Advanced Mathematical Methods for Chemistry
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Module - 04
Lecture - 01
Differential Equations, ODEs and PDEs, 1st Order ODEs, System of 1st Order ODEs

Now, we will start module 4 of this course and in this module I will be starting the discussion on differential equations which I will first talk in very general terms about differential equations - I will talk about ordinary differential equations and partial differential equations, their solutions and then I will talk about and then in this module we will discuss first order differential equations and their solutions. In the next module we will go to second order differential equations.

So, let us get started with this module. So, in the lecture today I will be talking about the different kinds of differential equations ordinary and partial differential equations and different kinds of ordinary differential equations and the types of solutions that you can have. So, we think of differential equations, when we think of differential equation we think of 2 kinds of differential equations.

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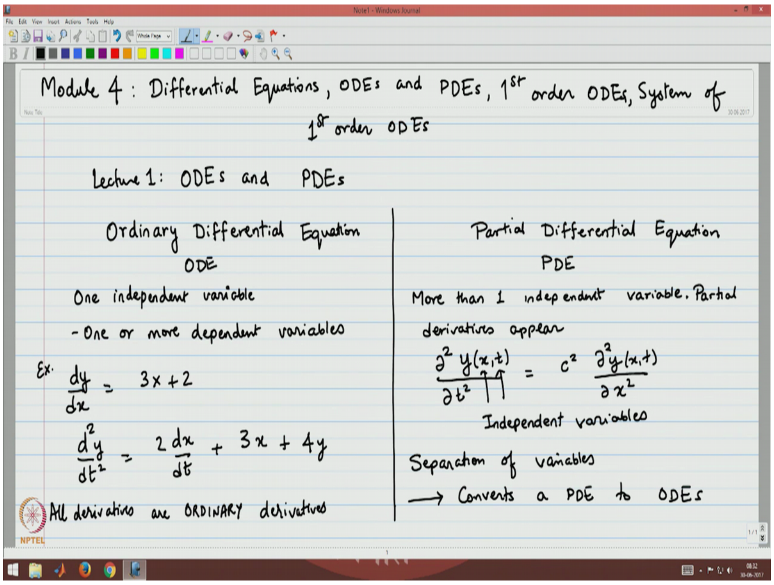

One is called an ordinary differential equation in which there is one independent variable, one independent variable and this can be either, for example, you can have and you can have one or more dependent variables. So, you can have something like d y by d x is equal to some function of x 3 x plus 2. So, these are examples you can have d square y by d t square is equal to 2 d x by d t plus 3 x plus 4 y. So, here the independent variable, in the second equation the independent variable is time whereas, the dependent variables are x and y and so on. So, importantly all derivatives are ordinary derivatives ordinary derivatives.

So, the differential equation typically contains derivatives and all the derivatives that appear in an ordinary differential equations or ordinary derivatives. A partial differential equation has more than one dependent, more than one independent variable and partial derivatives appear. For example, you could have something like d square y of x and t by d t square is equal to c square d square y of x t divided by d t square, you could have an equation like this.

So, so you see that partial derivatives appear and c is a constant and what this should be x. So, left hand side has per second partial derivative with respect to t right hand side has partial derivative with respect to x. Notice that there are 2 independent variables x and t. So, x and t are the independent variables, variables and so the dependent variable is a function of all the independent variables. So, when you take derivatives very naturally partial derivatives appear. What you can do is if you take a partial differential equation and you use a technique of separation of variables. So, this converts a PDE to ODE’s ordinary differential equations and usually if you have 2 independent variables then you will get 2 ordinary differential equations out of one second order, one partial differential equation.

So, what I want to emphasize is that the techniques to solve partial differential equations, once you have done separation of variables then the techniques to solve them are very similar to those used to solve ordinary differential equations. So, we will discuss partial differential equations in a later part of the course, but we can keep in mind that that many of the techniques that we use to solve ordinary differential equations they turn out to be very useful when you are working with, when you are also working with partial differential equations. So, now, let us look into ordinary differential equations in a little more detail.

