Principles and Applications of Electron Paramagnetic Resonance Spectroscopy
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Lecture – 05
Magnetic Moment in Magnetic Field – I

Today we are going to discuss what happens if we keep a magnet in a magnetic field, what sort of motions it experiences; to do that we need to know a little bit of vector algebra. So, let us recapitulate our understanding of vectors.

All the physical quantities that are come across are of two types, some are scalars some are vectors. Scalars are those which has only magnitude and no direction for example, temperature, mass, density. On the other hand vector quantities are those which have magnitude as well as direction for example, velocity, displacement, momentum. So, to denote a vector quantity we need to describe its direction and how do we describe the direction we need to have a definite coordinate system. So, the most popular coordinate system is of course, the Cartesian coordinate system and here it is.

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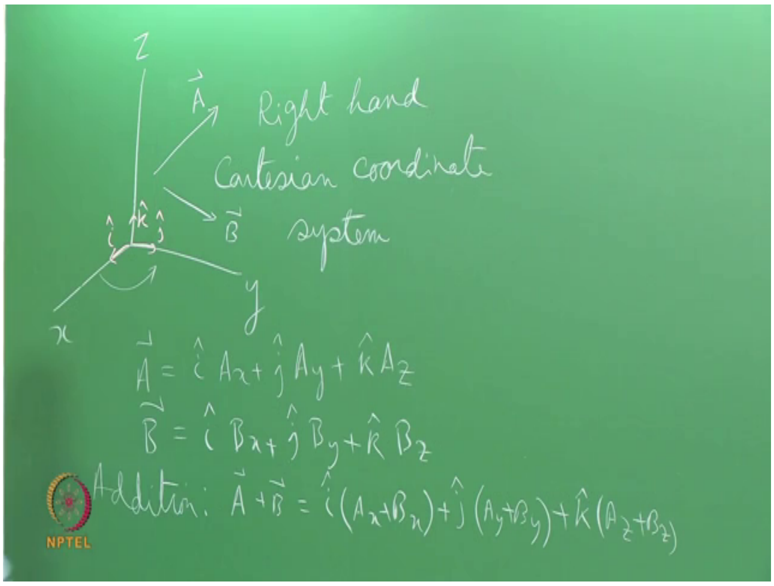

This is x, y and z notice the relative positions of this three coordinates x here and then you go anti clockwise direction to y and then this is the vertical direction. So, these direction is anti clockwise. So, we call this coordinate system by norm is this is the only coordinate system possible there are many others, but this is very common. What do we call the right hand coordinate system? If you hold your hand this way and bend this four fingers so that it goes from x to y then the thumb points the z direction that is why it is called right hand coordinate system. Here we define a unit vector in this three directions i, j and k magnitude is one and directions are along x y and z direction.

So, in terms of this I can write any vector let us say vector A written it noted in written in this fashion can be written as where A x, A y, A z are the three components. So, I could have another vector B for example, which could be similarly written as i B x, j B y, k B z. So, additional vector gives another vector which is defined to be vector where the components are added. So, this sum of the two vectors will be a sum of these two.

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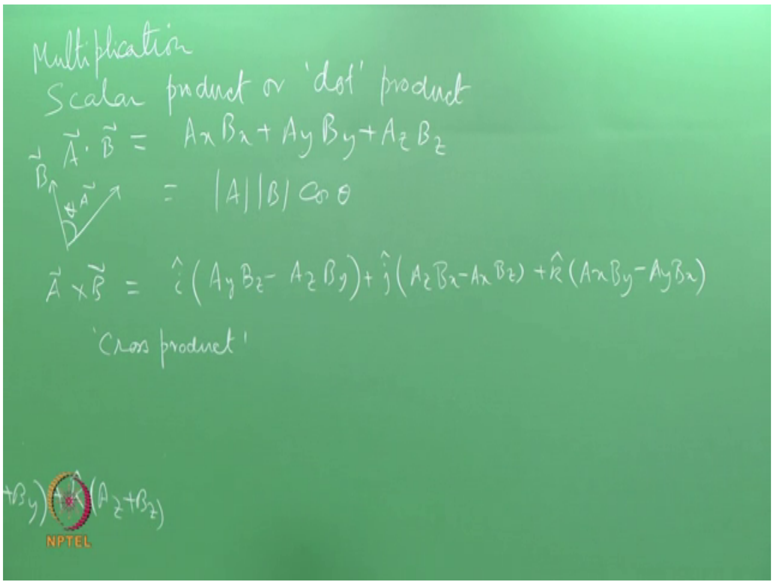

This is vector addition like multiplication. Multiplication is defined two ways one is called scalar multiplication where the resultant is a scalar quantity we write in this fashion A dot B is defined to be Ax Bx plus Ay By, Az Bz. Here we call it also a dot product because we use a dot symbol to write this. So, we can call it scalar product or a dot product.

