Molecules in Motion
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Lecture - 06
Kinetic theory of gases (Contd.)

Welcome, to the course on Molecules in Motion. So, in the last class we were discussing about the Maxwell's Boltzmann distribution of speed. What we had derived in the last class was the molecular speed distribution in one dimension; that means, in a particular direction x, we had derived the distribution function. So, we before we begin to this lecture we will summarize what we had talked about in the last class, so that we can carry it over in the present discussion.

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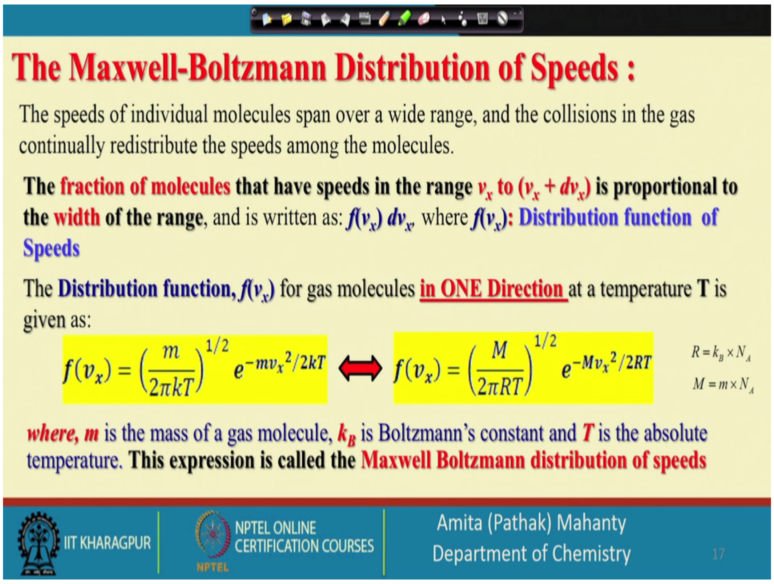

So, what we had discussed that the molecules of the gas molecules are having a wide range of velocities a particular molecule can have from c to c plus 1 c plus 2 c plus 3 a lot number of velocities associated with it and what happens is these molecules are continuing continuously undergoing collision and during these collision the speed of the molecules get redistributed.

So, the collisions in the gas continuously redistribute the speed in the gas molecules. So, to find out it is very difficult to find out the exact speed associated with the molecule, because they are continuously changing. So, what we had talked about we had talked about the probability density or from there we are going to talk about the fraction of molecule that may have a particular kind of speed. So, that is what we had derived we had talked about the probability density which can be translated to the fraction of molecules that have a speed in a particular range that is ranging between v x and v x plus delta v x.

This will be proportional to the width of the range velocity range and can be written as something like this it is written as f of v x into dv x, where we had taken the direction of the motion of the particles or the gas molecules were in the x direction and the distribution function was associated which we called and we derived the expression for was the f of v x, this is the x distribution function for the gas molecules at a given temperature. You see what we had derived we had the function of the this is the probability density function or distribution function we are going to call m by 2 pi k T to the power half into the velocity e to the power m v square by 2 kT, ok. This is the velocity which we had in the x direction and where m is the mass of the gas molecules and k is the Boltzmann constant.

Now, you know that when we have this small m and the big M can be if you are multiplying by the Avogadro the number in the numerator and the denominator then you get the R converted k converted to R, because k multiplied by Avogadro number gives you R and the small mN multiplied by the Avogadro number gives you the M.

So, these are the two forms we can have for the same distribution function.

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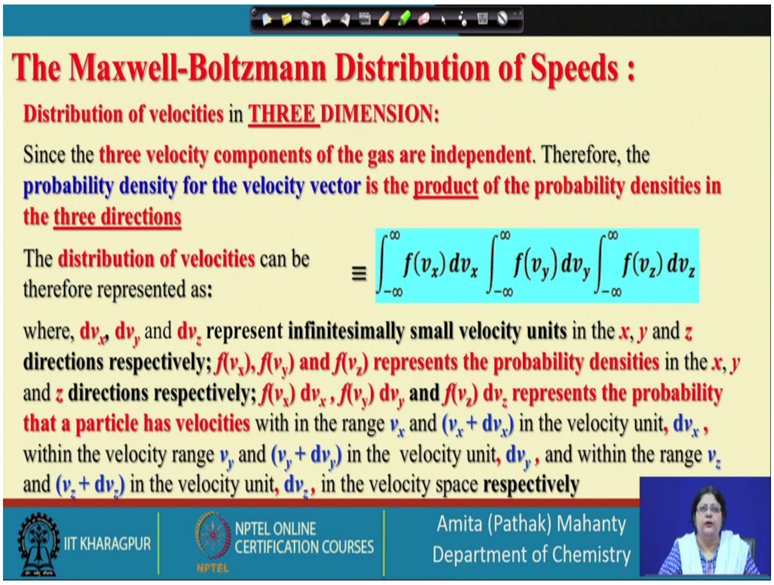

