Molecules in Motion
Prof. Amita Pathak Mahanty
Department of Chemistry
Indian Institute of Technology, Kharagpur

Lecture - 05
Kinetic theory of gases (Contd.)

Welcome to the course on Molecules in Motion. Last time, we had talked about the rms speeds of molecules. Today we are going to be continuing with the kinetic theory of gases.

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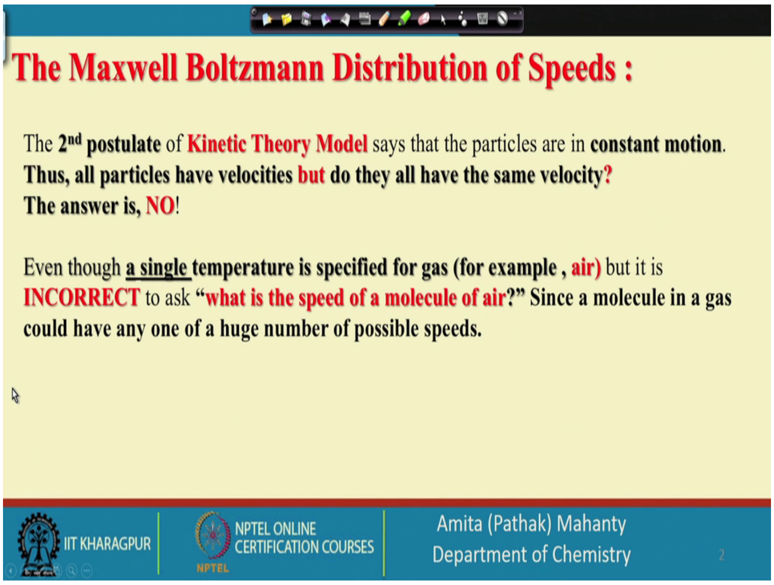

But, we are going to look into certain new parameters which is like when we talk about particular kinetic assumptions of the kinetic theory model we have discussed that we have taken assume that the molecules are in continuous motion.

So, this is this is the postulate we had. It is the assumption which we had. So, all these particles have some velocity, but do they the all particles have the same velocity? It is a question which we usually ask that when you are talking about velocity of the molecule they are they undergoing elastic collision and all the molecules or particles in the gas having the same energy. The answer is no. Suppose, you have a gas, suppose you have a gas or air in a container we know that the temperature of the gas will be what whatever we put in a thermometer and we look into the thermometer temperature and say a sign that this is the temperature of the gas.

And, that means, since temperature is proportional to the speed of the molecules which we can assume that the molecules are all supposed to have the same speed, but that is not. So, this is not. So, the molecules of the gas can have huge possibility huge large possibilities of speed. So, there is a distribution of speed which is existing and the particles which are moving they are moving randomly with different speed. Some can have very large speed. Some will have very small speed. Overall whatever you are seeing is the corresponding to the temperature of the gas, but individual molecules are having a different speed and contributing giving rise to a distribution of function distribution of possibilities of the values of the speed, ok.

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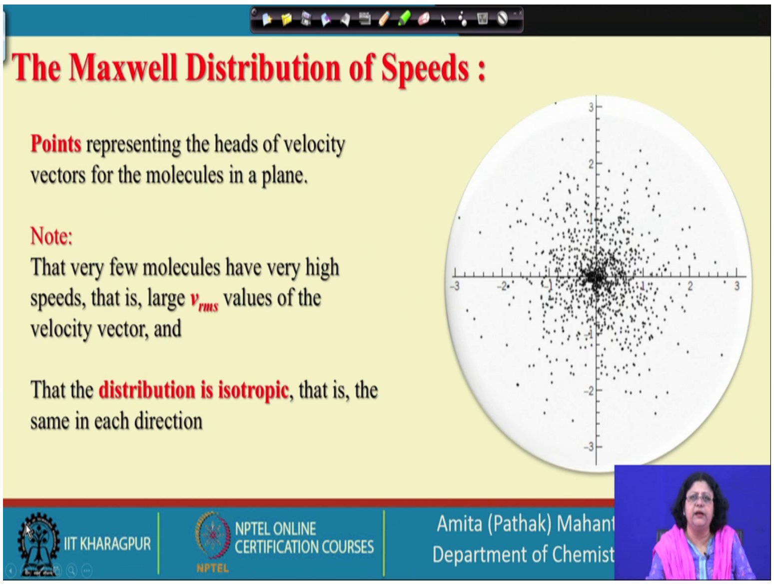

So, this being the condition we move into understanding how if we take as we have discussed previously, the molecules in the of the gas molecules which we are looking into they are all having velocities and these velocities are vector quantities and if we take a snapshot of suppose a container where we have n number of gas molecules, this will look something like this. You see these points are actually the vector heads of each of the components which we have in the x, y, z, ok.