Here this is also given by this vector let us say A and vector B if this angle is theta that this is given as magnitude of A, magnitude of B and cosine theta. Magnitude of vector is of course, a measure of the length of this, this is the magnitude of this, and this is similarly the magnitude of B and magnitude is defined this, this. Defined by Ax square

Ay square Az square. That is the magnitude. Similarly we can get the magnitude of B, here the scalar product as this magnitude.

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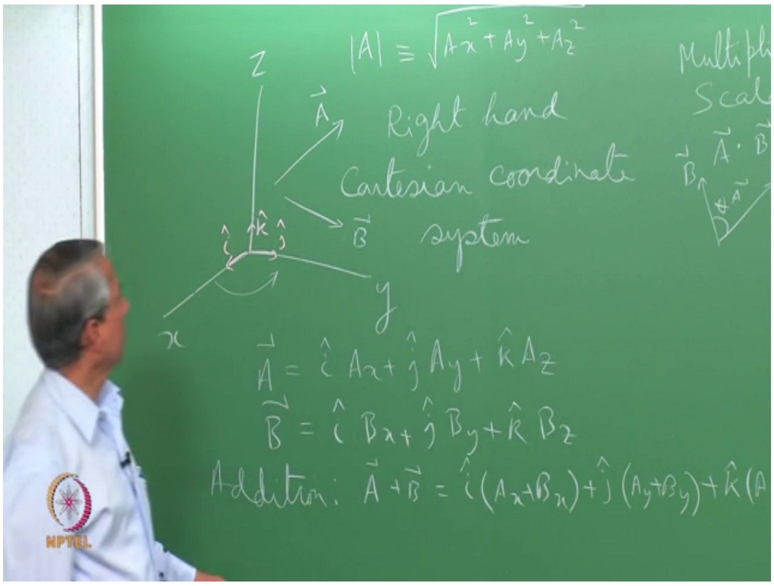

Another product is called the vector product where the resultant is a vector quantity that is written as A cross B written as i, we check this one. So, this is the vector product, it is also called cross product because we use a symbol cross to denote this one now. It is somewhat difficult to remember that you see I also cannot remember, so I had to use this note, but this is this can also be written as a determinant in this fashion i j k, Ax, Ay, Az into Bx, By, Bz if you expand the determinant it will give the same expression as this one.

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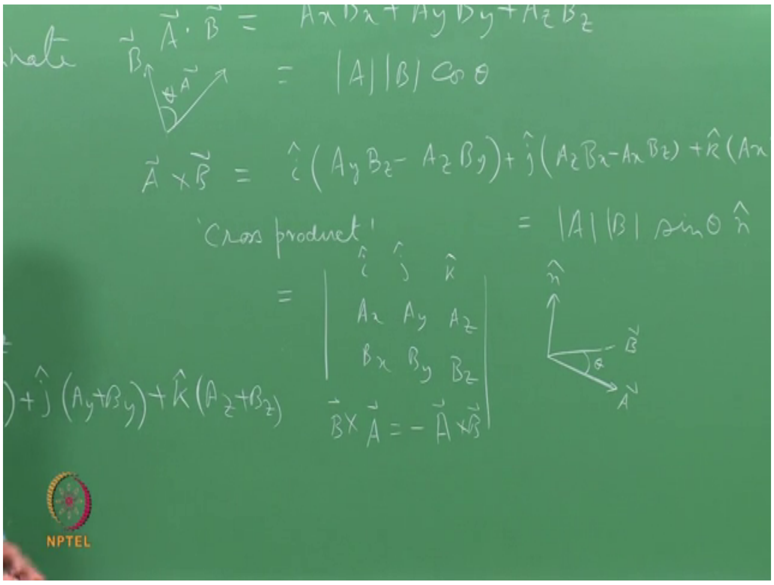

Now here in analogous manner the way you have written this one this can also be written as magnitude of A, magnitude of B times sin theta and let us say a unit vector in a direction this direction is defined to be the direction which is perpendicular to both vector A and vector B and follows the right hand rule. So, say this is A, this is B and this is theta. So, if I use my right hand this way. So, this angle this is the unit vector pointing is in this direction. So, when you go from A to B in the right hand fashion then the direction which perpendicular to both A and B is the direction given by this naturally if I change the direction of the multiplication make it B cross A that has to go from B to A. So, it comes in this fashion. So, this will therefore, be equal to minus of A cross B. So, you see the vector product the order is important.

