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Lecture - 3 Path Integrals and Schrodinger Equation

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So, in this lecture, I shall actually start with this question. The question is interesting, because I am supposed to be talking to you about the quantum chemistry; and the question is, do we understand quantum mechanics? But before I tell you why I ask such a question, let us look at this figure. And if you look at this figure, there obviously the… If you look at the marks left by this skier; this is something that is impossible. If you look at the figure, this is definitely impossible.

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And now with this, let us look at the original two-slit experiment that I started with. So, in the two-slit experiment, if you remember; if you keep only one slit open, you will get… If you keep only this slit open, you will get this pattern. While if you keep both the slits open, the pattern that you get is shown here. See you think point here. At that point, if you kept only one slit open; if you kept only one silt open, at that point, you will have electrons arriving. But, if you keep both the slits open, none of the electrons will arrive at that location. Therefore, the conclusion is that, any electron passing through the system knows that, I have kept both the slits open. That is why none of the electrons arrive at that location. So, any electron passing through the system is able to feel both the slits.

Now, in our classical ((Refer Time: 02:21)) if I had a particle, you see what is going to happen is that, the particle will pass through either one slit or the other slit. Now, electron does have characteristics of particles. But, in spite of that, this electron is able to know that, there are both the slits open that, we do have an explanation for that; the explanation is that, there is a wave and what is happening is that, the wave passes through both the slits and interferes destructively at that particular point. That is why the electron does not go.

But this is something that is very difficult for us to understand, for us human beings, because in our experience, there is nothing that is able to do such a thing. And therefore, how can the electron behave like that? That is something that is difficult for us to understand. And therefore, if you are asked, do you really understand it? And the answer is that, you see with our classical thinking, the only answer that we can have is, we do not really understand it. And in fact, R P Feynman, one of the great scientist of last century; he used to go around saying that, nobody understands quantum mechanics.

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So, the way you want – you or anybody has to imagine of the two slit experiment is that, the wave associated with the electron actually goes through both the slits. So, it is as if some part… I mean it is not really correct to say a part goes through here and another part goes through there, but it is something, is going through both the slits even if you have only one electron. And therefore, in quantum mechanics, this is actually the way things are happening even though such a thing will not happen in our real world; it is as if something has go on through one other slits and other, something associated with the electron has gone through both the slits. That is what is happening.

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So, then you have this very interesting question, how does an electron propagate? See if I was… If I have a chalk piece; imagine this chalk piece; if I drop it, I can actually see the path that it follows. So, I can use what is referred as Newtonian mechanics to describe its motion; I can actually see the path that the electron will… not the electron, but the chalk piece will follow. So, similarly, if suppose I wanted to describe the motion of this electron, how will I do it is the question that I want to answer. So, in order to do that, what I am going to do is... The answer is already given; it says through all possible paths… Let me explain this in a slightly better fashion.

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Imagine that instead of having a system of just one wall with two slits. Imagine that, I have three walls. So, these are three walls as we can see: 1, 2, 3; each one of them has two slits. And here is the electron; source of electrons is the electron produced there and it will pass through these system of slits. And imagine I want to think of the possibility or the probability that the electron will arrive at that location. How will I calculate this probability? In the case of the two-slit experiment, what I said was, to calculate the probability, I first have to calculate the wave function. To calculate the wave function, how will I do it? There are two possible ways in which the electron could go maybe through the first slit or through the second slit. Each possibility makes a contribution to the wave function. So, you have to first evaluate that contribution and then add the two contributions together to get the total wave function.

And then after that, you have to take the magnitude of that wave function and square it; you will get the intensity or the probability. So, if you extrapolated that at this situation, what will happen is that, you see the wave associated with the electron could have perhaps followed this green path; that is, definitely one possible path. This red path is another one; this blue path is yet another one. And if you think about it, you would realize that, there are 2 to the power of $3 - 8$ different possible paths that the wave associated with electron could have taken to arrive at that location.

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So, the contribution on the first one; imagine I have some wave; calculating it; I will calculate and call it psi 1. Then I will have a second path; I have not run all the paths; only three of them I have drawn. But, the paths are there. So, I will have to evaluate the second one – contribution on the second; and then up to psi 8, I will have to calculate and add them together to get the total. And once I get the total, I will take its magnitude and square it; I will get the probability or intensity if you like. I will speak a probability if I have only a single electron. But, if I have a beam of electrons, then I will speak of intensity. So, that is what happens in this case.

