**Laser: Fundamentals and Applications**
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**Lecture – 30**
**2nd order Non-linear optics**

Hello and welcome today is the last day of this week. We have been talking about the one important application of lasers which is Non-linear optics. We have learnt in the previous class under what condition one is stated to be in the non-linear optical regime. And we also learnt about the polarization that is created by intense light of laser and we learnt at this polarization created or induced.

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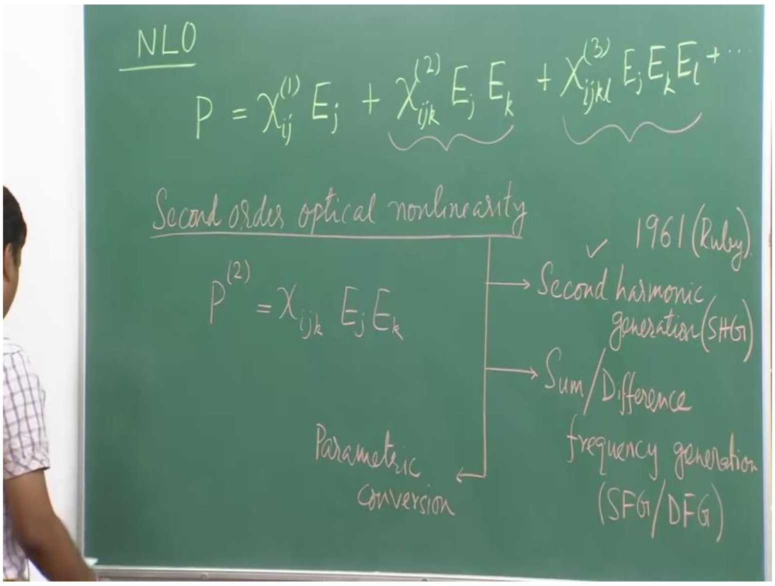


So, we saw that the polarization that is created is given by the electric field of the laser and the electric susceptibility of different orders. We learnt we further learnt that these susceptibilities are tensor quantities of different ranks. So, the proper way to write this expression is the following and remembers that P and E are the vectors I am not writing the arrows, but you should remember that and so on. This far we have learnt and we also saw that you know with a couple of examples where nonlinearity into optical properties is apparent taking a example of absorption and refractive index.

Now today we will be talking about one particular order of nonlinearity optical nonlinearity which is second order optical nonlinearity. Second order non-linear optics is probably the simplest, but most useful optical nonlinearity that people uses. Now from this expression you can find out about the order of different non-linear processes. So, the first term of course, this is the linear process, there is a first term process a linear process, and this is the theorem which corresponds to a second order non-linear optical process. If I write separately and I index this polarization by this term 2 which will signify this is a second order non-linear process, then I can write just isolate this. Now if I ask you this is the tensor you probably now agree you should because we have described why is it a tensor.

If I ask you what rank of tensor you should be also able to tell that because we have already learnt in you know plus 2 or in your undergraduate. So, this is a third rank tensor. It is good to know and this kies will have several components depending on what rank of tensor it belongs to. So, this is the third rank tensor, you will have total 27 components. That will come from different i j k indices which corresponds to the (Refer Time: 04:41) components.

Now, among these second different possible second order non-linear optical processes what we can have. So, the simplest yet most powerful is known as second harmonic generation. I have used this term quite a number of time and now we will be talking about this one in bit detail. And we call in short SHG Second harmonic generation. The second order non-linear optical process also consists of another process which is known as sum frequency generation also there can be difference frequency generation. So, Sum or Difference frequency generation, this is in short called SFG for the sum frequency generation and difference frequency generation is known as DFG.

Apart from this to most important processes there is another one process which is known as Parametric conversion is essentially optical parametric conversion. So, using these different second order non-linear optical processes we can convert frequencies and this is essentially not a part of this course, but you know this is not only use for frequency conversion, but this processes are you know associated with certain properties of molecules, they can you know tell us a lot about you know structure and property of different molecules and materials, and that is why these process has tremendous importance.

We will next, I will just mention here that the first process second harmonic generation, this was first observed in the year 1961 and it was done on Ruby laser. Ruby laser has fundamental wavelength of 694.3 nanometer and the second harmonic will be half of that, you can see I am calling it as an harmonic generation. This first term if both the frequencies are same, then the resultant new wave will be called a second harmonic wave, at the incident photon that is the E that we used to excide a material you know allow the you know material to interact with it is known as fundamental, and similarly if I go to this process and if I choose my all the electric fields that is e, j, k and l to be originating from the same frequencies.

Then the yield will be another harmonic and this is third order process it will be called a third harmonic and the process will be third harmonic generation all right. We will deal with the second order optical non-linear processes. We should know the expression of the second order non-linear optical processes and with a special emphasis on this process second harmonic generation.