So, you can see that the number of molecules which you are having there is a distribution of the vector heads which is the points which we are seeing in the vector space, this is the vector space velocity vector space. So, in here what we are seeing that few molecules are of having very high speed and these are coordinates which is going to be representing a very high speed and some are having very low speed. So, there is a distribution and this distribution is symmetric when the distribution is symmetric we call this as the isotropic distribution; that means, this there is a same type of distribution in the x, y and z direction, ok.

So, whatever we are talking about here, we are talking about the speed of the molecules, we are generally talking about the rms speed which is associated with the vectors.

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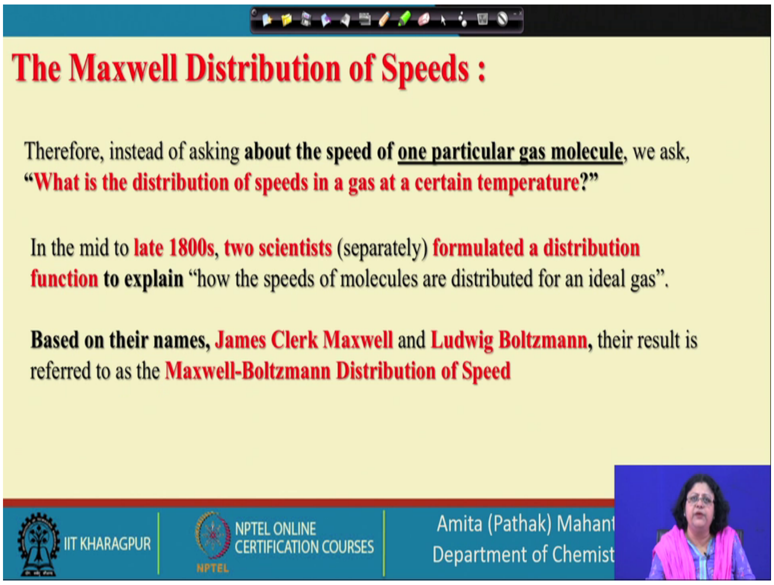

So, now what we look this distribution of a speed it is in; what particular manner will these particles be distributing having the different values of speed. As you can understand this is a random variable, a continuous random variable. Continuous random variable as you know if you have a studied probability and statistics is that a continuous function random function means random function can be either discrete or can be continuous.

Discrete functions are which have a definite value like you toss a a dice, toss a coin these are all having a limited possibilities of what the outcome may be, but when we are talking about a continuous distribution or the continuous function. That means, we have a large distribution of values which can be possibly assigned to that particular function. Suppose, we talk about the population of a country or per capita income of a particular um population of a our country what happens we talk the system which is the dynamic system is the country we are looking into. And this if you see the possibilities of the income which we have we can have a average income value assigned to a particular thing. But, when you look into the distribution you will see some are having a very high income some are very having very many schooling income someone even going with zeros income.

So, these are all a live examples of having a random function which is continuous and this continuous function was very beautifully explained by two scientists Maxwell and Boltzmann; James Clerk Maxwell and Ludwig Boltzmann.

Boltzmann these people renowned scientist and their contribution to the field of probability particularly Boltzmann's contribution to the field of probability is very high. It was there these two scientists independently started with the distribution function. Maxwell gave a distribution pattern a function along or formulated a distribution function to explain the how the molecules are distributed for a ideal gas. And this was modified by Boltzmann and this distribution hence we call it as Maxwell-Boltzmann distribution.

This distribution can be or in terms of the speed of the molecule all since speed is related to the temperature we can and a temperature is related obviously, the thermal energy is kT. So, we can always say that energy of system can be really having or will also have a distribution.

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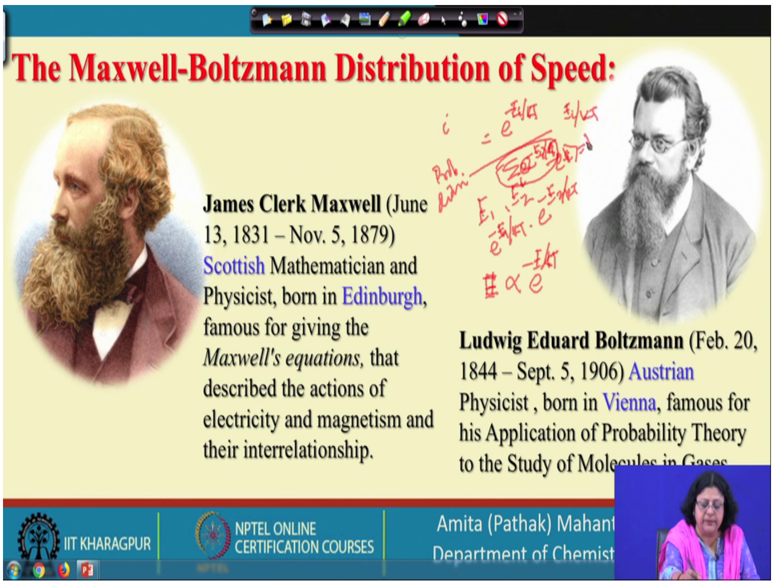

In fact, Boltzmann began with whatever things he proposed was based on energy calculations that was the time when we have this quantum mechanics coming in and the Newton classical mechanics was giving way to that quantum mechanics. And you see Maxwell was a mathematician, he was a Scottish and in around 19 in 1800 he was he had made contributions.