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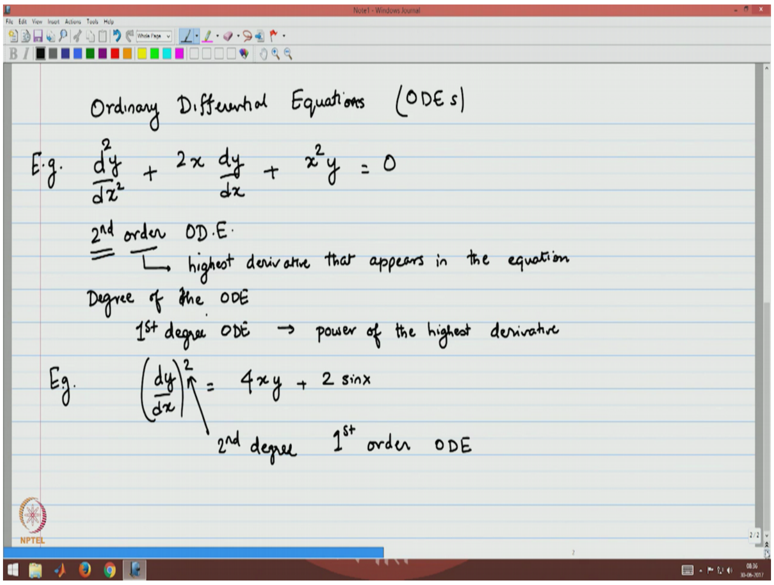

So, let us start ordinary differential equations ODE's. So, suppose I write an equation let me write let me write this equation d square y by d x square plus 2 x d y by d x plus x square y equal to 0. Now where this is an example, we would say since the, we would say this is a second order differential equation second order ODE. So, what is, so second order. So, the order of a differential equation is the highest a derivative that appears in the equation.

So, since the second derivative is the highest derivative that appears in this equation the order of this differential equation is 2. So, which say it is a second order differential equation. Now there is an, there is another term called the degree of the ODE, this is a first the above equation is a first degree ODE, first degree ODE. So, this is the power of the highest derivative. So, it is the power of the highest derivative. Now the highest derivative is the second derivative and you can see that you have second derivative at the first power. So, on the other hand if you have something like let us say d y by d x whole square is equal to 4 x y plus 2 sin x. This is another example this is degree equal to 2.

So, here you see that the highest derivative is 1. So, this is second degree first order, so the order is 1 because the highest derivative power is 1 ODE. So, these are, so one of the first things you will see in the differential equation is to see the order of the differential equation and the degree of the differential equation and in this particular module we will be mainly working with first order ODE's. Now next let us look at some other characteristics of differential equation.

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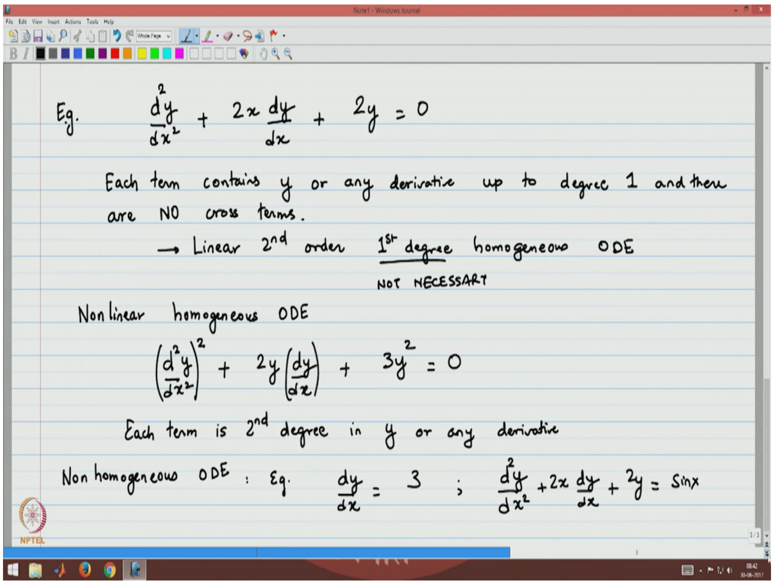

So, suppose I write a differential equation I will just write this in this form d square y by d x square this is plus 2 x d y by d x plus 2 y equal to 0.

Now, if you see each term contains y or any derivative up to order 1 and there are no cross terms. So, each term contains y, it either it contains either y raise to 1 or it contains d y by d x raise to 1 or d square y by d x square raise to 1 and there are no cross term there is no there is nothing like d y by d x square or d y d y by d x into d square y by d x square or y d y by d x nothing like that. So, there are no cross terms.