With this background now let us try to see what happens if we place a magnet in a magnetic field.

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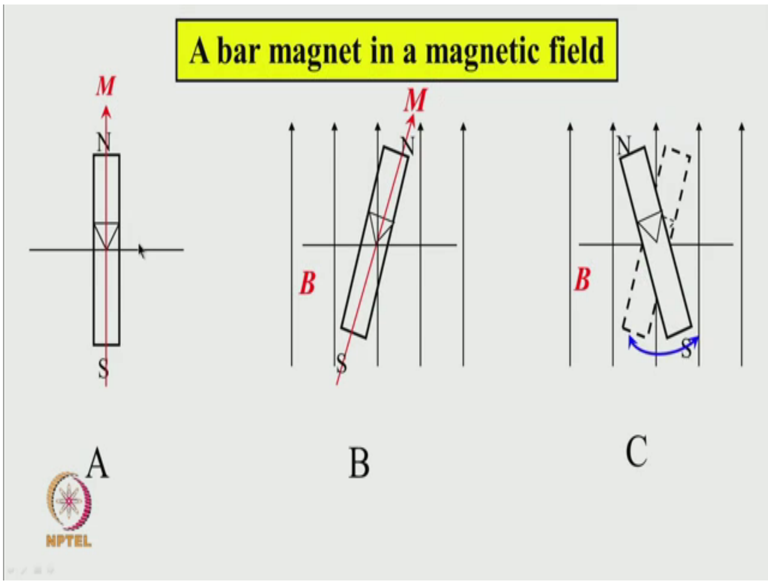

So, here this panel A shows a magnet it is a magnetic moment shown by the red line and with the letter M, this is the south pole and the north pole, this is let us say it is held at this pi word here. Now if I place this magnet in a magnetic field given with the black lines and the letter B and it is tilted at an angle then what happens? This will try to orient itself along the direction of the magnetic field. So, that the north pole points toward this direction south pole points towards this direction that will be the minimum energy to do that it has to experience a torque so that this can rotate this fashion.

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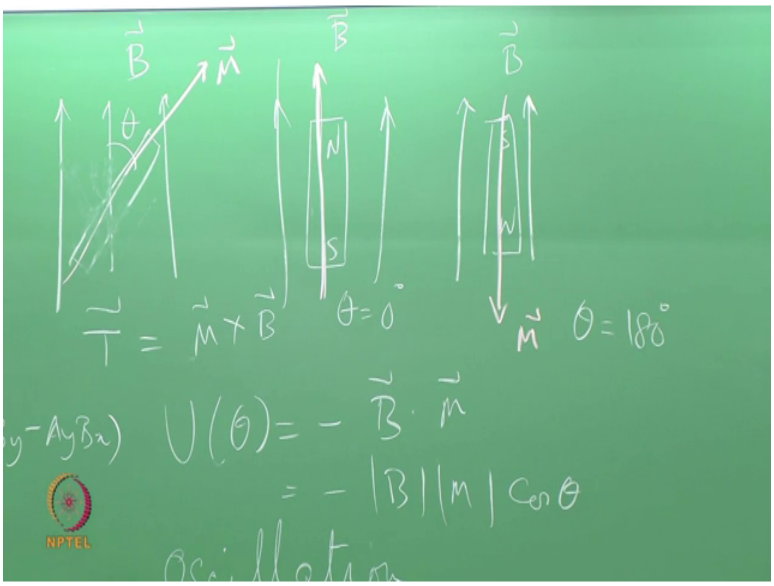

So, a magnet of this kind a bar magnet and placed a magnetic field will experience a torque. This is the magnetic field direction and this is the magnet. This is the magnetic moment. So, the torque will therefore; obviously, depend on the strength of this magnetic field and the strength of the magnetic moment and which way it is going to rotate is to given by a vector product. So, I write torque vector quantity is written as M cross B. So, this shows that the torque depends on the magnitude of this as well as magnitude of this one and also the how the angle is. Which direction the energy of this magnet becomes minimum when this points towards this so that this magnetic moment points in the same direction as this magnetic field direction and maximum energy will be when in the opposite direction this is A, this is B, magnetic field direction and the magnet points in the opposite direction.