But, suppose I say, now, I am going to think of even more complex experimental setups; suppose I had an arrangement like this, again these are all imaginary things. Nobody who is sensible will ever do these things. But, what has happened here is I have produced the electron at this location and I am going to calculate the wave function for it at that location. And I think probably 12 walls in between; and each one of them has 4 slits. So, if we have 4 slits on each wall and 12 walls, then what will happen? How many paths? The answer is 4 to the power of 12 parts are there. So, you have to think of all these 4 to the power of 12 paths. Somehow calculate the contribution from each one of them. After having calculated that, you will have to add all the contributions together; you will get the total wave function and take this square of wave function. That will give you the probability.

This is the prescription. Now, suppose you make it even more complex. One second; let me just go back. I think I did make a slight mistake; I suspect here is a system with 12 walls and 4 slits. The one that we are looking at earlier had more slits. Actually, this has how many? I think 8 slits and 12 walls. So, what we can do is, imagine you increase the number of walls as well as the number of slits. So, then what is going to happen is, you have the source here; you have the screen there; you have made many many many walls. These are all imaginary walls in between. And on each one of them, what you will do is, you will go on making slits. So, for example, these are the slits on the first one, second one – second, third and so on. So, then what will happen, you see you will have to calculate contributions from paths like this. This is one typical path eventually arriving at this location. So, I can imagine the limit, where I would have infinite number of walls; of course, I mean that is purely imaginary, not physically possible to have it.

Imagine I have infinite walls. And in each one of them, I am going to make more and more slits, so that the number of slits are progressed infinity. Then how many paths would I have to think of? Infinity to the power of infinity $-$ a huge number; all these paths are going to contribute to the wave function. So, you may have a path, which does this; something like that. Even that will make a contribution to the wave function. Of course, if you think about it physically, you know that the contribution from such a path may be small. But, in principle, it will also make a contribution.

So, I will do the ultimate; I will imagine that, I have infinite walls; in each one of them, I have made so many slits – large number – infinite number of slits, so that the walls themselves disappear. So, I say the walls are there; the slits are also there only in my mind. Correct? So, I have made so many slits that, the walls have disappeared. But, of course, the paths that I am thinking of – they have a real physical existence and they are all there. So, in the ultimate limit, where you have so many slits that, each wall has disappeared. What are you actually talking about? You are talking about the propagation of the electron in free space. And how does an electron in free space propagate? Here is the answer. If you have…

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If you start with an electron at this location; now, at this instant, I have an electron at this instant here. And imagine I want to calculate the probability that the electron will be there; maybe 10 nanoseconds later suppose. What should I do? I have to think of all possible paths that the electron could have taken. They will all start at this location now and the end to there after 10 nanoseconds. So, you will have to think all of these paths – this one, many many many paths. All these paths are possible. You have to think of anyone, all of them; each one will make a contribution to the wave function and you have to evaluate the contribution of any particular path or all possible paths and then perform a sum over all these paths. And that will give you the wave function. So, this is the meaning when I said an electron propagates through all possible paths, because all paths make a contribution to the wave function. This is to be compared with Newtonian mechanics.

See I told you, if have this particle here and if I drop it, I know exactly it follows only one path. But, that is not the way things are in quantum mechanics. If I have an electron here now and if you ask me where will it be after 10 nanoseconds; then I cannot actually tell you where it will be. All that I can do is to give you the wave function. What is the use of the wave function? The wave function will tell me the probability density that, it may be located at this point after 10 nanoseconds. That probability I can calculate. How will I calculate? I may think of a path, which starts here and goes there; another one, which again starts here now and goes there after 10 nanoseconds. I have to think of all the possible paths that the electron could have taken; evaluate the contribution from each; sum all these contributions. That will give me the total wave function. Take its magnitude and square it; that will give you probability density that it may be located at this point. That is the way it is.

