Laser: Fundamentals and Applications
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Lecture – 05
Calculation of Einsteins coefficient

Hello and welcome to the day 5 of first week. In the last class we started looking at to learn how laser works or in other word the Principle of laser action.

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In learning Principle of laser action we started looking at our interaction between light and matter taking simple example of an atom and taking only 2 levels 2 energy levels. So, constructing 2 level system and we learnt about different Optical transitions. What are the optical transitions that we learnt, we learnt about Absorption, we learnt about Spontaneous emission and also we learnt about Stimulated emission. We also learnt why there was a need to bring in this concept of stimulated emission which was done by none other than Einstein.

We also learnt about the rate laws governing these optical transitions this absorption and both the emission processes. After looking at their rates we brought in the principle of a detailed balance and thereby we equated the total rate of absorption to the total rate of emission which includes spontaneous and stimulated emission. What we had.

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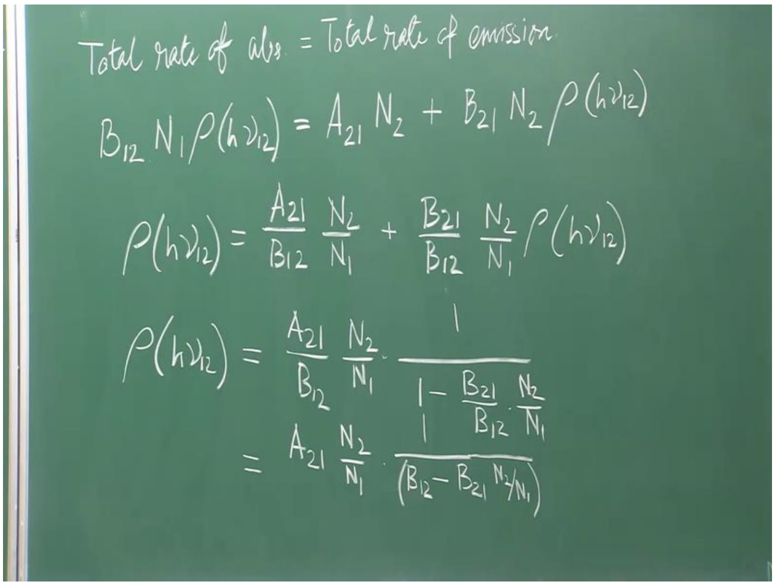

We had total rate of absorption equal to Total rate of a emission correct, we started looking into the mathematical relation.

We know what is the Total rate of absorption and that was given by B 1 2. So, just to remember this is state 1 and this is state 2, this having energy E 1 this having energy E 2 and the number of atoms are N 1 and N 2 in this 2 levels, we are considering the only 2 level systems. The absorption was given by B 1 2 multiplied by N 1 multiplied by rho h nu 1 2 that was our formulation and we equated it to the rate of spontaneous emission given by A 2 1 multiplied by N 2 plus the rate of stimulated emission that is given by B 2 1 N 2 into the photon density at frequency nu 1 2.

We are going to use this one to see how actually Einstein brought his model of blackbody radiation where he assumed the black body to be constituted of several such 2 level oscillators as we have shown here and see if we can come to Planck’s law which it should be. Now, if we rewrite this one as A 2 1 by B 1 2 N 2 by N 1 plus B 2 1 by B 1 2 N 2 by N 1 multiplied by rho.

Now what we need to do, we need to take this part on the left side and because both of them have this term rho and take this rho outside, that we can get a final expression for rho. If I do that will be equal to A 2 1 by B 1 2 multiplied by N 2 by N 1. Now I have taken this part on this side, this minus this part taking rho out of the bracket. I have one minus this part correct, now that part is now going to the denominator, then I have 1 by 1 minus B 2 1 by B 1 2 multiplied by N 2 by N 1.

If I just rewrite this one it will be A 2 1 and for the time being let us take this B 1 2 here, it will be N 2 by N 1 multiplied by B 1 2 minus B 1 2 into this quantity, it becomes B 2 1 into again N 2 by N 1 right this is equals to my rho. Now if I know a Boltzmann distribution of you know partitioning the number of atoms in different levels different energy levels then for our 2 level systems.

