

Laser: Fundamentals and Applications
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Lecture – 20
Mode Locking

Hello and welcome to the day 5 of 4th week of this lecture series. So, this week we have been talking about different pulsing techniques and we are talked about Q switching and cavity dumping. And we are going to talk about the third most widely used pulsing technique; that is mode locking.

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Mode-locking :

Earlier :

Standing wave condition

$$m\lambda = 2L$$

$$\nu = \frac{mc'}{2L}$$

gap between modes : $\Delta\nu$

$$\Delta\nu = \frac{c}{2L} = 10^8 \text{ Hz}$$

In solidstate (tunable) lang laser or dye laser, the emission linewidth is quite broad \Rightarrow No of modes falling under

ps - fs pulses

\downarrow

ultrashort pulses

\downarrow

used to probe ultrafast processes
(Chemical reaction / dynamics ...)

$m \rightarrow$ order of longitudinal modes

$L \rightarrow$ cavity length.

$c' = \frac{c}{n}$

So, we will be talking about mode locking; so, before we start talking about the principle of mode locking, let me tell you that this particular technique of generating pulsed output is used to generate very short pulse; when I say very short, it means approximately 10^{-12} to 10^{-15} seconds that is picosecond to femtosecond.

So, this generates picosecond to femtosecond pulses; so, this picosecond or femtosecond pulses this time duration is known as ultrashort pulses; and why do we need ultrashort pulses? Because there are so many processes that you know that take place in nature; in chemistry, in biology which happens really really fast; which are known as ultrafast processes.

So, this guy it probes; it is used to probe ultrafast processes where some chemical reaction or reaction dynamics etcetera. So, now with this knowledge we will start looking at how to mode lock a laser, how to generate this ultrashort pulses; starting from a continuous wave laser. So, in earlier classes we have learnt about the standing wave condition in a cavity. So, this we have already seen; so, earlier standing wave condition; what is the condition? That not any wavelength can be sustained in a cavity; there has to be a particular relation fulfilled.

So, that is given by $m\lambda = 2L$; where m is the order of mode particularly longitudinal mode. So, m refers to order of longitudinal modes, L is the cavity length. So, I can express this in terms of frequency also; so, if I write the frequency I can write $m c / 2L$. If I be very proper, then I should write instead of $m c$; I should write $m c'$, where c' is nothing, but the velocity of light divided by the reflective index of the medium.

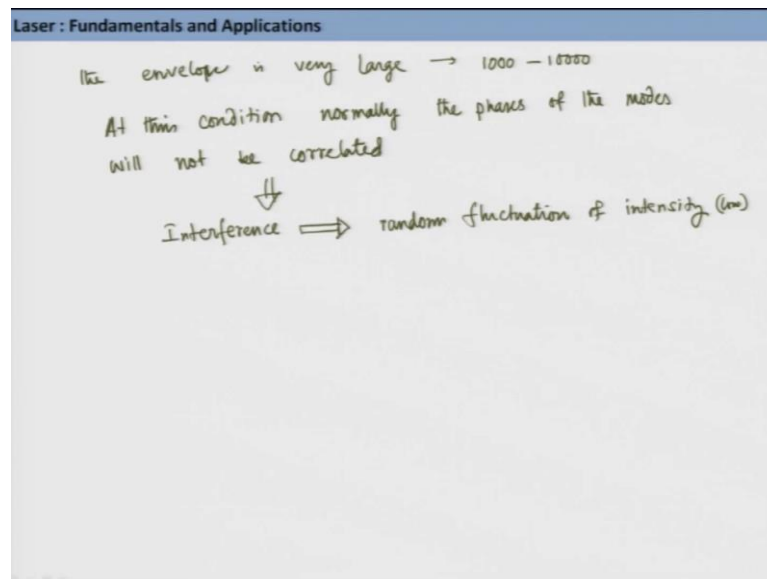
So, there are so many frequencies that can satisfy this standing wave condition within a cavity. And there will be separation between two successive frequencies. So, say ν_1 , ν_2 , ν_3 , ν_4 so on up to ν_m , so there will be a gap between ν_1 and ν_2 or ν_2 on to ν_3 ; what is that gap? That also we have learned. So, gap between modes is given by $\Delta\nu$ and $\Delta\nu$ is equal to $c / 2L$.

