Chemical Principles II
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Module 06
Lecture 33
Tutorial Problem - 04

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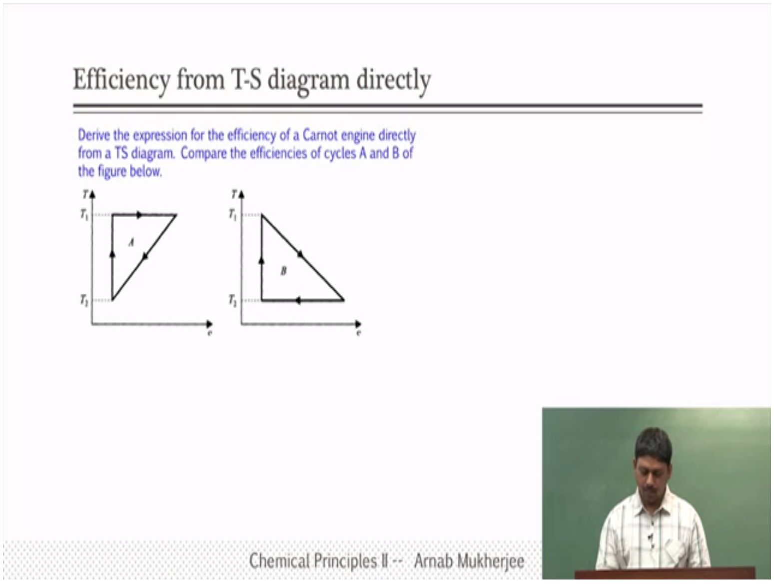

So let us continue with the problems that we were doing on the second law of thermodynamics. So here is a problem on the efficiency calculation from TS diagram directly. So the question says that derive the expression for the efficiency of Carnot Engine directly from TS diagram, compare the efficiencies of the cycle, A and B in the figure below. So before we actually can do that, we should calculate the efficiency of the Carnot Engine first from the TS diagram.

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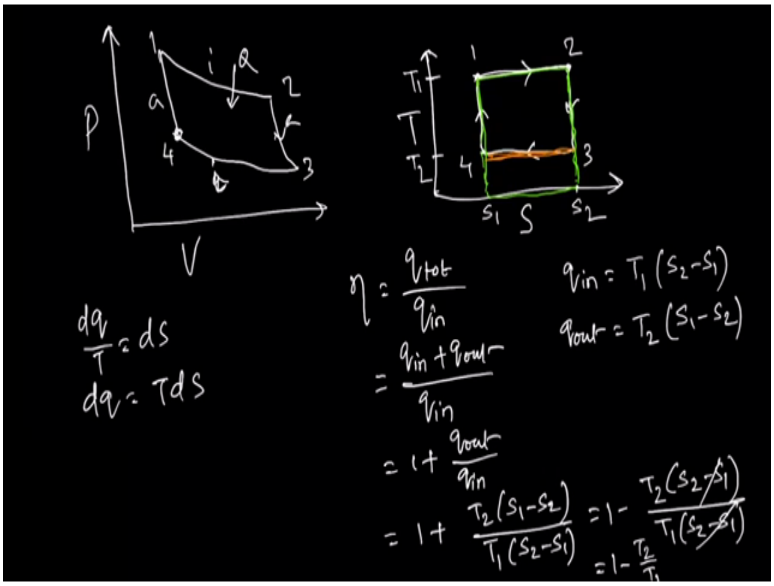

So for that we are going to first draw the PV and TS diagram. So first draw the PV diagram, so it is isothermal process, adiabatic expansion process, isothermal compression process and adiabatic compression process. Let us call it 1, 2, 3, 4 and we can write the corresponding TS diagram. So between the 1 and 2, temperature remains constant because in isothermal process however since it is an expansion process heat comes in in the reversible manner and the system's entropy increases, so it should be flat like this.