So, now, we are going to talk about what will be looking into when we are talking about the move because molecules are moving in all possible direction x, y, z and we are looking we had derived the expression for only one direction. So, how will the expression modify and how will the expression change when we are looking at the distribution in the three dimensions.

So for this, what we have to see the three velocity components of the gas which you have allow we are talking about the vec velocity space we are not talking about the x, y, z space, these are all velocity coordinates. So, what we are looking into the velocity components in x, y and z direction. So, the velocity components of this gas are they are mutually independent one does not depend on the other. Therefore, the probability density according to the laws of the probability the probability density of the velocity vector is the product of the probability densities in all the three directions. Means what we have? We have the probability densities that is we derived that as in the previous class the probability density was the f of v x d v x. So, this probability density in all of the three directions and they are extending from minus infinity to plus infinity.

So, if you want to represent the probability distribution in the three dimension what we are going to look into? We are going to look into the three components of the velocity which is x, y, z and for that we will look have to take the product of the probability densities in the three direction because they are independent variables, ok.

So, according to the laws of probability what we have to find out we want to find out the probability in on all the three direction then what we have to do is multiply the probability densities in each of obtain from each of the directions x, y and z. And, then the distribution function will be the velocity distribution function will look something like this is the you see that this probability density expression, where you have the dv is the infinites will small velocity unit along which we are going to see the change in x direction and the v the v of a function of a v x is the representation in the probability densities along x direction and what you have it is the in the range in the velocity of v x plus v x plus delta v x.

So, if the same thing is considered for the y direction and the x, z direction then we have already talked about that that it should be a product of that probability densities; so probability densities of x direction; sorry, in the x direction, in the y direction, and the z direction. We are taking the product of the probability densities and you can read in the lower part what I have written what each of the parameters represent.

Please we make careful note of each of the term I am using here; what is v x? What is v y? What is v z? These are the velocity coordinates along the x, y, z direction. What is dv x, dv y, dv dvz these are a small infinite small velocity unit in along the x, y and z directions, and what are these functions f of v x, f of v y, v f f of v z represent represents the probability densities along the x, y and z directions and that particles will have if we are if we are multiplying that v of x with the velocity interval or this were a very infinitesimally small velocity unit this product represents the probability that a particle have the velocity within the range v x plus v v x plus dv x, ok.

Similarly, this can be extended for the y and z understood. So, what we this product represents the function of v x or v z or v y into the corresponding and that will also to unit along the axis which is specified x, y or z this represents the probably that a particle will have the velocity in that particular range v x, y, z whatever plus v of that plus delta the infinitesimally small velocity unit.

So, this is the product we are going to be looking into and if you remember we had evaluated this function to be equal to 1 when we are talking about the one dimension and this the function was given from the Maxwell's distribution and we had a parameter constant.

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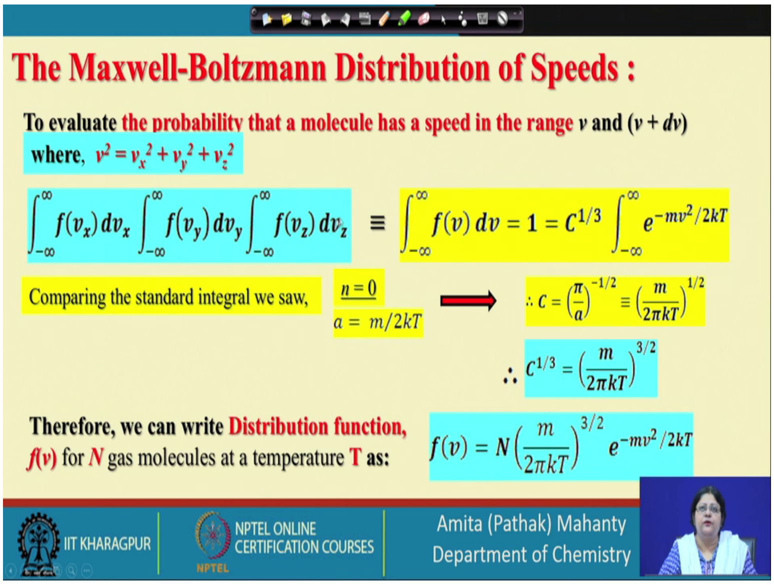

