Electrochemical Impedance Spectroscopy
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Lecture – 11
Assignment 02

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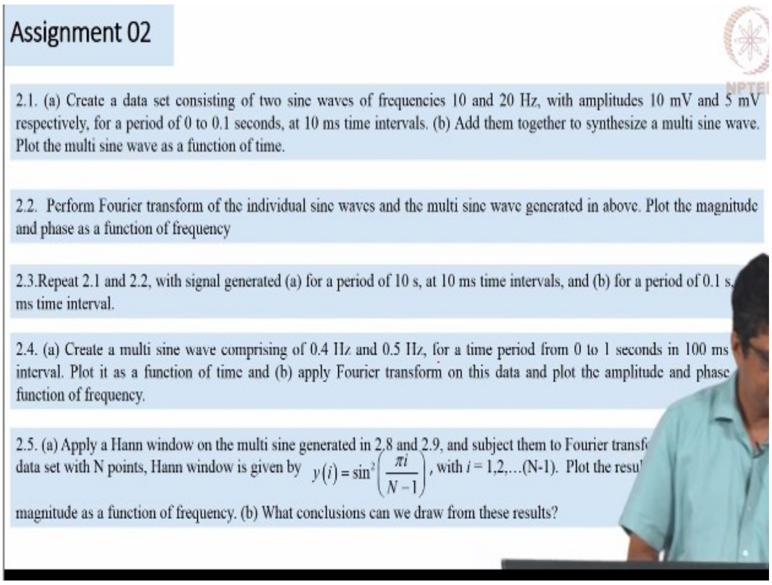

Today's assignment is split into multiple parts. First, I want you to synthesize a multi-sine data consisting of two sine waves, one at 10 Hz frequency and another is 20 Hz and with two different amplitudes. You can synthesize it for a period of 100 milliseconds and this is to be synthesized in discrete point intervals. It is 10 ms time interval. It is just to give you an idea of how a multi-sine would look like in the time domain. You can write your own program or you can use a software like Matlab to perform FFT. I want you to perform FFT so that you can learn about its ability to resolve this into different sine waves of these frequencies 10 and 20 Hz and also to have an idea as to how does the wave appear if you plot in time domain.

If you do the Fourier transform correctly you should be able to extract the magnitude. That is the transformed data should tell you that at 10 Hz you have 10 mV, and 20 Hz you have 5 millivolt in time domain data. And in this case, we have not given any phase, so it should get a phase of zero.

In the third part, what I have asked you is to synthesize data for a period of longer time. That is 10 seconds with the same time interval. Another part I have increased that time resolution or decreased the time interval, but the total time period is kept as 0.1 second that is the same as the original first case. So if I increase the duration and do FFT, what do I expect to see? If I increase the frequency of sampling and then do the FFT, what do I expect to see? I would get more number of harmonics in some cases. That means in the FFT I would plot it as 0, f_1, f_2, f_3, f_4 and so on. f_1 may be one frequency, f_2 will be twice that, f_3 thrice and so on. For example, 10 Hz, 20 Hz and so on. I might be plotting it as 1, 2, 3, 4 Hz and it ends at a particular limit, that is a finite length vector. We are given a time domain data at finite time intervals and for a particular duration.

When we do FFT what is the duration? What is the highest frequency you can get? Similarly, what is the resolution of this frequency? Do I get data at 0 Hz, 1 Hz, 2 Hz, 3 Hz, 4 Hz in this sequence or do I get data at 0 Hz, 0.1 Hz, 0.2, 0.3, 0.4 and so on until certain number of Hz? This depends on the time domain data and the frequency at which it is taken, what is the sampling frequency and what is the duration for which it is taken, but then I would like you to try that see for yourself.

In the next part, we have to create a multi-sine wave again comprising of two sine waves at a particular time interval, but the combined wave will not be a complete wave. In the sense it will not have an integer number of waves for the first wave and the second wave together. We get a multi-sine which is not a complete waste if I take the total period and then look at the individual wave's periods, the total period will not be an integer multiple of individual waves periods. That means I may have one and a half wave at 1 frequency. I may have 2 waves at another frequency. If this combined wave is subjected to FFT, what do I get? I will not get the correct phase and magnitude for each one of these. I want you to try and see. You will know that if the combine wave is not a complete wave for each of the individual components, then you will have spectral leakage.

Then the next part is to apply what is called *Hann window*. This is basically a trick to minimize the spectral leakage. But when you apply it, in certain cases, you will not be able to recover the original information. Lot of times you can apply it on set of waves and get the magnitude information correctly even though phase will be off by a little. But if the wave is really a partial wave, you will not be able to recover full information from this.

