Symmetry, Stereochemistry And Application
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Lecture No-55
2D Lattices and Space Groups

Welcome back to the course entitled symmetric stereochemistry and applications. In the first 3 lectures we have discussed about certain aspects of crystallographic symmetry different symmetry elements and then we have talked about the 1D lattices. So, in this lecture we will start with the possibilities of different 2D lattices and we will try to see how those 2D lattices should be drawn in a piece of paper as a 2 dimensional projection and how those symmetry elements are going to be applied in case of those 2D lattices.

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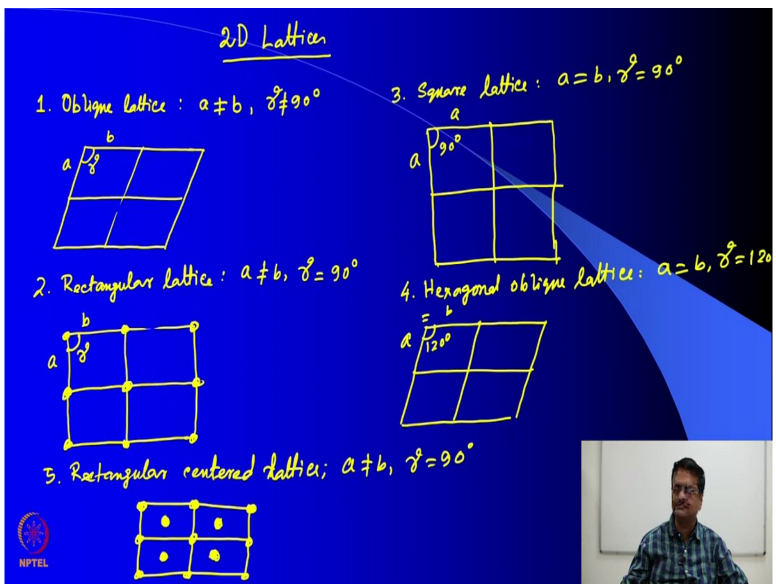

So, the 2D lattices can have different symmetries. So, the first one is an oblique lattice where a is not equal to b and the angle between a and b which is gamma not equal to 90 degree. So, the oblique lattice should be drawn like this. So, here it is a, b and the angle is gamma. Second one is a rectangular lattice where a is not equal to b. But gamma is equal to 90 degree. So, here the rectangular lattice can be drawn like this the third one is a square lattice which is characterized by a equal to b and the angle equal to 90 degree.

So, that should be drawn as a square and we have the individual portions which is equal the fourth one is a hexagonal oblique lattice where a is equal to b. But gamma is not 90 degree rather it is 120 degree. So, that lattice should be drawn like this where it is a and b are same and this angle is 120 degree. And the 5th type of lattice is a rectangular centered lattice where a is not equal to b gamma is equal to 90 degree.

But the lattice is now centered which means in all the previous cases you had atoms only at the corners. But in this particular case of rectangular center lattice you have objects at the corner plus the objects at the center of the lattice. So, this becomes a rectangular centered lattice right. So, these are 5 different 2 dimensional lattices that are possible just like 7 crystal systems here you have 5 different type of bravais lattices in case of 2 dimensional symmetry.

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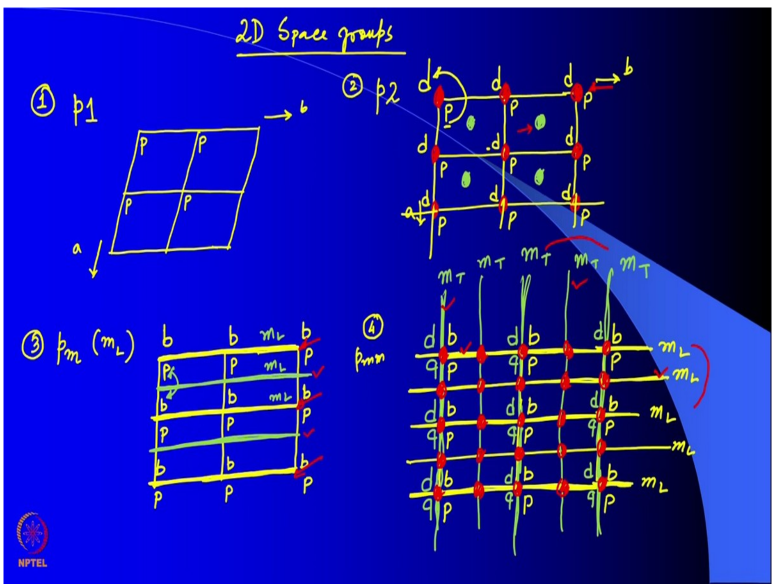

Now let us try to see what are the other what are the possibilities of different space lattices in 2D that is 2D space groups. So, of course the first one should be no symmetry which is indicated as P1 you see P means primitive and one means there is no symmetry that means there is only translational symmetry present. So if we draw the oblique lattice and you have the object p here this object is only translated along the direction of b and in the direction of a and that is it.