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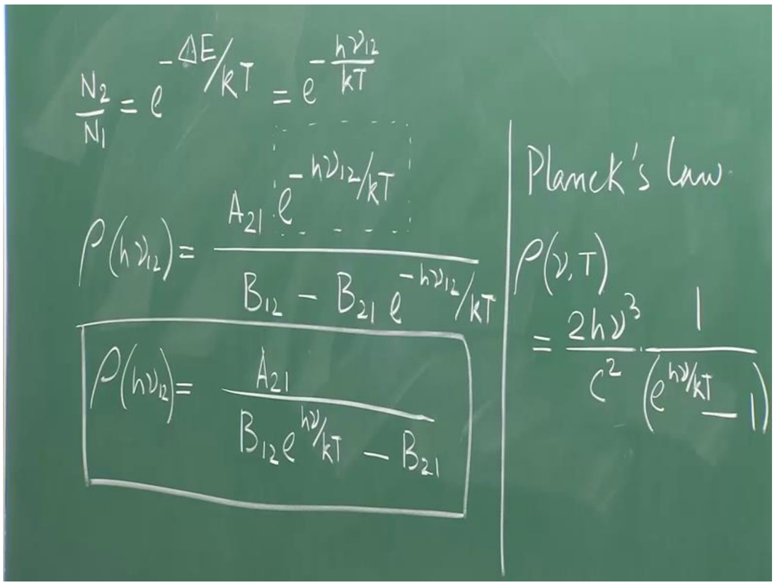

That we just discussed N 2 by N 1 is equal to e to the power minus delta E by kT, where k is the Boltzmann constant and in our case this delta E is E2 minus E1 which is equal to h nu 1 2.

I can write this one as e to the power minus h nu 1 2 divided by kT, if I take this value of N 2 by N 1 and replace it here I will get an expression in terms of that exponential term right. If I just rewrite here, again rho h nu 1 2 equal to if I look at there, I have A 2 1 multiplied by N 2 by N 1.

I will use this value, by kT then in the denominator what do I have I have B 1 2 minus B 2 1 and again multiplied by N 2 by N 1 which gives minus h nu 1 2. I will I can omit using this subscript 1 2 henceforth and I can just write a h nu.

We will know what nu it is actually right, at this point if I divide both numerator and denominator by this quantity e to the power minus h nu by kT then what do I get, I get A 2 1 divided by B 1 2 divided by e to the power minus h nu kT. It becomes e to the power plus h nu kT and I am not writing this 1 2 anymore this minus only B 2 1, this is my low value.

Einstein wanted to you know come to Planck's law , now he got this one, I will write it again h nu 1 2, now at this point you recall what Planck's law was, that we mentioned in the last classes the spectral density at a particular frequency that is our nu 1 2. That was given as it was given for a black body which is in thermal equilibrium at temperature T was given by 2 h nu cube by c square, where nu as usual you know the frequency of the particular light of the spectrum that we are considering c is the velocity of light h is the Planck constant multiplied by 1 upon e to the power h nu by kT minus 1, this was Planck's law.

If I want to compare what we what Einstein derived and what Planck had derived then we can see that they are very similar and actually this particular equation boil down to this Planck's law under certain conditions. What are the conditions let us look at that, let me wipe this part all right. As I was saying this will boil down to this expression when if you look at if B 1 2 becomes equal to B 2 1 then and if A 2 1 by B 1 2 or B 2 1 is a constant.

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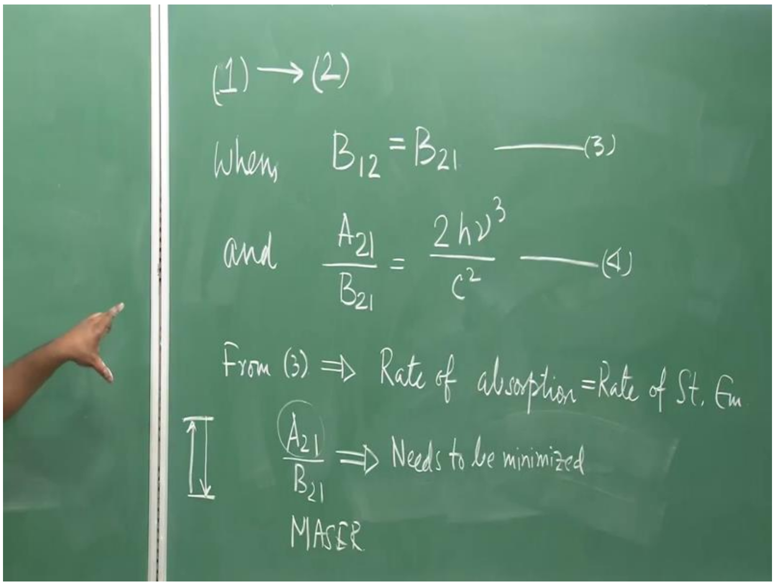

Let me mark this one, this expression I am putting as 1 and this Planck's one I am putting it as 2, then 1 goes over 2, the form 2 has when B 1 2 is equal to B 2 1 and A 2 1 divided by say B 2 1, these two are equal.