Now, what is the approximate number that also; one of those classes we have seen, that can be in the order of 10^8 hertz. This is just a number that you can easily figure out by given certain L values. So, this is a huge number; now the particular modes that will be within the you know laser output is determined by of course, the cavity length and also the spectral bandwidth, that emission bandwidth of the laser active medium. So, if you remember we have solved one problem using the example of hini laser; given a bandwidth and given a length of the cavity; how many modes can be accommodated within that spectral bandwidth.

So, if the spectral bandwidth of the active medium is small; then the number of modes also will be small there and if the bandwidth is very large we will have large number of modes. So, in certain lasers like particularly in solid state lasers, if I be even more particular then solid state tunable lasers, so, for example, ti sapphire; titanium sapphire or dye laser, we will talk about these different lasers in the coming week. So, this type of

lasers the emission line width is quite broad. If you are familiar with the fluorescence spectroscopy and if you have seen fluorescence spectrum of dye; say for example, rhodamine, you will know that this is fairly broad; they are fully theopproximais in order of tens of nanometers; which is really broad. So, within that particular broad spectral envelope; the number of modes will be also large.

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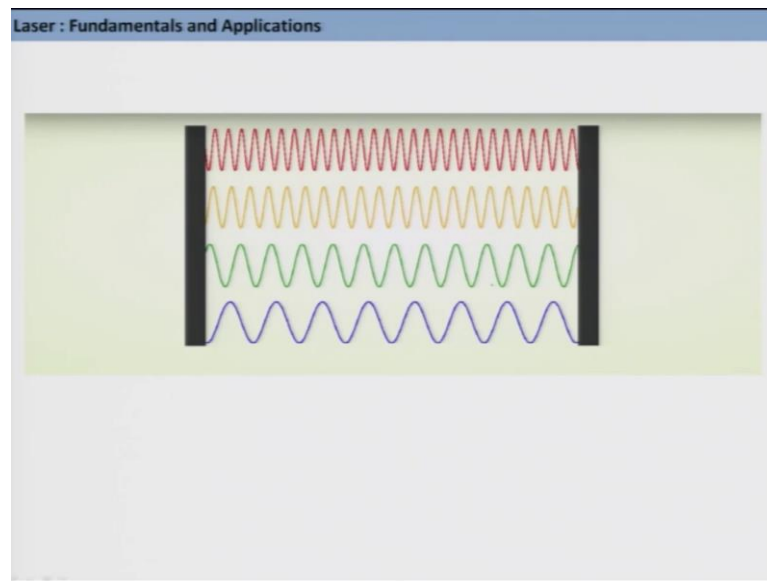
So, number of modes falling under the or within the envelope is very large. So, we have seen that in case of hini laser; with the 10 centimeter cavity length, number of modes within the spectral bandwidth is only 1; if I increase the cavity length to 30 centimeter, the numbers of modes are 3. So, compared to these for a laser system like dye laser; where spectrum is really broad, the emission spectrum is really broad; the number can be as large as 1000 to 10000 quite easily.

So, if I have so many modes there what may be my you know what can be the good thing or bad thing happening there this modes is individual modes will have their phases. So, it is not necessary that one mode will be related in terms of phase to another mode. So, what I am saying is that; at this condition normally the phase is of the modes will not be correlated. So, none of the modes are correlated with each other in terms of phase; what will happen then? So, each of the modes may contains several modes; now these photons belonging to different, different modes having different phases; they will ultimately

interfere and the result of this interference is to generate very random amplitudes of signal and the amplitude also will be very small in the level of fluctuations.

So, because of this there will be interference which will lead to random fluctuation of intensity and this intensity is also extremely low. So, if you know this is not desirable correct; so, what one can do is to try to bring some phase relation between different modes. So, now let us have a look at this situation pictorially.

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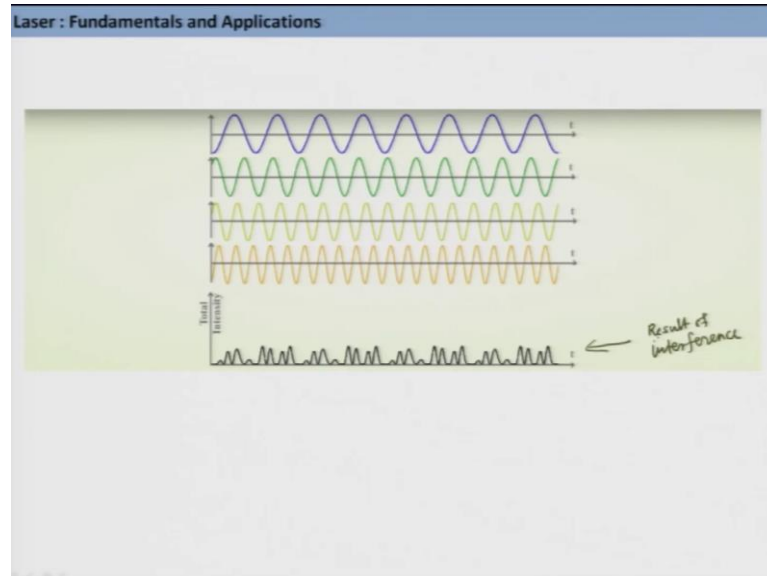