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So, these are shown in the slides actually. The point, where you produce the electron; the point, where… This is where you produce the electron or you start with the electron. That is where you are detecting the electron. So, in the element of infinite walls and infinite slits, you are actually speaking of free propagation; and then you find that, all the paths contribute. But, then of course, you will object to this; you see you… because I have not actually given you a prescription for evaluating the contribution from a particular path. See I just said evaluate the contribution from any given path. How will I evaluate the contribution from a particular path, is the next question. And for that, there is a prescription.

You see if you have this particular path or any paths, imagine electron is here; I am thinking of calculating the probability density at this point. You think of a path, which may be a straight line. That makes a contribution to the wave function. Or, maybe a path, which is say a parabola connecting the two. Even that makes a contribution. Therefore, you think of any particular path; I want to know how much is the contribution of that path. And the answer is that, if you give me this path; that means I know the position of the electron at any instant if it followed that path. The moment I say I have a path; that means I know the position of the electron at each instant of time along that path.

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So, if you say… It can be a simple one dimensional though I have been talking of the things in 3 dimensions; but let me say, I am thinking of a simple 1 dimensional situation, where the position of the particle is denoted by x. And the moment I say I know the path, it simply means that, I know x as a function of time. So, if I knew the position as a function of time, then what I can do, I can calculate the velocity of the particle, because velocity is obtained by differentiating this with respect to time. And if I knew the velocity, I can calculate the kinetic energy. Therefore, if I knew the position, I can actually calculate the kinetic energy of the particle. And not only that, you see maybe the particle is moving subjected to some potential. Therefore, I potentially say, is a function of position. So, it is possible for me to also calculate the potential energy of the particle if it followed that path. If it is following a definite path, I can always calculate its kinetic energy at each instant of time. Also, it is possible to calculate the potential energy of the particle at each instant of time.

And, if I can do that, I can take the difference between the two, not the sum, because some of you might have expected the sum to be there. But, that is not how it is. If you take the difference between the two and integrate it from the initial time to the final time; the limits are from initial time to the final time; this is something that is very familiar to physicists; it is referred to as action and it is denoted by the symbol S. See if you give me any path, it is possible for me to calculate the action associated with that path. And interestingly, what happens is that, the contribution of any path to the wave function in quantum mechanics is determined by this object, which is referred to as the action. If you give me any path, I will be able to give you a number, which is the action for that path. And once I get that number, what I will do is, I will take e to the power of i into S divided by h cross. What is i? i is square root of minus 1. I mean you may say this is very arbitrary. In fact, it is the arbitrary. In some sense, it may continue and then it will become clearer. So, given any path, what you do, you take e to the power of i S by h cross; h cross is the Planck's constant divided by 2 pi. It is going to occur in all over discussions.

And, this will determine the contribution of a particular path. And then if you have all the possible paths, what you will have to do is you will have to sum over all the paths – all the possible paths. But these paths have the condition that, they will all be starting at this location at the initial time maybe now. And then there are the final time maybe after 10 nanoseconds. That is the condition. So, you have to sum over all the possible paths. And this object is going to determine the wave function. Now, wave functions – if you have said it a little bit in quantum mechanics, then you know that, they have to be normalized. So, there will be a multiplicative factor there, which ensures that, the results are physically acceptable; acceptable in the sense that… See because wave function is related to probability, psi square will give you something, which may be referred to as probability density. And if you integrate it over the entire space, answer has to be 1. This is anticipating some parts of my lecture later. But, this is a normalization fact; that is, all that you need to know now; and you have to sum all the other possible paths.

So, that is the prescription. Now, as I said, you may ask me, where did I get this prescription from? You see this may be taken as a postulate. There is no other way in which this can be justified. This is... You can say this is how the nature is. This can make people unhappy, because you see I suppose to be doing science; and suddenly out of the hat, I pull out something like this, which cannot be derived. Unfortunately, this formula cannot be derived from anything which is more fundamental.

So, if I like, I can take this as a postulate of quantum mechanics and develop the subject of quantum mechanics there from. Now, as I said, maybe this will make you unhappy, but then I would like to remind you that, in Newtonian mechanics, when you start, you have studied force is equal to mass into acceleration. Have you studied a derivation of that? See what is happening is that, you have observed the field; and then from that observation, you would say this is how it is. Or, in classical electrodynamics, there are four equations; which again has abstracted out of experimental observations. And the good thing is once you assume these four equations, then you can make lots of predictions; and all of them in agreement with experiments. So, similarly, if you make this assumption, this you will have to think of it as an assumption. Once you make this assumption, then you can develop the whole subject of quantum mechanics from here.