His main contributions was the electricity and magnetism and their interrelationships and he gave the distribution of speed he was a physicist and he gave a distribution of speed which was later on modified by the Australian Austrian physicist which who was born in Vienna he was he is he is a father of the probability theory which we have for a gas molecule system. So, what he applied was the how the energy is going to be distributed in a in the particular he described or formulated the distribution pattern of energy of a for a particular dynamic system.

What he looked into was if I if you have say if you have a energy of a particular system the energy of the system he said the energy of the system was proportional to the probability of the energy of a molecule having a energy was proportional to e to the power x epsilon or e e to the power E by kT, this is the kT is the Boltzmann constant which he formulated. He said that the energy of that system was proportional to the exponential per term which he gave he said is arising from what do you call the product law of probability, ok.

So, if you have two systems if we if we have two systems with energy E 1 and say E 2 the energy of the system will be given as e to the power of a one system by kT into of the second system. So, the energies were summed up like something like this.

So, what you have and he is said if I have n number of if I have a i system the molecules of a i system then the distribution function can be given as e to the power divided by the summation of all the energies which the system i can have. This was the basis of the probability distribution which he called the energy probability distribution, the prob distribution for energy, ok. So, this is what he said this is e to the powers if you can read this e to the power e for the i-th system divided by kT summed over all the i's. So, what he did this summation he called he normalized this and this we called as p; this p normalized to 1.

So, this is how the Boltzmann distribution and energy looks like we are going to look into something else today and what is how is the distribution function going to be given is for a for a particular gas molecules are is what we are going to look into.

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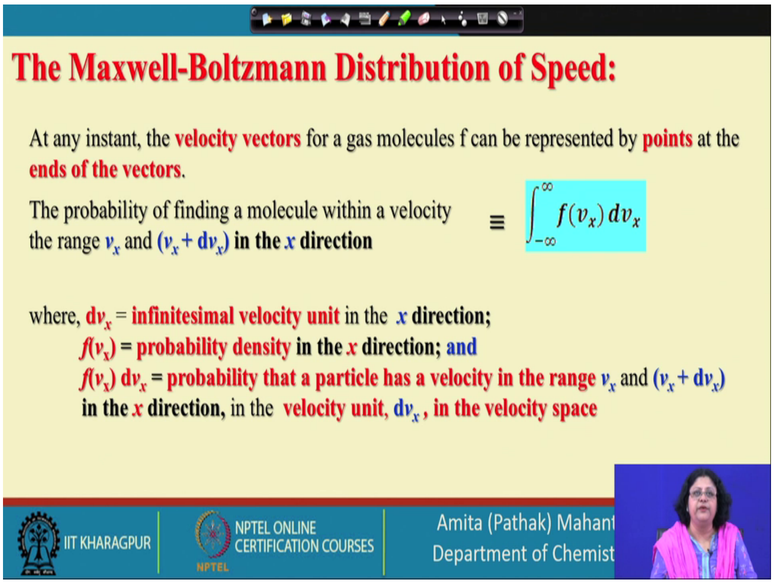

Suppose, we have at any instant a velocity vector represented by a point as I have shown you in the diagram whether you have a having a distribution of particles in a container a spherical container or a cubic container where the end of the vector was represented by a dot, ok. So, the probability of finding a molecule with us suppose they are moving in the x direction then the probability of finding the molecule in the direction in the range x v x plus v delta v x in the direction x is given by something like this. You integrate it over minus infinity to plus infinity.

The possibilities of extending from one infinity to another infinity in the x axis the x direction, and what is dv here? Dy dv here is the infinitesimal velocity unit in that x direction, ok, a small very small velocity unit which we are looking in the x direction. And, what is this? This phi of f x this is a function of velocity in the x component of velocity and this we call as the probability density along the x axis or x direction.

And, the product which we are getting the product of the function into the velocity unit in the x direction dv x is the probability that a particular particle has a velocity in the range this v x and v x plus the in infinitesimally small velocity unit dv, am I clear? What I am looking into what I what Maxwell's-Boltzmann distribution gives us in the crunch is it is it is gives you a range in which you find the velocity, ok. The probability that the particular particle is good to have a velocity in this particular range in x direction is what you get, and the velocity unit which you are looking in that in the velocity space is the dv x.

