**Advanced Mathematical Methods for Chemistry**

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**Module - 04**
**Lecture - 04**

# General Solution of a System of Linear 1st Order ODEs with Constant Coefficients

So, in the last lecture we saw how you could take system of linear first order homogeneous differential equations and write it in matrix form. You can write a system of linear differential equations that are non homogenous also in matrix form.

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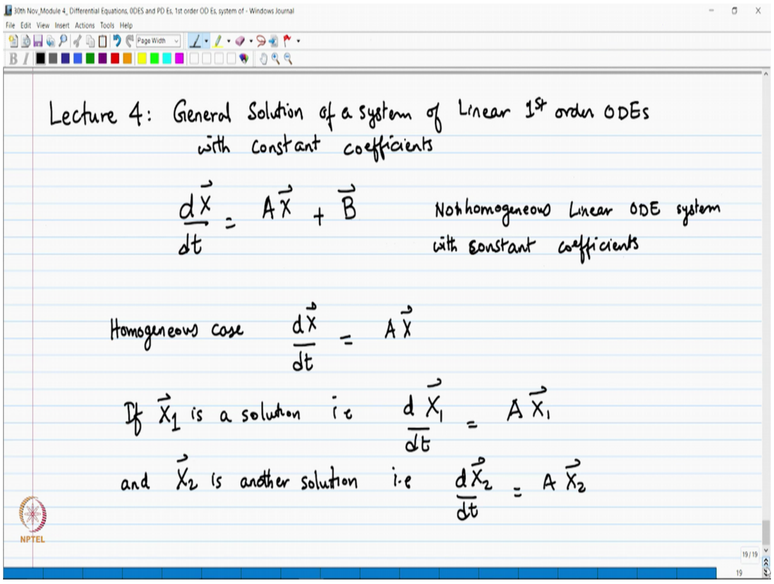


So, in that case what will happen is instead of having d X by d t equal to A x. So, this is homogeneous case, but in the case of non homogeneous equation you will have another term the another vector B. So, this is also a matrix equation and B is a column vector. So, B is some column vector with constant though those are the constants that make this equation non homogeneous.

So, this is non homogeneous linear ODE with linear ODE system it is actually a system of ODEs with constant coefficients. So, that would be the non homogeneous linear ODE system with constant coefficients. Now let us get back to the homogeneous system. So, the homogeneous case, you have d X by dt is equal to A x. Now if X 1 is a solution that is d X 1 by dt is equal to a X 1 and X 2 is another solution d X 2 by dt is equal to A X 2.

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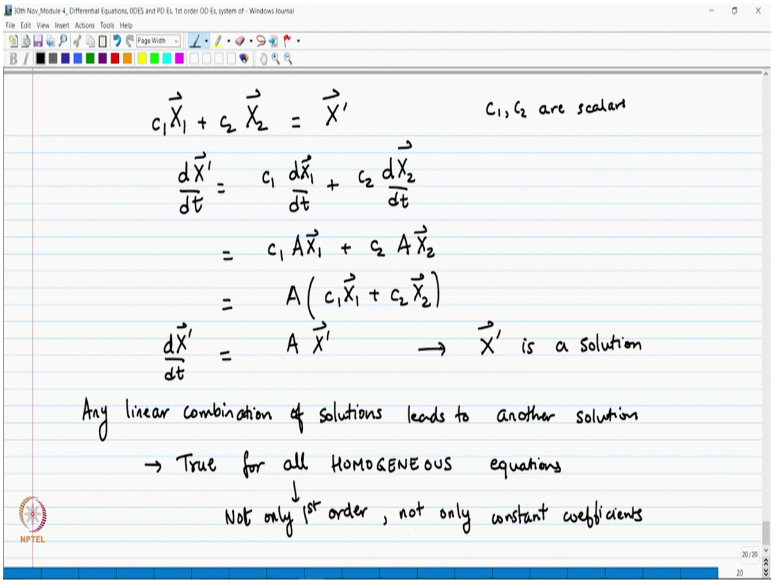


So, suppose you have 2 solutions X 1 and X 2 to this homogeneous differential equation then what you can easily see is suppose I take c 1 X 1 plus c 2 X 2 and now I put this in the differential equation I call this equal to equal to X prime . So, then you can see that d X prime by dt is equal to, I can write it as c 1 d X 1 by dt plus c 2 d X 2 by dt and I can write this as c 1 now d X 1 by dt is A X 1 plus c 2 A X 2, now A is the matrix c is just a scalar c 1 and c 2 are scalars. So, c 1 c 2 are scalars, A is a matrix, so then I can write this as I can take that a in front and I can write it as c 1 X 1 plus c 2 X 2.

So, all I did was I wrote c 1 ax a c 1 c 2 A X A c 2. So, I can do this only because c 1 and c 2 are scalars and, this is just A X prime. So, what you have is d X prime by dt equal to A X prime; that means, X prime is a solution. So, for homogeneous equations, any linear combination of solutions leads to another solution, this is true for all homogeneous equations, all homogeneous equations including those that do not have constant solution. So, all homogeneous equations and by all you mean not only first order not only constant coefficients.

So, if you have any homogeneous equation you can write the general solution as a linear combination of any 2 solution. You can write you can take linear combinations of solutions and get new solutions.