So, the energy in an interval condition like this where let us say this angle is theta this energy theta is given as B dot M. Why is this so easy to understand that this is nothing, but magnitude of B, magnitude of B times cosine theta. So, see when theta is 0 which is this and here theta is 180 degree, cosine theta gives you the minus 1. So, this energy go maximum and this goes minimum which is understandable why it should be. So, importantly that if I start with an arrangement of this kind then let it go they will try to count the minimum energy configuration which will be trying to align in this fashion, but then because it comes here it also acquire a sudden kinetic energy. So, it will not come to a rest it will go beyond this then when it is comes away from the equilibrium position the torque again will act in the direction such that it brings it back in the sense it will go back and forth. So, this is shown in this panel C here. So, this bar may not is going to oscillate back and forth if it is starting from arrangement of this kind given B.

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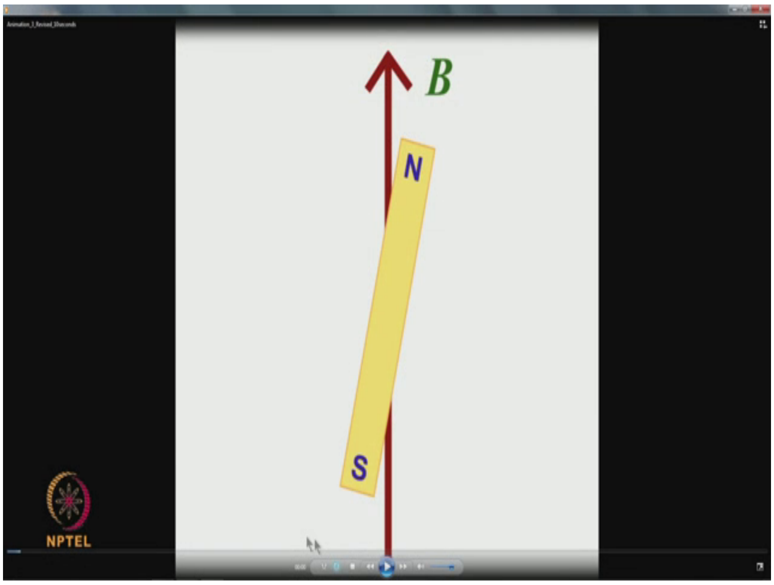

So, I have an animation here to show that, this magnet is kept in a magnetic field B and pointing at an angle to start with, now let us see how it looks.

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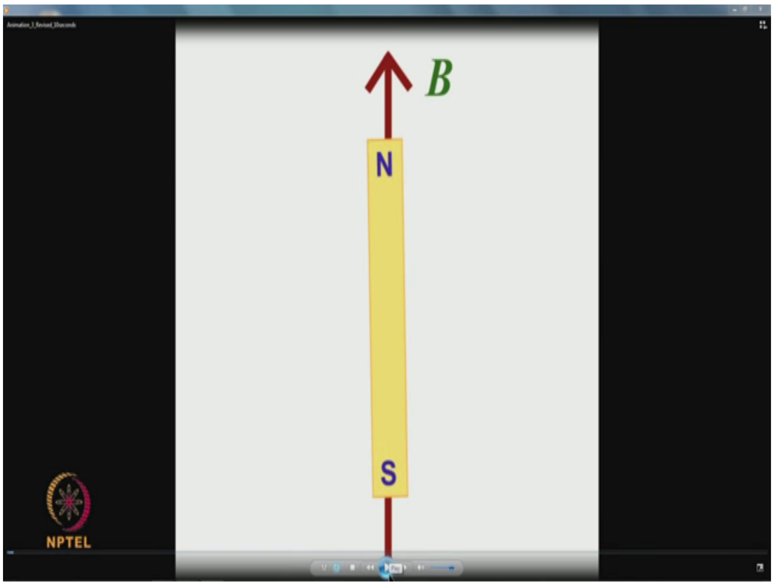

So, we see that this undergoes this sort of movement. So, in this is simple motion we call it oscillation. So, a bar magnet kept in a magnetic field will undergo to and fro motion which is oscillation of this kind.

Now, what happens if this magnet is not really a bar magnet that we have but the magnet of elementary particles like electrons or protons the magnetic moment comes from its motion of orbital in nature or spin angular momentum. So, we should see what sort of motion experiences if this magnet has an angular momentum to start with. So, to do that let us sort of first find out how the magnetic moment arises from the orbital motion of some charged particle.