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There is a nice analogy though this does not ((Refer Time: 22:50)) concern the subject of quantum mechanics itself; whereas, there will be lot of similarities between the way an electron behaves, quantum mechanical particle behaves and that of a drunken walker. Essentially, what I am saying is that, there is lot of similarities between the way an electron moves or lot of connections actually, not… I wish they should be completely similar. The mathematics used to describe the motion of an electron, is very similar to the mathematics that is used to describe the walking of a drunken person. That is the precise way to put it. You may wonder why; let me make clear. I suppose you are all familiar with Tintin comics; even if you are not familiar, it does not matter. This is a character in Tintin comics; his name is captain Haddock and he likes to drink.

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So, if suppose this person – captain Haddock – in the evening, he always gets drunk. So, imagine he goes to the pub maybe around 9 O'clock in the night; stays there till 10 O'clock in the night every day. And then 10 O'clock, he will come out of the pub and he is completely drunk; he wants to of course, go home, but he has absolutely no sense of direction. So, what he does is he will execute what is referred to as a random walk. Now, a mathematician will describe the random walk in discussion. He will say that, each step that the person takes each in a direction independent of the direction of the previous step. See if he takes a step like this, then after having taken the step, he has absolutely no memory in which direction he has to go. So, the next step will be independent of the previous step. So, that is the characteristics of a random walk.

Suppose I am going to study his walking. So, this is the bar and he comes out at 10 O'clock in the night. And on the first day, suppose I observe him; then maybe he would have taken such a path, which I suppose is not very likely, because he will be executing all kinds of walks. But, maybe this is one of the possible paths. And he will be at this location maybe perhaps after 1 hour. The next day if I observe him; and he will not definitely follow that path, but another one; and the third day, maybe another one; fourth day, another one and so on. Suppose I am persistent I have studied him, I have looked at his behavior for 1 year; then on the 366-th day, I am going to observe him let us say. And suppose one of you ask me, where will he be after 1 hour; will you able to answer the question? Definitely not; you can say, maybe there is a large probability that, he will be

at this area. The probability that he will be 10 kilometers away; that is 0. That kind of statements you can make. Therefore, it is like the electron. See if I had an electron here now and if you ask me, where will be after 10 nanoseconds; I can only give you the probability. Similarly, in this case also, I can only give you the probability.

Now, suppose I want to calculate the probability; how will I do is; suppose I want to calculate the probability. In fact, I want to calculate the probability that the electron… Not the electron, but captain Haddock is at this point; I want to calculate the probability density that the random walker is at that location after 1 hour. How will I do that? The answer is here in this picture. See you will say that, it is not very likely that, he would have followed perhaps a path like this; but maybe a path like this is more probable than that. Therefore, you see what should I do is, I should think of all the possible paths that he could have taken starting at the initial location and ending there at the final location. You have to think of all the possible paths. He could have taken in any one of them. What happens is that, some of them would be more probable than others. Therefore, you have to assign probabilities to different paths. And then for each path, you have to do a calculation of the probability and then sum over all the possible paths you are going to get the probability – total probability.

See this is very much analogous to the case of the electron. But, in the case of electron, there is a large difference – huge difference; here you are calculate… for each path, you are assigning a probability; whereas, in the case of the electron, for each path you are assigning not a probability, but a contribution to the wave function; psi 1, psi 2, psi 3, etcetera remember; they were wave functions. Therefore, for example, in here you have psi 1 plus psi 2 plus psi 3 etcetera until psi 8 perhaps. So, these things – wave functions – they are in general, complex; they may be positive or negative. Therefore, if you added complex numbers, you see the final answer can be 0.