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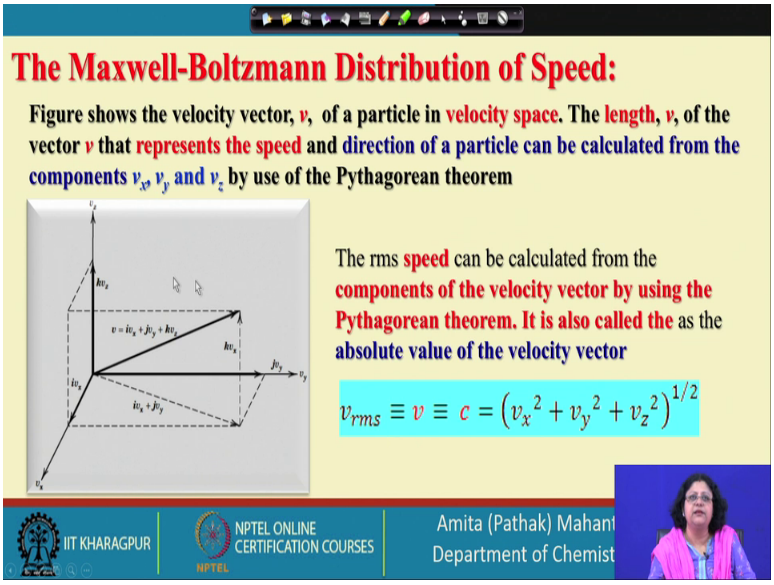

So, let us look at how it is this, how this is going to look? In a vector space if this is the vector space I do not know whether you can see it, here you have the y axis this is the x axis, is a z axis, this is a x axis. So, what we have? initially we had this point position of the particle and this position of the particle could be represented by the x, y, z coordinates into the i, j and k now we differentiate those point spatial coordinates with respect to time to get the velocity.

So, initially we had the r which was represented r as x into I plus y into j plus x z into k. Now, we differentiated that that was in spatial coordinates and this is a velocity coordinate. So, we are generating a new coordinate system, where we are looking into the differentiation of the with positions of the particle with time to get the velocity values and this velocity is as you know is a vector quantity is going to be v is in the other i-th from the a v x component into i z into the v z component plus k into v z component.

So, this is the velocity um a resultant velocity which we are going to get because it is a vector quantity, we have three components we have components of velocity which can be determined as you can see that the figure is showing the velocity vector and in the vector space and the length of the velocity vector is v and that represents the magnitude of that represents the speed and the direction of the particles is calculated from the components of the v x, v y and v z through the Pythagoras theorem.

This is what we had done previously the rms speed, we call this as the rms speed which is represented as v or sometimes as c calculated from the components of the velocity vector by using the Pythagoras theorem and sometimes it is also called as the absolute values value of the velocity. And, so, this v which we are looking into is nothing, but the square of the velocity component in this x direction, velocity component in the y direction and the velocity component in the z direction, squaring it up and taking a under root.

So, we use the Pythagoras theorem to find out the root mean square velocity which we have discussed in the previous classes.

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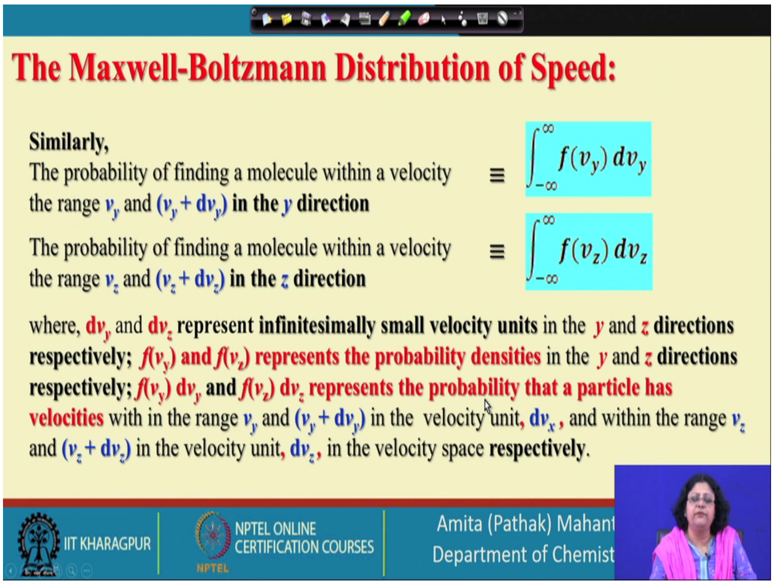

So, what we have we had we talked about in the how what will be the probability distribution or what will be the probability function in the x direction. Similarly, we can find the probability of finding a molecule within the velocity range of v s y and v y plus v z. This is the unit which we are adding. This is the velocity um infinitely small velocity unit in the z y direction.

Similarly, you can write that further and how do you represent that? Since the probabilities are going to be same in all the three directions so, I take it from minus infinity to a positive infinity and this is the function which is going to be similar to what we had in the x direction. Similarly, you have this for the probabilities along the z direction where you have the velocity range limited to v z and v z plus the infinitely small a velocity unit which is v dv z, ok.