So, we had a parameter constant which we had evaluated. I am not going to go into what we had done in the last class, but and if you remember if you go through the notes you will see that you had something like this for parameter this was the term which we had got from the Maxwell's distribution function and multiplied by C and this C we had evaluated equating the total probability of finding the particle in normalized to 1. So, this is what is representing that and we had evaluated the C value that time.

Now, when we are talking about the three dimension to evaluate the probability that a molecule is having a speed in the range say v plus v delta v. Now, what is this v we are looking into? We are looking into the v which v is what do you call the rms velocity which is a representation of the squares of the means along the x, y and z direction of the velocities.

So, this is the one which we are looking into when we are talking about when we are we when we talked about three dimension because having a unit which is going to be taking care of the x, y and z this is according to the Pythagoras theorem that we have derived the v square term which is representation of the rms in some books you will write you will see they have written instead of v you have written C. So, what we have here in this expression as v at in some books you will find this represented as C. So, that time it will read as f of C into dc equal to 1.

So, now what we are looking into we are saying that the total probability distribution should be along the x, y, z direction that probability distribution function functions along the x component, y component, z component can be represented by the probability of molecules in a speed range this which where this v and v plus delta v is the rms speed which we are talking about.

So, this can be represented like this? This is the rms speed which we are looking into. So, that is a normalized to 1. So, the total change in total integration of the velocity distribution, the probability distribution function it should be integrated from minus infinity to plus infinity because it is possible for the molecules to exist in all three directions in a 0 to plus to 0 to minus infinity.

So, we have to take the total limits as minus infinity to plus infinity and this is the distribution function into the interval into which we are looking into. So, this a normalized to 1, then that time when we are talking about the experiment expression for one dimension; if you remember this was only C, but now we are looking into the contribution to C by from each of the directions each of the component. So, the contribution along the v axis which is a contribution from x, y and z this should be 1 by 3. So, whatever expression we had derived for the value of C will be now raised to the power 1 by 3 ok.

So, remember what we had done we had done comparison with the standard integrals and the standard integrals were taken where it is a standard integral from minus infinity to plus infinity, these are handbook values. You can get it in any handbook values value, you just have to compare your expression with integral form to that of the one which is a given in the solution for which is given in the handbook.

So, we the solution which we had looked into was a matching our expression only when n equals equal to 0 and if a term was equal to m by 2kT. So, from there we had found out the C will be if I am taking this part of the expression. So, this to evaluate C I have to take it minus if the C was equal to pi by a; so I have to take minus pi by because I am going to take evaluate the value of C. So, it is equated to 1. So, I take it to that side. So, it becomes my 1 to the power minus half. So, this expression which we had this was for one dimension.

Now, for three dimension what we have we have the expression of the I am I am talking about whatever I am writing here it is a much simplified form, I did not derive how I got C equal to 1 by 3, but let us take it that it is going to be 1 by 3 because contributions from all of the three has to be taken into account. When we are taking along up component v, we are taking along only one of the contributions. So, I will take the one contribution not C by 1 by, but 1 by 3 C to the C to the power 1 by 3. So, C to the power 1 by 3 will be whatever expression we had got will be multiplied by 1 by 3. So, what we have here is m by 2 pi kT to the power 3 by 2.

So, this is the distribution this is the constant which we have got. Now, I substitute this constant in the expression I have for the distribution function. So, there therefore, we rewrite the distribution function for N molecules of the gas into at a given temperature equal to the v of f of v v is the rms speed which we are talking, into this is the total number of molecules we are now considering total number a being n into the expression which we had here we had the expression of C to the power 1 by 3. So, C to the power 1 by 3 is here and the rest of the term is as it is got from the expression we have for the distribution function.