So, let us consider a cavity and a particular mode; so, this green colour, this is one particular mode. So, here green colour it is shown one particular mode; so, let us call this is mode 1 in my this picture. Now, let us have a look at another possible mode; so, the mode is shown in orange colour. So, this is my mode 2 and if we keep going; then this blue one, this is mode 3. Let us have one more; this red colour mode 4, I can have several such modes; like I said like 10 to the power 3 to 10 to the power 4 number of modes. For simplicity, let us consider 4 which will be good enough to explain our problem.

Now, just expand this; let us expand this modes, right now I have shown it in a super imposed way. Now, if I separate them they will look like this; now if you just take a pause and have a look at all these different modes here and try to see if there is any phase relation. You will be seeing that there is no real phase relation; that is not that after a certain number of repetition; the modes are constructively interfering. Currently, the kind

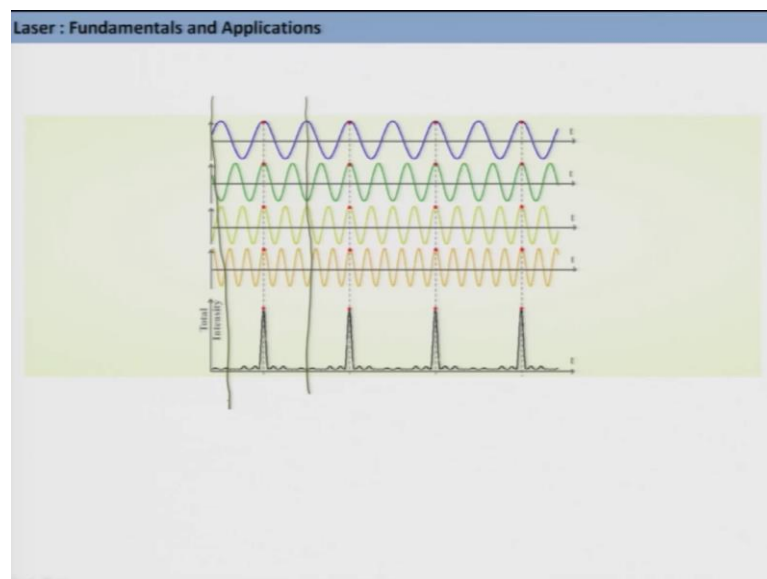
of modes that we have chosen; they will not do that, so the result will be a random intensity like a fluctuation, due to mostly destructive interference.

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So, this is what is shown here; so, this is the result of interference. So, what mode locking will do? Mode locking will bring in a phase relation between this different modes.

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So, now look this guys; so, you see here all of them are in phase. So, they are all will constructively interfere; now you keep going, you come here you see again same thing is

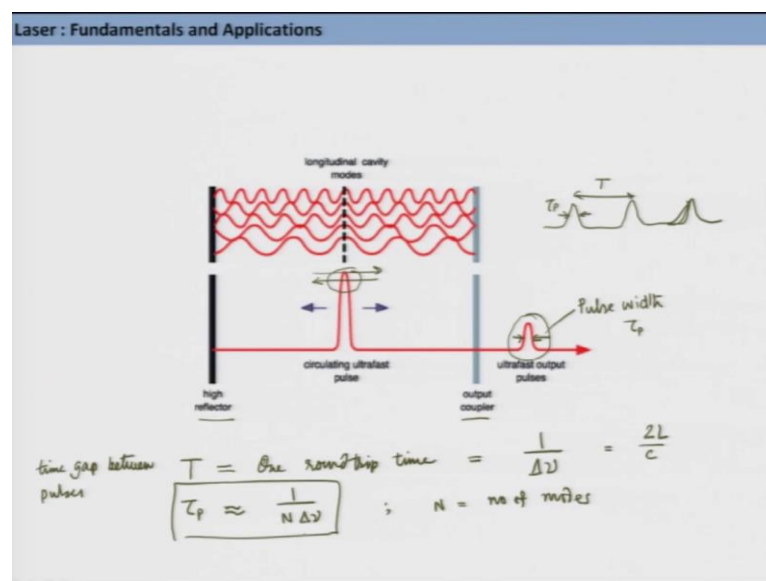
repeated; again here. So, if I am looking at the green one, so after every 3 pulses; a 3 waves they are repeating themselves, the condition of this constructive interference there.