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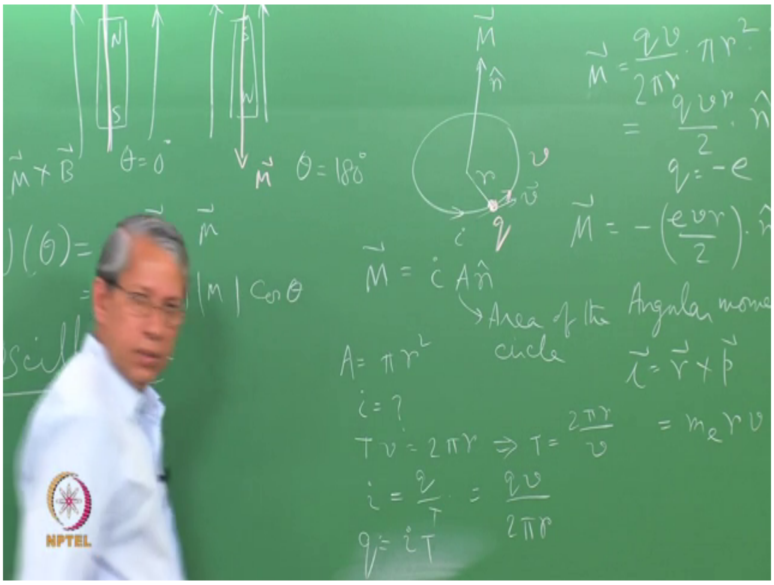

Let us say take a circular loop and certain charge q is moving in this direction at a velocity v. Then you know that movement of charge produces magnetic field, so it is a circle. So, this will produce a magnetic field and this is the magnetic moment of that say let us say this radius is r and the this moment of charge at a given velocity can be represented also by a current i. So, here the magnetic moment that this will produce is given as a little current times the area times direction of the magnetic movement is given by this, this is the direction of this. So, it is again the right hand rule that if it is moving in this way that that the moment will point in x direction say A is the area at the circle.

So, this is the magnetic moment that is going to be produced when the current I moves in a circle of radius r. So, the area A is equal to of course, pi r square. Now this charge is moving at a velocity v, so what is current i equal to how much. Let us say this charge takes time T to complete this circle, so if the velocity is v then T times v is equal to 2 pi r the current i is produced by the movement of charge q, so this is q by T, T is the time that this charge moves here in to make say one complete circle.

So, amount of charge that is flown through this will be i times T is equal to the charge. So, this is a current you nothing, but charge following passing at unit time. So, this times this gives that this gives T is equal 2 pi r y v. So, this gives q v y 2 pi r, then M becomes magnetic moment becomes I is equal to q v by 2 pi r times a is y I square times the direction of the unit vector here yeah, so this is equal to q v r by 2 and a. So, this is the expression which gives the magnetic a moment in terms of the charge velocity and the radius here and gives the direction.

Now if the charge happens to be the charge of an electron that is electron is moving in this fashion then the q becomes a negative charge on the electron. So, magnetic moment M is given by minus e v r by 2 times this is the magnetic moments. Now let us try to relate this magnetic moment to the angular momentum of this charge. So, how do I do that? The same way, so this is moving at a velocity v its direction.

So, angular momentum is one vector is equal to r cross p and p is nothing, but mass into the velocity. So, this is actually equal to mass of electron r e into the direction basically. So, r cross p is actually the same vector product. This gives that direction in the same direction as this n. So, I need some more space from this we find that M is equal to minus e 2 m e by l. So, this now connects the angular momentum of electron to the magnetic moment it produces in this is the relationship we are trying to arrive at.