Whereas, in the case of the random walker, what happens is that, you are adding not wave functions, but probabilities. Probabilities are constrained in that; they have to be real first of all; and the second thing is that, they are all positive, they are never negative. Therefore, if you add it to probabilities, you see they cannot cancel each other. And therefore, in the case of the random walker, there is no interference. Whereas, in the case of the electron, because you are adding together different contributions to the wave function, the wave function can cancel each other. And therefore, you have interference happening in the case of the electron. But, interference will never happen in the case of this random walker. And that is the major difference. But, otherwise, mathematically speaking, it is the same kind of object; and in fact, instead of S, which is there in quantum mechanics, the object that happens – they do not worry about that equation. It is not very understandable.

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But, what happens is that, instead of this action, what happens in the case of the random walker is, you have this object, which actually resembles the action. The contribution of any given path is determined not by the action, but something, which is similar to the action. And what happens is that, you have to take the exponent; you have to calculate the exponent of that. And so what I want to say is that, the mathematics. So, if a quantum mechanics and the mathematics; describing the motion of a random walker is very very similar. And of course, we are very familiar with random walks. Where do they occur in science? Answer is that, if you had a colloidal particle, which is immersed in a solution; and if you observe it under the microscope, then you will see that it executes random motion. It executes random motion much similar to the walking of a random walker or a drunken person.

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So, this is what is shown in this slide. This is the colloidal particle. It is big enough for you see under the microscope let us say. And then if you observe it, you will find that, it seems to follow some trajectory, which may be something like this and which resembles the random walk of a drunken person. And this is how a colloidal particle will diffuse in solution, because if you put a colloidal particle in solution… I wonder whether you are familiar with diffusion equations themselves. If you have a particle – colloidal particle put in a solution, what is going to happen is that, there are molecules of the medium surrounding it.

These molecules will come and hit the ((Refer Time: 31:38)) and all. The colloidal particle, at any given instant of time, you see there may be more particles hitting from this side than from the other side. And the momentum imparted in this direction would be more than the momentum imparted from the other direction; and then the particle will go in this direction. And when it has reached here, maybe the momentum of the particles imparting from this side is more than the momentum of particle imparting from this side. So, then the particle will go in that direction and then it will go like that randomly in direction. And that is what happens. This is essentially the thing that is responsible for diffusion of a gas or diffusion of a liquid or diffusion of colloidal particles in a solution.

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So, this we have already seen. And as I have told you, in the case of the electron, things are relatively bit weird, because you are not adding the probabilities, but instead you are adding the wave functions. And wave functions are actually complex. And therefore, they can cancel each other. The signs may be positive or negative. Therefore, there may be cancelations. Now, if he found likes, it is possible to develop the whole subject of quantum mechanics; you see starting from this equation.

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In fact, as I told you earlier, it is possible to take this as a postulate and develop the whole subject of quantum mechanics from this formula. That is possible, but mathematically, it is a little bit difficult. It is little bit involved. If anyone is interested, they should have a look at this book. See if you look into this book, you will find that, the subject of quantum mechanics is developed starting from this particular formula. The whole thing, the whole book is based upon this formula. It starts from here and develops everything or almost everything.

Now, mathematically speaking, this is the rather difficult. And I do not know if any chemistry, quantum chemistry books, where the approach is described. But, it is a very beautiful and very nice approach. Many of the physics book these days do make use of this approach. If you want to get the details of this approach, you will have to read this book or some of the good quantum mechanics books. For example, there is a book by R. Shankar – very very nice book entitled principles of quantum mechanics. It has a chapter on this approach. But, because it is mathematically involved, I shall not continue to discuss this.

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But, what I will do is, I will switch over to the usual procedure of introducing quantum mechanics, which is to introduce it as a set of postulates. So, the way one would go about is this. If you have wave phenomenon after all, we are seeing that, there are waves. So, if we have waves, the argument is that, there should be a wave equation, because people

are very familiar with waves; and waves are always described by wave equations. Therefore, if we have wave phenomenon, there must be a wave equation. And it so happens that, the wave equation that is appropriate for the description of motion of electrons or the wave equation of quantum mechanics, is an equation that was introduced by Schrodinger. This is Schrodinger. And matter waves obey his equation. So, we will see this equation later.