And then 2 to 3 is an adiabatic compression process, sorry it is an adiabatic expansion process, since it is an adiabatic process entropy remains fixed. However since it is an expansion process, then it cools down, so temperature decreases. So it will be like this, then 3 to 4 is just the opposite of 1 to 2. So this is 1, this is 2, this is 3 and this is 4 and we get 4 to 1 like this. Now what is the, in order to calculate the efficiency, we know the efficiency calculation, it is the Q total divided by Q input.

So we have to calculate Q total and Q input, so let us calculate that. And we know what is Q because we know that dQ by T equal to dS, so Q is, dQ is nothing but TdS. Therefore when we calculate the entropy for the heat input which is coming between 1 and 2 steps, the heat coming in should be area under the curve 1, 2. Let me draw that and show you. So the whole green region is the area corresponding to the heat input. And how do we get that? We can get that from the expression.

So let us say this temperature is T1, let us say this temperature is T2, this is entropy S1 and let us say this is S2. So then our Q in is going to be T1 multiplied by S2 minus S1. It is a positive quantity and S2 is greater than S1, so therefore Q in is a positive quantity. Now we have to do Q tot, so in order to get the Q tot, we know that it is Q in plus Q out. But this Q out also we can calculate, Q out is the, I will show in a different color, orange color. Q out is the area under this line and the temperature of that is T2, however it is a negative quantity because the entropy decreases there. So it is S1 minus S2, that is Q out.

So we can also calculate the efficiency using Q out and Q in as well, so we can do that, let us do that. So this is Q tot is Q in plus Q out, remember Q out itself is a negative quantity divided by Q in which is 1 plus Q out divided by Q in, equal to 1 plus, now Q out is T2, S1 minus S2 and that is T1, S2 minus S1. Taking the negative sign on the numerator we get T2 S2 minus S1 by T1 S2 minus S1 which cancels giving us 1 minus T2 by T1. So that is the efficiency of the Carnot Engine calculated directly from the TS diagram. Now given that we know how to calculate that, let us calculate the two problems that we were discussing. So let us go back to that and calculate that.

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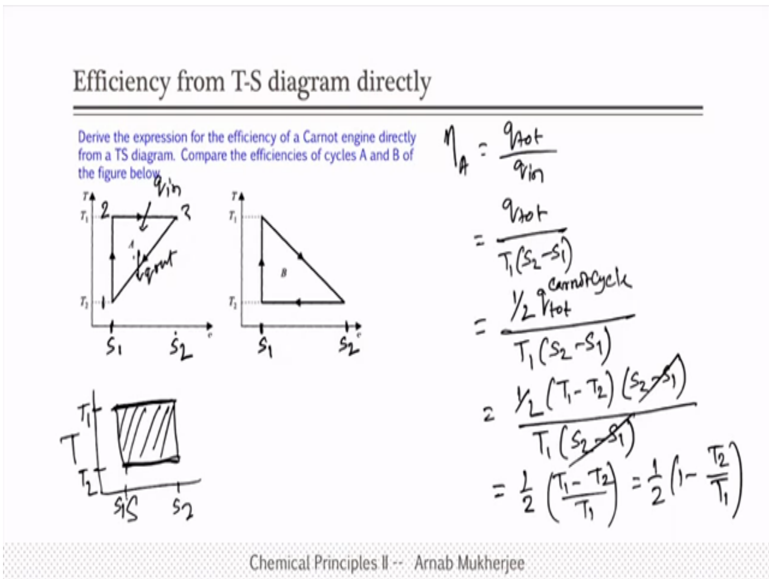

So for the first problem I will just mention the values. This is let us say S1 and this value corresponding to this value is S2, for this also S1 and S2. Temperature is also mentioned and everything is mentioned. Now let us calculate for cycle A. So for the cycle A as you can see, that if I start from here to here, there is no change in entropy. Temperature increases but entropy remains constant. So this is an adiabatic step where nothing happens, no heat input or output takes place.

