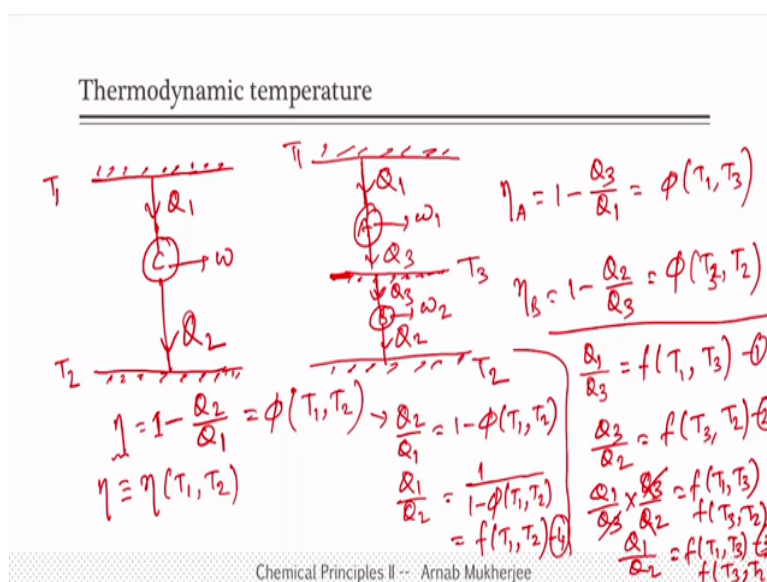


Chemical Principles II
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Module 05
Lecture 30
Thermodynamic Temperature

The fact that the Carnot engine has highest efficiency working between two temperature reservoirs gives rise to something call thermodynamic temperature, so we are going to talk about that.

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So remember that a Carnot engine working let us say between two temperature bath T_1 and T_2 where T_1 is high temperature bath and T_2 is the load temperature bath and let us say it takes in Q_1 amount of heat does some work and throws away Q_2 amount of heat. So this particular engine will have or you can will have the efficiency as $1 - \frac{Q_2}{Q_1}$, so where we know that Q_1 is the you know heat that is taken in from high temperature and Q_2 is heat that is thrown out to the low temperature.

So you see the efficiency η which is $1 - \frac{Q_2}{Q_1}$ depends on these two temperatures T_1 and T_2 , right so we can say that the efficiency is basically a function of two temperatures T_1 and T_2 because nothing else is a required to define the efficiency of a Carnot engine apart from that the heat taken in and heat you know thrown to two different temperatures reservoirs.

So we can call that this is particular efficiency some function let us say ϕ_1 of T_1 and T_2 , ok now let us construct another engine a combine system where two engine work simultaneously heat Q_1 is taken in from high temperature some work is being done and Q_3 is thrown to an intermediate temperature reservoir T_3 , so T_3 works as temperature reservoir low temperature reservoir for the upper engines let us call that A and then same Q_3 will be taken in by another engine called B which does some other work and finally Q_2 heat is thrown to the T_2 temperature reservoir.

So what we have done is that we replace one engine by two engines in and there is a temperature reservoir in between T_1 and T_2 temperature, ok. Since the heat cannot escape it takes in and give the same amount of heat to the next engine, so now this is a combine system, now if you calculate the efficiency of engine A what we are going to get $1 - \frac{Q_3}{Q_1}$ and this one is a function of T_1 and T_3 , right.

Similarly, η_B which is the efficiency of the second engine is $1 - \frac{Q_2}{Q_3}$ therefore we can say it is a function of T_2 so we are writing like first high and then low so T_3 and T_2 , ok. So now you see if I consider the final you know initial and final temperature differences then efficiency of this engine let us call it C, so efficiency of engine C will be same as that of the combine efficiency of the right hand side engine, ok.

So now if I rearrange a little bit here this particular equation let us say from here if I rearrange a little bit then what I get is that $\frac{Q_2}{Q_1}$ is equal to $1 - \phi(T_1, T_2)$, ok we can invert it and write $\frac{Q_1}{Q_2}$ so we are inverting just to say that heat input divided by heat output is $\frac{1}{1 - \phi(T_1, T_2)}$, right it is still a function of T_1 and T_2 , so let us call it $f(T_1, T_2)$ now finally what we got? We got for the engines C we got $\frac{Q_1}{Q_2}$ is equal to a function that is a function of T_1 and T_2 , ok.

