Chemical Principles II

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Module 03

Lecture 18

Adiabatic Reversible Work

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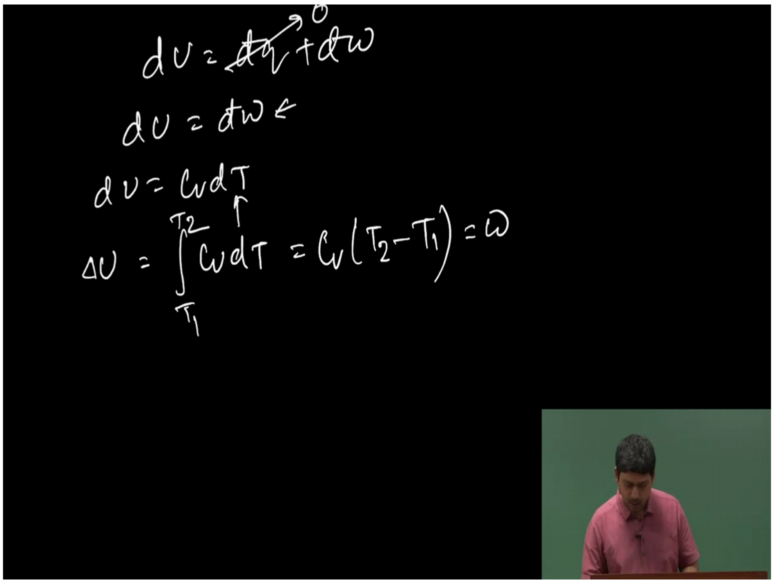

$$dU = \overset{0}{\cancel{dq}} + dw$$

$$dU = dw$$

$$dU = C_v dT$$

$$\Delta U = \int_{T_1}^{T_2} C_v dT = C_v(T_2 - T_1) = W$$

So now we are going to talk about adiabatic work, so what is an adiabatic work? Adiabatic work is where you have no exchange of heat and I am going to just derive that adiabatic work here, so dU equal to dQ + dW, in case of adiabatic dQ is equal to 0 so we are going to have dU equal to dW for adiabatic cases. Now we know that for an ideal gas we can write dU as C v dT now given temperature difference so in adiabatic case of course if there is a work done that means some value of W is there, then you see that C v because C v is taken to be constant, temperature is going to change and that is the reason that in adiabatic case the temperature changes.

How much is the work done that will depend on how much change in the temperature is happening. Let us say the temperature is changing from T1 to T2 so in that case total change in U will be integration of C v dT from T1 to T2 giving us C v T2 minus T1. And since this is the amount of work done so the work done in adiabatic case is nothing but C v T2 minus T1 that is when you write the work in terms of temperature. However if you want to write that work in terms of pressure and volume you are going to do a little bit more work okay and that is what you are going to do now so let us go back again.

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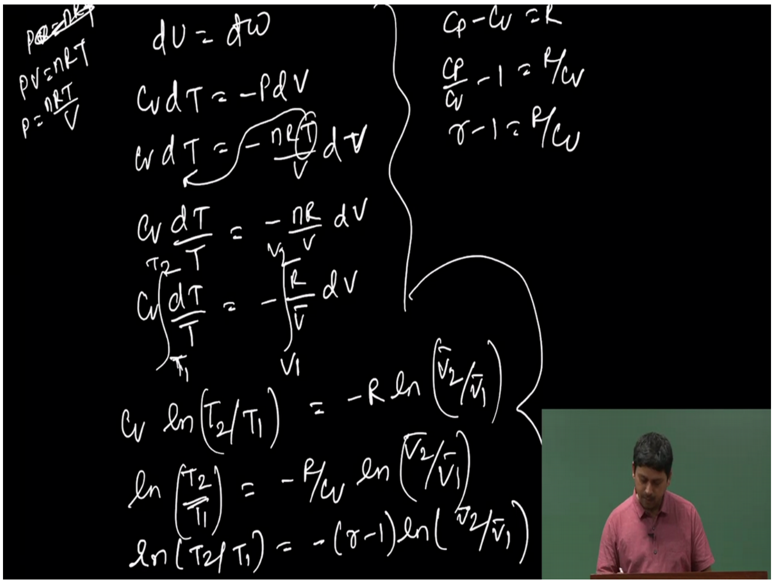