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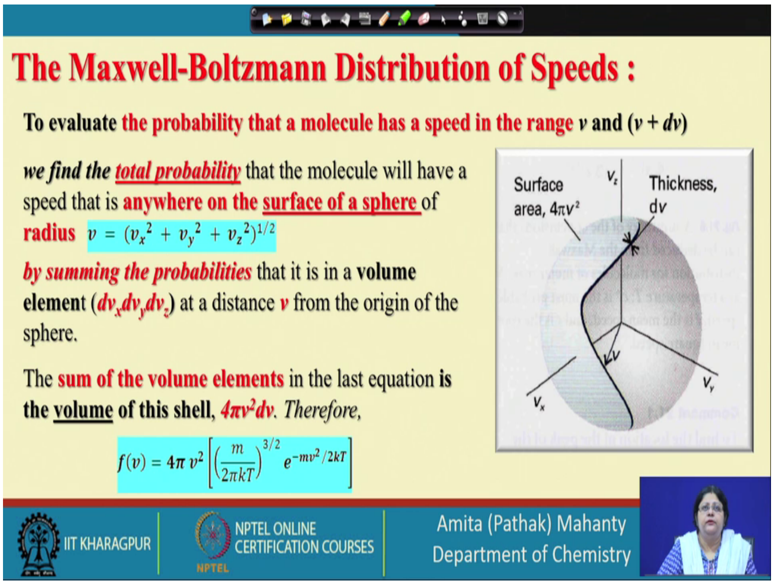

Now, what happens when we are looking into the probability distribution of molecular speed say in the v plus v plus delta v the limits or with within that range we have to find the total if we want to find out the total probability the molecules will have a speed that is anywhere on the surface of a sphere which sphere we have the sphere if you see here I was giving you a diagram we have to we are look what we are looking into the volume elemen element we are going to take into account we are looking into the volume element volume element means the dx, v d x, v d y, v d z the small units of velocity along the three axis,.

So, total probability of finding the total probability what we have to see we have to see that the molecules have a speed that is anywhere on the surface of a sphere this is the surface of a sphere which is having a radius of what? It is having a radius equal to the rms speed, ok. The radius is the square of the mean values of xyz direction of velocities to the power half. So, this is the rms speed is the radius into anywhere on the surface of a sphere having a radius v. By summing the probabilities that is in the volume element what I said just now vd x, vd y, vd z at a distance v from the origin of the sphere.

So, I am looking at a velocity component velocity rms velocity v which is corresponding to a sphere radius of a sphere and this radius of a sphere is such that you have the volume element corresponds to surfaces it is corresponding to a sphere surface a speed of a particle on the surface of the sphere which is having the radius v a v and this is nothing, but the sum of the probabilities in the volume element which we have taken as v x, v z and v y at a distance v from the origin of the sphere it is you need to visualize this to understand that we are looking into a surface like this where this is the distance we are looking into. So, this is if you are looking what is the surfaces going to be? Surface area is for a sphere what you have 4 pi r square, ok. So, this is the value we are looking into when we are talking about a surface.

So, the sum of the volumes elements in the in the last equation which we had, in the last equation means the one which we had talked about in the last class or we can talk about here also, these are the volume elements which we have. So, these are the three which we are looking into and then we have looked into the simplified that into equivalent to being dv which is the rms velocity. So, what we are looking into we are finding out the total probability that the molecules will have the speed anywhere on the surface of the sphere with the radius this that can be obtained by summing those probabilities that of the volume element this at a distance v from the origin of the sphere, ok.

The sum of the volume elements in the equation was given to be the volume total volume and this in this is the shell which is going to be 2 pi r square d v not r because r is we are look talking into the vector space. So, velocity space; so here the radius is also in velocity units. So, it will be 4 pi v squared dv, ok, the sum of the volume element in the in the expression dv dv a dv x, dv y, dv z can be taken to be the volume of a shell which is having this is the surface 4 pi v square dv.

And, then we can put that into the expression. So, this is the volume element which we have instead of dv dv dy dx we are writing the volume element and this is the distribution which we have already obtained for a distribution function.