So, such an equation is called a linear second order first degree homogeneous. Now, the first degree is actually redundant because we already said that it is linear. So, we do not need this is not necessary to say. So, each term is the, each term is first degree. So, this is a linear second order it is homogeneous because every term every term has at least one y. So, you can also have a non-linear homogeneous ODE. So, you could have an, you could have a differential equation that is homogeneous, but non-linear. So, for example, you could have something like d square y by d x square whole square plus 2 y d y by d x plus 3 y square equal to 0. So, this each term, each term is second order. So, each term is second order in y each term is second degree and why yeah, I should probably change this to degree and above.

So, each term is second degree in y or any derivative. So, this is y double prime that is d y d square y by d x square raise to power 2 this has 1 term with just y and 1 term with d y by d x. So, the total power of y or any of it is derivative is 2 this is y square. So, it is homogeneous because each term is of the same degree in y or any of it is derivatives. It is non-linear because it is not first order it is not it is not a degree 1 each term is not degree one. So, you could have a non-linear homogeneous ODE you could have a non homogeneous ODE.

Now, I will take a very very simple example of a non homogeneous ODE. So, suppose you say d y by d x equal to 3. Now, the left hand side highest y up to first power the right hand side does not have y up to any power, a more slightly more complicated example is you just take this homogeneous equation that you had plus 2 y and instead of 0 you put something you put let say you put sin x you put something that is that has no power and y and this would be an example of a non homogeneous ODE. So, you see that the left hand side is first degree in y or it is derivatives the right hand side here is 0th degree, here similarly the each term on the left hand side is first degree in y, but the right hand side is 0 th degree in y. And you can construct I mean there are many many different ways to construct non homogeneous ODE's.

So, these are some of the terms that come into that you will see often used to describe differential equations and it turns out that the way you solve a differential equation it changes depending on what kind of differential equation you have. So, now, let us look at the solution of the, let us look at certain kinds of solutions of the differential equation.

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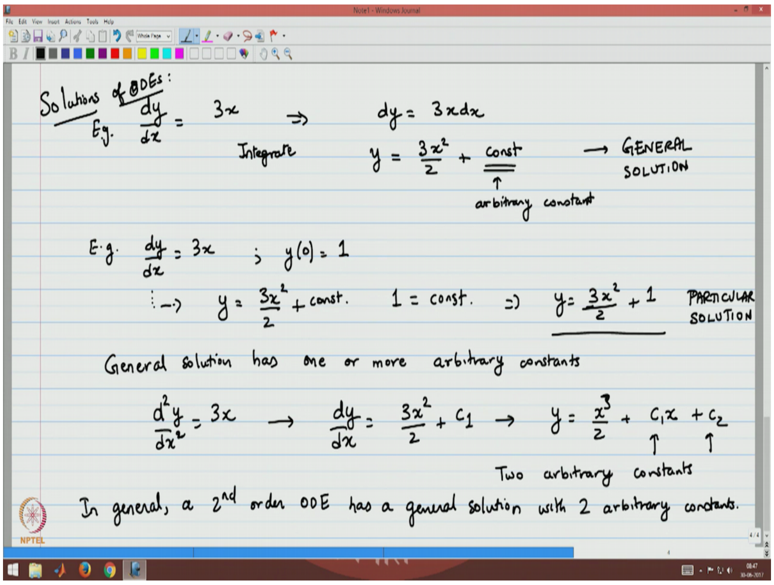

So, suppose I have a differential equation let say I take d y by d x equal to 3 x. So, this is solution, solutions of ODE's. So, example suppose I have d y by d x equal to 3 x then I can immediately write, you can write d y equal to 3 x d x and I can integrate both sides to get y is equal to 3 x square by 2 plus constant plus an arbitrary constant. So, this is arbitrary constant. And you can choose any value of this constant and you will get a solution. So, such a solution is called a general solution. So, this is the general solution of this differential equation and the arbitrary constant is you can choose any value of the constant and you will still get a solution.

Now, the other kind of solution, now suppose I have, suppose I have d y by d x equal to 3 x and you have told that y of 0 equal to 1 suppose you are told these two. So, then what you will do is you will follow exactly the procedure in the previous part and you will get y equal to 3 x square by, 3 x square by 2 plus constant then what you will say is y of 0, when x equal to 0 y equal to 0, so y equal to 1. So, you get that constant 1 equal to constant. So, this implies y equal to 3 x square by 2 plus 1 this is called a particular solution.