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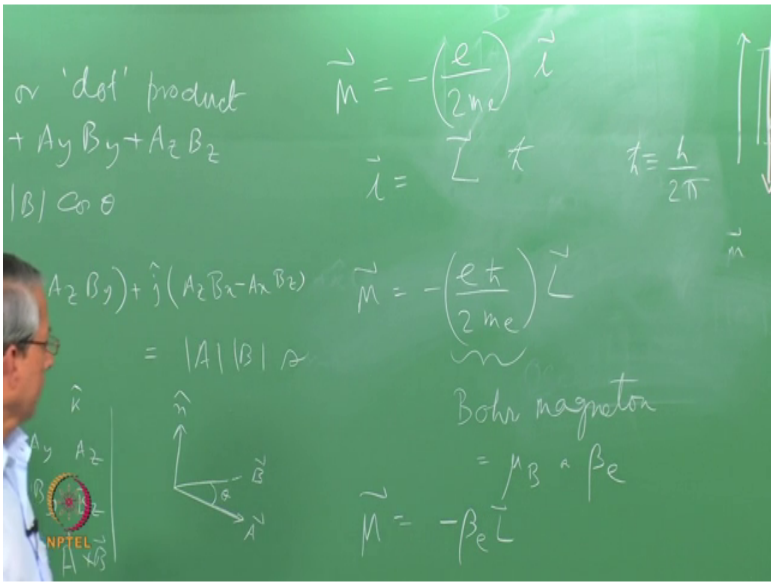

Now, in quantum mechanics the angular momentum l is actually measuring units of h by h cross which is actually equal to h cross is defined to be h by 2 pi. So, this is the unit angular momentum. So, this l I can write it as some dimensionless operator l times the unit angular momentum is this. So, in that unit now this M can be written as e h cross by 2 m e by l. So, here these are all fundamental constants. So, this constant is called the Bohr magneton which we have used earlier Bohr. So, this is usually written as mu of v symbolically or beta of e. So, this will be written as beta e l minus of mu yeah. What is the value of this Bohr magnetron? Since these are coming from the fundamental constants we can easily calculate them. Let us do it.

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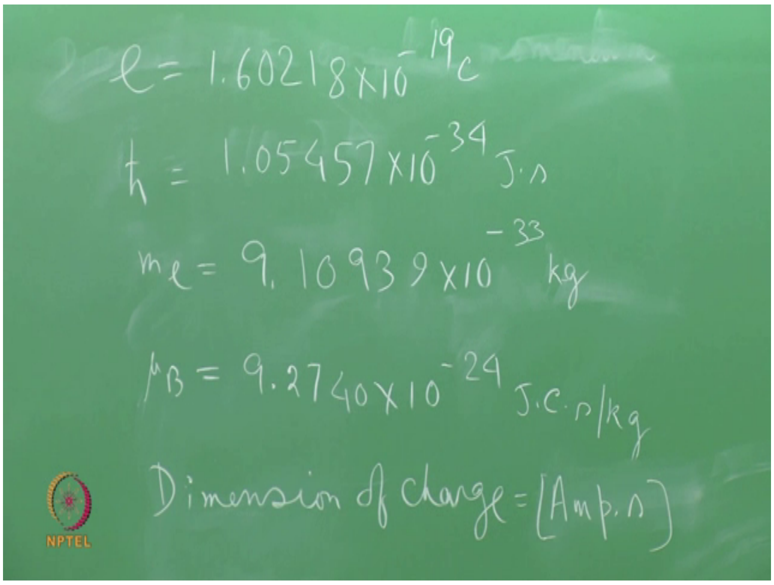

Electric charge e is 1.60218 in to 10 to the power minus 19 coulomb plants constant h cross by 2, 1.05457 into 10 to the power minus 34 Joule second and mass of electron 10 to the minus 33 kg. If you put all these things here then Bohr magneton mu B appears will come out to be 9.2 Joule coulomb second by kg Joule coulomb second per kg this all very well, but will let us try to change this unit to a little more should I say, quote and unquote usable unit which is will be proving very convenient for us. So, for that we do some dimensional analysis, dimension of charge ampere how do I write ampere in terms of some other quantities. For that if electric charge moves in a magnetic field that electricity experiences certain force that force is called Lorentz force. Force experienced by moving charges in a magnetic field it is a Lorentz force.

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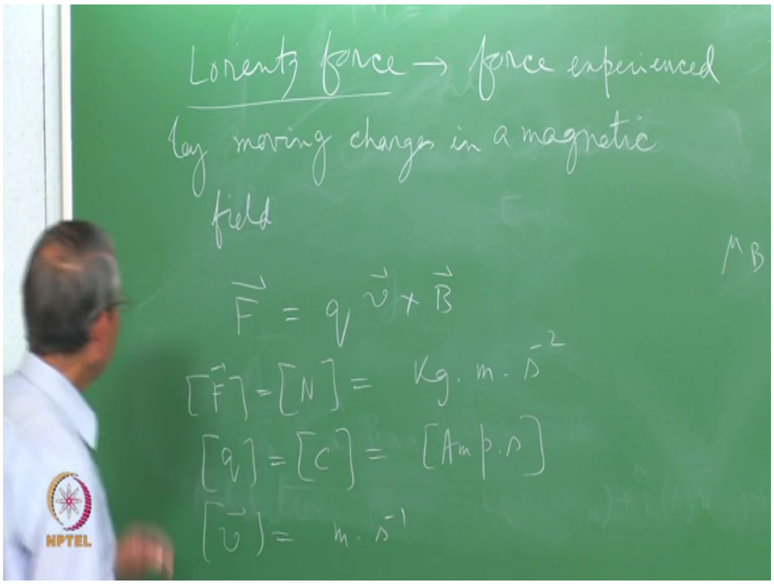

