

Chemical Principles II
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Module 05
Lecture 29
Carnot's Engine: The Most Efficient Engine

So we have discussed about four different engines and their efficiencies: Carnot engine, Stirling engine, then Gasoline engine and Diesel engine. Out of which two engines are external combustion engines which are like a Stirling engine and no Stirling engine is the only one that we have discussed and actually Carnot engine is not even a real one, so it is a hypothetical one and gasoline engine and diesel engines are the internal combustion engines we have discussed about the efficiencies of those.

However, there can be many other possible engines or possible thermodynamic cycles that you can construct and calculate their efficiencies.

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Carnot' Cycle: The most efficient engine

No heat engine operating between two given reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.

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But the important thing is that the Carnot engine which is a hypothetical one we can show that no heat engine operating between two given thermal reservoirs, reservoirs means basically having a fixed temperature can be more efficient than the Carnot engine operating between the same two reservoirs, so that means if an engine runs between two fixed temperature reservoirs then the efficiency of any of those engines cannot be more than the Carnot engine.

So that is what we are going to discuss today. So let us take about so you know that all real engines are irreversible engines, right. So let us take an example of an irreversible engine and an example of a Carnot engine. So typically the way we construct the process is that you have a high temperature reservoir from which a heat Q_H is taken let us say it is Q_H' and a Q_L' is ejected to the low temperature reservoir, so this is an irreversible engine and therefore I put I as indicator of that and let us say the work output of this particular engine is w .

So efficiency of this engine η_I will be $1 - \frac{Q_L'}{Q_H'}$. As I said in all these cases we are just taking the modulus rather than the sign so it will automatically come to be that. Let us take a Carnot engine within the same temperature difference and let us call it E which we have been define as a E, here also at different amount of heat is taken in let us say same work output is given in and a different amount of heat is ejected out.

So efficiency of this particular engine E which is a Carnot engine is $1 - \frac{Q_L}{Q_H}$. Now let us say for the argument sake that efficiency of the irreversible engine is greater than that of E say, so in that case what it means is that $1 - \frac{Q_L'}{Q_H'}$ is greater than $1 - \frac{Q_L}{Q_H}$. So how to write in a different way we can say that the efficiency of this engine irreversible engine is the work output by the heat input Q_H' let us say it is greater than that of the Carnot engine, so automatically we can see so remember that we have adjusted the heat input and output in such a way that the work is same, ok.

So you can always do that, we have kept the work same but efficiency is different so now efficiency have written in a simpler form w is a work output which is of course $Q_H' - Q_L'$ and the heat inputs are different, now once it is put it like that you can clearly see that here the Q_H' is since it is greater Q_H' should be smaller than Q_H , if the efficiency of the irreversible engine is higher than that of the Carnot engine, ok.

Now let us see that if that is the case what is going to happen, now let us combine the irreversible engine and Carnot engine in such a way that the irreversible engine should drive the Carnot engine that means the work that you are going to get out of this irreversible engine should be helping the opposite of the engine, ok. So we are going to put this particular engine in reverse and this is in the forward direction.

Now let us do that, so again T_H going to take Q_H' heat is going to produce w amount of work and going to through away Q_L amount of heat to the low temperature reservoir this

w amount of work now is going to go into the Carnot engine which is kept in reverse, so call it R the exactly opposite of this thing will happen here which means that Q_H will go back to the high temperature and Q_L is going to be taken in from the low temperature just the opposite when you put, so whenever you want to put an engine in reverse first construct the engine and then just reverse it, so exactly the signs will be reversed.

So here like Q_H is positive, w will be negative and then Q_L is negative, here Q_L is positive, w is positive and Q_H is negative, ok that is also indicated. So now see none of these is still violating any of the laws because the heat is transferred from the low temperature to high temperature with the help of the work, ok so I am here to maintain the efficiencies of this engine and efficiencies of this engine, everything is maintained.

Now the combined system is such that coming out as that so Q_L prime is getting thrown away and Q_L is taken in, so Q_L minus Q_L prime heat is taken in from the low temperature and without work being done Q_H minus Q_H prime is thrown in, right. Now we know that from this construction of the engine we know that Q_H prime minus Q_L prime is equal to w , we know from this engine is that Q_H minus Q_L is w equating them together we get Q_H (min) Q_H prime minus Q_L prime is equal to Q_H minus Q_L and therefore Q_H minus Q_H prime is equal to Q_L minus Q_L prime.

So which means that I can call it Q and this also will be Q they are equal which means that the heat is taken in from low temperature reservoir and thrown in to the high temperature, see sign is very important because if it turns out that heat is flowing from high temperature to low temperature reservoir then there is no violation of the second law of thermodynamics, so it is important that this is a positive quantity that is taken in from the low temperature and this is a negative quantity that is thrown into the high temperature and that is what is happening because we know that (big) the way we have done this that this is a positive quantity, right because we have seen that Q_H you look at this one now, so Q_H is larger than Q_H prime, so therefore a positive amount of heat is thrown into the high temperature reservoir.

So now this is in violation of Clausius second law of theorem, right. Clausius Statement of so this is a violation of Clausius Statement of second law, ok. So which means that our initial argument that irreversible engine can be higher than a reversible engine cannot be true because it would have if it were true then it would have violated the second law of thermodynamics, so that means no irreversible engine can have higher efficiency than a reversible Carnot engine.

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Equivalence of all reversible engines

Corollary: all reversible engines working between two temperature reservoirs have the same efficiency

$\eta_I \leq \eta_R$
 $\eta_{R1} \leq \eta_{R2}$
 $\eta_{R2} \leq \eta_{R1}$
 $\eta_{R1} = \eta_{R2}$

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Now question is that can one reversible engine be better than other reversible engine, now we are going to say that so corollary to this earlier argument is that all reversible engines working between two temperature reservoirs have the same efficiency, now how do we arrive at that? So remember our construction was like this that we have a high temperature reservoir from which an engine was taking an heat and throwing away heat and with that we have added an engine in reverse which took the heat from low temperature and thrive into the high temperature, ok.

Now we said that let us say in this particular case now we have both of them as reversible engine let us call it R 1 and let us call it R 2, of course R 2 is added in the opposite direction, ok so here R 1 is driving R 2 just like the way we have constructed it before, now following the same argument we can show that η_{R1} so we have what we have gathered from last example that η_I cannot be greater than the reversible one which means it will be either less or equal to the reversible one, the one on the right hand side.

Similarly following the same argument if we put it that way we can show that η_{R1} cannot be greater than the R 2 for the same reason that and irreversible engine was not being able to derive that so it cannot R 1 cannot be greater than R 2, now let us switch the places of R 1 and R 2, so this left hand side becomes R 2 which will derive now R 1 and we can because there is an R 1 and R 2 arbitrary η_{R2} cannot be greater η_{R1} .

Now you see when you combine both of them there is only one possibility that exist that η_{R1} is equal to η_{R2} which means that all reversible engines will have the same efficiency,

if it is not be the case then let us say if one of them is less efficient what will happen is that if one of them is less efficient we are going to put it on the right hand side and try to derive a reversible engine which you know will actually violate.

So now we see that the all reversible engines working between twos temperature reservoirs have the same efficiency, so that is very important efficiency when you talking about we are only talking about the engines which have same fixed two temperature from which it is taking in the heat and throwing away the heat however in case of diesel and gasoline engine we have seen that is not the case for diesel engine we had all four different temperatures and in the gasoline engine also we had different temperatures however for starling engines we saw that if using the region eta it was maintaining all the two different temperatures like the upper plate and the plate below and therefore it could have as close to real Carnot engine as possible.