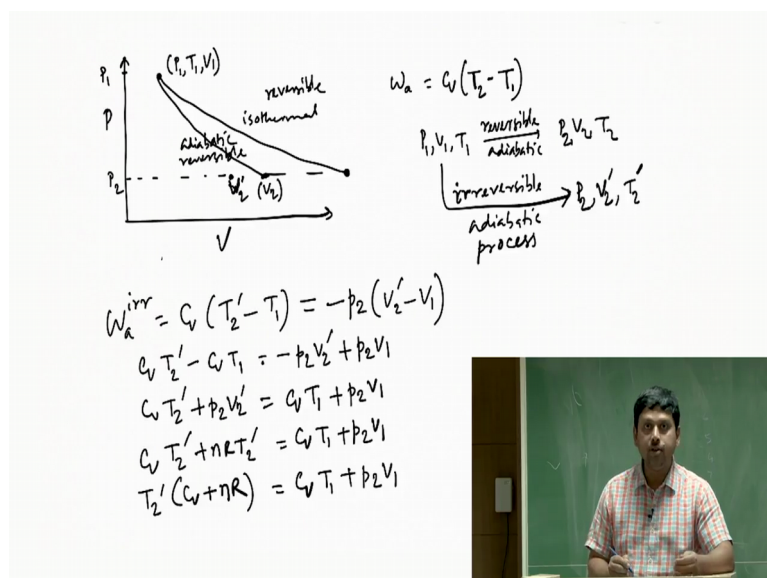


**Chemical Principles II**  
**Professor Dr Arnab Mukherjee**  
**Department of Chemistry**  
**Indian Institutes of Science Education and Research, Pune**  
**Module 03**  
**Lecture 19**  
**Adiabatic Irreversible Work**

Ok so you have seen how to calculate the work done for a reversible adiabatic processes, now we are going to talk today about how one can calculate the work done for irreversible adiabatic process and then we will compare the work done in irreversible and reversible manner for adiabatic processes and see that which work is more and which is less given that the final pressure is same for all the processes.

(Refer Slide Time: 0:50)



So now we have already discussed that for a PV diagram let us say our initial point is P1 V1 at this point and then let say the final point is somewhere P2 and I draw that by this line this is P1 pressure and this final pressure is P2 that is drawn by this line this dash line. Now if the process is carried out in isothermal manner, you know the final volume will be much more such that the work done is more than a corresponding reversible adiabatic process. So let us say this is isothermal reversible process and this is adiabatic reversible process. Now we would like to know that what will be the work done for irreversible adiabatic process given that the pressure final pressure is P2 and that in the reversible process as well.

So let us say that for reversible process the final volume is V2 but for irreversible process let us say the final volume is V2 prime. So then we know from the formal of work done in

adiabatic process is it is  $C_v dT = C_v \Delta T$  or let us say the final temperature for a reversible adiabatic process is  $T_2$  and initial temperature let us take as  $T_1$ , so let us say initial point is  $P_1, T_1$  and  $V_1$  and is going to  $V_2$  and  $T_2$  for reversible process so let us say  $P_1, V_1$  and  $T_1$   $P_2, V_2$  and  $T_2$  for a reversible adiabatic process and it is going to same pressure same final pressure but maybe a different volume  $V_2$  prime and different temperature  $T_2$  prime in irreversible adiabatic process, so we are going to calculate the work done in both and you are going to see that which work done is more and which is less.

Although we already know that work done for an isothermal reversible process is more than that of adiabatic process. Also we have shown that irreversible process in isothermal process has you know work done by the system is less than work done by the system when the process is reversible in an isothermal case, we are going to compare here only the adiabatic case ok. So let us start with the calculation of adiabatic work done and we know that adiabatic irreversible work done, so let us write that adiabatic irreversible work done will be  $C_v T_2$  prime minus  $T_1$  prime which is again of course same as minus  $P_2$  which is the final pressure and the volume is  $V_2$  prime minus  $V_1$ .

Now if you write down the equation will be  $C_v T_2$  prime minus  $C_v T_1$  equal to minus  $P_2 V_2$  prime plus  $P_2 V_1$ . And then we rearrange the equation, we will write  $C_v T_2$  prime plus  $P_2 V_2$  prime is equal to  $C_v T_1$  plus  $P_2 V_1$ . Now we know that from ideal gas equation of state that  $P_1 V_1$  equal to  $N R T_1$ ,  $P_2 V_2$  prime will be  $N R T_2$  prime so you can write  $C_v T_2$  prime plus  $N R T_2$  prime is equal to  $C_v T_1 + P_2 V_1$ , so we can take  $T_2$  prime common and write  $C_v$  plus  $N R T$  is equal to  $C_v T_1 + P_2 V_1$ .