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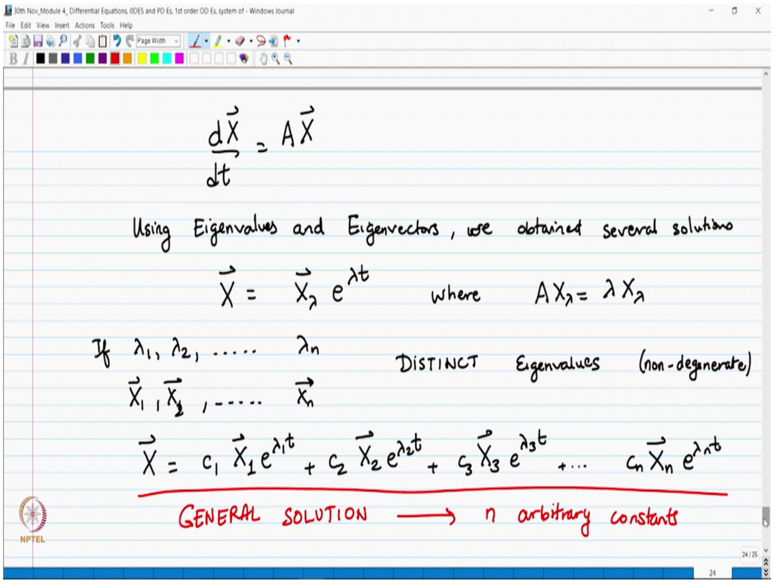


So, what we had was d X by dt equal to a times X and now if we take, if we what we saw was at using eigenvalues and eigenvectors we obtained several solutions. So, X we wrote as X lambda e raise to lambda t where A X lambda equal to lambda times X lambda. So, now the different, if lambda 1, lambda 2 up to lambda n and X 1, X 2 up to X n. So, these are distinct eigenvalue, distinct eigenvalues or you call them in quantum mechanics you will come with the term non degenerate. So, if these eigenvalues are distinct then I can write my X as a linear combination of each of these solutions. So, I can write it as c 1 times X 1 e raise to lambda 1 t plus c 2 times X 2 e raise to lambda 2 t plus c 3 times X 3 e raise 2 lambda three t and so on all the way up to c n times X n e raise to lambda n t.

So, this is the general solution and it has n arbitrary constants. So, remember if you just had if you just had single differential equation first order differential equation.

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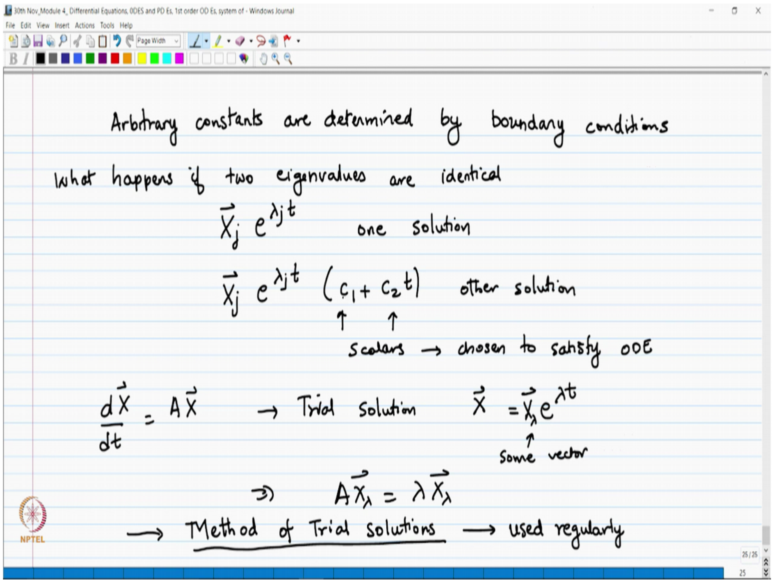


You would have 1 arbitrary constants, if you had a, if you had n first order differential equation the general solution will have n arbitrary constants and these arbitrary constants are determined by are determined by boundary conditions. Now this is the case when all the eigenvalues are distinct. So, you can do this when the eigenvalues are distinct.

Now, what happens if eigenvalues a degenerate? If 2 I will just take the example of 2 eigenvalues are identical; that means, when you when you solve for eigenvalues you find that 2 of the eigenvalues are the same this is possible. Now if you have 2 eigenvalues that are the same then how do you write the general solution? You may or may not be able to find 2 orthogonal eigenvectors.

So, if 2 eigenvalues are the same. So, let us say X j e raise to lambda j t, this is 1 solution. Other solution you can write in this form X j e raise to lambda j t times on c 1 plus c 2 t. So, c 1 and t c 2 are scalars. So, what is done in this case is you take. So, if 2 eigenvalues are identical then to find 2 solutions. So, 1 solution you take as X j e raise to lambda j t now lambda j appears twice. So, the second solution with corresponding to lambda j is written in this form. So, c 1 and c 2 are scalars and these are chosen to satisfy ODE. So, you basically you choose the appropriate values. So, that they satisfy the ODE. So, you try this form of solution and then you put in the equation. So, we will discuss all these in more details when we do some practice problems.

But basically the there is one other important message in this in this whole approach. So, this is see suppose you have d X by dt equal to A X we can imagine a trial solution X is equal to X equal to e raise to lambda t times some vector and you can imagine trying a solution of this form. So, this is some vector and what you can do is suppose you try a solution of this form then we find that ax lambda equal to lambda X lambda. So, what I wanted to say is that you can also think of method of trial solutions this is something that we will be used regularly.

So, in other words what I want to say is suppose you have a differential equation. So, it is d X by dt is some matrix times X you can imagine that you try a solution of this form and then you get the eigenvalue condition. So, that is one of the things that we did here was we tried a solution of this form and then finally, you will get some conditions for c 1 and c 2. So, the method of trial solutions is something that is used very regularly and in all differential equations not just first order differential equations and this is something that we will see on, we will see very often as we solve different kinds of differential equations.

I will end this lecture here. In the next lecture we will recap all the things that we have learnt in this module and then and then we will conclude with some practice problems.

Thank you.