$PV = nRT$

$P = \frac{nRT}{V}$

$dU = dW$

$C_v\, dT = -P\, dV$

$C_v\, dT = -\frac{nRT}{V}\, dV$

$C_v \frac{dT}{T} = -\frac{nR}{V}\, dV$

$C_v \int_{T_1}^{T_2} \frac{dT}{T} = -\int_{V_1}^{V_2} \frac{R}{\bar{V}}\, dV$

$C_v \ln(T_2/T_1) = -R \ln(\bar{V}_2/\bar{V}_1)$

$\ln\left(\frac{T_2}{T_1}\right) = -R/C_v \ln(\bar{V}_2/\bar{V}_1)$

$\ln(T_2/T_1) = -(\gamma-1)\ln(\bar{V}_2/\bar{V}_1)$

$C_p - C_v = R$

$\frac{C_p}{C_v} - 1 = R/C_v$

$\gamma - 1 = R/C_v$

So now we know that dU equal to dW for ideal gas or C v dT equal to minus PDV. Now for ideal gas we know that PV is equal to N R T and therefore I am going to replace P by N R T by V, so C v eT is equal to minus N R T by V dV but left-hand side I have a dT you know which takes care of the temperature change so I have to bring this T below here because there is a function of temperature C v dT by T is equal to minus N R by d V okay. Now I can write C v dT by T is minus R by V bar, V bar is V by N molar volume d V.

Now I integrate both sides, so when I am integrating from T1 to T2 what I am going to get is C v L N T so it was current actually T2 by T1 it was correct, right-hand side is minus R L N V2 by V1 we will put the bar. So now you see now I equate them both so in that case I am bringing C v down below. So L N T2 by T1 is equal to minus R by C v L N V2 by V1, and what is R by C v? So are by C v is, you know CP minus C v is equal to R right divide both sides by C v, you will get CP by C v - 1 equal to R by C v right, so R by C v is CP by C v - 1, and CP by C v is typically called Gamma so Gamma - 1 is R by C v. So we can now write that L N T2 by T1 is equal to minus Gamma - 1 L N V2 by V1 bar ok.

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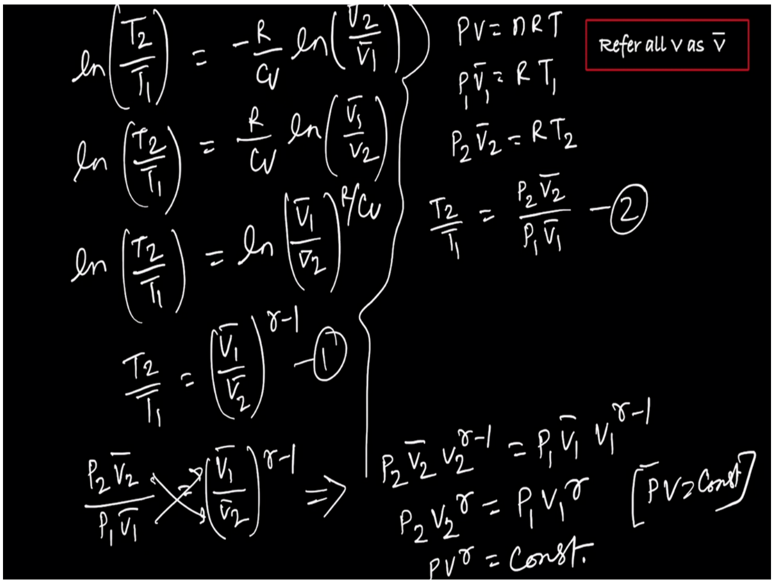

Refer all V as $\bar{V}$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{C_V}\ln\left(\frac{\bar{V}_2}{\bar{V}_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{R}{C_V}\ln\left(\frac{\bar{V}_1}{\bar{V}_2}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{\bar{V}_1}{\bar{V}_2}\right)^{R/C_V}$$

$$\frac{T_2}{T_1} = \left(\frac{\bar{V}_1}{\bar{V}_2}\right)^{\gamma-1} \quad -①$$

$$PV = nRT$$

$$P_1\bar{V}_1 = RT_1$$

$$P_2\bar{V}_2 = RT_2$$

$$\frac{T_2}{T_1} = \frac{P_2\bar{V}_2}{P_1\bar{V}_1} \quad -②$$

$$\frac{P_2\bar{V}_2}{P_1\bar{V}_1} = \left(\frac{\bar{V}_1}{\bar{V}_2}\right)^{\gamma-1} \Rightarrow$$