(Refer Slide Time: 5:39)

$$\begin{aligned}
 T_2' &= \frac{C_v T_1 + P_2 V_1}{C_p} \quad \text{--- (1)} \\
 W_a^{irr} &= C_v (T_2' - T_1) \quad \text{--- (2)} \\
 &= C_v \left\{ \frac{C_v T_1 + P_2 V_1}{C_p} - T_1 \right\} \\
 &= \frac{C_v}{C_p} \{ C_v T_1 + P_2 V_1 - C_p T_1 \} \\
 &= \frac{1}{\gamma} \{ P_2 V_1 - \gamma T_1 \} \\
 &= \frac{1}{\gamma} \{ P_2 V_1 - P_1 V_1 \} \quad \text{--- (3)}
 \end{aligned}$$

$$\begin{aligned}
 W_a^{rev} &= C_v (T_2 - T_1) \\
 &= C_v \left\{ \frac{P_2 V_2}{nR} - \frac{P_1 V_1}{nR} \right\} \\
 &= \frac{C_v}{nR} (P_2 V_2 - P_1 V_1) \\
 &= \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1) \quad \text{--- (4)}
 \end{aligned}$$

$$\begin{aligned}
 W_a^{irr} &= \frac{1}{\gamma} (P_2 V_1 - P_1 V_1) \\
 \frac{1}{\gamma} (P_2 V_2 - P_1 V_1) &> W_a^{irr} \\
 \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1) &> \frac{1}{\gamma} (P_2 V_1 - P_1 V_1) > W_a^{irr} \\
 W_a^{rev} &> W_a^{irr} \\
 &\text{expansion}
 \end{aligned}$$

Now we know that  $C_v$  plus  $nR$  is  $C_p$  or  $T_2'$  is  $C_v T_1$  plus  $P_2 V_1$  by  $C_p$ , so we got our final temperature  $T_2'$  for an irreversible adiabatic process. Now let us calculate the work done in adiabatic irreversible process will be  $C_v T_2'$  minus  $T_1$  which will be the same for again the reversible adiabatic process as well, but in case of reversible adiabatic process the temperature itself will be different final temperature when the pressures are same between reversible and irreversible processes. So let us do that then so  $C_v$ , now I will put the equation 1 in equation 2 and write that  $C_v T_1$  plus  $P_2 V_1$  by  $C_p$  minus  $T_1$   $C_v$  by  $C_p$  and  $C_v T_1$  plus  $P_2 V_1$  minus  $C_p T_1$  and  $C_v$  by  $C_p$  will be  $1/\gamma$  and you will get  $P_2 V_1$  minus and you know  $C_p$  minus  $C_v$  is  $nR$  so we will get  $nRT_1$ .

Now again using ideal gas equation of state we can get  $nRT_1$  is  $P_1 V_1$  so we get  $P_1 V_1$  so now this is the work done for an irreversible adiabatic process, let us compare that with reversible adiabatic process. So work done for reversible adiabatic process is again  $C_v T_2$  minus  $T_1$ , remember the pressure is same  $P_2$  between reversible and irreversible processes however, the temperature may or may not be different so we designated that by  $T_2$  for reversible and  $T_2'$  for irreversible processes and then converted everything in terms of uh known quantities for example  $P_1 V_1$  is known because that is the initial state and  $P_2$  is also known and  $\gamma$  is also known so therefore we have converted that way, now let us see what we get from this.