So you can write the same thing for above two, so from η_A we get heat input is $\frac{Q_1}{Q_3}$ is a function of T_1 and T_3 and from η_B the engine B what we get? The input is Q_3 output is Q_2 we get $f(T_3, T_2)$. Now you see one interesting thing happens I can multiply equation 1 and equation 2, so if I multiply equation 1 and equation 2 what I am going to get $\frac{Q_1}{Q_3}$ into $\frac{Q_3}{Q_2}$ is equal to $f(T_1, T_3)$ multiplied by $f(T_3, T_2)$ left hand side cancels and gives us $\frac{Q_1}{Q_2}$ is equal to product of two functions, ok.

Now this we get from here let us call that equation 3 and we already have from this equation 4 $\frac{Q_1}{Q_2}$, now 4 and 3 we can equate.

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$$\begin{aligned}
 C \Rightarrow \frac{Q_1}{Q_2} &= f(T_1, T_2) \\
 A+B \Rightarrow \frac{Q_1}{Q_2} &= f(T_1, T_3) f(T_3, T_2) \\
 f(T_1, T_2) &= f(T_1, T_3) f(T_3, T_2) \\
 \frac{\psi(T_1)}{\psi(T_2)} &= \frac{\psi(T_1)}{\psi(T_3)} \frac{\psi(T_3)}{\psi(T_2)} \\
 f(T_1, T_2) &= \frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)} \Rightarrow \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \\
 \frac{Q_1}{Q_2} &= \frac{T_1}{T_2}
 \end{aligned}$$

So what we have seen that from engine C we got Q_1 by Q_2 as a function of T_1, T_2 and from combine engine A and B we got Q_1 by Q_2 as $f(T_1, T_3)$ multiplied by $f(T_3, T_2)$, ok so equating these two we get T_1, T_2 is a product of $f(T_1, T_3) f(T_3, T_2)$. Now you see somehow this product or these functions is dropping out here even though we have introduced a temperature and that is possible if this function T_1, T_2 or any two temperature is a ratio of function of these two temperatures separately, let us say if I write an I can show you.

So this can be a function of $\psi(T_1)$ is a function of T_1 and this ratio is the overall function we can write the same way, you see if we write that the $\psi(T_3)$ cancels out, so f is basically f which is a function of two temperatures is basically a ratio of function of individual temperatures, right. So now what we learned from here is that and where did you get this? Now what is our $f(T_1, T_2)$? Our $f(T_1, T_2)$ is nothing but Q_1 by Q_2 , right and we got $f(T_1, T_2)$ as $\psi(T_1)$ by $\psi(T_2)$.

So this is the basic formulations of the thermodynamics temperature. So you see it is not exactly a temperature it is a function of temperature some function of temperature. Now kelvin used a linear function we can use some linear function with changes with T_1 and if you use a linear function or we can call this $\psi(T_1)$ itself as T_1 then we can say that the ratios of heat input output is related to the ratios of two thermodynamics temperatures.

You need only need one specific point in order to determine the other temperatures, so for example if you calculate Q_1 and Q_2 for a known temperature and the known temperature typically is taken to be the temperature of triple point of water which is 273 point 16, so this

is T triple point is known Q_1 , Q_2 will be measured and then you can get the value of T_1 , so that is we can measure the temperature given that we can calculate heat input and output.

So in real experiment as especially below 1 kelvin temperature where the gas see other the temperature that we learnt from you know zero law of thermodynamics is when we combine a property with the heat, so for example or with the temperature for example we say that how much is a value of $P V$ when the pressure is very low and how much is a value of $P V$ with respect to $P V$ at triple point that will give us a major of the temperature, that is in a gas thermometer.

Similarly for other cases for solids and a other you know mercury we calculate the property by looking at the changes in the volume or changes in the pressure and things like that. Here thermodynamics temperature we are calculating the temperature by calculating the heat input and output and that is possible because Carnot engine gives us that, remember that for ideal cases we exactly know that Q_1 by Q_2 in ideal gases case we know that Q_1 by Q_2 is exactly as T_1 by T_2 that is from our normal (therm) you know temperature definition itself it gives that in ideal gas however in general for non-ideal gases also we can write the ratios of heat input output as a function of the individual temperatures.

That is kelvin actually decided you know given this kelvin scale and one can see that when one of that will go to zero then let us say you know in order for the efficiency to go to 1 the one of the temperatures to go to zero and in that the heat output will be zero there will be no heat going out, so it becomes an adiabatic step itself. So when the isothermal and adiabatic becomes equal at that point your efficiency will become 1, ok.