Now, expression is this force experienced by the moving charge is given as the charge moving q velocity and the magnetic cross product of this one. So, here the dimension of force is Newton which is kg meter and second inverse square, charge q this is coulomb which is ampere second because current is nothing, but charge passing per second. So, charge becomes ampere into second. Velocity v is meter per second, so what we have here the force dimension is equal to dimension of this, this and this magnetic field dimension we keep it as it is.

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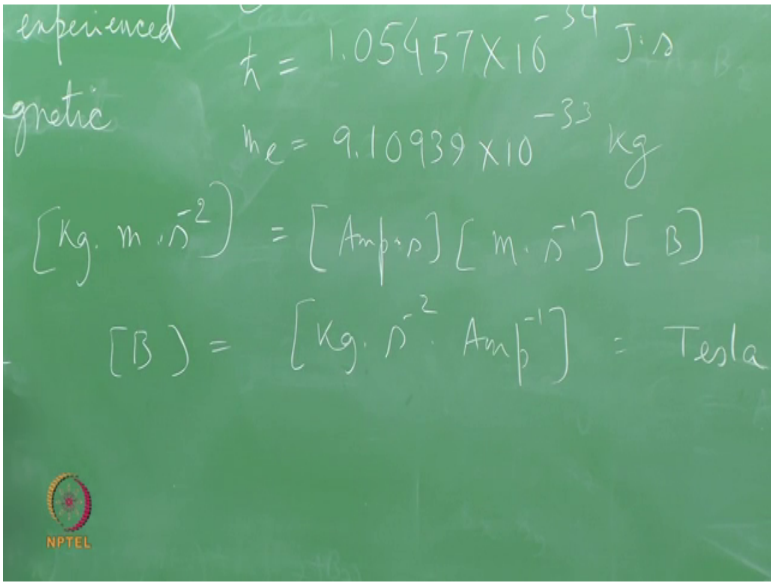

So, this gives me the kg meter second square this unit which is the force is equal to charge which is ampere second in the velocity meter per second and this dimension of magnetic field. So, this gives me the dimension of the magnetic field in terms of more familiar units, kg second square ampere inverse. So, this is the dimension of the magnetic field now let us call this Tesla and write it as T. So, this is simply a shorthand notation of this one. So, Tesla is this.

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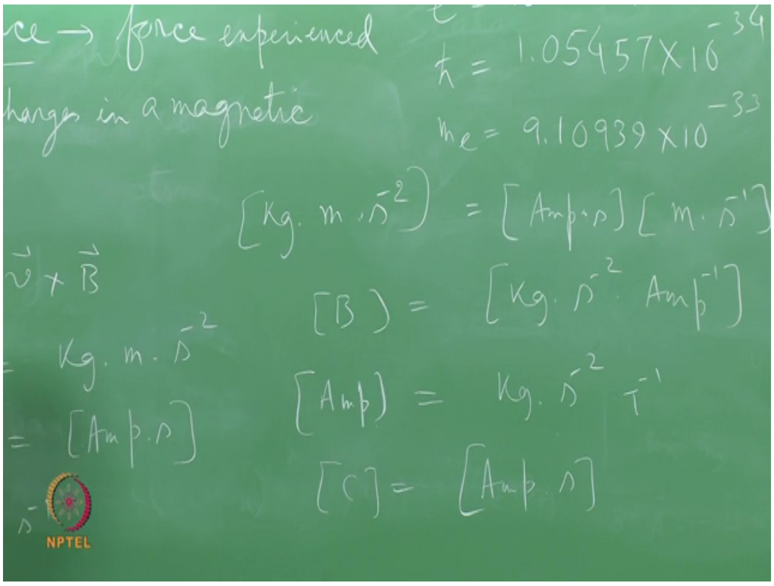

So, using Tesla now the dimension of ampere becomes kg second square and Tesla minus 1. Keeping in mind that coulomb charge is equal to ampere and second you can now get the dimension of mu B here that mu B we saw the dimension of Bohr magneton mu B is given as Joule coulomb second by kg.