So, the resultant is there will be constructive interference at certain point in time window; at a certain time there will be constructive interference. Everywhere else there will be destructive interference, so, what is happening? So, I have several modes oscillating with several frequencies; now I somehow lock all of them or bring in a condition, where all these modes, all these waves their phases are in unison.

So, therefore because they have different different frequencies, so when there will be unison like this; I will have constructive interference; everywhere else you can see this will interact with this guy or this guy; there will be definitely cause destructive interference. So, only here I will get intensity and everywhere else I see nearly 0 intensity; very good. So, this is already is like creation of a pulse.; so, if I look at it in this fashion, so I can see that how this constructive interference is giving rise to a solid intensity.

Now, if I can choose my cavity length in a way that I collect only up to this; what will happen? Within the cavity, I will have only 1 pulse; that is that can have round trip. So, if I can have that then this is what I will get.

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So, there are several modes here; so, similarly I have taken 5 modes here and the constructively interfere here; everywhere else they are destructively interfere and give 0 intensity. And this is my cavity comparison of high reflector and output coupler. So, if this guy; this particular pulse it moves back and forth, through the active medium, this only gets intensified more and more. And output coupler being partially transmissive; every round trip will throw some light out of the cavity.

So, the distance between two successive such pulses will be equal to one round trip time; this is very easy to understand. I have somehow created a phase relation between all the possible modes in the cavity, which particularly fall under the emission spectral bandwidth of the active medium. And then I have chosen the cavity in such a way, that the result of this constructive interference which is given by this peak here; will be the only 1 pulse within the cavity moving or making round trips. And in that case, every round trip will give me 1 pulse; whenever it comes and hits this output coupler, I get 1 pulse. So, the distance between two successive pulses is one round trip.

So, I said like this time period is the time gap between pulses T is essentially one round trip time. And one can actually show that this period is equal to $1/\Delta\nu$. So, in other words it will be $2L/c$ and also one can show that the result in pulse width. So, talking about this width; this is my pulse width and we denote it as τ_P and let me be with clear. So, if I have pulses like this; so, my drawing is a little bad here.

So, this is my τ_P and this distance is my T . So, this τ_P that is the pulse width; it can be shown that it is somehow related to $\Delta\nu$ and the number of modes in this fashion; where N is the number of modes that are taking part in this interference. So, from this relation what can we say about the nature of pulses?

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The envelope is very large $\rightarrow 1000 - 10000$

At this condition normally the phases of the modes will not be correlated

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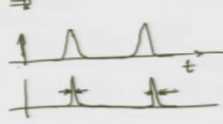
Interference \Rightarrow random fluctuation of intensity (am)

$$T = \frac{1}{\Delta \nu}$$

$$\tau_P \approx \frac{1}{N \Delta \nu} \Rightarrow$$

Let's take $N = n$

Now if $N = 2n$



So, let us quickly write down that one; so, T equals to 1 by $\Delta \nu$ and τ_P equals to 1 upon $N \Delta \nu$. So, this is like almost equal to; there are some constant term; like ϕ . So, from this relation what we can say is that for say, for number of mode let us take N equals to say small n and then I have a picture like; this is the intensity and let us just consider 2 successive pulses, there are pulses beyond.

So, this is my time axis; now if I double the number of modes; what will happen? So, from this relation we can see this will be essentially multiplied by 2 in the denominator. So, τ_P will be now half; so, what I have is nothing is happening to my time period, but my pulse width is totally changing. So, this is the effect of number of modes; so, for example, if I can increase the number of modes in the cavity by my own designs; by simply you know adjusting the cavity length. Then for more mode number of modes, my pulses will be shorter and shorter and shorter.

