Laser: Fundamentals and Applications
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Lecture – 06
Population inversion, 2-level system and 3-level system

Hello and welcome. Today is the first day of this week, and I hope so far you found these initial lessons a useful. So, what we were looking at the last class was the conditions to have laser actions. And what we did: we use the principle of little balance to equate the rate of absorption and rate of emission. And thereby we boil down to a condition where you know the Einstein's formalism matches with the Planck's one, and from there we got two useful relations the ratio of stimulated to spontaneous emission which says that is equals to 1; sorry stimulated emission to absorption. So, we found that the rate of stimulated emission is equal to the rate of absorption.

And we also saw why it is difficult to you know achieve laser at higher frequencies. And we ultimately came to the conditions at which I can or I cannot get a laser action.

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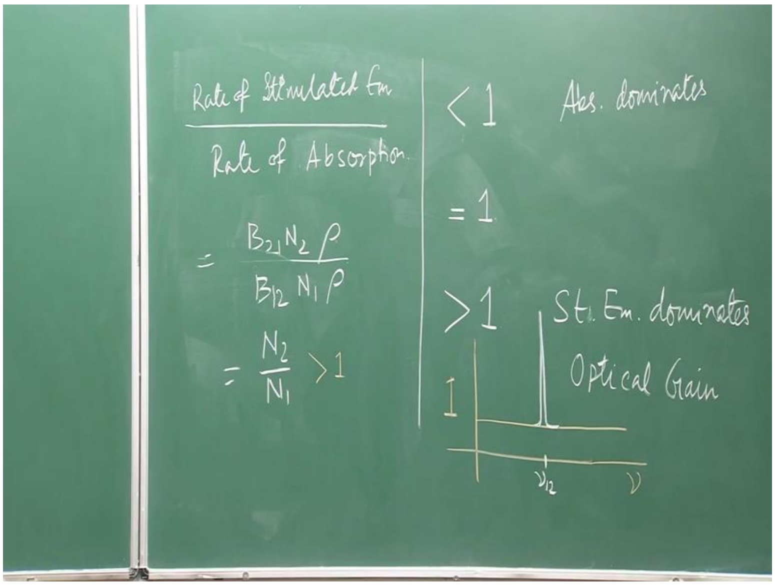

So, what we found out that the populations of the 2 states, we are still considering a 2 level model. So, for the condition where N 2 by N 1 that is the ratio of the population of atoms at state 2 and state 1 is greater than one, only then I can have a condition where stimulated emission dominates and I can achieve laser action.

So, suppose for time being let us assume that I can achieve this condition that is N 2 by N 1 greater than one. So, at this condition what do I see? What I will see can be shown graphically if I plot the intensity of light as a function of frequency. So, when I have a condition which is equal to this I have no net gain. So, whatever be the scale of intensity that is flat.

Now, our concern is at a particular frequency correct. So, let me is put this as nu one 2 as we have been you know using. So, here I do not have any kind of amplification. Now the condition when I have N 2 greater than N 1 then what I will have I am putting some number of photons and I am getting number of photons which is much higher. So, when I you know keep say x input and get n number of times of x as output, then I say I have gain here also I am giving photon and I am not getting anything out of it when I have this one.

So, my you know income and expenditure both are same no gain. So, if I come to this particular condition then what I will say, I have much more photon then I have already used. So, therefore, at that particular frequency where I am using I will see a net gain. So, this is not any other type of gain, but other you know it is called optical gain at this particular frequency, when we can achieve this condition N 2 by N 1 right.

Now, mathematically we got it, but how we can get this condition N 2 by N 1, if I can get that I can make a laser much not much. So, now, what is that is preventing me to have this condition right. So, if I can do it mathematically why cannot I actually do it? Simply, if I look at the equilibrium population distribution and if we again look at the Boltzmann statistics then what do I have this is equal to right.

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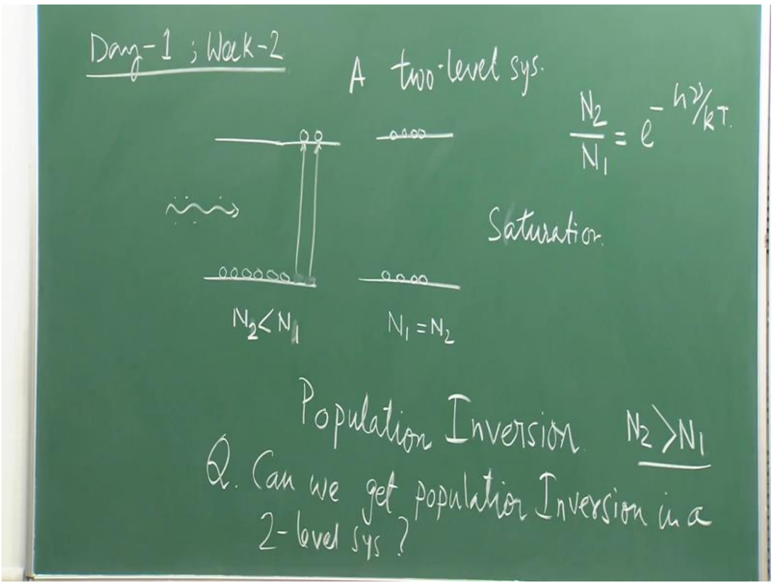