So, as you can see what are the each of the components are representing I hope you understand what I am trying to say that the velocities are distributed all throughout and this velocity distribution; since velocity is a vector quantity we are representing that in terms of the each point as a vector in the head of the vector, velocity vector as one point and we are looking into the range in which the probability of finding the molecules are.

What is the range? In the x direction, in the y direction and the z direction is the unit is in the range in which the speed is going to be existing and each of the component plus that speed plus the infinite in infinitely infinitesimally small velocity units, ok. So, this is the range in which we are looking into in the probability distribution.

So, what these are the v function into v why we z represents the probability density in the y and z direction and when you have this product of the velocity function with the unit velocity unit then you get the representation of the probability that a pop probability that a particle has a velocity in the range which we have specified in a y axis and along the z axis.

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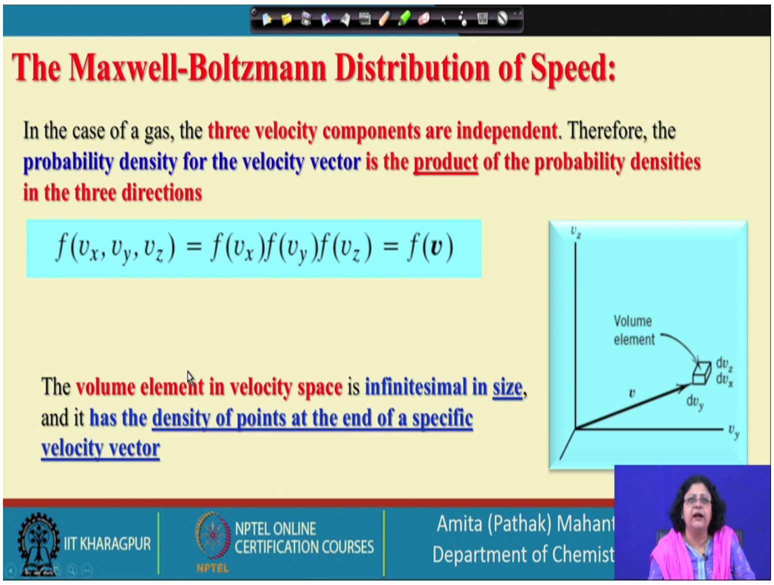

So, if you look at this you can imagine a small unit with a velocity vector represented, this is the head of the velocity which we are looking into this is the small unit which we are looking into which is having x, y, z and dv are the velocity units in this particular z, x and z direction. This is y this is x and this is z direction this is the volume element we are looking into and this is represented by the vector head which we are looking into.

So, in this case the three velocity if I have to look into all the three possibilities and then what we can represent the velocities since these are random variables and you have the laws of product of probabilities product law of probabilities then since all of these are equally probable in all the three directions and they are independent of each other; that means, one if the event is what whatever event we are talking about is independent of the others.

So, we can say that if I have this probability represented by this then it is actually the function product of the probabilities in the y axis along this is y z x axis, y axis and z axis that will be represented by from now as a unit which is representation of the resultant velocity vector v, ok. So, this volume element is the velocities is in the velocity space of in infinitesimal in size and the density of the points in this end is the specific vector head which we are looking into, ok.

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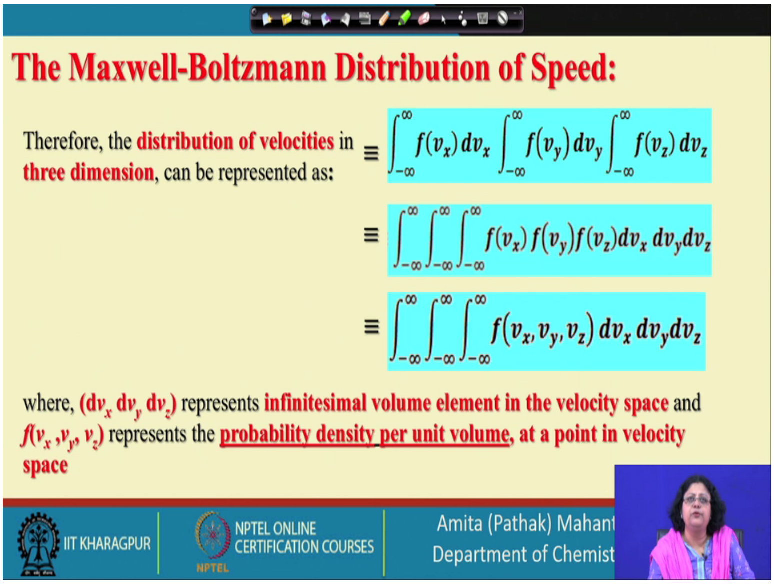

So, if I want to see in what whatever I just now said can be represented as something like this. The distribution in all the three dimensions is actually the product of the probability functions which we have taken. So, we simplify that and since this is a a probability and it is a product of the probabilities. So, I represent this by v x v function of v v a function of v in where v is the rms speed which we are looking and this is the unit velocity element which we are putting or representing the particle.