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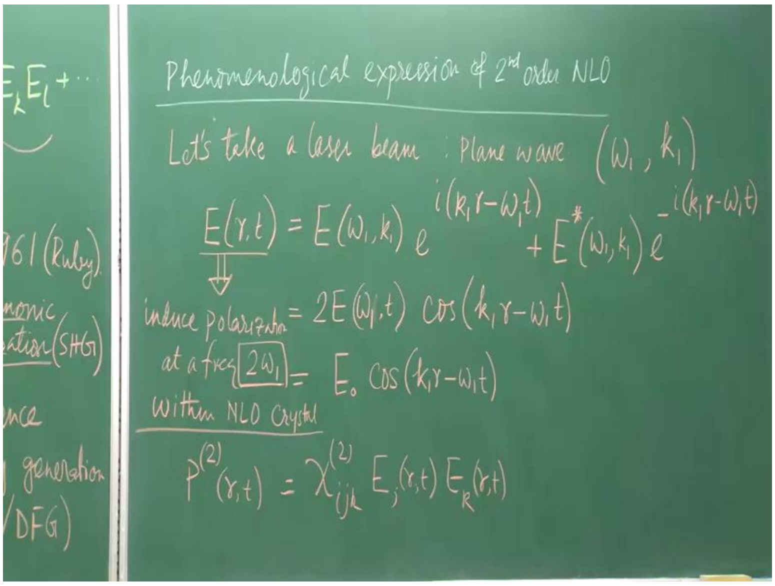


We will first look at the phenomenological expression of second harmonic generation. Phenomenological expression of second order Non-linear optical processes which involve laser radiation. So, let us consider, let us take this a laser beam and consider it to be a plane wave all of us are familiar with plane wave. So, we have learnt about it in high school physics and let us also assume that this fundamental laser light which is a plane wave is characterized by a frequency omega 1 and in associated wave vector k 1. So, these are the 2 things that characterize my fundamental laser beam.

The electric field associated with this, when I am sending it to any material, the electric field will be, I will specify the position and the particular time. This one I can express as a function of omega 1, k 1, right and I have the phase as k 1 and position as r; so k 1, r and the frequencies omega 1.

Now in general when I have to write I take the complex conjugate of the electric field is again omega 1, k 1, and complex conjugate of the other part that is minus k 1 r minus omega 1 t, here there is no confusion we already know this things. If I want to write this exponential terms in terms of cosign we can easily do that now I will just write the form. This is omega 1 t and in terms of cosign it will be k 1 r minus omega 1 t correct and I can club this together and I can write in terms of quantity e 0 and there remaining fine. This is the form of the electric field of our chosen laser light which is characterized by frequency omega 1 and wave vector k 1.

Now, this particular field this will induced what it will do this will induced polarization at a frequency 2 omega 1, because we are considering the process second harmonic generation essentially under the second order non-linear optical process. If I want to you know make it general then it will not be 2 omega 1, but it can be you know if I use 2 difference frequency say omega 1 and omega 2 then it will be in you know combination of omega 1 and omega 2.

So, is all I want to say that this particular field will induce polarization within a material which is suitable for non-linear showing non-linear optical phenomena and that polarization. This is not a general thing, but this is what we will end up with later on, this will create a non-linear polarization, within non-linear optical material say it is a non-linear optical crystal fine. So, that particular polarization which we are talking about is also a function of the position and the time and this one I have again the same way I j k and E keeping it most general and E k again remember E and P r vectors. So, this is nothing new we have already seen that.

I have the value for this electric field, so that is the electric field corresponding to the laser beam that is the fundamental light. Let us see; if I put this one here, what is the form of the second order non-linear polarization with a most general term.

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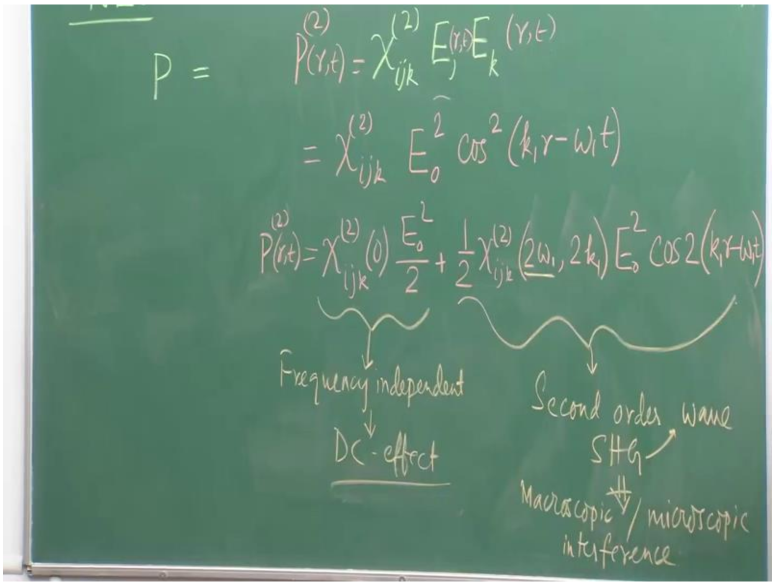


Let us just keep this part, essentially I have P 2 as a function of r and t is equals to this that is that equation particularly and these are also functions of r and t, both this one as well this one. Now what we will do, we will put this value of E there. So, if I do that what will I get I will get this following, I have and I have E 0 square, cosign square k 1 r minus omega 1 t correct, this is what we will get.