So, this is one trick than one can use to make an ultrashort pulses. First place, you create a phase relation between the number of modes; which will lead to constructive interference at one particular point and everywhere else distractive interference, select one particular pulse thus created to osculate in the cavity. This osculation will intensify only 1 pulse and that is quite narrow.

Then, now if you number of modes involved in this interference is much more then this pulse is even shorter. So, you can create shorter and shorter pulses by increase the

number of modes. So, at this point let us get an idea like how to estimate; what kind of pulse is that we can expect. So, let us take an example.

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An Example

Ti : saph. Band width $\sim 128 \text{ THz}$
 Cavity length $\sim 200 \text{ cm}$

$$\Delta\nu = \frac{c}{2L} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 2 \text{ m}} = 75 \times 10^6 \text{ Hz} = 75 \text{ MHz}$$

$$\text{Number of modes} = \frac{\text{Bandwidth}}{\text{gap between modes}} = \frac{128 \text{ THz}}{75 \text{ MHz}} = \frac{128 \times 10^{12} \text{ Hz}}{75 \times 10^6 \text{ Hz}} = 1.7 \times 10^6$$

$$\text{Time gap between pulses, } T = \frac{1}{\Delta\nu} = \frac{1}{75 \text{ MHz}} = 13.3 \text{ ns}$$

$$\text{Pulse width } \tau_p = \frac{1}{N \Delta\nu} = \frac{1 \times 13.3 \text{ ns}}{1.7 \times 10^6} \quad N = 5 \times 10^5$$

$$= 7.8 \times 10^{-15} \text{ s}$$

Limiting value of $\tau_p \approx 8 \text{ fs}$ For lesser no of modes τ_p will increase

And let us try to estimate, so the most common mode locking lasers is titanium sapphire. So, let us take a titanium sapphire laser; so, titanium sapphire is the active medium. So, what is done here? So before that let us give some numbers; the bandwidth of this one. Let us take the number that I have with me 128 terahertz and (Refer Time: 26:00) cavity length around 200 centimeter. Now, what I have to find out for this given bandwidth and the cavity length; what can be the pulse width, what is the shortest pulse width that is possible. So, we will have a limiting value of possible pulse width that is possible. So, how to find that out? So, we should be able to do it because we have done similar type of problem in other context. So, delta nu first is C by 2 L; which is divided by 2 into, so I did it in meters. So, this will give you approximately 75 into 10 to the power 6 hertz or 75 megahertz.

Now, if the distance between two pulses in frequency unity is 75 megahertz; then what is the number of modes in the cavity. So, the number of modes is; now I can easily find out I have this bandwidth. So, within this bandwidth how many such gaps are possible that will give me the number of modes. So, essentially it is given by the bandwidth divided by my gap between modes. So, this is 128 terahertz divided by the gap between mode is 75 megahertz.

So, this is equal to 120×10^{12} hertz; 75×10^6 hertz. So, this will give you total number of modes which I write the final value that is 1.7×10^6 . So, this many number of modes that are possible within the cavity; now what is the time gap between each pulses? So, I have found out $\Delta \nu$, so I can easily figure out what is T . So, time gap pulses that is $T = 1 / \Delta \nu$; that is equals to $1 / 75$ megahertz.

And if you calculate this one, the final value will be 13.3 nanosecond and finally, what is the pulse width? Pulse width τ_P is equal to $1 / N \Delta \nu$. So, N is this number of mode; so, $1 / 1.7 \times 10^6$; so, this is the number of modes correct and $1 / \Delta \nu$ we have just found that is equals to 13.3 nanosecond; so, 13.3 nanosecond.

So, this will give you a value of around 7.8 is into 10^{-15} second. So, roughly I can say this is around 8 femtosecond; see this I have a 200 centimeter long cavity and having a particular bandwidth of 128 terahertz. I calculated how many number of modes are possible, I calculated what is the gap between the you know pulses and ultimately I found out the pulse widths and the pulse width turns out to be 8 femtosecond.

This is really really short, so in reality if the number of modes are less than this theoretically possible number of modes. So, if suppose instead of say 1.76; if we have the number of modes equals to say like 5×10^5 ; of course, this value will increase. So, for lesser number of modes τ_P will increase so; that means, this is my limiting value of τ_P . So, I can see that if I can create a phase relationship between the modes of cavity, I can create pulses not only create pulse, but we can create really ultra short pulses as I just showed. So, we will stop here today and in the following class we will look at what are the possible ways, practical ways to bring in this phase relationship in within the modes and create more block laser.

Thank you very much for your attention.