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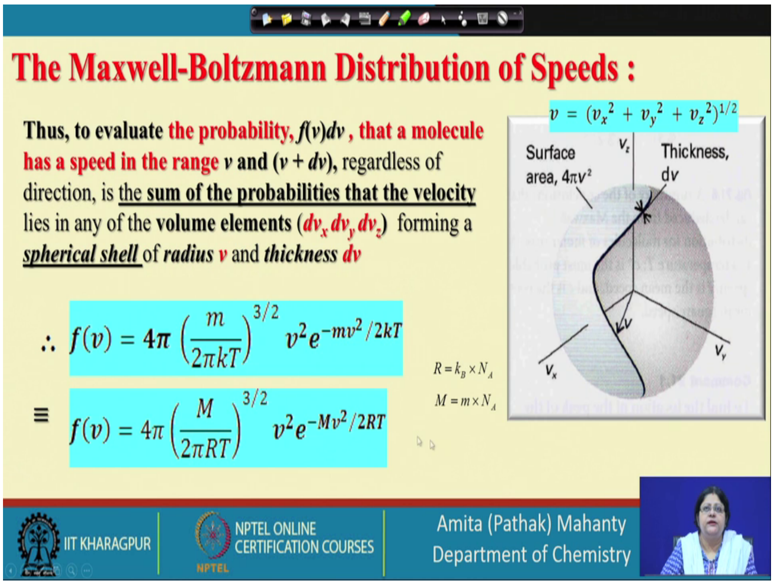

So, now what we are looking into see thus to evaluate the probability the f of v into dv that is the total that is the molecules with having a speed in the v range v plus v plus dv regardless of which direction we are talking about, the sum of the probabilities that the velocity lies in any of the volume element v x, v y, v z forming a spherical sphere of radius v with thickness d v.

So, you understand what we are looking into we are looking into the proba we are trying to evaluate the probability f of v into dv that the molecules have a speed in the range v plus v plus delta v, regardless it does not matter which direction we are taking. The sum of the probabilities of the velocities lies in any of the volume elements which we are looking into v x,v y and v z forming a spherical shell with radius v and thickness dv.

So, the shell which we are looking into it this shell is located at a distance v from the center and this shell thickness is we are replacing x, y, z in here in this place by a unit dv this is going to be the thickness of the shell. So, this is if we are looking in three dimension this is this is region in which we are finding out the probability.

So, the function probability function now becomes you take the v inside. So, 4 pi is the volume, from the volume element we have got the expression because it is a spherical surface we are considering. So, v square it is coming from the radius surface area for the sphere is 4 pi v square into dv is what we are looking into the thickness of volume element of the spherical unit which is the shell. So, this is going to be v square into the term which we already have.

So, we can represent this in another form in terms of the capital an R if you know multiplied by m na in the numerator and the denominator. So, what we expression now we have this is the Maxwell's distribution function in the three dimensional space, where we have considered that probability we have evaluated the probability of v of x dv with where this v is the rms speed that the molecule has in the range v plus delta v plus delta v regardless of the direction we are taking the sum of the probabilities that the vector lies in one of the volume elements x, y, z anywhere along a spherical shell which is having a thickness of dv and the radius of that speric sphere is v, ok.

I hope you understood how we derive this it is if you need you need to put your mind to understand this, but at least the logic of a how we obtained you can understand if you meticulously go through the derivation.

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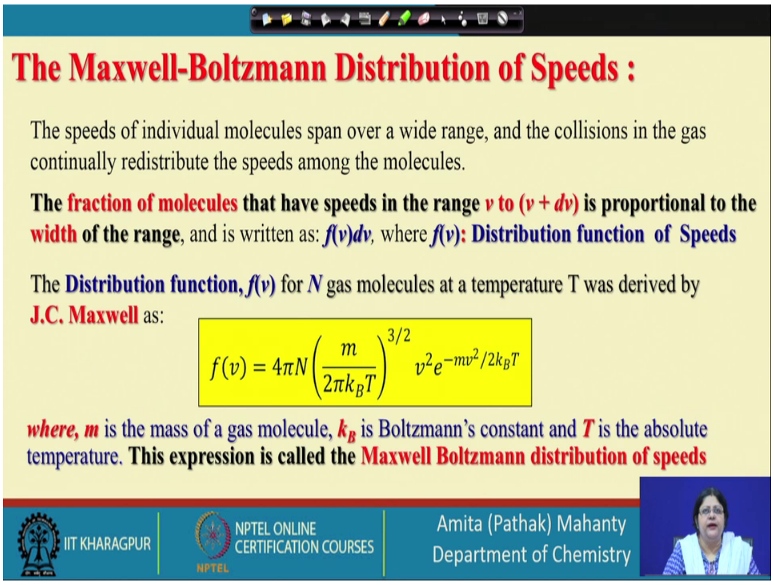