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 $\frac{\partial}{\partial t}P(\mathbf{x},t)=D\nabla^2P(\mathbf{x},t)$ $i\hbar\frac{\partial}{\partial t}\Psi({\bf x},t)=-\frac{\hbar^2}{2m}\nabla^2\Psi({\bf x},t)$

Now, I said that, in the case of a colloidal particle, which undergoes diffusional motion, there is mathematically speaking the motion of the Brownian particle. And the motion of the electron, are mathematically speaking similar. And the motion of a Brownian particle is actually described by what is referred to as a diffusion equation. Again, I will not go into the details of this equation. But, in the case of a Brownian particle, you are concerned with the probability of finding the particle at a particular location x at the time t. So, this is the kind of equation that a diffusing particle will obey. And this D there is known as the diffusion coefficient. So, we know that, in the case of Brownian motion, this kind of diffusion equation… Do not worry about this equation, if you are not familiar. But, there is an equation, which describes a diffusional motion of a particle; and it is that equation.

Now, I have argued that, mathematically, diffusion and the motion of an electron in quantum mechanics are similar. So, if diffusion is governed by such an equation, then it is very natural that, for the electron also, there should be an equation, which resembles that. And that equation is nothing but the Schrodinger equation. It is this equation. You can see the similarity if you look at it carefully. This is for a free particle – particle moving freely in space; you have d by dt of probability here. You have the diffusion coefficient; then you have del square. Del square involves differentiation with respect to position. And you can see that, in the case of electron, you do not have the probability, but instead you have the wave function. And the wave function changes with time. How does it change? It changes. According to this equation, instead of the diffusion coefficient there, you have this say, minus h cross square by 2m and then of course del square; there you have... here also you have del square. So, this is a structure... Mathematically speaking, at least the structure of it is very similar to the case of diffusional motion.

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So, the basic equation of quantum mechanics is actually this equation, which is the Schrodinger equation. In comparison with what I had written earlier, we will look at this in more detail later. In comparison with what I had written earlier, there is an additional term. This is the potential that the particle feels. Earlier I was ((Refer Time: 39:23)) of a free particle, which is not subjected to any potential. But, if you have a potential, this is the equation that it obeys. And not only that, if you know the wave function at any initial instant of time, it is possible to use this equation. If you know the wave function psi at an initial position x at the time 0, then it is possible to use this equation and find its value at

the position x itself if you want at a later time. Or, at any position, we can calculate at a later time. That is the idea. But, we will see more details of this later.

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My introductory lectures; I said first 3 or 4 lectures will be in introductory in nature. This is the last one. So, it is natural that, I should give you some references. The last part of what I have told you; those are the things that we will be discussing in more detail later. And therefore, things were not clear to now. It will become clear as we proceed. But, let me give you some references to this part. There are very very nice, very beautiful books; you should have a look at them; they are non-mathematical; I have selected only nonmathematical books. The first book is actually by G. Gamov. It is called Tompkins in Paperback. This is a book that was written in the… I do not remember the years, but probably 1930s. Between 1930 and 1940, this book was written.

Gamov was a Russian scientist, who migrated to the west during the communist era. And he wrote series of articles in one of the British newspapers. And these articles were collected together into the form of a book. So, the book is very interesting if you have not read; or, not seen it or read it. The story is this actually. It is kind of friction. Mr. Tompkins is a clerk in an office in London. He has absolutely no interest in physics. But, it so happens that, his girlfriend is the daughter of the physics professor at the university. So, the physics professor gave some lectures – some popular lectures on quantum mechanics, relativity and so on. And just to please his girlfriend and hopefully the future father-in-law, he goes and listens to the talks of this professor. And as I said, he has absolutely no interest in physics. The professor speaks about quantum mechanics, relativity theory and so on. Tompkins does here a few things, but dozes off while he is listening. So, he goes to sleep. And when he is sleeping, he dreams. And the subject matter of the book are his dreams.

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So, in our universe, the value of h cross is approximately 10 to the power minus 34 Joules second; and this is extremely small – 10 to the power minus 34. Therefore, if you think of a wave or if you think of a particle, the wave length associated with that particle will be lambda, which is equal to the h divided by the momentum. So, because the value of h is h cross and h is quite small, the wave associated with any particle is very very very small. And if you think of the uncertainty principle – delta x into delta p; I told you what it is; greater than or equal to h divided by 4pi. This number is very very small. So, the effect of the answer in the principle simply cannot be seen in our world. In our world meaning in our everyday life. In our everyday life, we do not see any effect of these things.