Now as you can see, this point to this point, so let us write it as 1, 2, 3. So 1, 2 and 3, so between 1, 2 entropy is 0 and 2 to 3 this is where entropy increases at constant temperature, so it is basically an isothermal expansion process in which heat comes in. So our Q in is this, and obviously 3 to 1 is our Q out. Now how do we calculate the Q out? This is our Q out. How do we calculate the efficiency? So efficiency of the engine A we can write again as Q tot by Q in. Q in is same as what was there for the Carnot Cycle which is T1 multiplied by S2 minus S1. But what is Q tot?

Now you can see interestingly that the Q tot is from the Carnot Cycle was the area enclosed by the rectangle. So if we remember that, when we added in the TS diagram, Q tot is this one. Q in is area under this curve, Q out is area under this curve, so therefore Q tot was the area enclosed by the rectangle. And you can see this is half of that, so here the Q tot is half of Q tot, sorry this should be Q tot, Q tot half of Q tot of the Carnot Cycle divided by T1 S2 minus S1. So now what was the Q tot of the Carnot Cycle? Again from here also you can see T1, this is T2, this is S1 and this is S2. So therefore it is T1 minus T2 multiplied by S2 minus S1 divided by T1 S2 minus S1. S2 minus S1 cancels giving us half T1 minus T2 by T1. It is half 1 minus T2 by T1. So we get exactly half the efficiency of the Carnot Cycle.

And the reason is clear because input is the same as that of Carnot Cycle for engine A. However the total work output is half that of the Carnot Cycle because in Carnot Cycle we get the whole rectangle as the output but here we are getting half the rectangle as the output, work output, so therefore the efficiency will be half that of the Carnot Cycle. Now let us talk about the engine B. Let me remove the A just to get more space.

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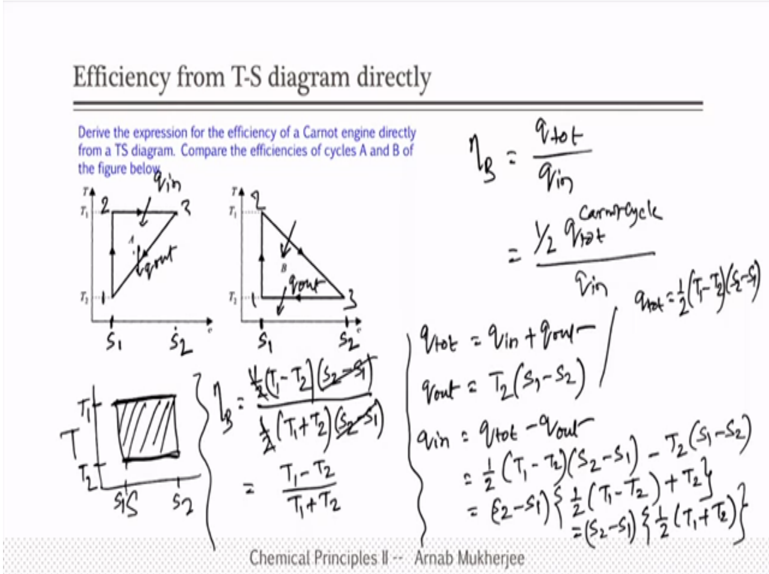

So we can see that for engine B again it is half that of the, so let us write that engine B efficiency. Again engine B efficiency is Q tot by Q in. Now Q tot again is half that of the Q tot of Carnot Cycle. But what is the Q in? So we have to understand where the heat comes in. Heat comes in through this path or this section, so 1, 2 and 3, so heat comes from the process 2 to 3 because 3 to 1 is the process in which heat goes out, Q out, so Q in is only 2 to 3 because 1 to 2 there is no heat exchange happening as you can see clearly.

So what is the value of 2 to 3? So we can calculate that again by knowing that the value of Q out. So Q tot we know is Q in plus Q out. And we know the Q out from Carnot Cycle expression itself that Q out is T2 S1 minus S2, it is a negative quantity remember. So therefore Q tot, we need Q in right? So therefore Q in is Q tot minus Q out. And what is Q tot? Q tot is half that of the Carnot Cycle which is T1 minus T2, S2 minus S1 minus, what is Q out? Q out is T2 S1 minus S2.