So, to summarize what we have we have the fraction of molecules that have the speed in the range v plus v delta v is proportional to the width of the range, and where here we are writing the f of v dv this is the width of the range and what is then the distribution function distribution function is the f of v. This is the f of v for n gas molecules at temperature T can be given by the expression here, as you see 4 pi n, this is the expression was previously also. Therefore, one dimension only change you have is here it was 1 by 2, now you have 3 by 2, taking into account that now we are talking about rms speed not a when the speed along any particular one direction.

So, the contribution is taken from all the three. So, c the constant which we have evaluated was 1 by it will raised to the power 1 by 3. So, from there we get this expression. And, then you have this velocity unit also coming from the volume element v square is coming from the volume element because this was the volume element we are looking into. 4 pi v square into dv. So, this expression is also coming from that and then the expression of the Maxwell's Boltzmann relation which is always which we are dealing with from the beginning.

As and you can you can convert this into the you can replace kB by r this can replace kB by r m here m here by capital M if you are multiplying by the Avogadro number n a in the numerator and the denominator then if you have Avogadro number multiplication into molecular mass then you can get capital M and here kB into the Avogadro number will give you r, similarly for this then you can get the expression in terms of capital M and r.

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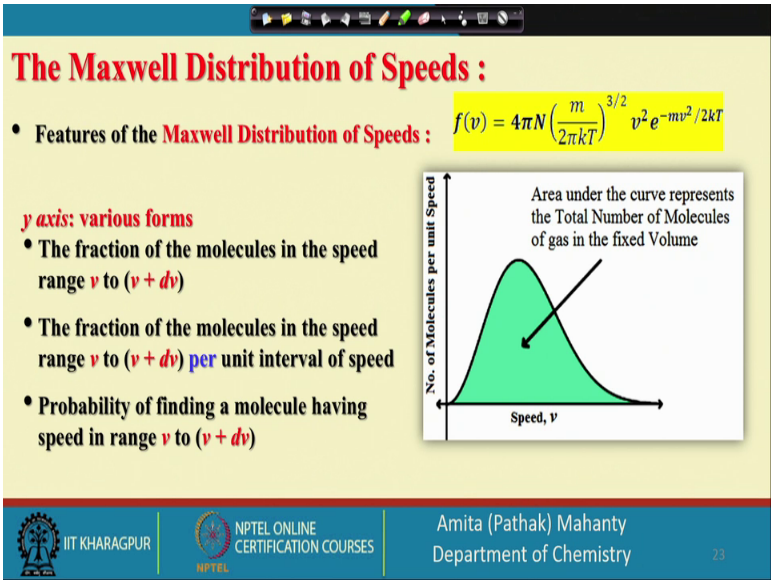

So, now we are talking about the distribution of speed which we have already derived. So, what we have derived is the features we are going to look into before we discuss in detail what is the parameters which are going to contribute, let us make sure what we are whether we understand what we are looking into. What is this x axis? What is this y axis? We had now got a distribution function that we are going to plot and in the y axis we will have the distribution function represent the probability of finding a molecule within range whichever range we have taken into account, v plus dv plus v or we can and some books you will write c it is the fraction of molecules in the speed range v plus dv plus delta v.

So, or when at the at times the unit may be changed it can be a fraction of molecules in the speed range this per unit interval of speed. So, per unit interval of speed can be for this also for this also. So, per unit interval of speed can be also measured, but whatever we are looking into the distribution function is if you is going to look something like a Gaussian function and this area whatever area we are looking into is the area of the curve represent total number of gas molecules in the in the system which we are looking into. So, that should the volume should be a constant. So, the total probability also should be equal to 1.

So, this is what we are going to embark on in the next lecture. So, we the Maxwell distribution function which we have here, we are going to see the features of the Maxwell's distribution of speed and whether we can extend that to energy is what we are going to take up in the next lecture.

Thank you.