So, there is no other symmetry element present. So, we call it as P1. The second one is the next higher symmetric is P2 which means there is a 2 fold axis of rotation and as soon as you have a restriction of 2 fold axis of rotation then the angle that gamma angle has to become 90 degree. So, this can be there in case of square and rectangular lattices. So, suppose I am drawing a rectangular lattice I will have it divided into parts.

So, these are 4 lattices and suppose I have a letter p which is translated as usual like this in both the directions both along b and along a. Now if you apply a symmetry element at this point which I am drawing as red this is a 2 fold axis which is passing through the corner of the unit cell I should also draw the translational objects here as well as this line can continue like this. So, now if we apply this 2 fold symmetry to each of those objects.

The letter p this letter p when gets rotated about 180 degree it comes here this goes there this comes here similarly you have the objects like this. So, now what do we see we see that the object here and object there they are also related by another 2 fold which has got generated at the center of the unit cell. So this is a representation of a 2D lattice P2 then what happens if we have the third one which we write as Pm.

So, by now you should understand that this m means a mirror plane and this mirror can be a longitudinal mirror or a transverse mirror. So, in general we apply this as a longitudinal mirror to start with. So, here again we need to have a lattice and I have a letter p which is translated in both the directions both inside and outside in itself like this and suppose this is my mirror plane which I am trying to draw as a bold line.

So, this is a mirror the longitudinal mirror. So, when you have a longitudinal mirror these objects get reflected as I am drawing now right. So, the translation related objects are here and their mirror images are inside the units. So, now what do we see is that between these 2 that exist a mirror plane here as well. So, 2 parallel longitudinal mirrors mL and mL gave rise to a third longitudinal mirror in the midway between the original 2 mirrors.

So, this is the third possible space group or third possible 2 dimensional space group which is Pm and represented like this. Let us see the fourth one we write it as Pmm. Now when I write 2 m's it essentially means that there are 2 mirror planes which are perpendicular to one another and one is longitudinal and the other one is a transverse mirror. So, when you try to draw such unit cell we need to apply both the mirrors one after another.

So, I am drawing it slightly bigger. So, that we can easily have these mirrors displayed inside the unit cell. So, I am taking for simplicity the same letter p and drawing the corresponding translation related objects these are all one unit translation related and then we have the original mirror planes which are here the ones which I am making in bold the lower ones and then we draw the longitudinal mirror related objects.

Now since this is mm we have transverse mirrors also which run perpendicular to our longitudinal mirror that I am drawing now. So, if we draw the corresponding symmetry related object let me erase this and write it somewhere else otherwise you will not have enough space we apply transverse mirror on both the objects and draw these green objects. You see when you draw it yourself you will see that it is not that difficult to draw and you can draw these diagrams very easily if you practice.

So, these yellow ones are related by longitudinal mirror and these green ones are again, related by the longitudinal mirrors. But yellow and green are related by transverse mirrors. So, the green and yellow is related by transverse mirror, green green and yellow yellow are related by longitudinal mirror. So, what we see here is in the middle parallel to the longitudinal mirror there is a new mirror plane that has got generated.

Similarly in the middle between the 2, transverse mirror a new mirror plane has got generated. So, 2 parallel mirror planes give rise to one more mirror plane parallel to the original mirror and what we further see we see that the point of intersection of 2 mirrors is a two-fold axis. So, every point of intersection of 2 mirrors which are perpendicular there are 2 fold axis. So, these are the additional symmetry elements that are generated because of the presence of some other symmetry element.

And this particular feature that you are observing is new because in molecular symmetry we did not have any other symmetric being generated other than what is present within the molecule.

So, here we are applying 2 or more symmetry elements we are drawing the symmetry related objects and then we are finding that the symmetry related objects are further related by some other symmetry. So, in the case of P2 you have seen that these 2 folds gave rise to a new set of 2 folds.

In case of Pm you saw those mirrors gave rise to a new set of mirrors here these 2 m L's gave rise to a new mm these 2 m T's gave rise to a new m T and the point of intersection of m T and m L gave rise to a two-fold. So, in crystallography whatever symmetries are present can give rise to the other symmetry elements and that becomes very important in crystallography to find what additional symmetry elements are formed because of a certain amount of symmetry that is being applied.