I will do this, I will take S2 minus S1 common giving me half T1 minus T2. Since I took this thing, it will be plus T2 giving me S2 minus S1 multiplied by half T1 plus T2. So we got Q in, now we have everything available to us. So let us write it here. We have some space here.

So eta of B is Q out or Q tot is given here, yeah Q tot I have not written here but okay, we can write it many ways. We can write 1 plus Q in by Q out also. But let us write it by Q tot. So again we will just write Q tot value which is half T1 minus T2, S2 minus S1, half T1 minus T2, S2 minus S1 divided by Q in which is here, half T1 plus T2, S2 minus S1. S2 minus S1 cancels, half cancels giving us T1 minus T2 by T1 plus T2. So now that we got half T1 plus T2, S2 minus S1, we will write that way also.

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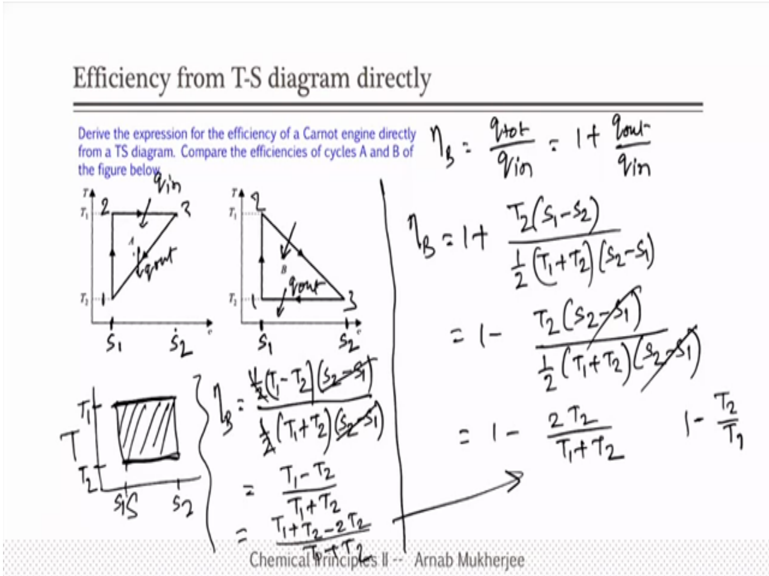

So efficiency Because can also be written as Q tot by Q in or 1 plus Q out by Q in. And then 1 plus and Q out how much we got? Half, Q out is not half, Q out we just got, right? That it is T2, S1 minus S2 and this one we got as half T1 plus T2, S2 minus S1. We take a minus, so we get T2, S2 minus S1 by half T1 plus T2, S2 minus S1, giving us 1 minus 2 T2 by T1 plus T2. Now at the same we can easily check that. We can just take T1 plus T2, so here we can just take T1 plus T2 minus 2T2 by T1 plus T2 and we are going to get this one.

So this is the efficiency of the engine B. Now you see, is this efficiency be more or less than Carnot Engine? So we have seen here obviously that the efficiency is less than Carnot Engine. Why? Because the input is same, output is half. But here output, heat out is same but heat in, is it more or is it less by this way? Is this efficiency, so what is the Carnot efficiency? 1 minus T2 by T1. Now is it more or less than this thing? The output is also less, and the input is it more or is it less? Of course, it will depend on, so Q input is half, T1, T2 S2 minus S1. So since T1 is greater than T2, obviously the input is also less than the Carnot input.

So total is also less than the Carnot input. Now it all depends on the value of T1 and T2, whether it will, this construction will have more or less than the efficiency of the Carnot Engine. Let us say for example, we can take arbitrarily close temperature, let us say both of them are very close to T1, then it will have efficiency 1. So when they will come closer and closer, then, no, no, so if T1 equal to T2, what will happen? No, T1 it will be 1 minus 2T2 by 2T. It will be 0, yeah, so they will come closer and closer of course.