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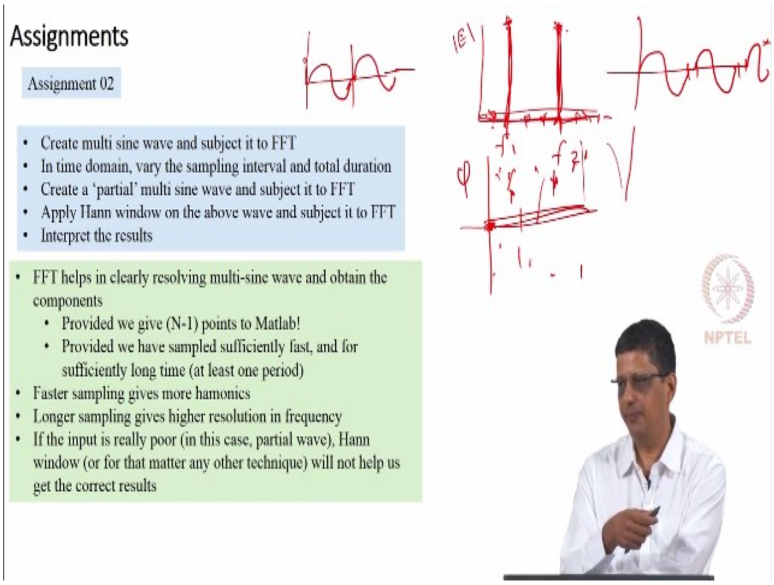

Second (question), I had asked you to create multi-sine wave and do FFT. Some of you have created single sine wave and multi sine wave and you have done FFT on both which is fine, but I would be happy if you just do the FFT on the multi sine wave. If your sine wave contains frequencies f_1, f_2, after doing the FFT, the magnitude data should look like spikes at the corresponding frequencies. Depending on what the magnitude of amplitudes are, they may look equal or they may not; whatever value we are giving here , [maybe 5 mV, 10 mV], and they should be exact. If they are not exact, there is something wrong in what you have done.

Something wrong in what you have done; you may or may not be able to identify it; that is fine. But I would like you to write that "yes, it is supposed to come at 5 mV, but it is not coming at 5"; you may think it is (due to) round off error, you might think it is (due to) some (other) error; but you should notice that (and) write that down. In time domain, I asked you to vary the sampling frequency and the total duration. I had given you specifically to do it for 10 millisecond, 5 millisecond, 100 seconds [whatever number]. Do the FFT, then I would like you to interpret the data and tell "what is the difference if we change the number of samples", or "change the total duration of the data acquisition". Then I asked you to create a partial sine wave; that is, you have about 0.4 Hz and 0.5 Hz. For 0.5 Hz you will need 2 seconds for it to complete a full cycle, and for 0.4 Hz you will need a little more than that. You will need 2.5 seconds to complete full cycle. You add them together, but you take the data only for 1 second. You are really giving partial wave, you are not giving the full wave. If you do FFT with that data, you will not get proper data (in frequency domain). You can apply Hann window, you can apply other windows, [although in this example I have asked you to apply

Hann window], (but) you will not get what you are looking for. (i.e. You will not get the correct magnitude and phase). What you are looking for is , ideally , a frequency set where it says 0.4 (Hz), 0.5 (Hz), *this* is the magnitude, *this* is the phase. If it tells that (those values) correctly, it is great. You would not get it, because we are not supplying proper wave.

Hann windowing helps in some cases, when there is some spectral leakage because of providing incomplete wave, but that incomplete wave there means, “I am applying 1, 1.5 or 2 (cycles) somewhere there I am stopping. Instead of stopping at (the end of) 2 (cycles), I may stop at 1.75 (cycles). (Then applying Hann window will help). If you do not even give one full wave, if I give half wave, these tricks will not help” That is the interpretation that I was looking for. You will not even get 0.4 Hz, 0.5 Hz in the FFT. I think you would have, if it is 1 second you would have got 1 Hz, 2 Hz, 3 Hz and so on. But in the best-case scenario what we look for is 0.4, 0.5 Hz.

When you are doing FFT in the Matlab, the way that is implemented, you should give 1 point less, i.e. (N-1), so that it is going to connect (the next point and form) the continuous line. If you give all the points (i.e. N points), the resulting wave is not a good sine wave, and that results in incorrect phase and magnitude. Because it is actually taking a slightly distorted sine wave and it is doing FFT on that. This assumes that you are given at least one period and they are giving enough sampling (frequency). Generally, if you have sufficient number of samples and no spectral leakage, you will get the correct result, as long as the magnitude in that frequency is greater than zero. If the magnitude is close to zero, it is zero because of round off error it may be close to zero, otherwise it will be exactly zero in other frequencies where we are not having any signal. Phase will be random, so it is like saying “a point here it is zero, but it is not exactly 0, it is very close to 0, because of truncation error”. It will have a phase, but that phase can be 10°, 90°, 70°, 180° or any number from 0° to 360°. It can be a large number in the phase, but it does not mean this is a significant number, because the magnitude is close to zero. Ideally, what you should do is whenever magnitude is very low compared to the signal, whatever phase you are getting there artificially set it to zero or force it to zero. Only for the frequencies where the magnitude is significant for those frequencies show the phase in the plot. Then you can present this is the FFT data, this is the magnitude, this is the phase. Do not show a phase with everything present everywhere (i.e. scattered all around the plot) and (conclude, incorrectly that) phase is becoming random because FFT cannot do phase. FFT can (get) the phase, but just that (the phase value) is not (meaningful) when the magnitude is not significant.