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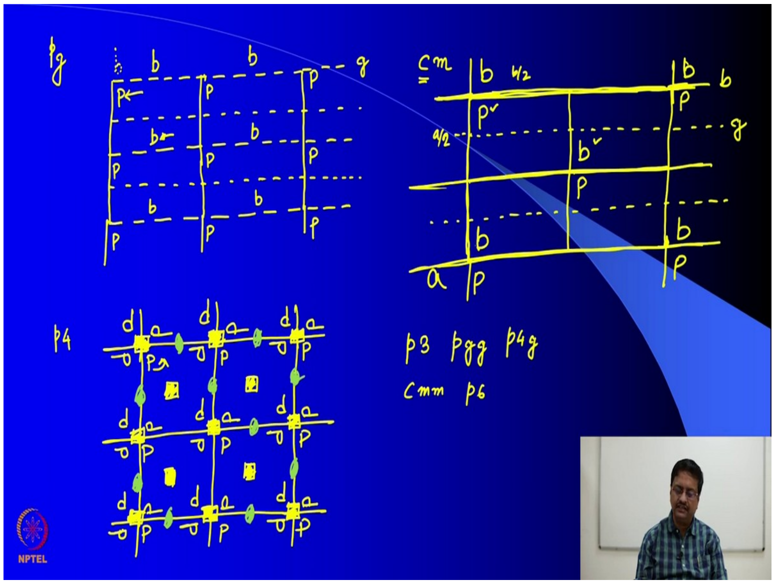

Now let us try to draw some other 2 dimensional point groups I am not going to draw everything because it will be too much for this course we will draw a few for example Pg which essentially means a primitive lattice with a glide plane. So, when you say that there is a glide plane of course we draw one side as a dashed line and the other side has a bold line and then draw these separations which essentially means that there are 4 lattices.

So, if you have a letter p here the translational objects I am first drawing now these translational objects will be having the symmetry g which is the glide plane here. So, technically the reflection should have come here. But because of this being a glide it moves halfway around this similarly you have the glide related objects here and the glide related objects there. So, now what is the relationship between this object and that object once again these 2 objects are also related by a glide here.

So, what we see is in many cases almost all the cases the presence of one symmetry element gives rise to some other symmetry element in the unit cell let me draw another one which is P4. What does it mean P4 it means it is a primitive lattice and has a 4 fold symmetry. So, what is the necessity for a 4 fold symmetry that a should be equal to b then only by 90 degree rotation one will fall on the other. So, it has to be a square lattice. So, now if I have my object as usual and then I have a 4 fold axis at the corner of the unit cell like this what I should have is that there should be 4 objects around this 4 fold axis and it should look very interesting.

So, when you rotate this object 90 degree about the axis which is perpendicular to the plane of projection it would look like this another 90 degree rotation will look like that another 90 degree rotation it will look like this. So, if you practice drawing this it, it will not be so difficult you may be thinking that it is very difficult to draw. But it is not that difficult to draw when you do it yourself 90 degree rotation every time rotates the object.

So, what has happened here is that because of these 4 fold axis a new 4 fold has got generated in the middle as you can easily see and additionally at the middle point of the edge a twofold has got generated at the midpoint of all the edges right. So, here P4 has given rise to new twofold symmetry. So, there are many such 2 dimensional point groups you can refer again back to F.A. Cotton's textbook on chemical applications of group theory chapter 11.

And you will be able to see the projections of all of them I will do one more here because that makes it sort of complete when we say Cm, Cm means there is a centering lattice centering and a mirror plane. So, if I have and a lattice like this and i am dividing it into 4 parts and I am saying that this edge length is b this edge length is a. So, this half of length is b by 2 this half of length is a by 2. So, when I say it is a C center lattice I have a object at every corner associated with the corner point and half translated along a half translated along b and I should have an object here.

So, whatever is represented is a lattice which is C lattice that means it is a C centered it is a centered lattice. Now when you apply the middle planes which are here the Cm is the longitudinal mirror which is this one you get the mirror related objects right. So, what is the relationship between this object and that object? If you see carefully this is nothing but reflection and half translation you are reflecting about about a mirror which is a glide.

And about that glide plane you have the objects related to the glide. So, here you have the glide plane that is present in case of Cm. So with these representations I would like to leave it here for you to draw it yourself and draw other 2 dimensional point groups which are like P3 then you can draw Pgg. These are the things which you should draw yourself P4g Cmm and maybe P6 these things you should practice drawing yourself and understand how these symmetry elements are forming.

So, what we have seen here is that in case of crystallographic symmetry the symmetry elements are in general outside the object and those symmetry elements are responsible for generating the new symmetry related object in the unit cell. And in case the object has a particular symmetry and that symmetry element coincides with the symmetry of the lattice then only half or one third or one fourth of the object is possible to be in the asymmetric unit. So, with this I conclude this lecture and we will continue in the next lecture from here.