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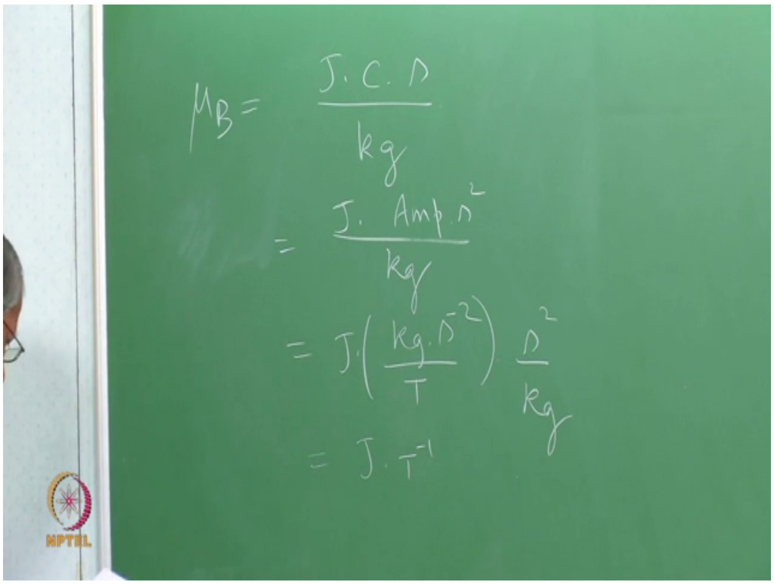

Now, if we replace the dimension of charge by ampere second this gives Joule ampere second square by kg. Now we replace the dimension of ampere that is the dimension of by this Joule kg second square by Tesla second square kg this is the dimension of ampere that we have seen. So, this gives this dimension of Bohr magnetron to be Joule per Tesla. So, as I was saying this is sort of easier unit to use or more convenient unit to use because if I know the magnetic moment of a particle in units of Bohr magnetron then I can multiply the magnetical strength to this, to find out the energy in Joule because Joule per Tesla is here. So, if I multiply by Tesla I get that energy in Joule very easily.

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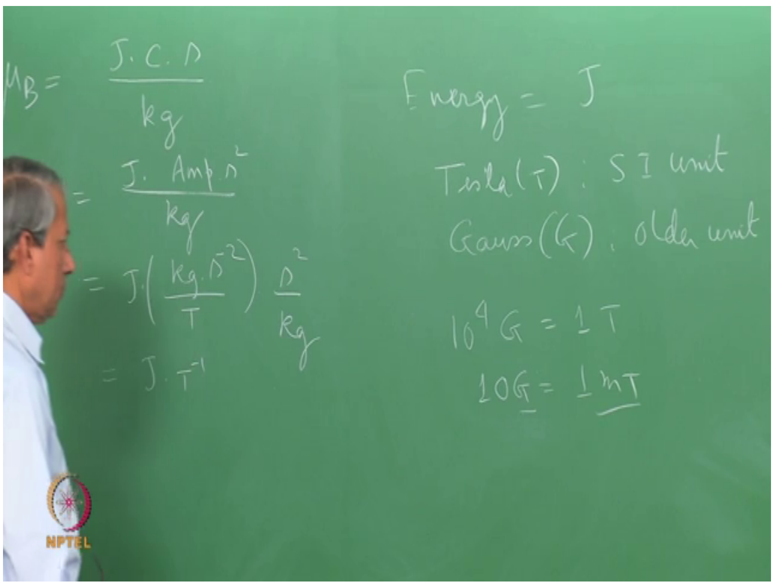

Now, this Tesla is a modern unit that is it is SI unit. The older unit of magnetic field strength was guans written as g and the relation between the two is 10 to the power 4 Gauss is 1 Tesla you as the Tesla is a modern unit it is expected that everybody should use it as much as possible, but scientific literature has usage of these as well as this one. So, we will continue to use both of them interchangeably in our lecture. So, this also is equivalent to saying that 10 Gauss is equal to 1 mini Tesla I prefer any coupling constant for example, are denoted as either mini Tesla or Gauss.

So, at this stage let us take a break and we will continue a discussion of the introduction of magnetic moment in a magnetic field in a subsequent lecture.