Then, that of course it will not work, that is right. So that means if the T1 and T2 come arbitrarily close together, the efficiency will go towards 0, that is fine. Efficiency will go towards 1 only when the T2 will go to 0, that is also fine according to Carnot this thing. Now the question is that whether this one will have more or less the efficiency of the Carnot Engine. So this is 1 minus 2T2 by T1 plus T2, right?

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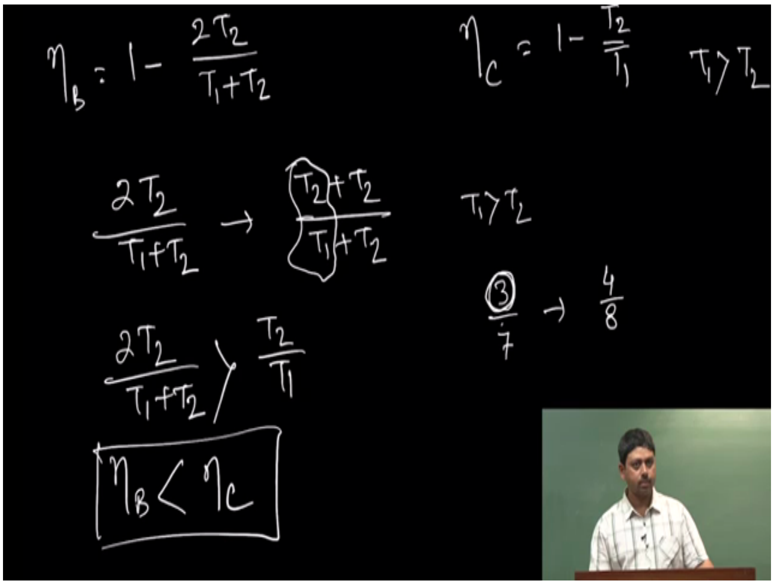

So this is the efficiency of the engine B. And efficiency of a Carnot Engine is 1 minus T1 by T2. Now the difference is that I am adding, sorry it should be T2 by T1 because T1 is larger than T2. Yeah, we can do some analysis here. We can say that look, this one is, let us analyze this quantity, 2T2 by T1 plus T2. This comes from T2 plus T2 by T1 plus T2. Now you see this, I will just write it properly. This is the ratio that is there in the Carnot. We are adding T2 value. Now since T1 is larger than T2, by adding the same amount T2 on numerator and denominator, so for example I am talking about let us say 3 by 4, 3 by 7, I add 1 to both of them. We are going to get 4 by 8.

Now adding 1 is more, will have more value, so is 4 by 8 bigger or 3 by 7 bigger? 4 by 8 bigger, right? So adding this one will have larger effect. So this quantity, 2T2 by T1 plus T2 will be larger than T2 by T1. Therefore efficiency of B will have lesser efficiency than C, that is my analysis of that.

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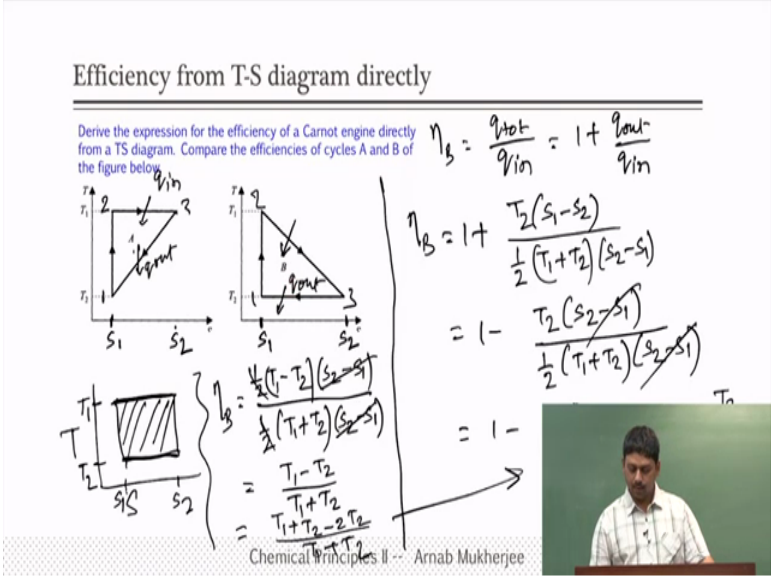

This will have half the efficiency, this will have slightly lesser efficiency depending on the value.