So, normally what I can see that N 2 is very very very less than N 1, and I want N 2 be much higher than N 1 correct N 2 should be greater than N 1 in order to have laser action.

So, suppose I have again 2 state models, and I have several atoms here suppose I have taken total 8 atoms. Now in the step one I come with some photons. So, there are you know many photons I am sending in and 2 of these are going from here to here fine. So, they make a transition here this one makes a transition here. So, I essentially have this one and this to our non existing here. So, after sending these photons I get this condition.

Now, if I can achieve a condition where half of the atoms are in this state and half of the atoms are in the excited state then that will be something like this. So, I have 1 2 3 4, 1 2 3 4 this is a saturated condition. So, I have so called. So, this was initially all the molecules are grounds taken very few in the excited state that was my equilibrium condition that is governed by Boltzmann distribution. Then by putting light more and more light I say I can somehow achieve a condition which is like this where I have equal population in both the states and I get a condition which is known as saturation.

Now, I have to go here I have in this case I have N 1 less than N 2, here I have N 1 equals to N 2. Now I need to go to N 1 is less than N 2 sorry I am here I am I made a mistake this should be N 2 less than N 1 right. So, now, I want N 2 to be greater than n one. So, I want to actually invert the population. So, this is my equilibrium population distribution and what I want? I want to invert these populations so that now my state 2 contains more number of atoms then state one and this is known as population inversions right. So, this condition N 2 greater than N 1 is essentially a population inversion condition, ok.

So how can I get in a population inversion? Now here we will describe you know in detail that whether or not we can find population inversion in this system, which is essentially a 2 level system. So, can we? So, the question is can we get population inverse population inversion in a 2 level system. So, that is a question and we will try to answer that. So, we are going to answer that question so; obviously, we will take the same 2 level model, their rates and all the details.

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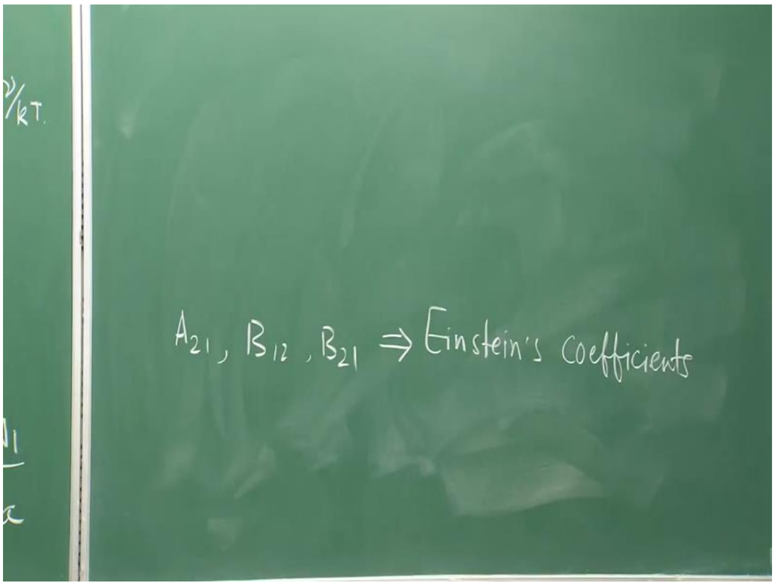

By the way I think I have missed out one thing here that we are talking about this A21, B 12, B 21 these rate constants they are known as Einstein’s coefficient. I do not remember if I have mentioned this one, but this A and B is they are known as Einstein coefficient you will many time come across in the books also is written as Einstein’s A coefficient Einstein’s B coefficient and so on. So, you should definitely know this one. So, if I have not mentioned it. So, you know now all right. So, let us go back to our discussion.