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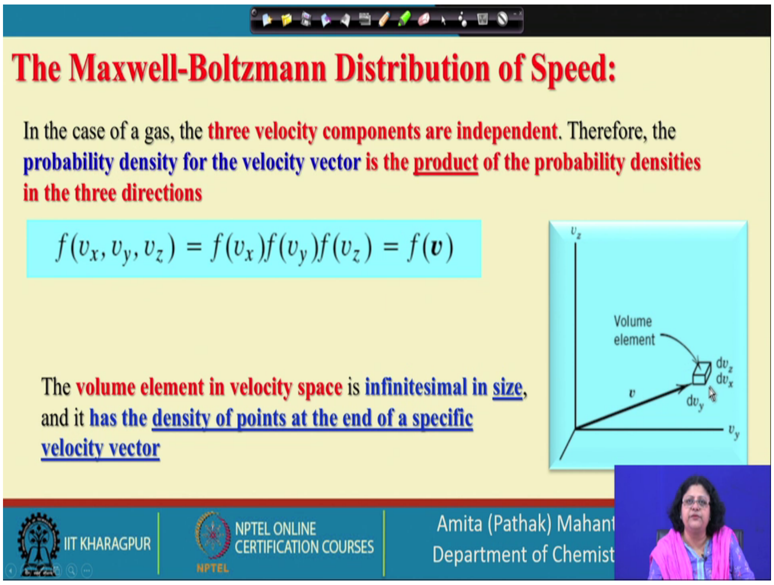

So, where dv x, dv y, dv z represents the volume element which we just now saw this is the volume element which we are looking into. So, and this is how we are going to look into the whole picture and where this is the velocity and this is represented by the velocity head. So, this product of the velocities can be given something like this which you can represent this like this and then you come to this form, right. This is the after the product what you look into is this, the function of v which is the rms speed, ok.

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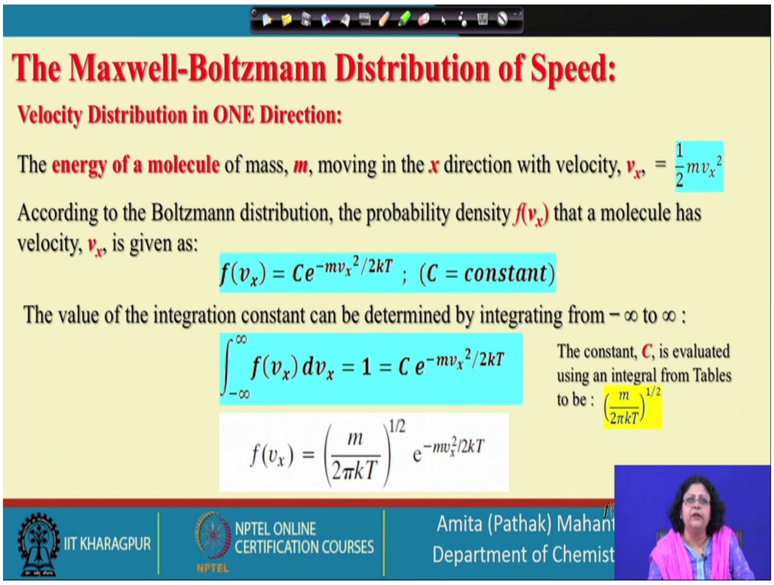

So, now we look into the if I want to look into the Maxwell's distribution of speed suppose the energy of a molecule which is having a mass m moving in the x direction with a a velocity v x we know the we are looking into just we are going to just look into one dimension and then we can look into the all the three direction when we take the component. First let us see in the one the dimension.

So, in one dimension if I look then I can only write one component of the velocity that is the v x. So, v x if according to the Boltzmann distribution which I have just now written down it is going to be if this is the energy term I said it is going to be energy divided by the kT and if this is the energy associated with one molecule then the f of v x will be some constant into e to the power the energy divided by kT and what is the energy is the half mv square and where m x is the direction we are looking. So, we have to looking into the v the v x component of a velocity.

So, this constant which we have this is the equation which is following the Maxwell's-Boltzmann energy distribution, right. This is the probability distribution and then probability density of the molecules in the v z, v x direction which we have represented by the v x of f of v x into some constant into the function which we had talk talked about in just a few minutes ago this is the energy term which we are looking into, ok. I said that it is a exponential term exponential in terms of energy by kT and the energy which we are looking into is the kinetic energy which is half mv square which where x is the component v x component we are looking into, since we are looking into only one direction.

Now, what we have to do we have to find out the distribution how to then get the value of the integration we. First let us find out what the c is we do not have the full function of the probability density. This is the probability density the full function is not possible to understand until we know what the value of c is.