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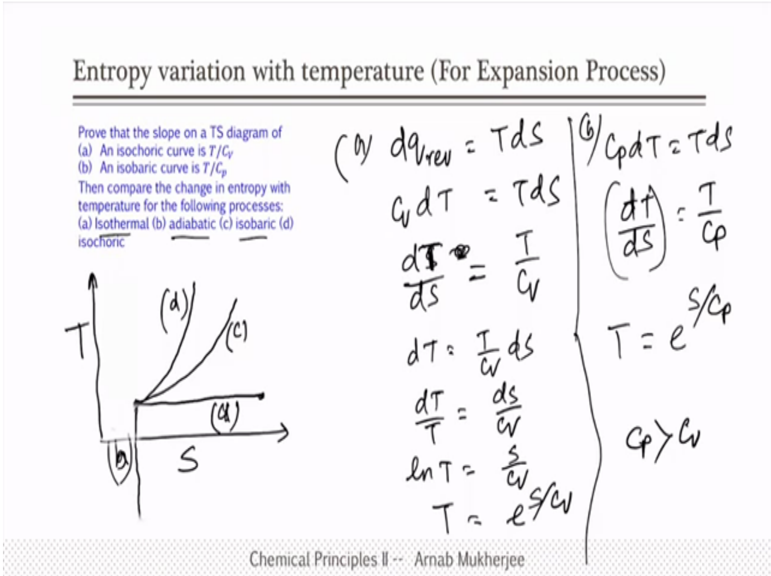

Prove that the slope on a TS diagram of an isochoric curve is T by Cv and Isobaric curve is T by Cp. So we know that d Q reversible is TdS. Now what is dQ for a constant volume process, isobaric process? Is Cv dT, So Cv dT is TdS or dS is, or dT dS rather, dT dS is T by Cv. So that is proven. And similarly Cp dT is TdS for isobaric process. So dT by dS is T by Cp. So this is A and this is B, proof of A and B. This is the slope, right? When you calculate the slope, you will take the differentiation. So we got that.

But now the second question is that then, compare the change in entropy with temperature for the following processes. So let us draw that, temperature and entropy. And an isothermal process, how does the temperature change in the isothermal process? Does not change, right? This is the isothermal process, it does not change at all. Temperature does not change, entropy changes. How does it change for an adiabatic process? Temperature changes but entropy does not change, so this is adiabatic process, this is isothermal process.

How does it change in an isochoric and isobaric process? For that we need to take the help of this one. So dT equal to T by Cv dS or dT by T equal to dS by Cv. So ln T is S by Cv. So T is e to the power S by Cv. So temperature as goes as e to the power S by Cv in case of isobaric process. Similarly temperature will be e to the power S by Cp in case of, sorry this is isochoric process, this is isobaric process. Which will go in a steeper manner? Both are exponential but which will go in steeper manner?

We know that Cp is greater than Cv, right? So therefore S by Cp is a smaller number, S by Cv is a bigger number, because Cv is small. So therefore isobaric process will go exponentially, isochoric process will be even higher, with higher exponential it will move. So this is isobaric process, this is c and isochoric process, this is d. Did you understand this one? So these are the four ways that, this is isothermal process a, I am just denoting by a, b, c, d on the top. And this is b, do not confuse a by adiabatic. So a is denoting is isothermal, b is denoted by adiabatic, c is isobaric and d is isochoric.