$$P_2\bar{V}_2 V_2^{\gamma-1} = P_1\bar{V}_1 V_1^{\gamma-1}$$

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$$

$$PV^{\gamma} = \text{Const.}$$

$$[PV = \text{Const}]$$

If I will have a little bit more space here I will write it so how much we got? We got that L N T2 by T1 was minus R by C v L N V2 by V1, this was we had and we show that this is nothing but are by C v L N, this was bar V1 by V2 okay, which means L N T2 by T1 is equal to L N V1 by V2 to the power R by C v and we know that R by C v okay we will do that R by C v later on. So now when we have this equation both sides L N is there then we can write T2 by T1 is equal to V1 by V2 to the power R by C v which we have calculated to be Gamma – 1, so this is the equation we have but we cannot forget the 1st law of thermodynamics sorry we cannot forget the equation of ideal gas.

What was that? PV is equal to N R T that is always there or PV power equal to RT which means if I have T1 then I have P1 V1 bar and if I have T2 then I have P2 V2 right. Now let us take the ratio of T2 by T1 what I am going to get, P2 V2 bar by P1 V1 bar okay, so let us put equation 2 in equation 1 so T2 by T1 I am going to replace there so P2 V2 bar by P1 V1 bar is equal to V1 bar by V2 bar to the power Gamma – 1. Now let us take V2 to the power Gamma - 1 on the left and now let us cross multiply ok. What we are going to get, P2 V2 bar into V2 to the power Gamma - 1 is equal to P1 V1 bar V1 to the power Gamma – 1 or Gamma minus 1 correspondence like P2 V2 to the power gamma equal to P1 V1 to the power gamma.

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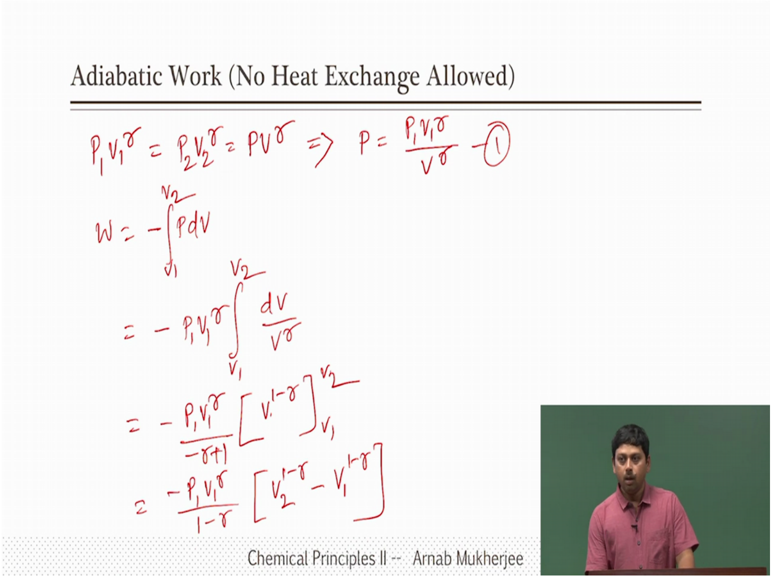

Adiabatic Work (No Heat Exchange Allowed)

$$P_1V_1^\gamma = P_2V_2^\gamma = PV^\gamma \Rightarrow P = \frac{P_1V_1^\gamma}{V^\gamma} \quad ①$$

$$W = -\int_{V_1}^{V_2} PdV$$

$$= -P_1V_1^\gamma \int_{V_1}^{V_2} \frac{dV}{V^\gamma}$$

$$= -\frac{P_1V_1^\gamma}{-\gamma+1}\left[V^{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{-P_1V_1^\gamma}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

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So for an isothermal case remember what was the case, PV is equal to constant but for an adiabatic case we get PV to the power gamma equal to constant that is the difference between isothermal and adiabatic case. So now that we have derived that let us write it here, so what we have seen that P1 V1 to the power gamma is equal to P2 V2 to the power gamma equal to PV to the power gamma in general. Once that is the case we can always write then pressure as, in general pressure we can express as P1 V1 to the power gamma divided by V. Now we are going to calculate the PV way the work done so which is minus PDV integration V1 to V2 and we are going to replace the pressure from this expression which is.