Now, how to find out the value of c? What we know that if I have to integrate this function this is this is the velocity this is the probability density into that if I multiply that of the um the say the wall in the element which is the velocity element along the v z axis it is dv x. So, I multiply that into this is the density function. So, this is the distribution function we get of a once we multiply the probability into the distance velocity unit which we are looking not distance, sorry it is the velocity space we are looking into.

So, this is the velocity element through which we are going to have a the probability density multiplied and which will be extending to minus infinity to plus infinity and this should be normalized to get value of 1. If this is normalized to get the value of all the probability functions that need to be or normalized to and when we say normalized; that means, the product of the probability density into that of the velocity element should be equal to 1, when if we are talking about in the x direction extending minus infinity to plus infinity.

So, on integration of this we can find out the distribution and the distribution will look something like this you just have a look m by 2 pi kT raised to the power half and this is the energy term which we are already having. So, how did we evaluate this? How did we get this distribution function?

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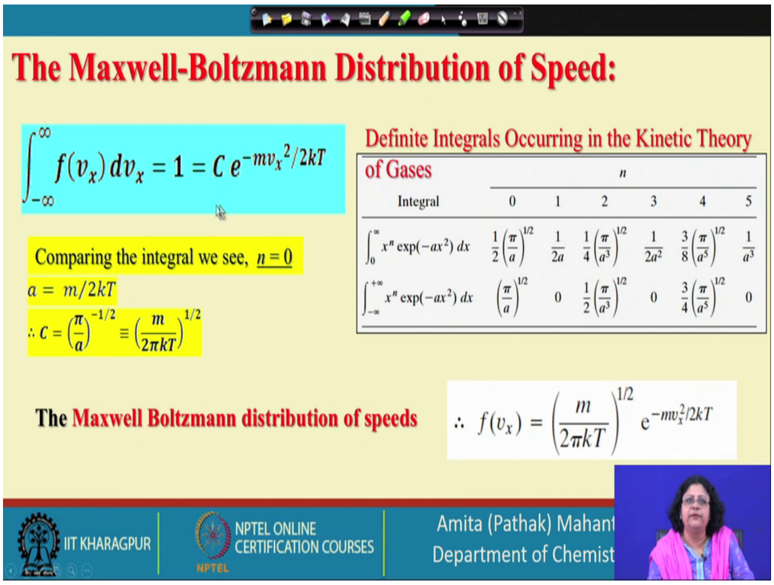

If you go through the books you will see this is the function which we have, right. Now, a this is a in a definite integral occurring which is for a kinetic theory of gases you have handbook values which you can get for the various functions of a various integral values in a definite integral values extending for various values of n.

So, if you look into this you will see that if I compare this expression of mine with that of a integration here then I can see that the x square term is there, the x square some x is the variable here, here v x is the variable this is matching, but I do not have anything in front of it. So, this will be n equal to 0. If I put n equal to 0, this becomes 1, then my equation is resembling that of the definite integral condition.

So, if n is equal to 1, what will be the result this is the result if you put n equal to 1. So, if you put n equal to 1, in the definite integral then this reduces to something similar to what we have here, right. This is the function which we have. So, this is the function which is integrated over minus infinity to plus infinity. So, I have a function which is e to the power minus v x square by 2 kT. Now, what are the parameters which is a variable which is a v x is the variable. So, other parameters like n by 2 kT is the constant.

So, if I look into this expression then here I have a into x. So, the a must be having some value. What is the value of a here? The a value is nothing, but m by 2 kT, right. Now, look at the x expression of the solution of the integral definite integral from minus infinity to plus infinity when e to the power x square is there dx. So, this is a in x terms our variable is obviously, v x. So, if I have this a definite integral similar to what I have here I should be writing dx and integral of this, ok.

This is supposed to be integrated over minus infinity to plus infinity and there should be a dv x term here, ok. So, if I compare this with this one, now you see this is the value I should get for n equal to 0 because I do not have any component in terms of x. So, if I do that I can get a I am C the constant is equal to pi by a to the power minus a half. How? Because, this is equal to 1; so I will take the entire thing into right hand side and find out the value a left hand side and find out the value of C.

So, this if I go take it to other side will become minus half and if I substitute the value of a here, then I get a expression like this. Now, I substitute this value of c in this expression and I get the final value of the Maxwell's Boltzmann distribution, clear?