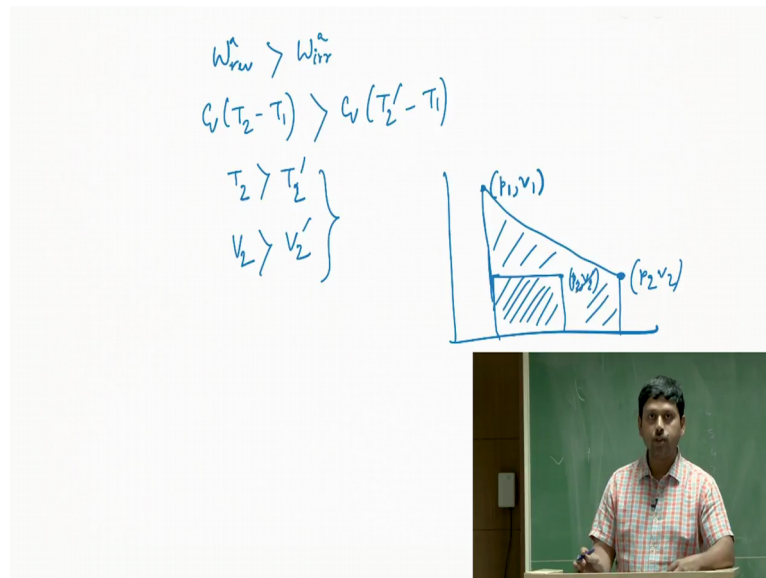
Now  $C_v T_2$  minus  $T_1$  we can say that that  $T_2$  is again we can use ideal gas equation of state and write  $T_2$  as  $P_2 V_2$  so you know  $PV$  equal to  $nRT$  so by  $nR$  minus  $P_1 V_1$  by  $nR$ . Now

$nR$  I can take it out and I can write  $C_v$  by  $nR$  and then write  $P_2 V_2$  minus  $P_1 V_1$ . Now what is  $nR C_v$  by  $nR$ ? So we know that  $C_p$  minus  $C_v$  is equal to  $nR$  so if I divide by  $C_v$  both sides then I will get  $C_p$  minus  $C_v$  by  $C_v$  equal to  $nR$  by  $C_v$  so which means  $1 - 1$  by  $\gamma$  is  $nR$  by  $C_v$  or let say I just backup this step and we write, we can invert that and we want  $C_v$  by  $nR$  right so we can do the inverse of that and we can write,  $C_p$  minus  $C_v$  is  $nR$  we know, we have to divide by  $C_v$  so it should be  $C_v$  correct, so  $C_p$  minus  $C_v$  by  $C_v$  equal to  $nR$  by  $C_v$  and which means that this  $C_p$  by  $C_v - 1$  equal to  $nR$  by  $C_v$  or  $\gamma - 1$  equal to  $nR$  by  $C_v$ .

So  $C_v$  by  $nR$  is  $1$  by  $\gamma - 1$   $P_2 V_2$  minus  $P_1 V_1$ , now we got the expression for reversible adiabatic process and irreversible adiabatic process, now let us compare between them and see which is more and which is less. Now once you compare that we see that, let us start with irreversible adiabatic process. Now we see that irreversible adiabatic process is  $1$  by  $\gamma$   $P_2 V_1$  minus  $P_1 V_1$ , now if I replace  $V_1$  by  $V_2$  since  $V_2$  is larger in volume that will be a bigger quantity. So you can write  $1$  by  $\gamma$   $P_2 V_2$  minus  $P_1 V_1$  is greater than the irreversible adiabatic work done. Now we can replace  $\gamma$  by  $\gamma - 1$  which will increase it further so you can write  $1$  by  $\gamma - 1$   $P_2 V_2$  minus  $P_1 V_1$  is larger than  $1$  by  $\gamma$   $P_2 V_2$  minus  $P_1 V_1$  which is larger than adiabatic irreversible process.

Now you see compare that with the above expression from equation number 2, so this is same as equation number 2 and so therefore this is same as work done in a reversible process, so therefore work done in a reversible process is greater than work done in irreversible process given that both the processes reach the final pressure. And this is truly for the expansion process why, because we have already assumed that  $V_2$  is larger than  $V_1$  so we are only talking about an expansion work.

(Refer Slide Time: 11:37)



So only for the expansion work in a reversible process adiabatic work done will be greater than work done in an irreversible process. And we know it is  $C_v(T_2 - T_1)$  will be greater than the  $C_v(T_2' - T_1)$ , which means  $T_2$  will be greater than  $T_2'$ . So the temperature will be lowered more in case of adiabatic processes and therefore we can say that final volume  $V_2$  that will be obtained in a reversible process will be because we know that  $V$  is proportional to  $T$  right because  $PV = nRT$  so  $V$  is proportional to  $T$ , when the pressure is constant then  $V_2$  will be more than  $V_2'$ .

So the volume also will be more final volume will be more in case of reversible process than in irreversible process. So starting with  $P_1, V_1$  let us say if you reach a volume  $V_2$  then the adiabatic volume will be somewhat less than that, it will be  $P_2, V_2'$  and the work done for an adiabatic process in a irreversible way will be this much that is work done for an ideal reversible process will be that much ok so that is very stuffs