Now P1 V1 to the power gamma is a constant right, and this expression is wrong so P V to the power gamma so it will be gamma ok. You see from this one I am getting it is P1 V1 to the power gamma by V2 the power gamma that was the thing so now I am getting the correct one dV by V2 the power gamma that is I now. Now minus P1 V1 to the power gamma when I integrate the Gamma what I am going to get, minus Gamma plus 1 by V2 the power 1 minus Gamma going from V1 to V2, so minus P1 V1 to the power gamma by 1 minus Gamma V2 to the power 1 minus Gamma minus 1 minus gamma V1 to the power 1 minus Gamma.

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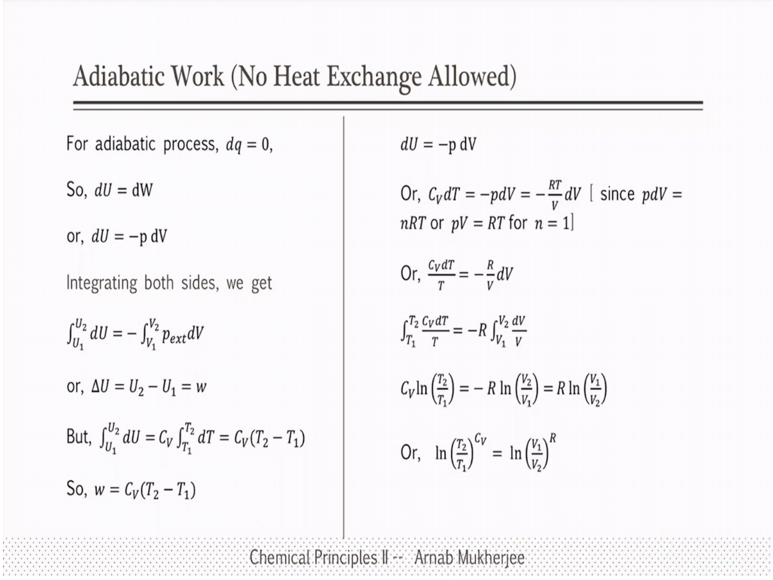

Adiabatic Work (No Heat Exchange Allowed)

For adiabatic process, $dq = 0$,

So, $dU = \mathrm{dW}$

or, $dU = -\mathrm{p}\,\mathrm{dV}$

Integrating both sides, we get

$\int_{U_1}^{U_2} dU = -\int_{V_1}^{V_2} p_{ext} dV$

or, $\Delta U = U_2 - U_1 = w$

But, $\int_{U_1}^{U_2} dU = C_V \int_{T_1}^{T_2} dT = C_V(T_2 - T_1)$

So, $w = C_V(T_2 - T_1)$

$dU = -\mathrm{p}\,\mathrm{dV}$

Or, $C_V dT = -pdV = -\frac{RT}{V} dV$ [since $pdV = nRT$ or $pV = RT$ for $n = 1$]

Or, $\frac{C_V dT}{T} = -\frac{R}{V} dV$

$\int_{T_1}^{T_2} \frac{C_V dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V}$

$C_V \ln\left(\frac{T_2}{T_1}\right) = -R\ln\left(\frac{V_2}{V_1}\right) = R\ln\left(\frac{V_1}{V_2}\right)$

Or, $\ln\left(\frac{T_2}{T_1}\right)^{C_V} = \ln\left(\frac{V_1}{V_2}\right)^R$

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And we can see the same expression is written here so we can see that what is the correct one?

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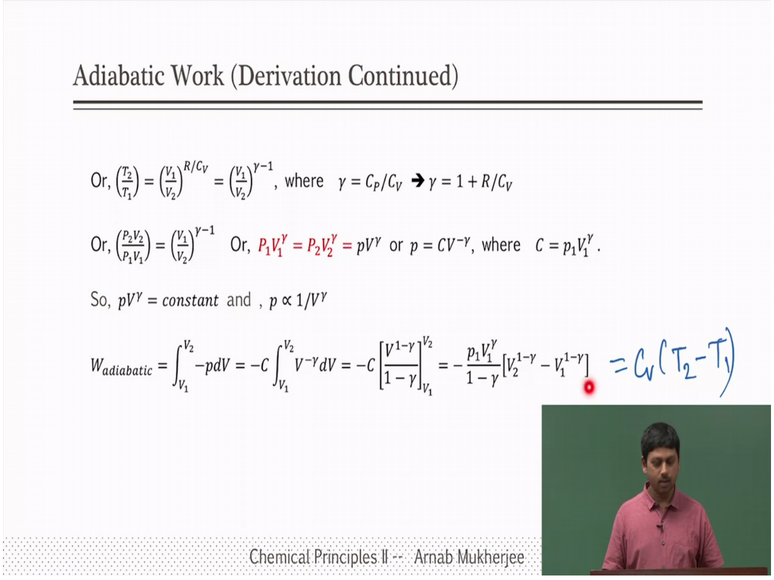