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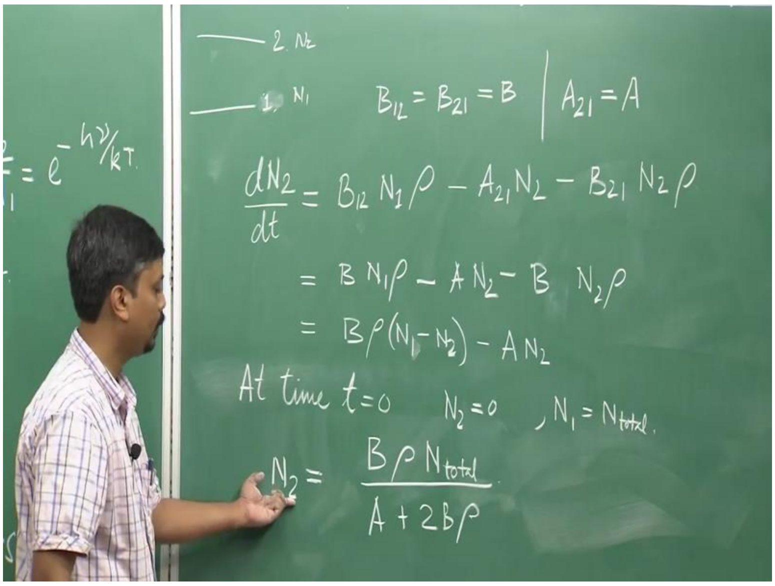

So, we are still considering this 2 level system, one let me call simply as one and 2 without bothering about the bracket notation and N 1, N 2. Now in order to answer this question we will formulate in this way because we are going to deal with this population of state 2, N 2 we have to see how we can make N 2 greater than N 1 that is our aim is.

So, let us see what is the change in population of state 2. So, from the basic knowledge of kinetics we can easily for you know get that. So, how do I write that I write d N 2 function of dt, by dt. Now this N 2 is also time dependent. So, when I am shining light and then as a function of time it evolves you know N 2 changes now how it is changing. So, how this one is populated it is populated by absorption process correct and how it is depopulated by the emission process.

So, the absorption processes will be you know will come with positive sign and the emission processes emission rates will come with negative sign. So, if I write this one what do I have first absorption. So, B 12, N 1 into rho I am not writing that rho within bracket h nu 1 2 anymore for simplicity minus spontaneous emission A 2 1, N 2 minus B 21 N 2 rho. So, from previous discussion we already know that this rate that is absorption rate is equal to the rate of stimulated emission.

So, I have this condition B 12 equals to B 21, if you remember the previous class you have already seen this correct. So, if I take this B 12 equals to B 21 equal to be something as B, and suppose I write A 2 1 to be just A. Then what I get? I get B N 1 rho minus A N 2 minus B 21 N 2 rho sorry I am supposed to remove this one. So, it is B. So, what I have is B rho N 2 minus sorry N 1 minus N 2 minus A N 2 correct.

Now, this is a differential equation and we need to solve it. So, what are the conditions that we have in our hand? So that I can solve this one, at time t 0 at time t equals to 0, my N 2 is equals to 0 we all agreed right when there is no photon we start giving the photon at time t when there is no photon there is no population of the state 2. So, N 2 is 0 and if there are total n number of molecules, and if I call it like n total then essentially at that time my N 1 is equals to n total [FL]. So, what I have this. So, I can get this expression.

Now, what do I have for N 2 is A is the rate constant for spontaneous emission right. So, A is for spontaneous emission. Spontaneous emission is always there it does not require any photon or anything in vacuum if you keep it, it will emit because there is a lifetime of the state it will come back to the ground state by emitting photons and that is what is spontaneous emission. So, no matter what spontaneous rate spontaneous emission is always there. So, the rate constant is nonzero. So, therefore, I can say A is greater than 0 always this constant you know this condition is there. So, I have here this B rho N total by this one. So, if I let me write it over here.

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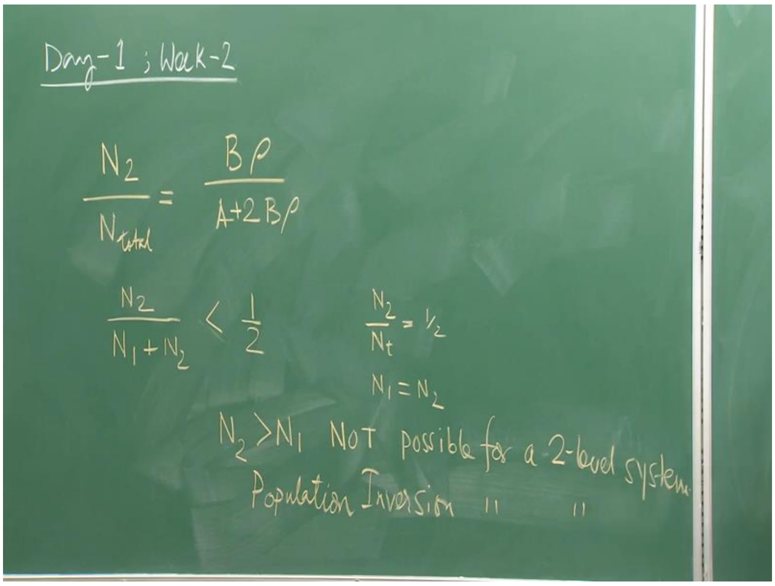

So, from that expression if I get N 2 by N total that is equal to B rho by A plus right.