But, suppose you had a universe, where the value of h cross is not 10 to the power of minus 34; but suppose it is 1 Joules seconds, then what will happen? You will be able to see quantum phenomena with your eyes. And he dreams of a world in which value of h cross – this is of the order of 1 Joules second, so that they can see wave phenomena

happening in his everyday life. So, one of the things that Gamov describes is this. He has a car; he puts the car into the car shed in the night; closes the door; and then goes to sleep in his room. Now, there is a typical quantum phenomenon, which is referred to as tunneling. So, the next day, he goes to sleep; the next day, he comes out of his room and finds that, the car has behaved like a wave. And when it behaves like a wave, it actually can go pass through even barriers.

So, it passes through the door, which is closed of course, and comes to the outside leaving the door intact. So, this is a typical quantum phenomenon, which can be observed if the value of h cross is not so small, but maybe 1 Joule second. And of course, in chemistry, there are many many effects of this kind of behavior, which is referred to as tunneling. He also… I mean if you look into the book by Gamov, you will find that, he also describes relativity theory. I will not go into description of that; some part… little bit of chemistry for example, combination of chlorine and sodium and so on. So, please do have a look at this.

Then, there is another book, which is that one; it is Robert Gilmore. He has written a book, which is called Alice in Quantum Land. You know this story, where Alice goes to the wonderland. But, here instead of going to the wonderland, she goes to the quantum land and sees quantum phenomenon. There is another one written by Gribbin; very nice book, very strongly recommended. It is available to purchase in most book house. In fact, in the airport, I sold the book today; not today, but yesterday. So, this is called Schrodinger's Cat. There is a very famous paradox, which is referred to as a Schrodinger's cat paradox; referred to Schrodinger's cat paradox. The book contains a description of this paradox; but not only the paradox, the whole of quantum mechanics is described in a non-mathematical language. So, very very nice book.

Then, this is… This book is by Cropper. It is the biography of a people like Schrodinger, Planck, Icenberg, who have been involved in development of quantum mechanics. And not only he gives you biography, but he also gives you an introduction to the subject itself; written very nicely. You see if you have cats, then naturally you will have kittens. And that is the subject of… I mean that is the title of this book by Gribbin. See since quantum mechanics was formulated, people have been trying to disprove it or trying to find exceptions to quantum mechanics. And so people have been trying to all kinds of experiments; trying to find conditions in which quantum mechanics is not obeyed. And

as of today, nobody has been able to find any exceptions. So, this gives you a history of all these experiments that have been carried out till 1995.

This book by Gamov is very old; and therefore, it was rewritten by somebody else called Stannard. And it is called The New World of Tompkins. So, it is necessary for you to read either this or that. I spoke about Feynman. There is a very nice and very beautiful book written by Feynman. The spelling in the name is not correct. The title is QED; QED stands for quantum electro dynamics. But, you should not be frightened by the name. This is a book, which is very readable and very understandable. I am sure you will enjoy reading this book. This is a little bit more mathematical book. Only if you are mathematically minded, this book is advised; otherwise, it is not such a great book.

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You should also look at this website, which I have referred to earlier. This is the famous quotation that I have shown you earlier. He said that, it is safe to say that, nobody understands quantum mechanics. And you also look at this quotation, which is from Wheeler. Wheeler happened to be the PhD supervisor of Feynman. And finally, I would also like to add that, there is a book by… There are 3 books by R. Venkataraman available in India; beautifully written books these actually. There are 3 of them. I will give you the reference to it maybe tomorrow perhaps. Very nice books, not expensive; it is very readable.

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So, here is the last slide. I have spoken only for 3 hours and it is surely not enough to convince you that, it is such a beautiful subject. But, I hope by the end of the course, which will be roughly 40 lectures, I hope you will be convinced that, quantum mechanics is actually a very beautiful subject. What you should do is you should think of the physics; you should try to understand the subject in the physical fashion rather than approaching it mathematically. Once you have understood the physics, then understanding the mathematics actually becomes easy. I think I should stop now.