Adiabatic Work (Derivation Continued)

Or, $\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{R/C_V} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$, where $\gamma = C_P/C_V \rightarrow \gamma = 1 + R/C_V$

Or, $\left(\frac{P_2V_2}{P_1V_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ Or, $P_1V_1^{\gamma} = P_2V_2^{\gamma} = pV^{\gamma}$ or $p = CV^{-\gamma}$, where $C = p_1V_1^{\gamma}$.

So, $pV^{\gamma} = constant$ and , $p \propto 1/V^{\gamma}$

$$W_{adiabatic} = \int_{V_1}^{V_2} -pdV = -C\int_{V_1}^{V_2} V^{-\gamma} dV = -C\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2} = -\frac{p_1V_1^{\gamma}}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right] = C_V(T_2 - T_1)$$

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This is the derivation that we have already done. Okay so this is the now expression for so you see this is same as that of C v T2 minus T1, do not forget that that is same as C v T2 minus T1. So if you have this information of temperature you use this C v T2 minus T1 formula, if you do not have the information of temperature you have to use this complicated formula ok.

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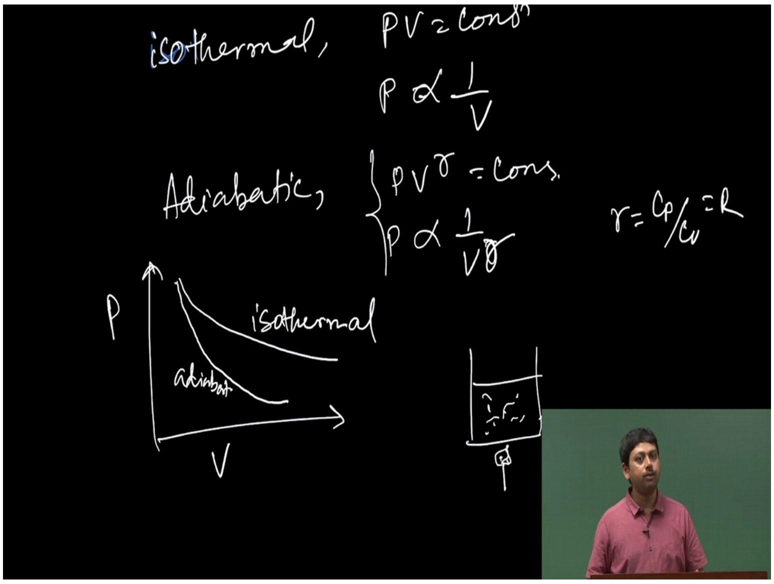

So now obviously when you compare between isothermal and adiabatic work, for an isothermal PV is equal to constant and therefore P goes as 1 by V right. For adiabatic case adiabatic processes PV to the power gamma is constant so therefore P goes as 1 by V2 the power gamma and Gamma is CP by C v okay which is R in case of ideal gas and it will be like more than 1, values will be more than 1 positive quantity. So when I plot isothermal and adiabatic what do you think the profile will look like? Let us say this is isothermal, will the adiabatic be higher or lower? Just compare these 2 expressions and tell me, in one case it is just going 1 by V and another case it is going 1 by V2 the power gamma.

Since gamma is larger than 1, imagine Gamma is 2 so it is decreasing like you know much faster so if gamma is greater than 1 then it will be always lower, so this is isothermal and this is adiabatic. Adiabatic work will be always the pressure falls in adiabatic processes will be always lower than isothermal processes. So this is the mathematical reason that one is going by 1 by V and another is going by 1 by V2 the power gamma, what will be the physical reason for this?

