime A prime in such a situation turns out to be m value becomes 3, so it is 3 times A B. Now, this defines a unit cell, because now we now define our unit cell in terms of these periodically repeating patterns. And this periodically reporting pattern will now have to repeat in all two dimensions to generate the two-dimensional lattice. So, the basic unit cell therefore can be defined as a quantity which has a direction A, and a direction B in two dimensions. And the angle between them is not necessarily 90 degrees, it could be any angle.

So, this therefore defines one possible type of conventional unit cell, which we can have with a compound which is or a or a material which is displaying a 2-fold rotation axis. Likewise, when the value of 1 minus m is minus of 1, and we have a cosine delta value of minus of half, the value of delta can be calculated and therefore n turns out to be three. This is a 3-fold rotation axis in which case the value of B' A' is 2 times AB. And the possible type of lattice, which can be generated is A equals B gamma is equal to 120 degrees.

Now, I would like you to dwell upon the fact that when such a situation occurs, when we have a 3-fold rotation axis. And then the B'A' turns out to be 2 A B in what way we are saying that the conventional units mesh should be A equals B and gamma one 120 degrees. This is something, which I would not leave it to your imagination.

The clue for this finding out what it is left in the Esher's diagram, which we saw earlier in the previous classes. Go to the Esher's diagram which showed the 3-fold axis symmetry. If you see the 3-fold axis symmetry, you will see that the unit cell which you can define in that situation will be something like A equals B gamma is equal to 120 degrees.

So, I want you to figure out likewise. The case when the 1-m value becomes 0, the corresponding value of m times t, which is B' A' will be AB. And this represents a 4-fold rotation, and the lattice will be a square lattice. We have also given an example of this kind in the earlier class in the Esher's diagram So, the idea is now that you have to go back and forth to the Esher's diagram and bring in the mathematical relation, which we have derived here to see one to one corresponding.

And the value of m is equal to 1, we get a 6-fold rotation axis. And you see now A B is given as 0, I want you to find out why it is turns out to be 0. And in which case A is equal to B and gamma is 120 degrees, it again defines a cell which is referred to later we were going to talk about plane lattices. These now in fact define the plane lattices, these 1, 2, 3, 4 types of plane lattices, we will get.

In fact, we get a total of five plane lattices, we will discuss it when we bring in the discussion on different types of lattices that are possible in crystalline materials. At this juncture we are now trying to tell you in this particular example of this slide that in this particular slide, what we are in stating is to show that the number of rotation axis that are possible are restricted to 1, 2, 3, 4 and 6 in crystalline objects which are the periodicity. And the same logic can be extended to three dimensions.

And therefore, the rotation axis are 1, 2, 3, 4 and 6 only, and no other rotation axis is possible. This can be easily seen in a physical situation. Suppose, you are given a 5-fold type tiles, and you are asked to fill this entire floor of this room, you will see that it is not possible to do a closed packing. As we have been discussing earlier all our diagrams in the Esher's pictures are close packed. There is hardly any space that is left between them.