I can take any one of them, here is equal to 2 h nu cube by c square if you use these 2 you will see there is this equation 1 and 2 are identical. There are very interesting thing to notice here suppose I mark this one as 3 and this one as 4 then from 3 what do I see B 1 2 and B 2 1 are equal. From equation 3 it tells me what is B 1 2 B 1 2 is the Rate constant for absorption and B 2 1 is a Rate constant for Stimulated Emission.

In other word I can say the Rate of absorption is equal to the Rate of Stimulated Emission. This is an wonderful outcome what it tells is that at a given condition, if I have certain population of state 1 and state 2 and if I cover with a light that has a frequency such that h nu matches with the delta e. Then the amount the or the number of molecule that will go from a state 1 to state 2 will be equal to the number of molecule coming down from this one to this one their rates will be same.

This is what you see here and actually if you think about it this can be a problem in having laser action why we will see it in a while just remember laser deals with the stimulated emission and it actually you know based on the amplification of the stimulated emission right. The radiation coming out due to the stimulated emission if you can amplify you can get a laser.

If some other process has the same probability of happening as that of the stimulated emission then that suddenly poses some problem to you. If we look at this equation 4 what does it tell me A 2 1 is the rate of spontaneous emission B 2 1 is the rate of stimulated emission. I want to have my system to give more stimulated emission and minimize the spontaneous emission because I want to you know utilize the stimulated radiation correct. What I need to have if I need to really get a laser that this quantity A 2 1 by B 2 1 this ratio, this needs to be minimized. So, how can I do that I can either make this stimulated emission sorry spontaneous emission rate constant very low and making B 2 1 that is a stimulated emission rate constant very high.

How can do that we will see that very soon now just look at this right hand side of this expression what do I have here, I have constants except the quantity nu. So, nu is more meaning this ratio is more, I do not want this ratio to be high I want this ratio to be low.

If I go and increase this frequency, if I go from say microwave to infrared to visible to ultraviolet to x rays. The probability of spontaneous rate an emission will be much more compared to stimulated emission, thereby my job will be very very difficult to achieve laser action at that high frequency compared to a lower frequency.

And if you remember my first class where I mentioned this that you know getting to lace at a higher frequency is a very challenging job and that is one of the reason that people struggle to get lasing action in the optical frequency and you remember at the microwave wavelength region or microwave frequency region this kind of amplified emission of you know stimulated radiation was already achieved and that was called MASER right. Maser was already there, laser or optical Maser that term that time people used came much later because it is not easy to handle this ratio as you increase this one. So, what I said in the first class is now well explained all right.

We have learnt to some extent about the stimulated emission and you know how we can possibly increase the stimulated emission, now just let us consider at this point the 2 level system once again.

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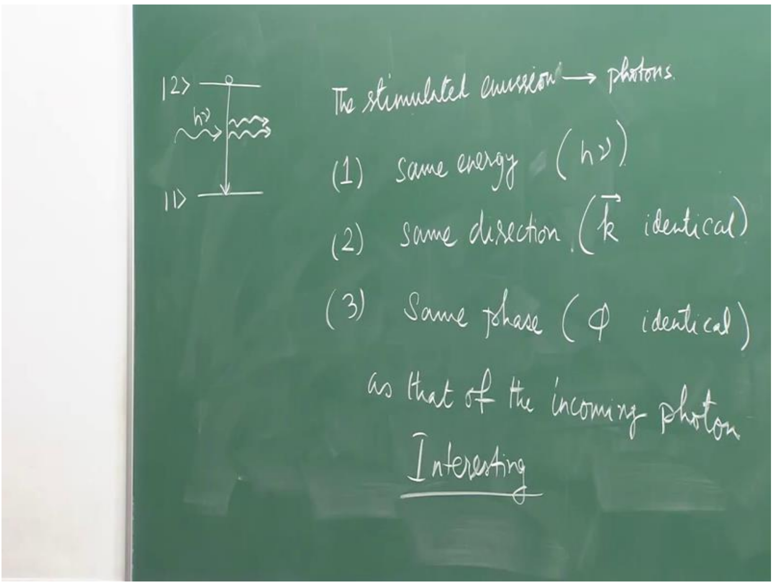

So, what we do we let us consider a system where the molecule is you know excited here and then I am coming with a photon matching with this energy gap and this comes down emitting 2 photons.

What are the properties of these photons that are coming out as stimulated emission that we can you know get from these behavior and learning whatever you know we discussed so, far. The first thing that we can say that for the system the stimulated emission will give me photons with or will give me the photons that will have same energy as that of the incoming photon, this is same h nu.