The physical reason for this is that in isothermal case let us say you have a gas system and that is going to expand but in order to keep the temperature you have a burner kept or a heat reservoir is kept there so heat is being supplied continuously so because of expansion, pressure falls, but because heat goes inside it does not fall as much. But in adiabatic case system there is no heat going inside so pressure is falling according to the expansion of volume alone, there is no compensation coming from heat outside and that is the reason for isothermal processes the pressure is less compared to that of adiabatic processes and that is shown by this particular nice looking graph where you have not only 2 dimensions but also 3 dimensions.

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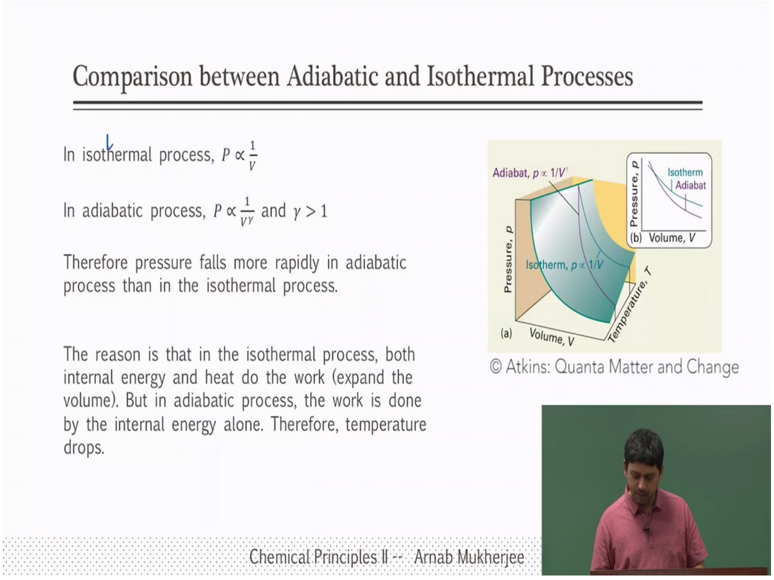

Comparison between Adiabatic and Isothermal Processes

In isothermal process, $P \propto \frac{1}{V}$

In adiabatic process, $P \propto \frac{1}{V^{\gamma}}$ and $\gamma > 1$

Therefore pressure falls more rapidly in adiabatic process than in the isothermal process.

The reason is that in the isothermal process, both internal energy and heat do the work (expand the volume). But in adiabatic process, the work is done by the internal energy alone. Therefore, temperature drops.

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The 3rd dimension says that you know as if there is small change in temperature then how it look like so that is going to different so that one can study, and the reason I have already mentioned.

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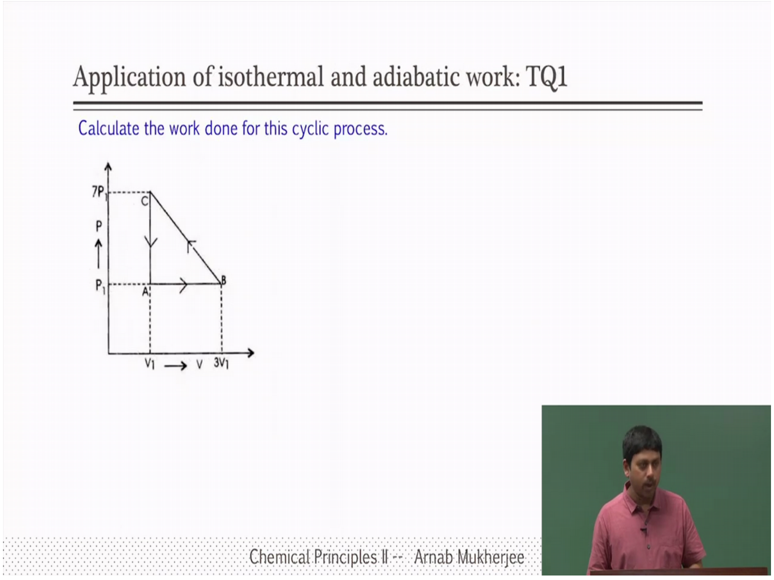

Application of isothermal and adiabatic work: TQ1

Calculate the work done for this cyclic process.

Okay so now I think we have crossed the time but we are going to discuss a few problems or calculations about how to do sudden works and things like that in tomorrow’s class, so we will stop.