Now at this point if I use the trigonometric identity, what is the trigonometric identity. So, cos square x equals to 1 plus cosine 2 x, this if I use this and we can rewrite this in a but simpler form. I will just write the final form plus and this is a function of the harmonic frequency as well as the wave vector multiplied by E 0 square cosine, this is the final form of my p. So, if I just want to rewrite this one, this is my P of r t and this is the second order non-linear polarization.

There is a very interesting thing to note here and, let me rub this one off. So, here this expression for the polarization, non-linear polarization, second order non-linear polarization, has 2 parts, one is this first one and second one. The first one is Frequency independent that is why this is 0. There are 2 parts coming from this equation, this one is Frequency independent while this one is describing the result of a polarization created at the frequency which is twice that of the fundamental frequency, fundamental frequency is omega 1 and this is a function of the double of the frequencies. This part is essentially the Second order wave or we can call it essentially the SHG wave, which is characterized by this double frequency double the fundamental frequency and this one which is frequency independent is known as DC effect because it is frequency independent.

Now we have to elaborate with this part for the second harmonic generation process all right. This second harmonic wave that will be created in this way that you know can interfere either positively or negatively mean constructively or destructively. So, and that can happen you know either microscopically or macroscopically. This guy will lead to either macroscopic if we are talking about a large system like a crystal and for you know individual molecules it will be microscopic right macroscopic or microscopic interference and that can be constructive as well as destructive. If there is, you know destructive interference of course it is a problem now the thing is the fundamental wave it creates the polarization that second harmonic frequency and thus the second harmonic wave will be created.

This fundamental and second harmonic they will pass through the crystal which I have a physical object say it is a crystal, I am sending the fundamental and then the fundamental is moving and in the, you know tube process I have created the second harmonic. The second harmonic wave also will propagate through the crystal, now while doing you know making this propagation through the material after the creation of second harmonic wave this fundamental and second harmonic wave can you know pass in an out of phase.

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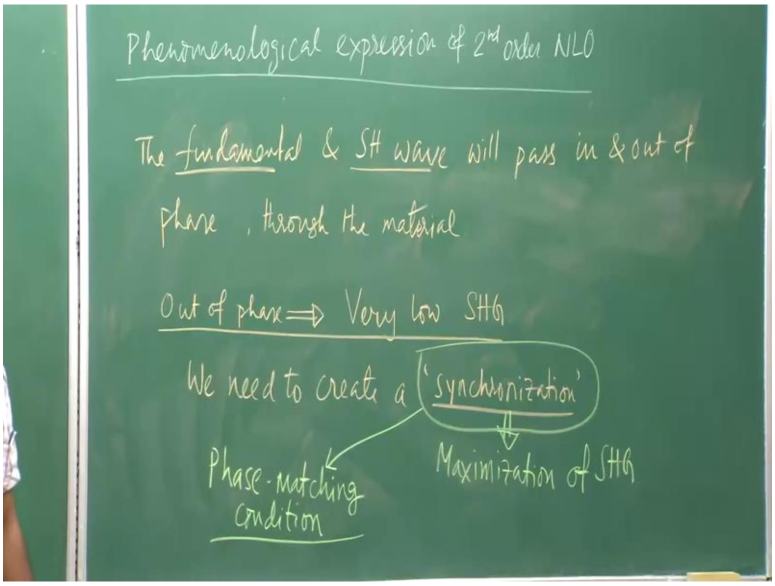

The fundamental and SH wave will pass in and out of phases through the material say a non-linear optical crystal. Now if there in phase well and good if there are out of phase then what we will happen for out of phase movement what I will get, I will get very low SHG. So, one moment I will just go back here if you have any confusion why I am saying this one as SHG, I will just clarify you know for half a minute this part refers to the second order polarization no doubt about that and a frequency which corresponds to double the fundamental frequency and that is what is an harmonic particularly second harmonic process and when you have both the incident wave having the same frequency this is what happens and that is also characterized by this E 0 square and that is why we have this process as second harmonic.

Coming back here, I cannot let this happen, what I need to do is, we need to create a synchronization, a synchronization between the phase velocities of the fundamental and SH wave. So, why do we need to do that because there is a something called dispersion write and because of this dispersion I can have this you know out of out of phase relation between those and can lead to destructive interference. I cannot let that happen and I need to create a synchronization between those phase velocities all right, and if I can do the synchronization you can if I can have the synchronization, that will essentially lead to maximization of second harmonic generation, not too difficult to understand here right. We have seen some kind of similar situation when we discussed about the you know laser process also when we talked about the you know creation of one particular mode developing high intensity of a particular mode in the laser.

This synchronization of phases phase velocities is it has a particular name and which is very very important in case of non-linear optics and this necessity of having a synchronization between the phases of fundamental and harmonic waves is known as phase matching condition without having a Phase matching condition fulfilled there will be no appreciable non-linear optical process, be it second harmonic generation, sub frequency generation or any other non-linear optical processes.

So, in the next class we will be looking at this Phase matching condition what is a necessity little bit more elaborately of Phase matching condition how to get that Phase matching condition all those things.

And I thank you for your attention.