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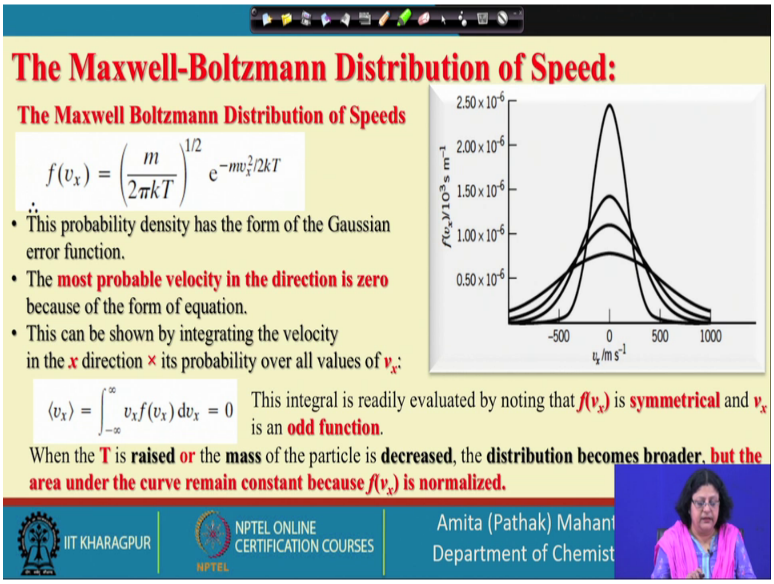

So, what we have here we can see that the Boltzmann distribution and speed is the this probability density it has the form of the Gaussian error it is going to be symmetric occur across the point zero and it is extending to minus infinity to plus infinity along the x axis.

The most probable velocity in the direction is zero, you see most probable is supposed to be the peak it is going to be zero because of the form of the equation and this can be shown by integrating the velocity along x axis into the probability of over all values along the v x. So, this is the velocity which we have this is the velocity component into a probability of all over the overall the values of x this is what we have is this function into the v x; v x is what; the velocity component in the x direction.

If we integrate this over minus infinity to plus infinity and equate it to zero, then what you get is we can evaluate and function find out what is the in the probability density and since it is symmetric which is a as obvious from the graph it is going to be a symmetric form and it is a odd function, and if I if you see in this plot if you raise the temperature or the mass of the particle what happens do you have this expression, you have raised the temperature or the mass what happens the distribute becomes more broad. And the area under the come curve always becomes v becomes v of a function of v v is a vx in the normalized form.

So, all the particles which you have talking about, suppose these are of various temperatures they are these are representing various and that distribution plot for various temperatures, so, if I have a distribution plot for various temperatures in integrated from minus infinity to plus infinity what I get the shape of the curve changes because it is normalized. The total area of the curve under the curve should be constant because that is going to be representing that a total number of molecules. Total number of molecules are not changing only their speeds are changing.

The speeds because the speeds are changing because of the change in temperature you have a different type of a distribution the total area though will remain the same because it is normalized to 1. The probability will be always equal to 1. So, if you are looking into this that all the values which we are having is going to be within this area, but depending on the temperature or when we increase the temperature what happens, increase the temperature or decrease the mass of the a gas particles which we are looking into, then the distribution becomes more broader, the height will be reduced and it will become more broader. When you are decreasing the temperature, what happens; then this becomes more steeper and the area under the curve remains the same.

So, you can understand if you have the same area of the under the integration, the area will be changing if we are having a number of particles which are having a higher speed, if you are increasing the temperature this is the velocity axis. So, if you are increasing the temperature this curve will be having a tail extended and become broader because number of particles having higher velocity will be increasing. But, if you are having a lower temperature then what you have you have a temperature which is a value which is relatively lower in the velocity, but most of them will be in a in x under this curve, all of them will be under this curve, ok.

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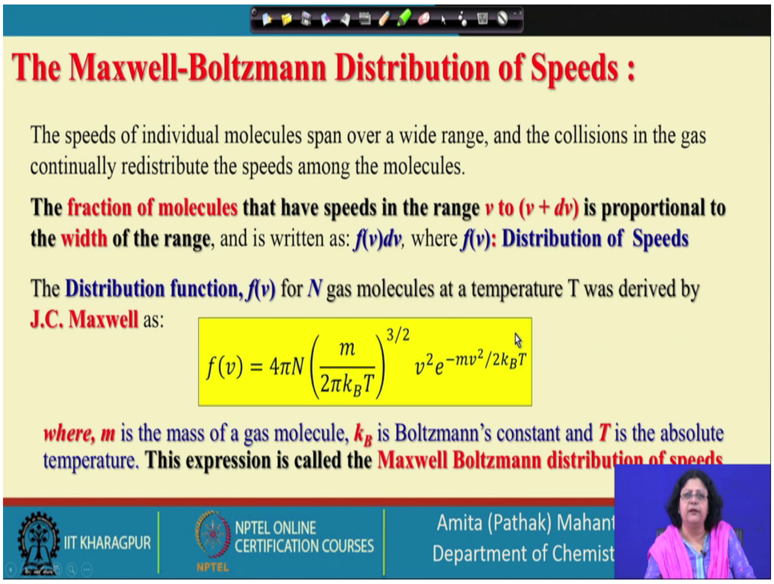

So, I think today we can we cannot go beyond this. So, we will talk about the other components and energy distribution in the next class.