From this interaction, if the photon is coming, this will go it will also you know take the outgoing photon that is actually you know coming due to emission along with it, their direction is the same, the outgoing photon will have the same direction. If I write the direction of the photon by the wave vector k then k is same as that of the incoming photon and this photon is coming and interacting with this instantaneously the light is being emitted and the original light is also going. There is no option for the emitted light to have a different phase; that means that both this photon both of these photons will have the same phase or they will be coherent.

Suppose there are many atoms excited here and I am coming with n number of incoming photon and there are 2 n number of outgoing photos then those 2 n photons will be coherent. Third is the same phase, phi is identical, same energy, same direction, same phase as that of the incoming photon. These are a set of a very wonderful properties right. These properties are of our interest because if I can use a stimulated emission with these properties and we can amplify it then we get a source of light which has a very high energy and the property as written here that is they are of same frequency.

There will be only one frequency of light coming out; that means, there will be monochromatic, they will have same direction. They will have directionality, they will act as a beam and they will be coherent. This is an wonderful thing right, these are all interesting. So, coming back to the question that; after learning this stimulated emission what is the condition when I can achieve laser action? So, we saw that this rate of stimulated emission correct.

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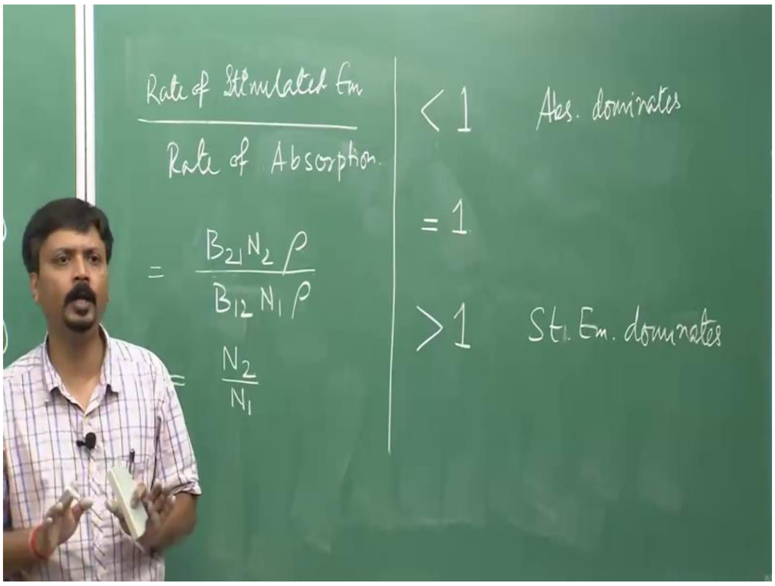

Stimulated Emission by rate of Spontaneous Emission this I can write it like B 2 1 N 2 rho. I am not using that h nu 1 2 just for simplicity. So, B 2 1 sorry by spontaneous emission that is A 2 1 N 2 sorry rate of I am sorry. So, I should write this is not spontaneous emission; but this is a Absorption. We have to minimize the spontaneous emission, but also I have to do something with this ratio of a rate of a Stimulated Emission to that of the Absorption because I want to have more stimulated emission and if absorption and simulated emission are equal then I will never be able to achieve the condition of lasing right lasing means which lays or when it lyses.

Let me correct this one, this one is B 1 2 N 1 rho, from that expression 3 we have seen from Einstein's formulation that absorption and emission for stimulated emission rates are equal. Therefore, it becomes like essentially N 2 by N 1, now when these is less than 1 this ratio. Rate of Stimulated Emission by Rate of Absorption which is a ultimately is N 2 by N 1, if it is less than 1 then what do I have, I have a system where absorption dominates.

Here Absorption dominates and I have no luck with laser action right I have to get stimulated emission using huge amount which I will amplify and get to you know get a laser. But here if the absorption dominates then most of my light will be only used for absorption I will never be you know able to get maximum output or stimulated emission.

If it is equal to one then again my situation is same that the rate of you knows absorption and the stimulated emission remains the same. So, I have a condition of saturation, the number of photons that I am putting it is not leading to anything because the number of photons going up number of photon coming down due to stimulated emission are exactly equal, there is no net gain.

This is the term that will be very very important to you I must gain something right. So, what I am you know spending and my earning should be much more than that then only there is a you know ultimately we can get light output which is much more energetic if I have to supply more than I get then there is you know it has no importance.

So, these 2 conditions is not going to lead me anywhere close to having laser action, but if I have a condition such that N 2 by N 1 is greater than 1 what I have a condition where Stimulated Emission it dominates. And the moment stimulated emission dominates my job is pretty much done I have got a system which gives me much more stimulated emission and I can take that stimulated emission amplify it and get a laser output.

So, we will stop here today. And in the next class we will go into the more detail of how laser can work how can I utilize this condition to get a laser.

Thank you very much.