So, the solution of an ODE you can either have a general solution or a particular solution. So, what you should keep in mind is that the general solution has one or more arbitrary constants, whereas a particular solution has no arbitrary constants. Now, when will it have here we saw that it has 1 arbitrary constant when can you have more than one arbitrary constant quite easy to see suppose I had d square y by d x square equal to 3 x. Suppose I had something like this then what I would do is I would integrate once and I would get d y by d x is equal to 3 x square by 2 plus constant I will call it c 1, then I integrate it again and I get y is equal to x cube by 2 plus c 1 x plus c 2. So, now, I have 2 arbitrary constants. So, in general a second order ODE has a general solution with 2 arbitrary constants.

So, if you have a third order ODE then you would have a general solution with 3 arbitrary constants and you need 2 conditions to determine both these arbitrary constants. So, in a first order ODE you could determine this arbitrary constant with just one condition y of 0 equal to 1, in this case you need more than 1 condition. So, that is about general and particular solutions.

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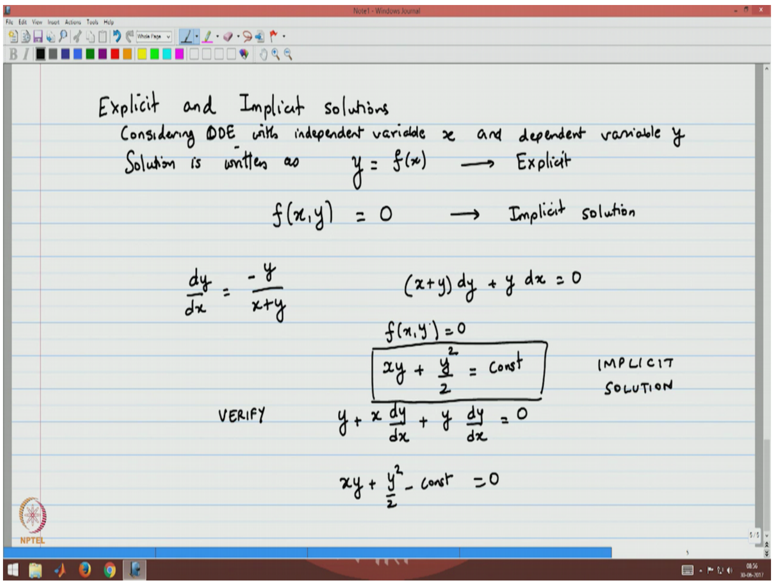

Now, the next thing that I want to say is about the explicit and implicit solutions. Suppose the solution is written as y is equal to something some function of x. So, this is called an explicit solution.

So, here we are considering ODE with independent variable x and dependent variable y. So, then this is an explicit solution. So, when you can write y on the left hand side and you can write equal to some function then it is an explicit solution. In other cases sometimes you cannot write it in this form sometimes you have a solution of the differential equation might have a form where basically y and x cannot be separated out you will might get f of x y equal to 0, might get some function of x y equal to 0 implicit solution. We will see soon some examples of explicit and implicit solutions. So, and you will encounter these quite often during this course.

In particular suppose you had d y by d x let us say equal to x by x plus y something like this then you know d y by d x equal to y by x plus y then if you try to solve this. So, finally, what you will get is something like x plus y d y plus y d x equal to 0 and it turns out that this can be solved, but when you solve it you will not be able to explicitly separate x and y, but there is a way to solve this and when you solve this you will get a solution that has the form that has the form f of x y equal to 0. We will see the solution of this when we are dealing with the methods to solve ODE's.

So, in this particular case you we will show you how to work this out, but the solution can be written as x y plus y square by 2 equal to constant. I have not shown you, but you can you can verify that this is a solution by suppose I differentiate with respect to x what I will get is, so verify you we are going to verify that this expression is actually a solution of this equation. So, if I differentiate with respect to x what I will get is y plus x d y by d x plus y d y by d x is equal to 0 and you can see that this will give us this expression.

Now, if you see the solution this solution is what is called an implicit solution, this is an implicit solution. So, in other words we wrote some function of x y equal to constant, or you can write or you can write as x y plus y square by 2 minus constant equal to 0. So, what we wrote as some function of x y equal to 0. So, you can have these implicit and explicit solutions to differential equations and we will see I mean you will encounter both of these during various courses. So, f of x y equal to 0.

So, I will conclude this lecture here for now and in the next lecture we will look more closely at methods to solve first order ODE's.

Thank you.