Now, when A is very very less, it comes to something like, so on the other hand say if I write N 1 plus into. So, it will become approximately; well A is very small right. So, it will be around half. So, or less than half; the maximum value is half now does it tell you something we drew the you know this 2 levels and we started exciting it and we got to a condition, when we had a saturated condition right. So, we had a condition like here if I show. So, 4 atoms here, 4 atoms here and we call that is a saturated. So, here if you have equal population of this store state, essentially in 2 by total number of atoms is half that is maximum possible ratio of N 2 by N total I cannot get more to more than that. So, essentially when N 2 is equal to N 2 by when N 2 by n total is equal to half; that means, N 1 equals 2 N 2 which is essentially this condition.

So, because A is always positive; essentially what I have this one is always less than half. So, forget about going to N 2 by N 1 greater than 1 I have N 2 much less than 1 N 1 all the time for. So, N 2 greater than N 1 this is not possible for a 2 level system. N 2 by N 1 greater than 1 this condition is not fulfilled in a 2 level system or in other word population inversion is not possible for a 2 level system this is an important relation.

So, I know that if I have to get a laser I must fulfill this condition into greater than N 1 or I must have a population inversion, and after seeing everything I figure out that well I cannot achieve a population inversion in A 2 level system. So, I have answered the question that I have asked few minutes back. Now then question is if not 2 level, whether a three level system can give rise to population inversion let us look at it. So, let me let me ask the question.

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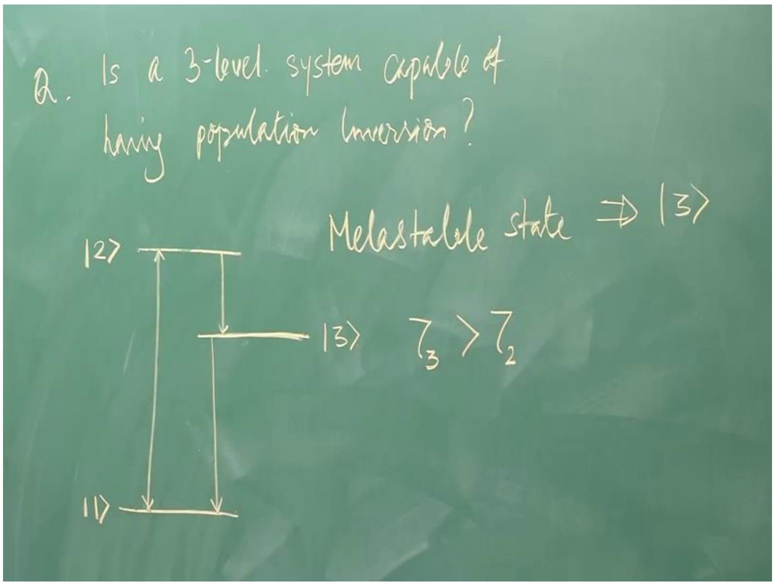

So, question is two level system cannot now is a three level system capable of having population inversion. So, if I draw three level system just for clarity I am drawing in this fashion, this has nothing to do the actual energy level picture. So, I have state 1 I have state 2 and I have state 3.

Now, what are the possibilities that we have here? We can have absorption as well as emission from this state, I can have spontaneous emission I can have stimulated emission and if I just consider these 2 system it is a fairly 2 level system. So, I know everything about it what I should know I am already learnt it. Now between these 2 level what I can have I am going sequentially. So, I you know hit the system with light I shine light the molecule goes here. So, at that condition either it can come down here or it can come down state three all right. So, I can have. So, the single headed arrow means there is a spontaneous emission and from here I can have spontaneous emission I can have stimulated emission.

Now, whether the stimulated emission will dominate over spontaneous emission, we will tell me whether I can have a laser using three level system. So, let me answer you first that whether this three level system can act as a laser, can it give population inversion; answer is yes it can under certain conditions. So, what are those conditions? That is this level 3 which is lying below level 2 must have a lifetime.

So, if I write tau 3 as the lifetime of state three then tau three should be much greater than tau 2. So, in other word the lifetime of the state 2 is much shorter. So, it is short lived state whereas, this is much long live and this kind of state is known as metastable state. So, my state 3 is a metastable state. And also this state 3 should not be a dark state. So, it should not be a dark state it. So, it should be able to emit radiatively. So, state 3 should not be a dark state.

So, at this condition I can actually achieve population inversion, how? That we will discuss in the following class, till then goodbye.

Thank you very much.